Demo 3

This demonstration shows that the implementation can also successfully isolate the roots of the large Wilkinson polynomial with $n = 20$. This polynomial is defined as

$$p = \prod_{i=1}^{20}(x - i)$$

and is analysed on the interval $[0, 25]$. All three algorithms successfully find all roots with the standard datatype `double` with precision $\varepsilon = 0.001$. Regarding the results see page 66, page 208 and page 311.

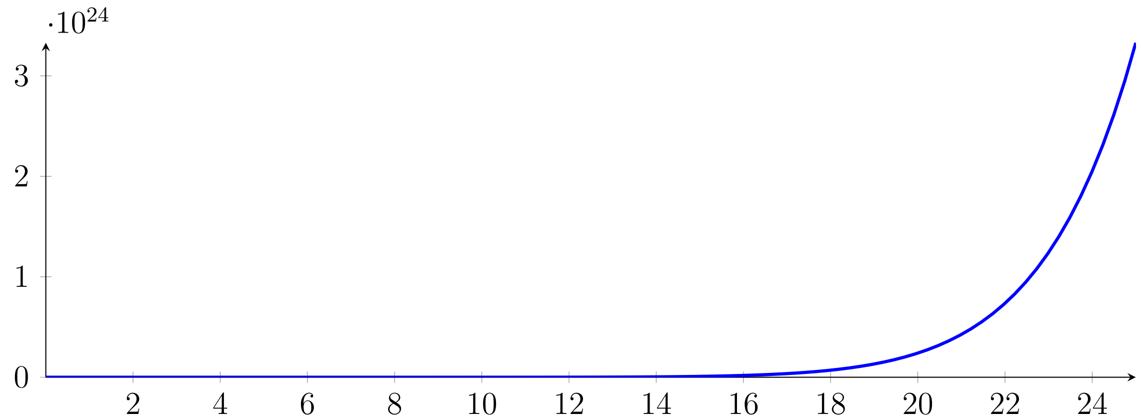
Contents

1 BezClip Applied to the Wilkinson Polynomial

$$1X^{20} - 210X^{19} + 20615X^{18} - 1.25685\cdot 10^6 X^{17} + 5.33279\cdot 10^7 X^{16} - 1.67228\cdot 10^9 X^{15} + 4.01718\cdot 10^{10} X^{14} - 7.56111\cdot 10^{11} X^{13} + 1.13103\cdot 10^{13} X^{12} - 1.35585\cdot 10^{14} X^{11} + 1.30754\cdot 10^{15} X^{10} - 1.01423\cdot 10^{16} X^9 + 6.30308\cdot 10^{16} X^8 - 3.11334\cdot 10^{17} X^7 + 1.20665\cdot 10^{18} X^6 - 3.59998\cdot 10^{18} X^5 + 8.03781\cdot 10^{18} X^4 - 1.28709\cdot 10^{19} X^3 + 1.38038\cdot 10^{19} X^2 - 8.75295\cdot 10^{18} X + 2.4329\cdot 10^{18}$$

Called BezClip with input polynomial on interval $[0, 25]$:

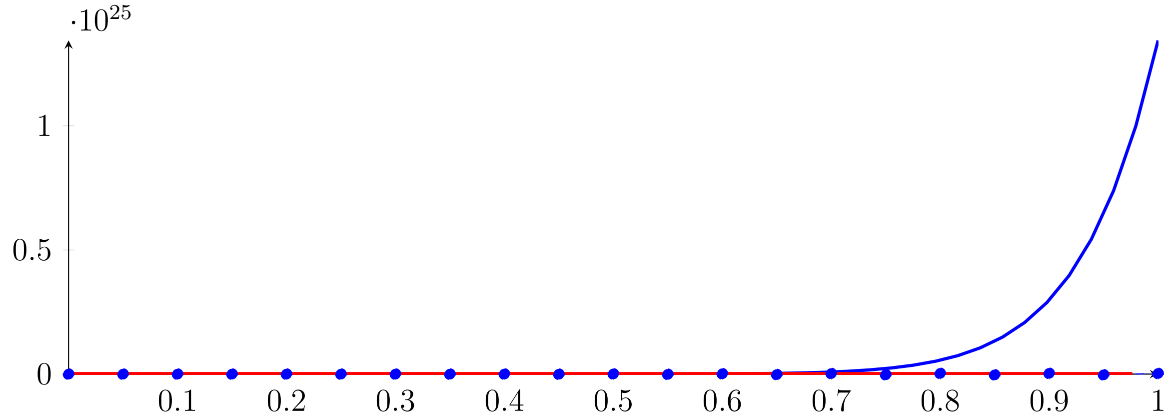
$$p = 1X^{20} - 210X^{19} + 20615X^{18} - 1.25685\cdot 10^6 X^{17} + 5.33279\cdot 10^7 X^{16} - 1.67228\cdot 10^9 X^{15} + 4.01718\cdot 10^{10} X^{14} - 7.56111\cdot 10^{11} X^{13} + 1.13103\cdot 10^{13} X^{12} - 1.35585\cdot 10^{14} X^{11} + 1.30754\cdot 10^{15} X^{10} - 1.01423\cdot 10^{16} X^9 + 6.30308\cdot 10^{16} X^8 - 3.11334\cdot 10^{17} X^7 + 1.20665\cdot 10^{18} X^6 - 3.59998\cdot 10^{18} X^5 + 8.03781\cdot 10^{18} X^4 - 1.28709\cdot 10^{19} X^3 + 1.38038\cdot 10^{19} X^2 - 8.75295\cdot 10^{18} X + 2.4329\cdot 10^{18}$$



