Demo 5

This demo contains the calculation procedures to generate the results shown in Appendix A of the lab report.

1 Degree Reduction and Raising Matrices for Degree 5

$$M_{5,4} = \begin{pmatrix} \frac{251}{252} & \frac{-113}{504} & \frac{1}{12} & \frac{-13}{504} & \frac{1}{252} \\ \frac{5}{252} & \frac{565}{504} & \frac{-5}{12} & \frac{65}{504} & \frac{-5}{252} \\ \frac{-5}{126} & \frac{65}{252} & \frac{5}{6} & \frac{-65}{252} & \frac{5}{126} \\ \frac{5}{126} & \frac{-65}{252} & \frac{5}{6} & \frac{65}{252} & \frac{-5}{126} \\ \frac{-5}{252} & \frac{65}{504} & \frac{-5}{12} & \frac{565}{504} & \frac{5}{252} \\ \frac{1}{252} & \frac{-13}{504} & \frac{1}{12} & \frac{-113}{504} & \frac{251}{252} \end{pmatrix} \qquad M_{4,5} = \begin{pmatrix} 1 & \frac{1}{5} & 0 & 0 & 0 & 0 \\ 0 & \frac{4}{5} & \frac{2}{5} & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{5} & \frac{3}{5} & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{5} & \frac{4}{5} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{5} & 1 \end{pmatrix}$$

$$M_{5,3} = \begin{pmatrix} \frac{121}{126} & \frac{-3}{7} & \frac{1}{6} & \frac{-2}{63} \\ \frac{8}{63} & \frac{37}{42} & \frac{-3}{7} & \frac{11}{126} \\ \frac{-1}{9} & \frac{16}{21} & \frac{1}{21} & \frac{-2}{63} \\ \frac{-2}{63} & \frac{1}{21} & \frac{16}{21} & \frac{-1}{9} \\ \frac{11}{126} & \frac{-3}{7} & \frac{37}{42} & \frac{8}{63} \\ \frac{-2}{63} & \frac{1}{6} & \frac{-3}{7} & \frac{121}{126} \end{pmatrix} \qquad M_{3,5} = \begin{pmatrix} 1 & \frac{2}{5} & \frac{1}{10} & 0 & 0 & 0 \\ 0 & \frac{3}{5} & \frac{3}{5} & \frac{3}{10} & 0 & 0 \\ 0 & 0 & \frac{3}{10} & \frac{3}{5} & \frac{3}{5} & 0 \\ 0 & 0 & 0 & \frac{1}{10} & \frac{2}{5} & 1 \end{pmatrix}$$

$$M_{5,2} = \begin{pmatrix} \frac{23}{28} & \frac{-3}{7} & \frac{3}{28} \\ \frac{9}{28} & \frac{2}{7} & \frac{-3}{28} \\ 0 & \frac{9}{14} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{9}{14} & 0 \\ \frac{-3}{28} & \frac{2}{7} & \frac{9}{28} \\ \frac{3}{28} & \frac{-3}{7} & \frac{23}{28} \end{pmatrix} \qquad M_{2,5} = \begin{pmatrix} 1 & \frac{3}{5} & \frac{3}{10} & \frac{1}{10} & 0 & 0 \\ 0 & \frac{2}{5} & \frac{3}{5} & \frac{3}{5} & \frac{2}{5} & 0 \\ 0 & 0 & \frac{1}{10} & \frac{3}{10} & \frac{3}{5} & 1 \end{pmatrix}$$

$$M_{5,1} = \begin{pmatrix} \frac{11}{21} & \frac{-4}{21} \\ \frac{8}{21} & \frac{-1}{21} \\ \frac{5}{21} & \frac{2}{21} \\ \frac{2}{21} & \frac{5}{21} \\ \frac{-1}{21} & \frac{8}{21} \\ \frac{-4}{21} & \frac{11}{21} \end{pmatrix} \qquad M_{1,5} = \begin{pmatrix} 1 & \frac{4}{5} & \frac{3}{5} & \frac{2}{5} & \frac{1}{5} & 0 \\ 0 & \frac{1}{5} & \frac{2}{5} & \frac{3}{5} & \frac{4}{5} & 1 \end{pmatrix}$$

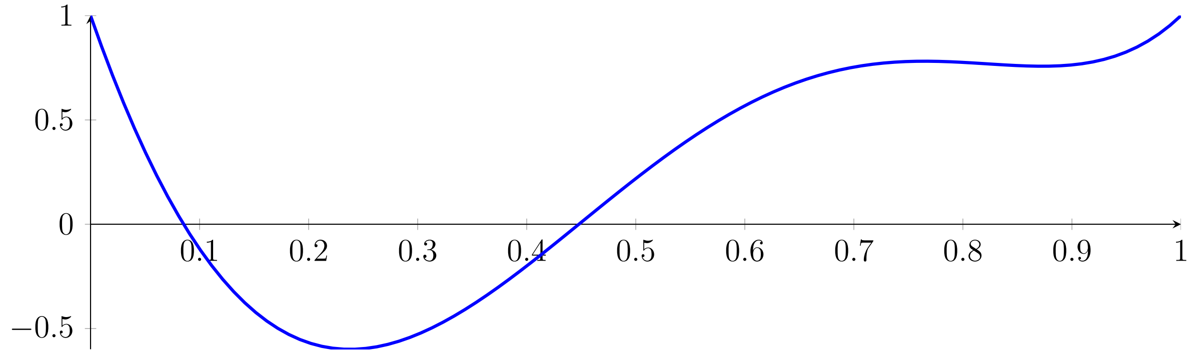
$$M_{5,0} = \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \\ \frac{1}{6} \\ \frac{1}{6} \\ \frac{1}{6} \\ \frac{1}{6} \end{pmatrix} \qquad M_{0,5} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

2 QuadClip Applied to a Polynomial of 5th Degree with Two Roots

$$25X^5 - 35X^4 - 15X^3 + 40X^2 - 15X + 1$$

Called QuadClip with input polynomial on interval $[0, 1]$:

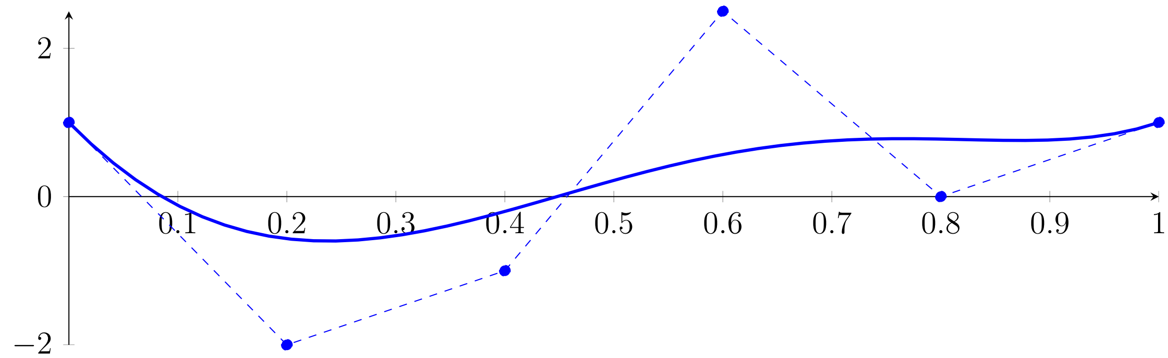
$$p = 25X^5 - 35X^4 - 15X^3 + 40X^2 - 15X + 1$$



2.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 25X^5 - 35X^4 - 15X^3 + 40X^2 - 15X + 1 \\ &= 1B_{0,5}(X) - 2B_{1,5}(X) - 1B_{2,5}(X) + 2.5B_{3,5}(X) + 0B_{4,5}(X) + 1B_{5,5}(X) \end{aligned}$$



Best approximation polynomials of degree 0, 1, 2, 3 and 4:

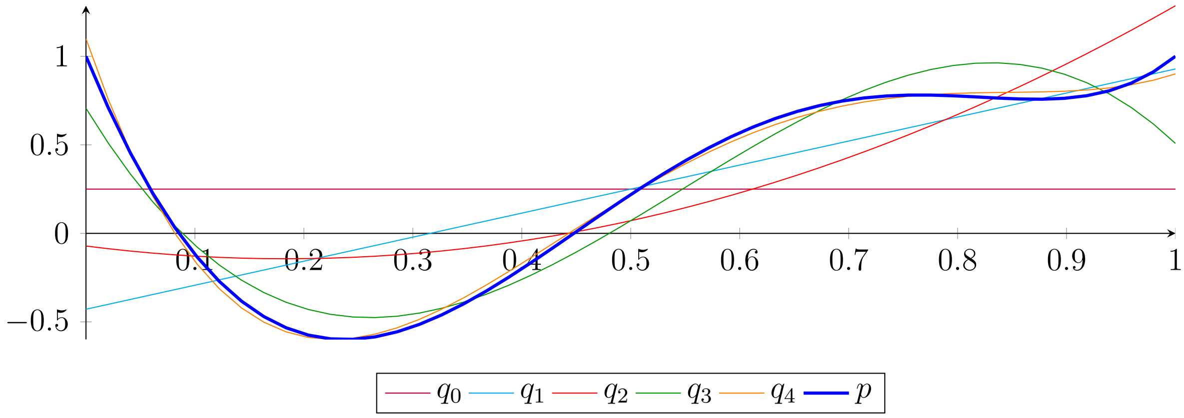
$$\begin{aligned} q_0 &= 0.25 \\ &= 0.25B_{0,0} \end{aligned}$$

$$\begin{aligned} q_1 &= 1.35714X - 0.428571 \\ &= -0.428571B_{0,1} + 0.928571B_{1,1} \end{aligned}$$

$$\begin{aligned} q_2 &= 2.14286X^2 - 0.785714X - 0.0714286 \\ &= -0.0714286B_{0,2} - 0.464286B_{1,2} + 1.28571B_{2,2} \end{aligned}$$

$$\begin{aligned} q_3 &= -15.5556X^3 + 25.4762X^2 - 10.119X + 0.706349 \\ &= 0.706349B_{0,3} - 2.66667B_{1,3} + 2.45238B_{2,3} + 0.507937B_{3,3} \end{aligned}$$

$$\begin{aligned} q_4 &= 27.5X^4 - 70.5556X^3 + 60.8333X^2 - 17.9762X + 1.09921 \\ &= 1.09921B_{0,4} - 3.39484B_{1,4} + 2.25B_{2,4} + 0.394841B_{3,4} + 0.900794B_{4,4} \end{aligned}$$



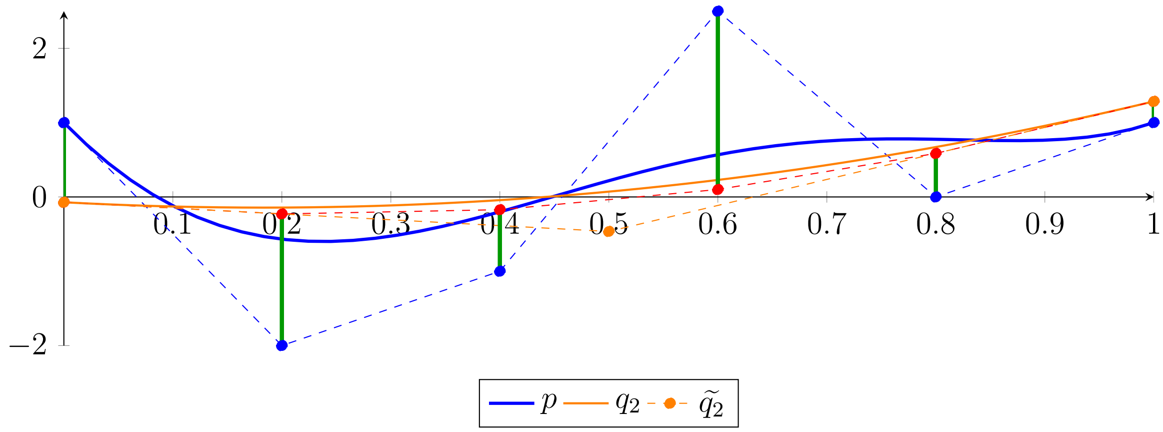
Degree reduction and raising matrices:

$$M_{5,2} = \begin{pmatrix} 0.821429 & -0.428571 & 0.107143 \\ 0.321429 & 0.285714 & -0.107143 \\ 1.249\cdot 10^{-16} & 0.642857 & -0.142857 \\ -0.142857 & 0.642857 & 0 \\ -0.107143 & 0.285714 & 0.321429 \\ 0.107143 & -0.428571 & 0.821429 \end{pmatrix} \quad M_{2,5} = \begin{pmatrix} 1 & 0.6 & 0.3 & 0.1 & 2.17604 \\ 8.10463\cdot 10^{-15} & 0.4 & 0.6 & 0.6 & 0. \\ 4.996\cdot 10^{-15} & -1.59872\cdot 10^{-14} & 0.1 & 0.3 & 0. \end{pmatrix}$$

Degree reduction and raising:

$$q_2 = 2.14286X^2 - 0.785714X - 0.0714286$$
$$= -0.0714286B_{0,2} - 0.464286B_{1,2} + 1.28571B_{2,2}$$

$$\widetilde{q}_2 = -1.18767\cdot 10^{-12}X^5 + 2.52388\cdot 10^{-12}X^4 - 1.79037\cdot 10^{-12}X^3 + 2.14286X^2 - 0.785714X - 0.0714286$$
$$= -0.0714286B_{0,5} - 0.228571B_{1,5} - 0.171429B_{2,5} + 0.1B_{3,5} + 0.585714B_{4,5} + 1.28571B_{5,5}$$



The maximum difference of the Bézier coefficients is $\delta = 2.4$.
Bounding polynomials M and m:

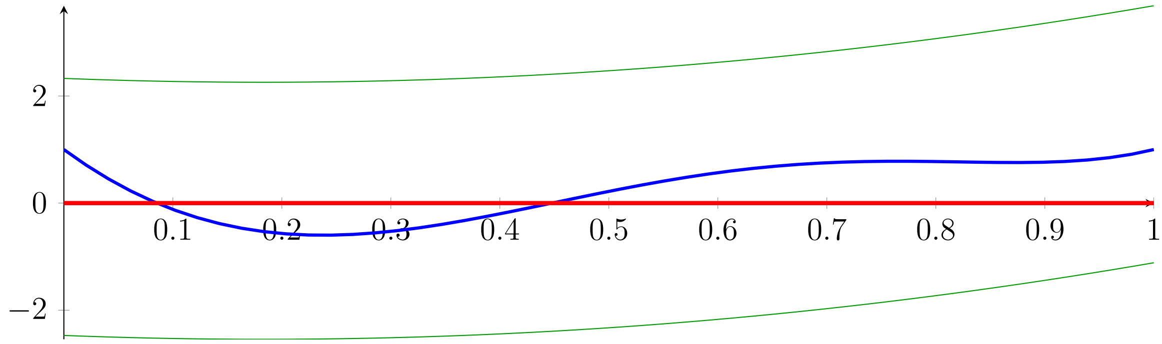
$$M = 2.14286X^2 - 0.785714X + 2.32857$$

$$m = 2.14286X^2 - 0.785714X - 2.47143$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{-0.906136, 1.2728\}$$

Intersection intervals:



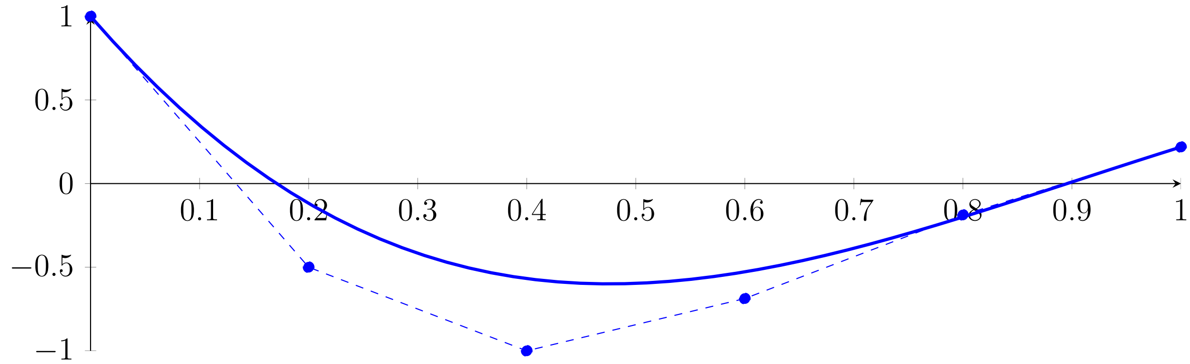
$[0, 1]$

Longest intersection interval: 1
$\Longrightarrow$ Bisection: first half $[0, 0.5]$ und second half $[0.5, 1]$