1.1 Recursion Branch 1 for Input Interval $[0, 25]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 9.09495\cdot 10^{27} X^{20} - 7.63976\cdot 10^{28} X^{19} + 2.99988\cdot 10^{29} X^{18} - 7.31583\cdot 10^{29} X^{17} + 1.24164\cdot 10^{30} X^{16} - 1.55743\cdot 10^{30} X^{15} + 1.49652\cdot 10^{30} X^{14} - 1.12669\cdot 10^{30} X^{13} + 6.74145\cdot 10^{29} X^{12} - 3.2326\cdot 10^{29} X^{11} + 1.24696\cdot 10^{29} X^{10} - 3.86898\cdot 10^{28} X^9 + 9.61774\cdot 10^{27} X^8 - 1.90023\cdot 10^{27} X^7 + 2.94592\cdot 10^{26} X^6 - 3.5156\cdot 10^{25} X^5 + 3.13977\cdot 10^{24} X^4 - 2.01108\cdot 10^{23} X^3 + 8.62735\cdot 10^{21} X^2 - 2.18824\cdot 10^{20} X + 2.4329\cdot 10^{18} \\ &= 2.4329\cdot 10^{18} B_{0,20}(X) - 8.50828\cdot 10^{18} B_{1,20}(X) + 2.59576\cdot 10^{19} B_{2,20}(X) - 7.05801\cdot 10^{19} B_{3,20}(X) + 1.73511\cdot 10^{20} B_{4,20}(X) - 3.8964\cdot 10^{20} B_{5,20}(X) + 8.05451\cdot 10^{20} B_{6,20}(X) - 1.54188\cdot 10^{21} B_{7,20}(X) + 2.74637\cdot 10^{21} B_{8,20}(X) - 4.56922\cdot 10^{21} B_{9,20}(X) + 7.12322\cdot 10^{21} B_{10,20}(X) - 1.04331\cdot 10^{22} B_{11,20}(X) + 1.43886\cdot 10^{22} B_{12,20}(X) - 1.87204\cdot 10^{22} B_{13,20}(X) + 2.30149\cdot 10^{22} B_{14,20}(X) - 2.67735\cdot 10^{22} B_{15,20}(X) + 2.95071\cdot 10^{22} B_{16,20}(X) - 3.08413\cdot 10^{22} B_{17,20}(X) + 3.06005\cdot 10^{22} B_{18,20}(X) - 2.88452\cdot 10^{22} B_{19,20}(X) + 2.5852\cdot 10^{22} B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{6.70466e-05, 0.976368\}$$

Intersection intervals with the x axis:

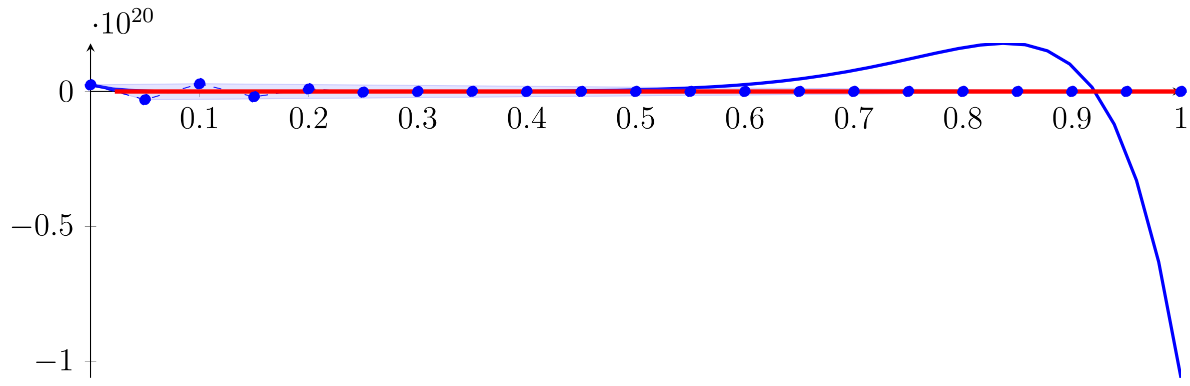
$$[6.70466e-05, 0.976368]$$

Longest intersection interval: 0.976301
$\Longrightarrow$ Bisection: first half $[0, 12.5]$ und second half $[12.5, 25]$

1.2 Recursion Branch 1 1 on the First Half $[0, 12.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 8.67362\cdot10^{21}X^{20} - 1.45717\cdot10^{23}X^{19} + 1.14436\cdot10^{24}X^{18} - 5.58154\cdot10^{24}X^{17} + 1.89459\cdot10^{25}X^{16} - 4.75291\cdot10^{25}X^{15} + 9.134\cdot10^{25}X^{14} - 1.37536\cdot10^{26}X^{13} + 1.64586\cdot10^{26}X^{12} - 1.57842\cdot10^{26}X^{11} + 1.21774\cdot10^{26}X^{10} - 7.5566\cdot10^{25}X^{9} + 3.75693\cdot10^{25}X^{8} - 1.48455\cdot10^{25}X^{7} + 4.603\cdot10^{24}X^{6} - 1.09863\cdot10^{24}X^{5} + 1.96236\cdot10^{23}X^{4} - 2.51385\cdot10^{22}X^{3} + 2.15684\cdot10^{21}X^{2} - 1.09412\cdot10^{20}X + 2.4329\cdot10^{18}$$
$$= 2.4329\cdot10^{18}B_{0,20}(X) - 3.03769\cdot10^{18}B_{1,20}(X) + 2.84349\cdot10^{18}B_{2,20}(X) - 1.9749\cdot10^{18}B_{3,20}(X) + 9.58506\cdot10^{17}B_{4,20}(X) - 2.63073\cdot10^{17}B_{5,20}(X) - 9.0343\cdot10^{15}B_{6,20}(X) + 3.44399\cdot10^{16}B_{7,20}(X) - 5.41351\cdot10^{15}B_{8,20}(X) - 4.28958\cdot10^{15}B_{9,20}(X) + 1.09675\cdot10^{15}B_{10,20}(X) + 6.89924\cdot10^{14}B_{11,20}(X) - 1.57583\cdot10^{14}B_{12,20}(X) - 1.3719\cdot10^{14}B_{13,20}(X) + 1.13888\cdot10^{13}B_{14,20}(X) + 2.83586\cdot10^{13}B_{15,20}(X) + 3.54186\cdot10^{12}B_{16,20}(X) - 4.9643\cdot10^{12}B_{17,20}(X) - 2.0514\cdot10^{12}B_{18,20}(X) + 5.37337\cdot10^{11}B_{19,20}(X) + 8.1419\cdot10^{11}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.0222362, 1\}$$

Intersection intervals with the x axis:

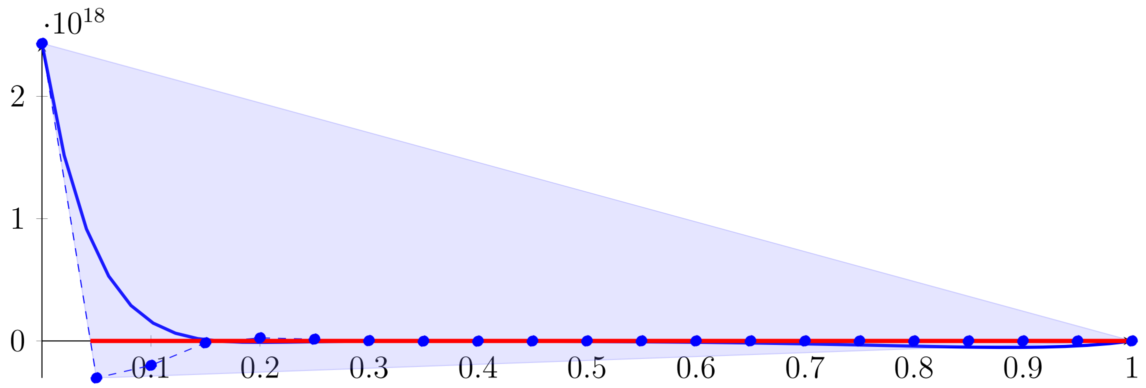
$$[0.0222362, 1]$$

Longest intersection interval: 0.977764
$\Longrightarrow$ Bisection: first half $[0, 6.25]$ und second half $[6.25, 12.5]$

1.3 Recursion Branch 1 1 1 on the First Half $[0, 6.25]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 8.27181{\cdot}10^{15}X^{20} - 2.77933{\cdot}10^{17}X^{19} + 4.3654{\cdot}10^{18}X^{18} - 4.25837{\cdot}10^{19}X^{17} + 2.89091{\cdot}10^{20}X^{16} - 1.45047{\cdot}10^{21}X^{15} + 5.57495{\cdot}10^{21}X^{14} - 1.6789{\cdot}10^{22}X^{13} + 4.01822{\cdot}10^{22}X^{12} - 7.70713{\cdot}10^{22}X^{11} + 1.1892{\cdot}10^{23}X^{10} - 1.4759{\cdot}10^{23}X^{9} + 1.46755{\cdot}10^{23}X^{8} - 1.15981{\cdot}10^{23}X^{7} + 7.19218{\cdot}10^{22}X^{6} - 3.43321{\cdot}10^{22}X^{5} + 1.22647{\cdot}10^{22}X^{4} - 3.14232{\cdot}10^{21}X^{3} + 5.39209{\cdot}10^{20}X^{2} - 5.47059{\cdot}10^{19}X + 2.4329{\cdot}10^{18}$$
$$= 2.4329{\cdot}10^{18}B_{0,20}(X) - 3.02394{\cdot}10^{17}B_{1,20}(X) - 1.99746{\cdot}10^{17}B_{2,20}(X) - 1.55733{\cdot}10^{16}B_{3,20}(X) + 2.51263{\cdot}10^{16}B_{4,20}(X) + 1.43711{\cdot}10^{16}B_{5,20}(X) + 2.36483{\cdot}10^{15}B_{6,20}(X) - 1.91069{\cdot}10^{15}B_{7,20}(X) - 1.81457{\cdot}10^{15}B_{8,20}(X) - 7.4091{\cdot}10^{14}B_{9,20}(X) - 3.15634{\cdot}10^{13}B_{10,20}(X) + 1.92739{\cdot}10^{14}B_{11,20}(X) + 1.62719{\cdot}10^{14}B_{12,20}(X) + 7.31276{\cdot}10^{13}B_{13,20}(X) + 9.11723{\cdot}10^{12}B_{14,20}(X) - 1.65546{\cdot}10^{13}B_{15,20}(X) - 1.79828{\cdot}10^{13}B_{16,20}(X) - 1.06656{\cdot}10^{13}B_{17,20}(X) - 3.51597{\cdot}10^{12}B_{18,20}(X) + 5.61716{\cdot}10^{11}B_{19,20}(X) + 1.86285{\cdot}10^{12}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.0444724, 0.999994\}$$