2.2 Recursion Branch 1 1 on the First Half $[0, 0.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 0.78125X^5 - 2.1875X^4 - 1.875X^3 + 10X^2 - 7.5X + 1$$
$$= 1B_{0,5}(X) - 0.5B_{1,5}(X) - 1B_{2,5}(X) - 0.6875B_{3,5}(X) - 0.1875B_{4,5}(X) + 0.21875B_{5,5}(X)$$



Best approximation polynomials of degree 0, 1, 2, 3 and 4:

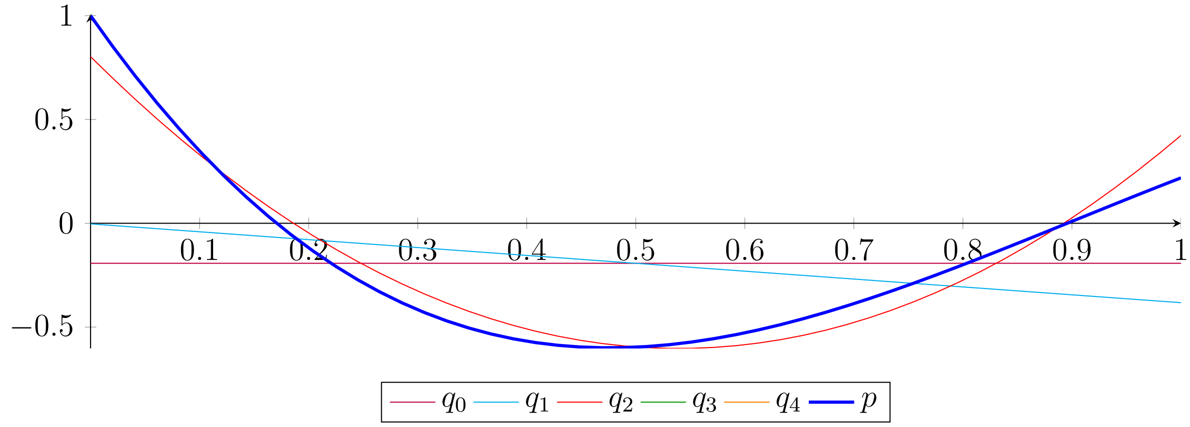
$$q_0 = -0.192708$$
$$= -0.192708B_{0,0}$$

$$q_1 = -0.379464X - 0.00297619$$
$$= -0.00297619B_{0,1} - 0.38244B_{1,1}$$

$$q_2 = 4.83259X^2 - 5.21205X + 0.802455$$
$$= 0.802455B_{0,2} - 1.80357B_{1,2} + 0.422991B_{2,2}$$

$$q_3 = -4.07986X^3 + 10.9524X^2 - 7.65997X + 1.00645$$
$$= 1.00645B_{0,3} - 1.54688B_{1,3} - 0.449405B_{2,3} + 0.218998B_{3,3}$$

$$q_4 = -0.234375X^4 - 3.61111X^3 + 10.651X^2 - 7.59301X + 1.0031$$
$$= 1.0031B_{0,4} - 0.895151B_{1,4} - 1.01823B_{2,4} - 0.268911B_{3,4} + 0.21565B_{4,4}$$



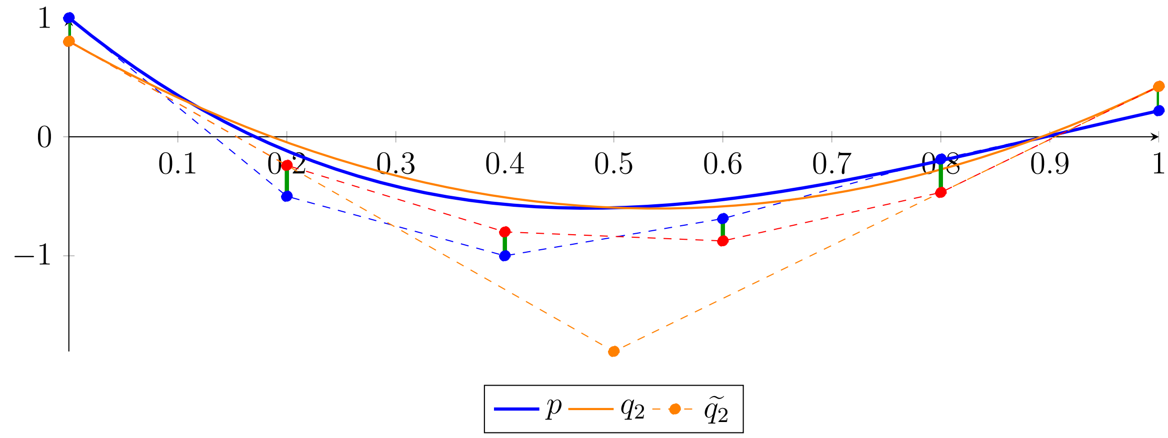
Degree reduction and raising matrices:

$$M_{5,2} = \begin{pmatrix} 0.821429 & -0.428571 & 0.107143 \\ 0.321429 & 0.285714 & -0.107143 \\ 1.249{\cdot}10^{-16} & 0.642857 & -0.142857 \\ -0.142857 & 0.642857 & 0 \\ -0.107143 & 0.285714 & 0.321429 \\ 0.107143 & -0.428571 & 0.821429 \end{pmatrix} \quad M_{2,5} = \begin{pmatrix} 1 & 0.6 & 0.3 & 0.1 & 2.1760 \\ 8.10463{\cdot}10^{-15} & 0.4 & 0.6 & 0.6 & 0 \\ 4.996{\cdot}10^{-15} & -1.59872{\cdot}10^{-14} & 0.1 & 0.3 & 0 \end{pmatrix}$$

Degree reduction and raising:

$$\begin{aligned} q_2 &= 4.83259X^2 - 5.21205X + 0.802455 \\ &= 0.802455B_{0,2} - 1.80357B_{1,2} + 0.422991B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q_2} &= 1.25711{\cdot}10^{-12}X^5 - 3.15137{\cdot}10^{-12}X^4 + 2.76557{\cdot}10^{-12}X^3 + 4.83259X^2 - 5.21205X + 0.802455 \\ &= 0.802455B_{0,5} - 0.239955B_{1,5} - 0.799107B_{2,5} - 0.875B_{3,5} - 0.467634B_{4,5} + 0.422991B_{5,5} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.280134$.
Bounding polynomials M and m:

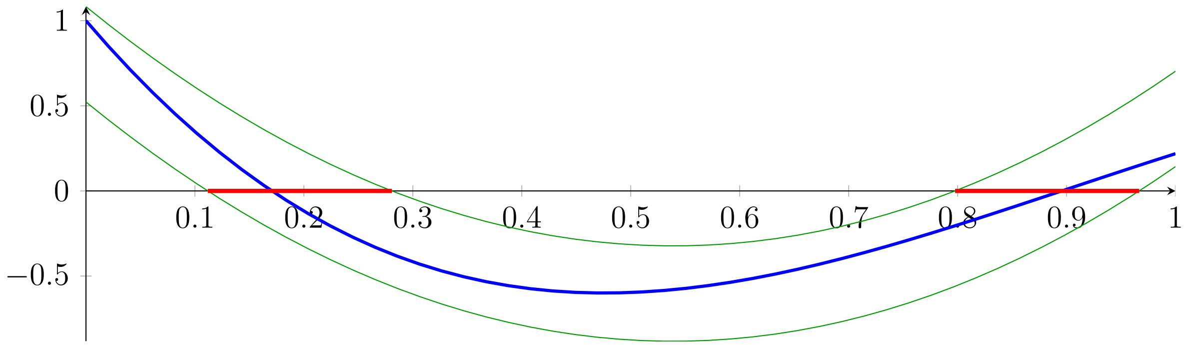
$$M = 4.83259X^2 - 5.21205X + 1.08259$$

$$m = 4.83259X^2 - 5.21205X + 0.522321$$

Root of M and m:

$$N(M) = \{0.280835, 0.797687\} \qquad N(m) = \{0.111804, 0.966718\}$$

Intersection intervals:



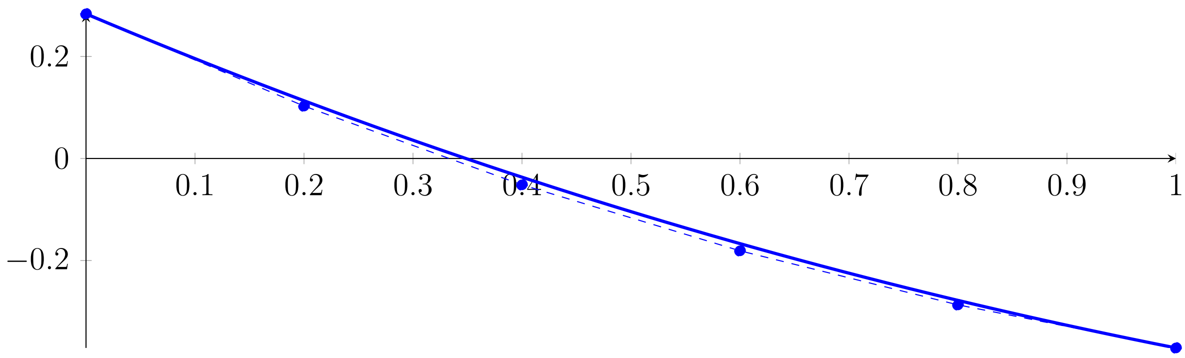
$[0.111804, 0.280835]$, $[0.797687, 0.966718]$

Longest intersection interval: 0.169031
$\Longrightarrow$ Selective recursion: interval 1: $[0.0559021, 0.140418]$, interval 2: $[0.398843, 0.483359]$,

2.3 Recursion Branch 1 1 1 in Interval 1: $[0.0559021, 0.140418]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0001078X^5 - 0.0014292X^4 - 0.0133082X^3 + 0.26337X^2 - 0.903613X + 0.283521 \\ &= 0.283521B_{0,5}(X) + 0.102799B_{1,5}(X) - 0.0515868B_{2,5}(X) \\ &\quad - 0.180966B_{3,5}(X) - 0.286956B_{4,5}(X) - 0.371351B_{5,5}(X) \end{aligned}$$



Best approximation polynomials of degree 0, 1, 2, 3 and 4:

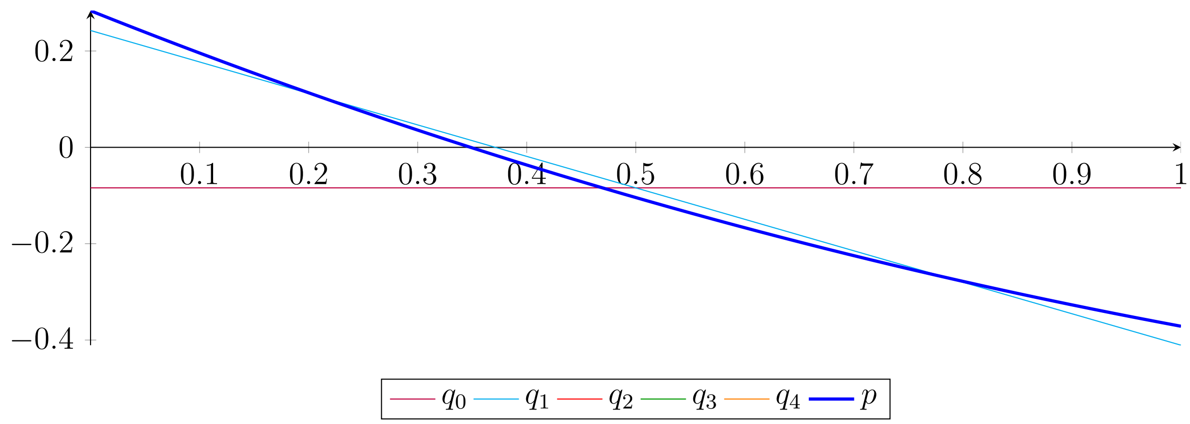
$$\begin{aligned} q_0 &= -0.08409 \\ &= -0.08409B_{0,0} \end{aligned}$$

$$\begin{aligned} q_1 &= -0.653287X + 0.242553 \\ &= 0.242553B_{0,1} - 0.410733B_{1,1} \end{aligned}$$

$$\begin{aligned} q_2 &= 0.24115X^2 - 0.894437X + 0.282745 \\ &= 0.282745B_{0,2} - 0.164473B_{1,2} - 0.370542B_{2,2} \end{aligned}$$

$$\begin{aligned} q_3 &= -0.0158671X^3 + 0.264951X^2 - 0.903957X + 0.283538 \\ &= 0.283538B_{0,3} - 0.0177807B_{1,3} - 0.230783B_{2,3} - 0.371335B_{3,3} \end{aligned}$$

$$\begin{aligned} q_4 &= -0.0011597X^4 - 0.0135477X^3 + 0.26346X^2 - 0.903626X + 0.283522 \\ &= 0.283522B_{0,4} + 0.0576154B_{1,4} - 0.124381B_{2,4} - 0.265855B_{3,4} - 0.371352B_{4,4} \end{aligned}$$



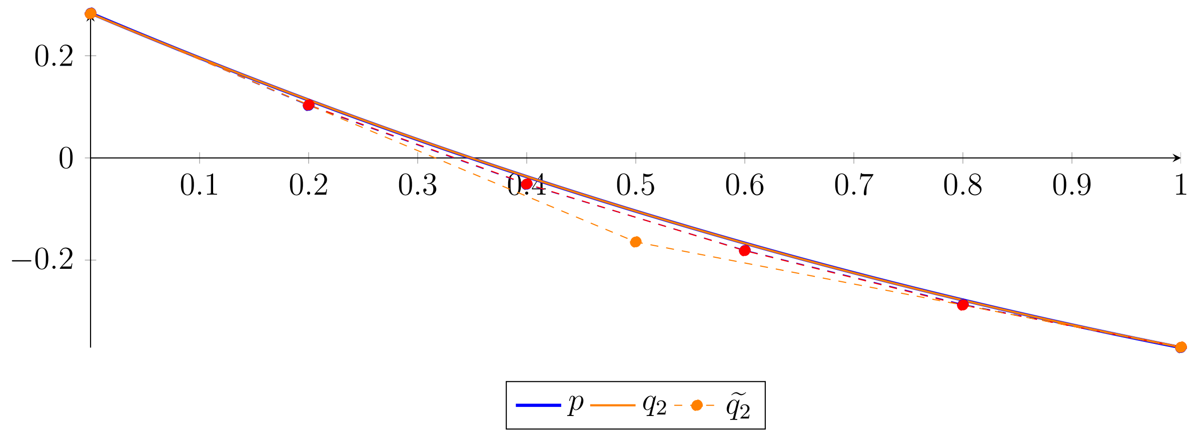
Degree reduction and raising matrices:

$$M_{5,2} = \begin{pmatrix} 0.821429 & -0.428571 & 0.107143 \\ 0.321429 & 0.285714 & -0.107143 \\ 1.249\cdot10^{-16} & 0.642857 & -0.142857 \\ -0.142857 & 0.642857 & 0 \\ -0.107143 & 0.285714 & 0.321429 \\ 0.107143 & -0.428571 & 0.821429 \end{pmatrix} \quad M_{2,5} = \begin{pmatrix} 1 & 0.6 & 0.3 & 0.1 & 2.1760 \\ 8.10463\cdot10^{-15} & 0.4 & 0.6 & 0.6 & 0 \\ 4.996\cdot10^{-15} & -1.59872\cdot10^{-14} & 0.1 & 0.3 & 0 \end{pmatrix}$$

Degree reduction and raising:

$$\begin{aligned} q_2 &= 0.24115X^2 - 0.894437X + 0.282745 \\ &= 0.282745B_{0,2} - 0.164473B_{1,2} - 0.370542B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 5.04929\cdot10^{-13}X^5 - 1.11688\cdot10^{-12}X^4 + 8.40994\cdot10^{-13}X^3 + 0.24115X^2 - 0.894437X + 0.282745 \\ &= 0.282745B_{0,5} + 0.103858B_{1,5} - 0.0509147B_{2,5} - 0.181572B_{3,5} - 0.288114B_{4,5} - 0.370542B_{5,5} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.00115826$.
Bounding polynomials M and m:

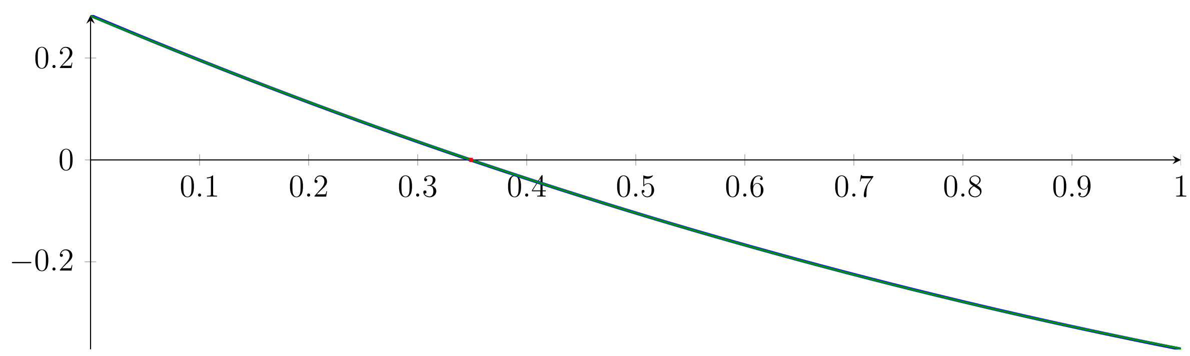
$$M = 0.24115X^2 - 0.894437X + 0.283903$$

$$m = 0.24115X^2 - 0.894437X + 0.281587$$

Root of M and m:

$$N(M) = \{0.350539, 3.3585\} \qquad N(m) = \{0.347349, 3.36169\}$$

Intersection intervals:



[0.347349, 0.350539]

Longest intersection interval: 0.00319018

$\Longrightarrow$ Selective recursion: interval 1: [0.0852585, 0.0855281],

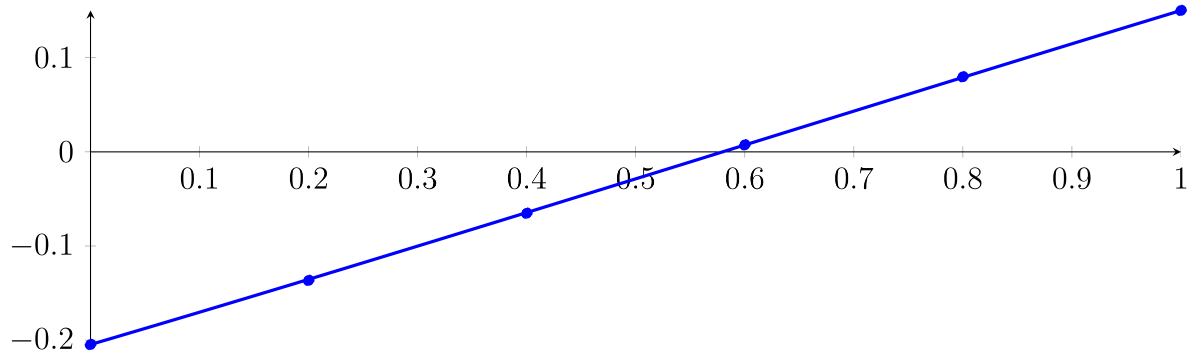
2.4 Recursion Branch 1 1 1 1 in Interval 1: [0.0852585, 0.0855281]

Found root in interval [0.0852585, 0.0855281] at recursion depth 4!

2.5 Recursion Branch 1 1 2 in Interval 2: [0.398843, 0.483359]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 0.0001078X^5 + 0.00075793X^4 - 0.0187558X^3 + 0.0321977X^2 + 0.340572X - 0.204667\\ &= -0.204667B_{0,5}(X) - 0.136552B_{1,5}(X) - 0.0652183B_{2,5}(X)\\ &\quad + 0.00746004B_{3,5}(X) + 0.0797585B_{4,5}(X) + 0.150213B_{5,5}(X)\end{aligned}$$



Best approximation polynomials of degree 0, 1, 2, 3 and 4:

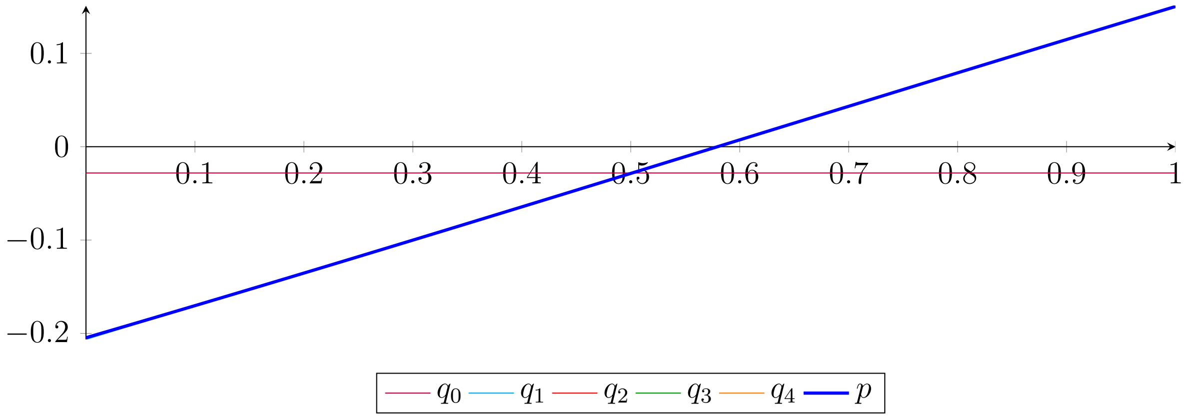
$$\begin{aligned}q_0 &= -0.0281677\\ &= -0.0281677B_{0,0}\end{aligned}$$

$$\begin{aligned}q_1 &= 0.356573X - 0.206454\\ &= -0.206454B_{0,1} + 0.150119B_{1,1}\end{aligned}$$

$$\begin{aligned}q_2 &= 0.00555581X^2 + 0.351017X - 0.205528\\ &= -0.205528B_{0,2} - 0.0300196B_{1,2} + 0.151045B_{2,2}\end{aligned}$$

$$\begin{aligned}q_3 &= -0.0169405X^3 + 0.0309666X^2 + 0.340852X - 0.204681\\ &= -0.204681B_{0,3} - 0.0910635B_{1,3} + 0.0328762B_{2,3} + 0.150198B_{3,3}\end{aligned}$$

$$\begin{aligned}q_4 &= 0.00102743X^4 - 0.0189954X^3 + 0.0322876X^2 + 0.340559X - 0.204666\\ &= -0.204666B_{0,4} - 0.119527B_{1,4} - 0.0290056B_{2,4} + 0.0621478B_{3,4} + 0.150212B_{4,4}\end{aligned}$$



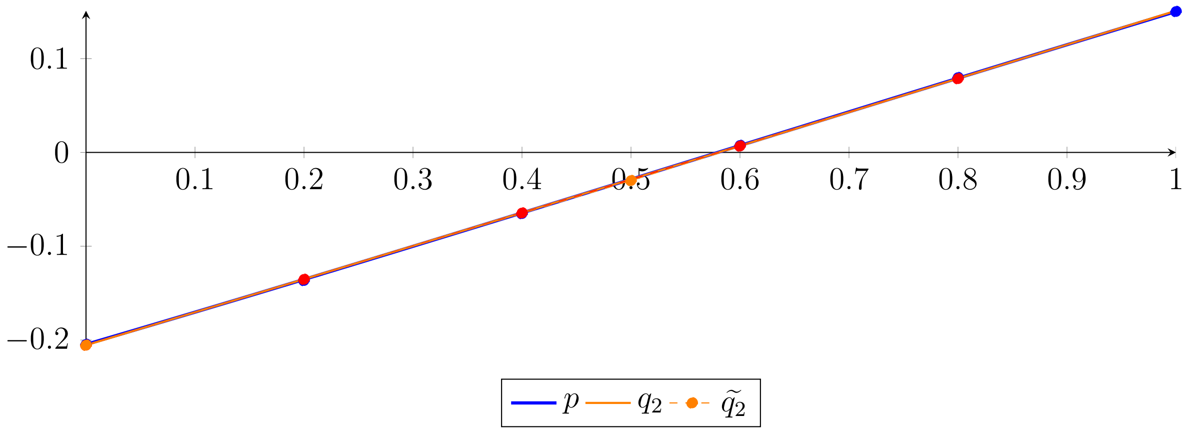
Degree reduction and raising matrices:

$$M_{5,2} = \begin{pmatrix} 0.821429 & -0.428571 & 0.107143 \\ 0.321429 & 0.285714 & -0.107143 \\ 1.249\cdot10^{-16} & 0.642857 & -0.142857 \\ -0.142857 & 0.642857 & 0 \\ -0.107143 & 0.285714 & 0.321429 \\ 0.107143 & -0.428571 & 0.821429 \end{pmatrix} \quad M_{2,5} = \begin{pmatrix} 1 & 0.6 & 0.3 & 0.1 & 2.17604 \\ 8.10463\cdot10^{-15} & 0.4 & 0.6 & 0.6 & 0. \\ 4.996\cdot10^{-15} & -1.59872\cdot10^{-14} & 0.1 & 0.3 & 0. \end{pmatrix}$$