Intersection intervals with the x axis:

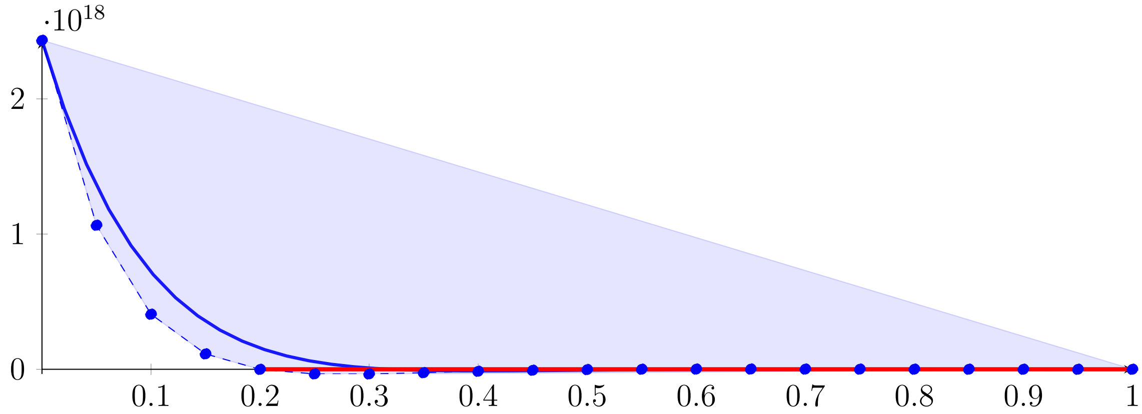
$$[0.0444724, 0.999994]$$

Longest intersection interval: 0.955522
$\Longrightarrow$ Bisection: first half $[0, 3.125]$ und second half $[3.125, 6.25]$

1.4 Recursion Branch 1 1 1 1 on the First Half $[0, 3.125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 7.89961{\cdot}10^{9}X^{20} - 5.30084{\cdot}10^{11}X^{19} + 1.66534{\cdot}10^{13}X^{18} - 3.24889{\cdot}10^{14}X^{17} + 4.41119{\cdot}10^{15}X^{16} - 4.42649{\cdot}10^{16}X^{15} + 3.40268{\cdot}10^{17}X^{14} - 2.04944{\cdot}10^{18}X^{13} + 9.8101{\cdot}10^{18}X^{12} - 3.76324{\cdot}10^{19}X^{11} + 1.16132{\cdot}10^{20}X^{10} - 2.88261{\cdot}10^{20}X^{9} + 5.73262{\cdot}10^{20}X^{8} - 9.061{\cdot}10^{20}X^{7} + 1.12378{\cdot}10^{21}X^{6} - 1.07288{\cdot}10^{21}X^{5} + 7.66545{\cdot}10^{20}X^{4} - 3.9279{\cdot}10^{20}X^{3} + 1.34802{\cdot}10^{20}X^{2} - 2.7353{\cdot}10^{19}X + 2.4329{\cdot}10^{18}$$
$$= 2.4329{\cdot}10^{18}B_{0,20}(X) + 1.06525{\cdot}10^{18}B_{1,20}(X) + 4.07092{\cdot}10^{17}B_{2,20}(X) + 1.13863{\cdot}10^{17}B_{3,20}(X) - 7.70051{\cdot}10^{14}B_{4,20}(X) - 3.41333{\cdot}10^{16}B_{5,20}(X) - 3.47444{\cdot}10^{16}B_{6,20}(X) - 2.52167{\cdot}10^{16}B_{7,20}(X) - 1.49942{\cdot}10^{16}B_{8,20}(X) - 7.22308{\cdot}10^{15}B_{9,20}(X) - 2.31656{\cdot}10^{15}B_{10,20}(X) + 2.94801{\cdot}10^{14}B_{11,20}(X) + 1.37334{\cdot}10^{15}B_{12,20}(X) + 1.56871{\cdot}10^{15}B_{13,20}(X) + 1.33924{\cdot}10^{15}B_{14,20}(X) + 9.67327{\cdot}10^{14}B_{15,20}(X) + 6.03998{\cdot}10^{14}B_{16,20}(X) + 3.14379{\cdot}10^{14}B_{17,20}(X) + 1.13755{\cdot}10^{14}B_{18,20}(X) - 7.46015{\cdot}10^{12}B_{19,20}(X) - 6.82353{\cdot}10^{13}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.199664, 0.999972\}$$

Intersection intervals with the x axis:

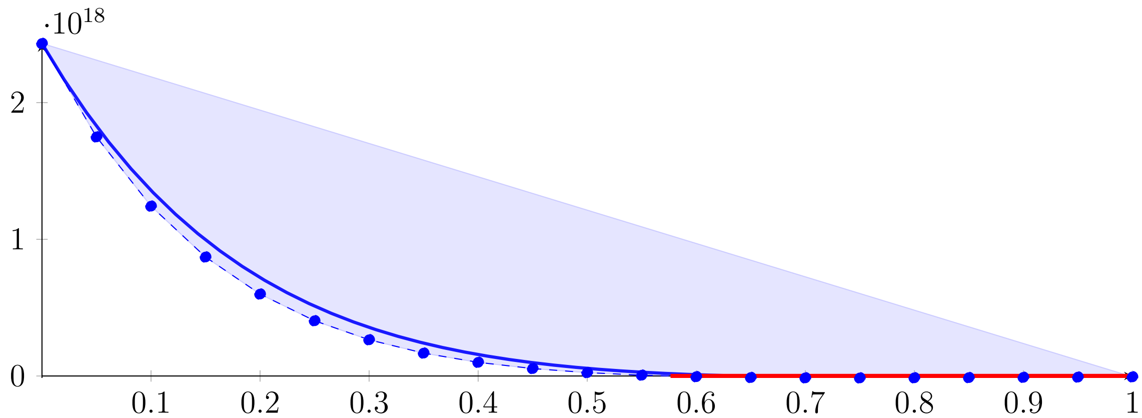
$$[0.199664, 0.999972]$$

Longest intersection interval: 0.800308
$\Longrightarrow$ Bisection: first half $[0, 1.5625]$ und second half $[1.5625, 3.125]$

1.5 Recursion Branch 1 1 1 1 1 on the First Half $[0, 1.5625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -8.46356 \cdot 10^{7} X^{20} - 1.83419 \cdot 10^{8} X^{19} - 5.89672 \cdot 10^{9} X^{18} - 1.44753 \cdot 10^{8} X^{17} - 9.10891 \cdot 10^{9} X^{16} \\ & - 1.29397 \cdot 10^{12} X^{15} + 2.06942 \cdot 10^{13} X^{14} - 2.50213 \cdot 10^{14} X^{13} + 2.39489 \cdot 10^{15} X^{12} - 1.83753 \cdot 10^{16} X^{11} \\ & + 1.13411 \cdot 10^{17} X^{10} - 5.63011 \cdot 10^{17} X^{9} + 2.2393 \cdot 10^{18} X^{8} - 7.07891 \cdot 10^{18} X^{7} + 1.7559 \cdot 10^{19} X^{6} - 3.35274 \\ & \cdot 10^{19} X^{5} + 4.79091 \cdot 10^{19} X^{4} - 4.90987 \cdot 10^{19} X^{3} + 3.37006 \cdot 10^{19} X^{2} - 1.36765 \cdot 10^{19} X + 2.4329 \cdot 10^{18} \\ = & \, 2.4329 \cdot 10^{18} B_{0,20}(X) + 1.74908 \cdot 10^{18} B_{1,20}(X) + 1.24263 \cdot 10^{18} B_{2,20}(X) + 8.70475 \\ & \cdot 10^{17} B_{3,20}(X) + 5.99447 \cdot 10^{17} B_{4,20}(X) + 4.04086 \cdot 10^{17} B_{5,20}(X) + 2.64953 \cdot 10^{17} B_{6,20}(X) \\ & + 1.67278 \cdot 10^{17} B_{7,20}(X) + 9.9902 \cdot 10^{16} B_{8,20}(X) + 5.44408 \cdot 10^{16} B_{9,20}(X) + 2.46418 \\ & \cdot 10^{16} B_{10,20}(X) + 5.87625 \cdot 10^{15} B_{11,20}(X) - 5.2528 \cdot 10^{15} B_{12,20}(X) - 1.12129 \cdot 10^{16} B_{13,20}(X) \\ & - 1.37757 \cdot 10^{16} B_{14,20}(X) - 1.41949 \cdot 10^{16} B_{15,20}(X) - 1.33428 \cdot 10^{16} B_{16,20}(X) - 1.1813 \\ & \cdot 10^{16} B_{17,20}(X) - 9.99781 \cdot 10^{15} B_{18,20}(X) - 8.1465 \cdot 10^{15} B_{19,20}(X) - 6.40794 \cdot 10^{15} B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.576401, 0.997373\}$$

Intersection intervals with the x axis:

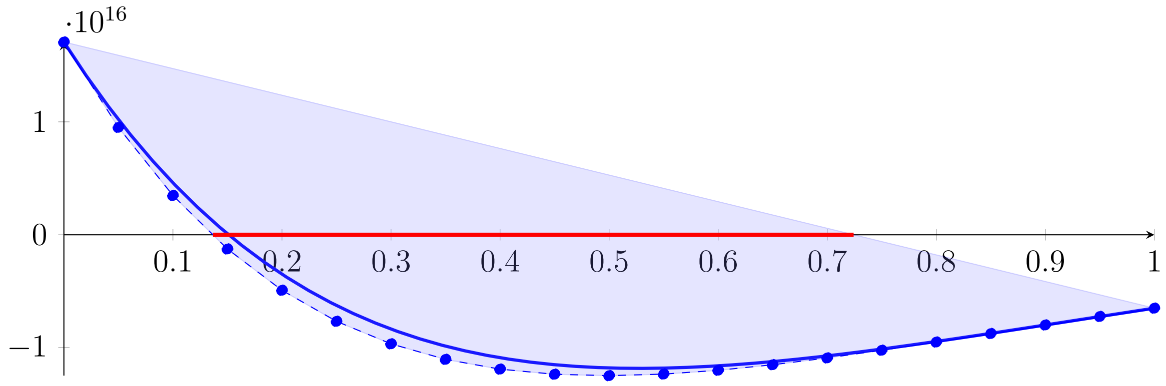
$$[0.576401, 0.997373]$$

Longest intersection interval: 0.420973
$\Longrightarrow$ Selective recursion: interval 1: $[0.900626, 1.5584]$,