Degree reduction and raising:

$$\begin{aligned} q_2 &= 0.00555581X^2 + 0.351017X - 0.205528 \\ &= -0.205528B_{0,2} - 0.0300196B_{1,2} + 0.151045B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 5.67324\cdot10^{-14}X^5 - 1.83464\cdot10^{-13}X^4 + 1.99285\cdot10^{-13}X^3 + 0.00555581X^2 + 0.351017X - 0.205528 \\ &= -0.205528B_{0,5} - 0.135325B_{1,5} - 0.0645657B_{2,5} + 0.00674878B_{3,5} + 0.0786189B_{4,5} + 0.151045B_{5,5} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.00122773$.
Bounding polynomials M and m:

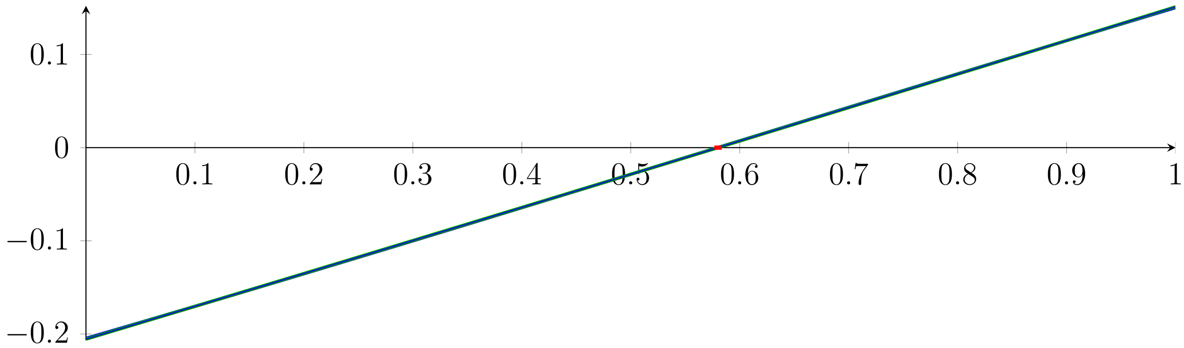
$$M = 0.00555581X^2 + 0.351017X - 0.2043$$

$$m = 0.00555581X^2 + 0.351017X - 0.206756$$

Root of M and m:

$$N(M) = \{-63.7569, 0.576759\} \qquad N(m) = \{-63.7637, 0.583628\}$$

Intersection intervals:



$[0.576759, 0.583628]$

Longest intersection interval: 0.00686912
$\Longrightarrow$ Selective recursion: interval 1: $[0.447588, 0.448169]$,

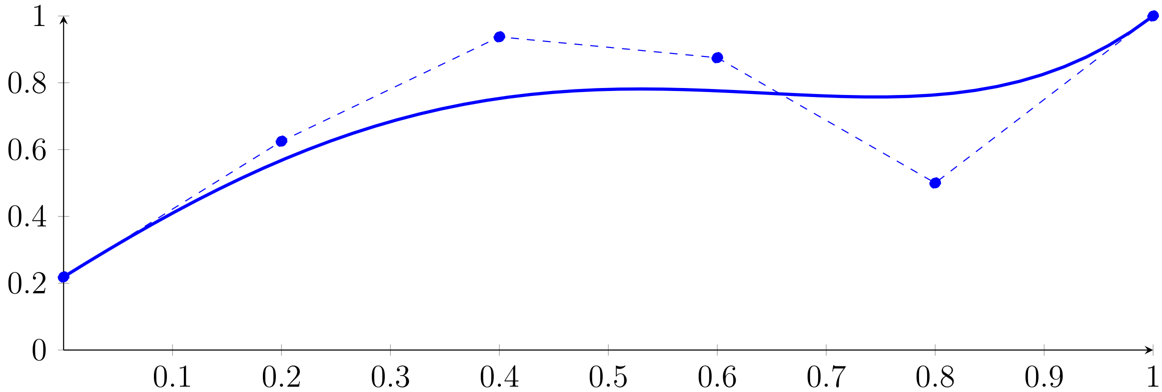
2.6 Recursion Branch 1 1 2 1 in Interval 1: $[0.447588, 0.448169]$

Found root in interval $[0.447588, 0.448169]$ at recursion depth 4!

2.7 Recursion Branch 1 2 on the Second Half $[0.5, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 0.78125X^5 + 1.71875X^4 - 2.8125X^3 - 0.9375X^2 + 2.03125X + 0.21875$$
$$= 0.21875B_{0,5}(X) + 0.625B_{1,5}(X) + 0.9375B_{2,5}(X) + 0.875B_{3,5}(X) + 0.5B_{4,5}(X) + 1B_{5,5}(X)$$



Best approximation polynomials of degree 0, 1, 2, 3 and 4:

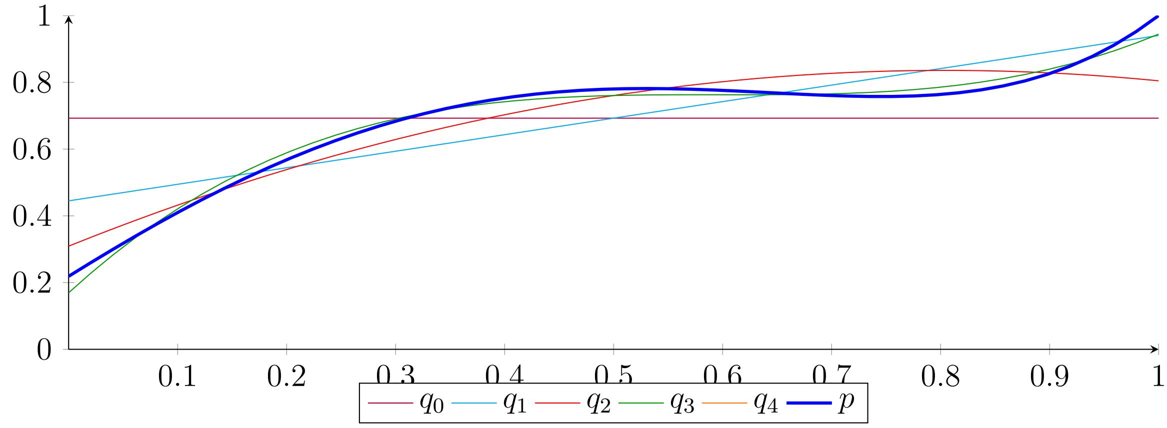
$$q_0 = 0.692708$$
$$= 0.692708B_{0,0}$$

$$q_1 = 0.495536X + 0.44494$$
$$= 0.44494B_{0,1} + 0.940476B_{1,1}$$

$$q_2 = -0.814732X^2 + 1.31027X + 0.309152$$
$$= 0.309152B_{0,2} + 0.964286B_{1,2} + 0.804688B_{2,2}$$

$$q_3 = 2.79514X^3 - 5.00744X^2 + 2.98735X + 0.169395$$
$$= 0.169395B_{0,3} + 1.16518B_{1,3} + 0.491815B_{2,3} + 0.944444B_{3,3}$$

$$q_4 = 3.67187X^4 - 4.54861X^3 - 0.286458X^2 + 1.93824X + 0.22185$$
$$= 0.22185B_{0,4} + 0.706411B_{1,4} + 1.14323B_{2,4} + 0.395151B_{3,4} + 0.9969B_{4,4}$$