1.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.900626, 1.5584]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &1.25941{\cdot}10^{7}X^{20} - 7.29698{\cdot}10^{7}X^{19} + 3.69107{\cdot}10^{8}X^{18} - 1.61684{\cdot}10^{9}X^{17} + 7.55795{\cdot}10^{9}X^{16} - 6.01574\\ &{\cdot}10^{9}X^{15} + 2.33971{\cdot}10^{9}X^{14} - 1.15071{\cdot}10^{9}X^{13} + 3.695{\cdot}10^{10}X^{12} - 5.17348{\cdot}10^{11}X^{11} + 6.60409{\cdot}10^{12}X^{10}\\ &- 6.70634{\cdot}10^{13}X^{9} + 5.38613{\cdot}10^{14}X^{8} - 3.38268{\cdot}10^{15}X^{7} + 1.63264{\cdot}10^{16}X^{6} - 5.90068{\cdot}10^{16}X^{5}\\ &+ 1.53584{\cdot}10^{17}X^{4} - 2.70691{\cdot}10^{17}X^{3} + 2.90287{\cdot}10^{17}X^{2} - 1.51163{\cdot}10^{17}X + 1.70696{\cdot}10^{16}\\ = &1.70696{\cdot}10^{16}B_{0,20}(X) + 9.51143{\cdot}10^{15}B_{1,20}(X) + 3.48109{\cdot}10^{15}B_{2,20}(X) - 1.25886\\ &{\cdot}10^{15}B_{3,20}(X) - 4.91419{\cdot}10^{15}B_{4,20}(X) - 7.66273{\cdot}10^{15}B_{5,20}(X) - 9.65786{\cdot}10^{15}B_{6,20}(X)\\ &- 1.10314{\cdot}10^{16}B_{7,20}(X) - 1.18964{\cdot}10^{16}B_{8,20}(X) - 1.23493{\cdot}10^{16}B_{9,20}(X) - 1.24724\\ &{\cdot}10^{16}B_{10,20}(X) - 1.23353{\cdot}10^{16}B_{11,20}(X) - 1.19968{\cdot}10^{16}B_{12,20}(X) - 1.15062{\cdot}10^{16}B_{13,20}(X)\\ &- 1.09047{\cdot}10^{16}B_{14,20}(X) - 1.02264{\cdot}10^{16}B_{15,20}(X) - 9.49945{\cdot}10^{15}B_{16,20}(X) - 8.74667\\ &{\cdot}10^{15}B_{17,20}(X) - 7.9865{\cdot}10^{15}B_{18,20}(X) - 7.23361{\cdot}10^{15}B_{19,20}(X) - 6.49943{\cdot}10^{15}B_{20,20}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.136721, 0.724238\}$$

Intersection intervals with the x axis:

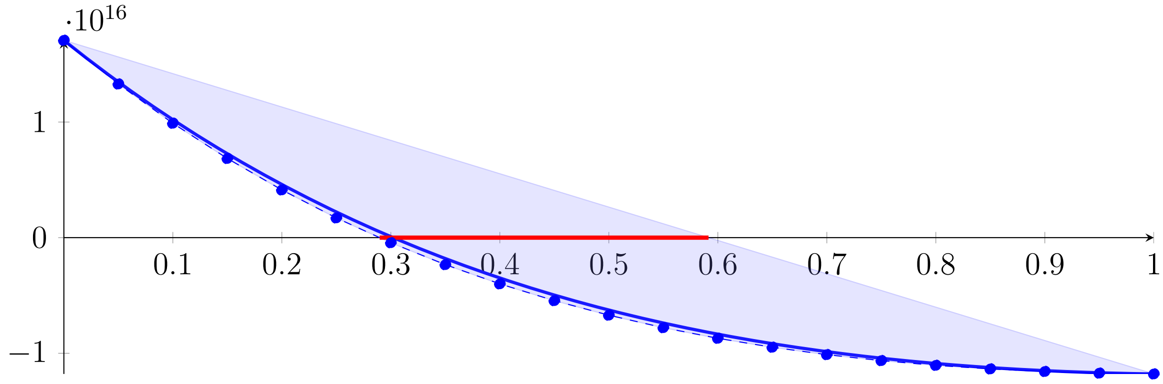
$$[0.136721, 0.724238]$$

Longest intersection interval: 0.587518
$\Longrightarrow$ Bisection: first half $[0.900626, 1.22951]$ und second half $[1.22951, 1.5584]$

1.7 Recursion Branch 1 1 1 1 1 1 1 on the First Half $[0.900626, 1.22951]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 5.14907 \cdot 10^{6} X^{20} - 5.24182 \cdot 10^{7} X^{19} + 8.53351 \cdot 10^{7} X^{18} - 8.13955 \cdot 10^{8} X^{17} + 2.73171 \cdot 10^{9} X^{16} - 1.94848 \cdot 10^{9} X^{15} + 2.43568 \cdot 10^{8} X^{14} - 3.03568 \cdot 10^{8} X^{13} + 2.35312 \cdot 10^{8} X^{12} - 8.61971 \cdot 10^{8} X^{11} + 6.14794 \cdot 10^{9} X^{10} - 1.31164 \cdot 10^{11} X^{9} + 2.10397 \cdot 10^{12} X^{8} - 2.64272 \cdot 10^{13} X^{7} + 2.55099 \cdot 10^{14} X^{6} - 1.84396 \cdot 10^{15} X^{5} + 9.59898 \cdot 10^{15} X^{4} - 3.38364 \cdot 10^{16} X^{3} + 7.25718 \cdot 10^{16} X^{2} - 7.55816 \cdot 10^{16} X + 1.70696 \cdot 10^{16}$$
$$= 1.70696 \cdot 10^{16} B_{0,20}(X) + 1.32905 \cdot 10^{16} B_{1,20}(X) + 9.89338 \cdot 10^{15} B_{2,20}(X) + 6.84854 \cdot 10^{15} B_{3,20}(X) + 4.12826 \cdot 10^{15} B_{4,20}(X) + 1.70673 \cdot 10^{15} B_{5,20}(X) - 4.40152 \cdot 10^{14} B_{6,20}(X) - 2.33483 \cdot 10^{15} B_{7,20}(X) - 3.9982 \cdot 10^{15} B_{8,20}(X) - 5.44971 \cdot 10^{15} B_{9,20}(X) - 6.70746 \cdot 10^{15} B_{10,20}(X) - 7.78826 \cdot 10^{15} B_{11,20}(X) - 8.70773 \cdot 10^{15} B_{12,20}(X) - 9.48037 \cdot 10^{15} B_{13,20}(X) - 1.01196 \cdot 10^{16} B_{14,20}(X) - 1.0638 \cdot 10^{16} B_{15,20}(X) - 1.1047 \cdot 10^{16} B_{16,20}(X) - 1.13572 \cdot 10^{16} B_{17,20}(X) - 1.15787 \cdot 10^{16} B_{18,20}(X) - 1.17205 \cdot 10^{16} B_{19,20}(X) - 1.17909 \cdot 10^{16} B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.289749, 0.591451\}$$

Intersection intervals with the x axis:

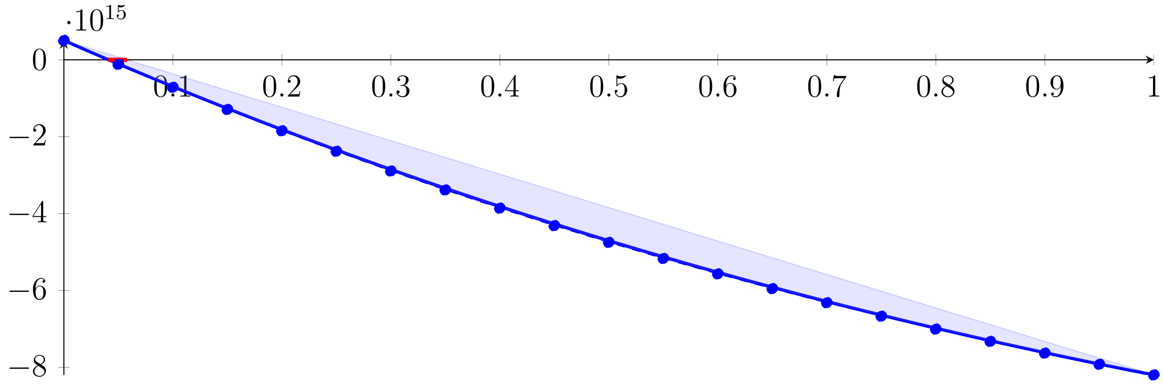
$$[0.289749, 0.591451]$$

Longest intersection interval: 0.301702
$\Longrightarrow$ Selective recursion: interval 1: $[0.99592, 1.09514]$,

1.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.99592, 1.09514]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 4.0672 \cdot 10^{6} X^{20} - 2.99481 \cdot 10^{7} X^{19} + 1.01182 \cdot 10^{8} X^{18} - 5.59769 \cdot 10^{8} X^{17} + 2.54684 \cdot 10^{9} X^{16} - 1.81259 \cdot 10^{9} X^{15} + 5.99535 \cdot 10^{8} X^{14} + 1.72201 \cdot 10^{8} X^{13} + 1.42496 \cdot 10^{9} X^{12} + 2.13099 \cdot 10^{7} X^{11} + 3.39512 \cdot 10^{8} X^{10} + 4.99681 \cdot 10^{6} X^{9} + 1.29088 \cdot 10^{8} X^{8} - 4.98894 \cdot 10^{9} X^{7} + 1.555 \cdot 10^{11} X^{6} - 3.61036 \cdot 10^{12} X^{5} + 5.98808 \cdot 10^{13} X^{4} - 6.62976 \cdot 10^{14} X^{3} + 4.33021 \cdot 10^{15} X^{2} - 1.2423 \cdot 10^{16} X + 5.03561 \cdot 10^{14}$$
$$= 5.03561 \cdot 10^{14} B_{0,20}(X) - 1.1759 \cdot 10^{14} B_{1,20}(X) - 7.1595 \cdot 10^{14} B_{2,20}(X) - 1.2921 \cdot 10^{15} B_{3,20}(X) - 1.84661 \cdot 10^{15} B_{4,20}(X) - 2.38004 \cdot 10^{15} B_{5,20}(X) - 2.89293 \cdot 10^{15} B_{6,20}(X) - 3.38582 \cdot 10^{15} B_{7,20}(X) - 3.85923 \cdot 10^{15} B_{8,20}(X) - 4.31366 \cdot 10^{15} B_{9,20}(X) - 4.74962 \cdot 10^{15} B_{10,20}(X) - 5.1676 \cdot 10^{15} B_{11,20}(X) - 5.56808 \cdot 10^{15} B_{12,20}(X) - 5.95152 \cdot 10^{15} B_{13,20}(X) - 6.31838 \cdot 10^{15} B_{14,20}(X) - 6.66911 \cdot 10^{15} B_{15,20}(X) - 7.00415 \cdot 10^{15} B_{16,20}(X) - 7.32393 \cdot 10^{15} B_{17,20}(X) - 7.62885 \cdot 10^{15} B_{18,20}(X) - 7.91935 \cdot 10^{15} B_{19,20}(X) - 8.19581 \cdot 10^{15} B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.0405345, 0.0578848\}$$