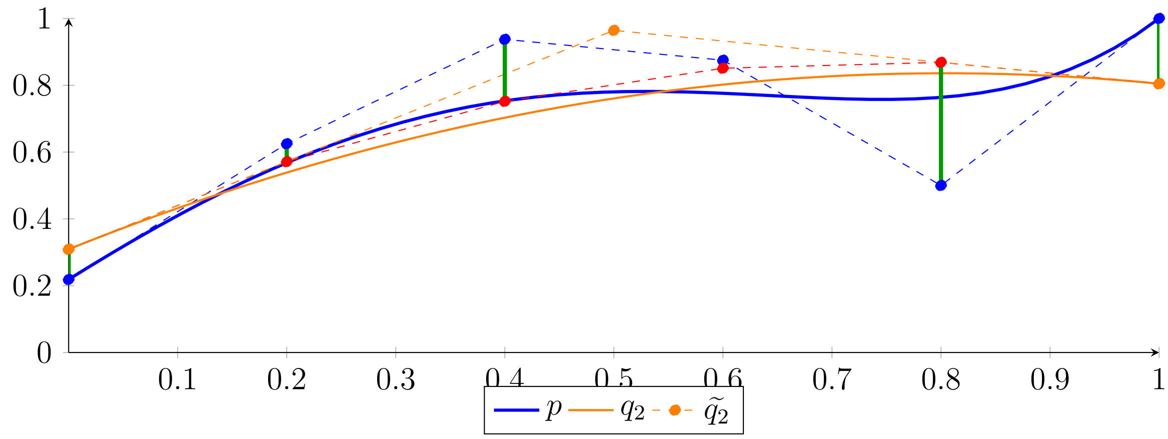
Degree reduction and raising matrices:

$$M_{5,2} = \begin{pmatrix} 0.821429 & -0.428571 & 0.107143 \\ 0.321429 & 0.285714 & -0.107143 \\ 1.249\cdot 10^{-16} & 0.642857 & -0.142857 \\ -0.142857 & 0.642857 & 0 \\ -0.107143 & 0.285714 & 0.321429 \\ 0.107143 & -0.428571 & 0.821429 \end{pmatrix} \quad M_{2,5} = \begin{pmatrix} 1 & 0.6 & 0.3 & 0.1 & 2.17604 \\ 8.10463\cdot 10^{-15} & 0.4 & 0.6 & 0.6 & 0. \\ 4.996\cdot 10^{-15} & -1.59872\cdot 10^{-14} & 0.1 & 0.3 & 0. \end{pmatrix}$$

Degree reduction and raising:

$$\begin{aligned} q_2 &= -0.814732X^2 + 1.31027X + 0.309152 \\ &= 0.309152B_{0,2} + 0.964286B_{1,2} + 0.804688B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= -3.16791\cdot 10^{-12}X^5 + 7.47069\cdot 10^{-12}X^4 - 6.11289\cdot 10^{-12}X^3 - 0.814732X^2 + 1.31027X + 0.309152 \\ &= 0.309152B_{0,5} + 0.571205B_{1,5} + 0.751786B_{2,5} + 0.850893B_{3,5} + 0.868527B_{4,5} + 0.804688B_{5,5} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.368527$.
Bounding polynomials M and m:

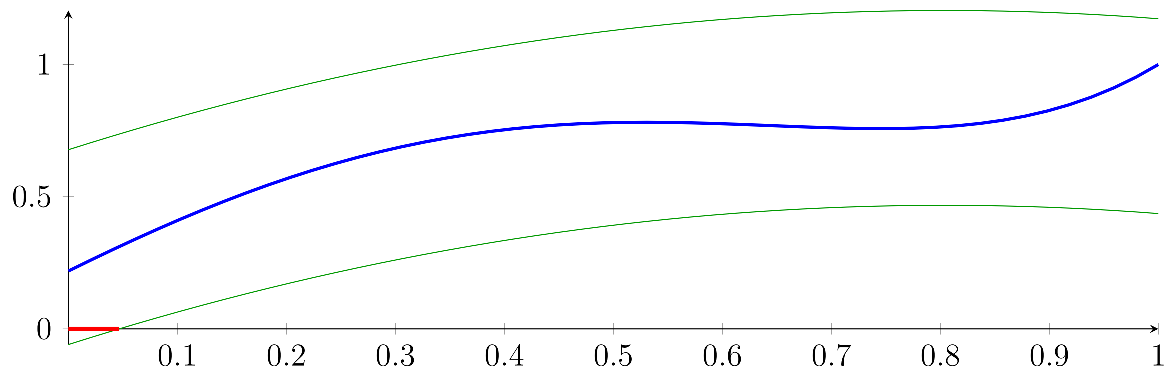
$$M = -0.814732X^2 + 1.31027X + 0.677679$$

$$m = -0.814732X^2 + 1.31027X - 0.059375$$

Root of M and m:

$$N(M) = \{-0.411774, 2.01999\} \qquad N(m) = \{0.0466695, 1.56155\}$$

Intersection intervals:



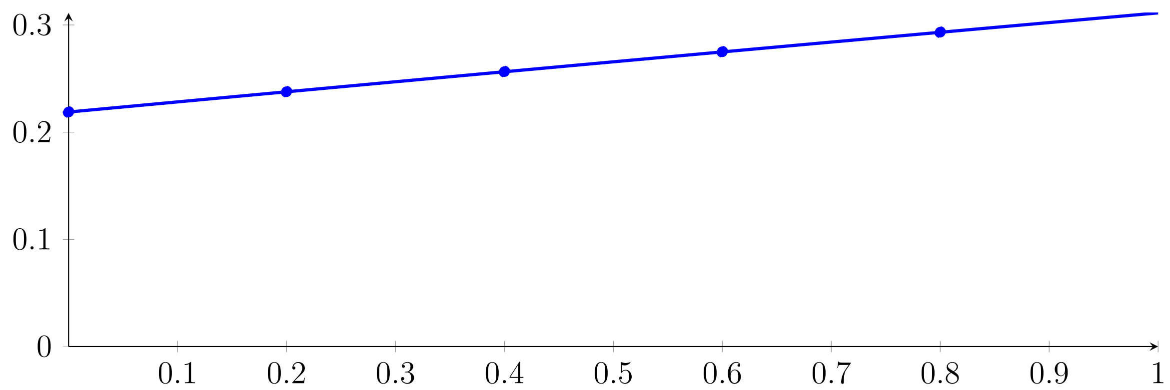
[0, 0.0466695]

Longest intersection interval: 0.0466695
$\Longrightarrow$ Selective recursion: interval 1: [0.5, 0.523335],

2.8 Recursion Branch 1 2 1 in Interval 1: [0.5, 0.523335]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 1.72964 \cdot 10^{-07} X^5 + 8.15351 \cdot 10^{-06} X^4 - 0.000285885 X^3 - 0.00204191 X^2 + 0.0947974 X + 0.21875 \\ &= 0.21875 B_{0,5}(X) + 0.237709 B_{1,5}(X) + 0.256465 B_{2,5}(X) \\ &\quad + 0.274987 B_{3,5}(X) + 0.29325 B_{4,5}(X) + 0.311228 B_{5,5}(X) \end{aligned}$$



Best approximation polynomials of degree 0, 1, 2, 3 and 4:

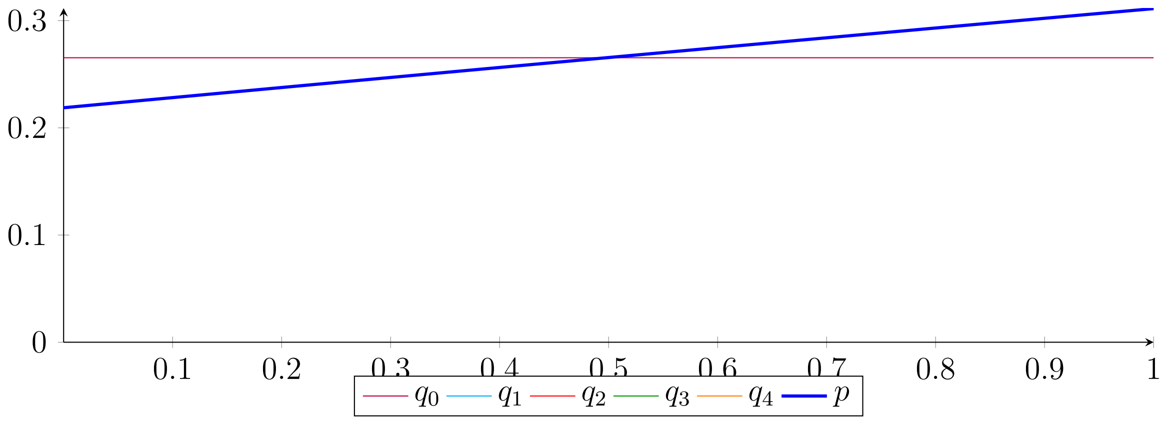
$$\begin{aligned} q_0 &= 0.265398 \\ &= 0.265398 B_{0,0} \end{aligned}$$

$$\begin{aligned} q_1 &= 0.0925048 X + 0.219146 \\ &= 0.219146 B_{0,1} + 0.311651 B_{1,1} \end{aligned}$$

$$\begin{aligned} q_2 &= -0.00245645 X^2 + 0.0949613 X + 0.218736 \\ &= 0.218736 B_{0,2} + 0.266217 B_{1,2} + 0.311241 B_{2,2} \end{aligned}$$

$$\begin{aligned} q_3 &= -0.000269098 X^3 - 0.00205281 X^2 + 0.0947998 X + 0.21875 \\ &= 0.21875 B_{0,3} + 0.25035 B_{1,3} + 0.281265 B_{2,3} + 0.311228 B_{3,3} \end{aligned}$$

$$\begin{aligned} q_4 &= 8.58592 \cdot 10^{-06} X^4 - 0.000286269 X^3 - 0.00204177 X^2 + 0.0947974 X + 0.21875 \\ &= 0.21875 B_{0,4} + 0.242449 B_{1,4} + 0.265808 B_{2,4} + 0.288756 B_{3,4} + 0.311228 B_{4,4} \end{aligned}$$



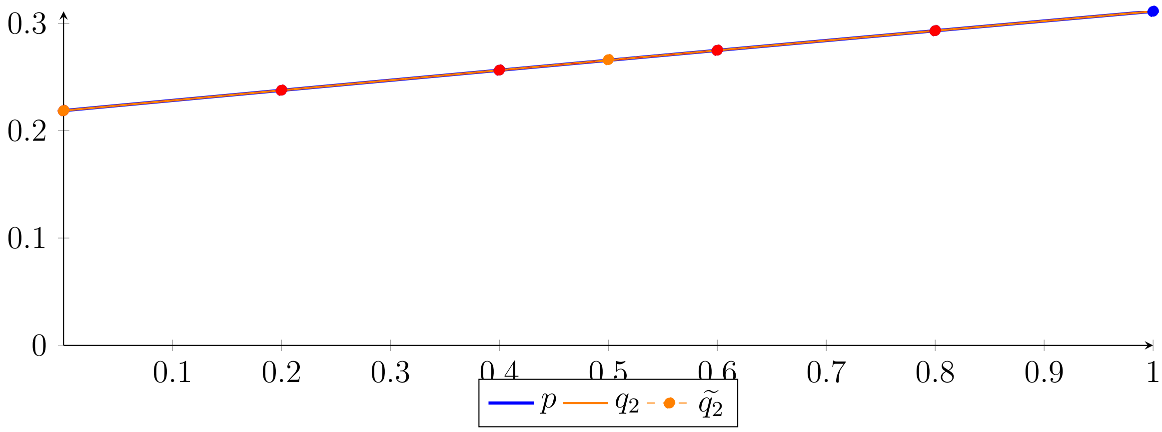
Degree reduction and raising matrices:

$$M_{5,2} = \begin{pmatrix} 0.821429 & -0.428571 & 0.107143 \\ 0.321429 & 0.285714 & -0.107143 \\ 1.249{\cdot}10^{-16} & 0.642857 & -0.142857 \\ -0.142857 & 0.642857 & 0 \\ -0.107143 & 0.285714 & 0.321429 \\ 0.107143 & -0.428571 & 0.821429 \end{pmatrix} \quad M_{2,5} = \begin{pmatrix} 1 & 0.6 & 0.3 & 0.1 & 2.17604 \\ 8.10463{\cdot}10^{-15} & 0.4 & 0.6 & 0.6 & 0. \\ 4.996{\cdot}10^{-15} & -1.59872{\cdot}10^{-14} & 0.1 & 0.3 & 0. \end{pmatrix}$$

Degree reduction and raising:

$$\begin{aligned} q_2 &= -0.00245645X^2 + 0.0949613X + 0.218736 \\ &= 0.218736B_{0,2} + 0.266217B_{1,2} + 0.311241B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= -1.17323{\cdot}10^{-12}X^5 + 2.76473{\cdot}10^{-12}X^4 - 2.26041{\cdot}10^{-12}X^3 - 0.00245645X^2 + 0.0949613X + 0.218736 \\ &= 0.218736B_{0,5} + 0.237729B_{1,5} + 0.256475B_{2,5} + 0.274976B_{3,5} + 0.293232B_{4,5} + 0.311241B_{5,5} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.92014{\cdot}10^{-05}$.

Bounding polynomials M and m:

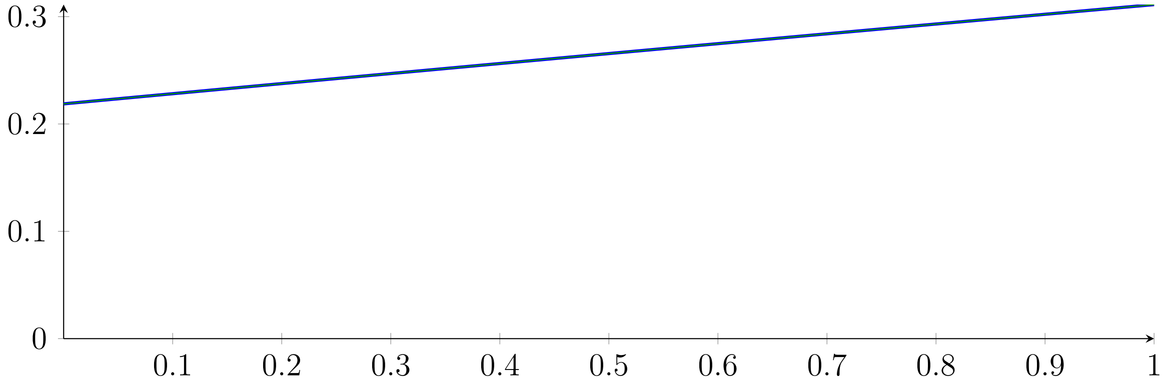
$$M = -0.00245645X^2 + 0.0949613X + 0.218756$$

$$m = -0.00245645X^2 + 0.0949613X + 0.218717$$

Root of M and m:

$$N(M) = \{-2.18062, 40.8385\} \qquad N(m) = \{-2.18026, 40.8381\}$$

Intersection intervals:

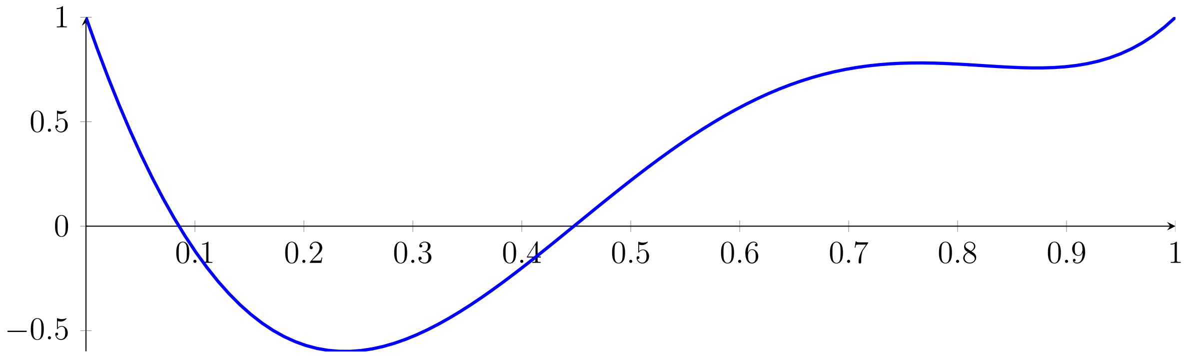


No intersection intervals with the x axis.

2.9 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 25X^5 - 35X^4 - 15X^3 + 40X^2 - 15X + 1$$



Result: Root Intervals

$$[0.0852585, 0.0855281],\ [0.447588, 0.448169]$$

with precision $\varepsilon = 0.001$.