Intersection intervals with the x axis:

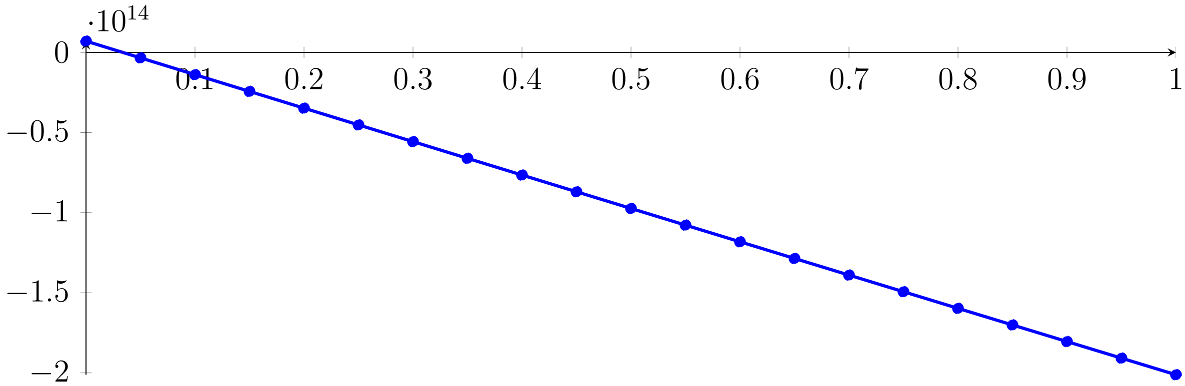
$$[0.0405345, 0.0578848]$$

Longest intersection interval: 0.0173503
$\Longrightarrow$ Selective recursion: interval 1: $[0.999942, 1.00166]$,

1.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.999942, 1.00166]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = {} & 84805.9X^{20} - 642943X^{19} + 2.18649{\cdot}10^{6}X^{18} - 1.15583{\cdot}10^{7}X^{17} + 5.14762{\cdot}10^{7}X^{16} \\ & - 3.74281{\cdot}10^{7}X^{15} + 1.24619{\cdot}10^{7}X^{14} + 2.59571{\cdot}10^{6}X^{13} + 2.75628{\cdot}10^{7}X^{12} - 125970X^{11} \\ & + 6.72627{\cdot}10^{6}X^{10} + 50519.2X^{9} + 179114X^{8} - 15443.4X^{7} + 18017.3X^{6} - 6480.19X^{5} \\ & + 5.36063{\cdot}10^{6}X^{4} - 3.41232{\cdot}10^{9}X^{3} + 1.27944{\cdot}10^{12}X^{2} - 2.09509{\cdot}10^{14}X + 7.07074{\cdot}10^{12} \\ = {} & 7.07074{\cdot}10^{12}B_{0,20}(X) - 3.40469{\cdot}10^{12}B_{1,20}(X) - 1.38734{\cdot}10^{13}B_{2,20}(X) - 2.43354 \\ & {\cdot}10^{13}B_{3,20}(X) - 3.47906{\cdot}10^{13}B_{4,20}(X) - 4.52391{\cdot}10^{13}B_{5,20}(X) - 5.56809{\cdot}10^{13}B_{6,20}(X) \\ & - 6.6116{\cdot}10^{13}B_{7,20}(X) - 7.65443{\cdot}10^{13}B_{8,20}(X) - 8.6966{\cdot}10^{13}B_{9,20}(X) - 9.73809 \\ & {\cdot}10^{13}B_{10,20}(X) - 1.07789{\cdot}10^{14}B_{11,20}(X) - 1.18191{\cdot}10^{14}B_{12,20}(X) - 1.28585{\cdot}10^{14}B_{13,20}(X) \\ & - 1.38974{\cdot}10^{14}B_{14,20}(X) - 1.49355{\cdot}10^{14}B_{15,20}(X) - 1.5973{\cdot}10^{14}B_{16,20}(X) - 1.70098 \\ & {\cdot}10^{14}B_{17,20}(X) - 1.80459{\cdot}10^{14}B_{18,20}(X) - 1.90814{\cdot}10^{14}B_{19,20}(X) - 2.01162{\cdot}10^{14}B_{20,20}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.0337492, 0.033956\}$$

Intersection intervals with the x axis:

$$[0.0337492, 0.033956]$$

Longest intersection interval: 0.000206812
$\Longrightarrow$ Selective recursion: interval 1: $[1, 1]$,

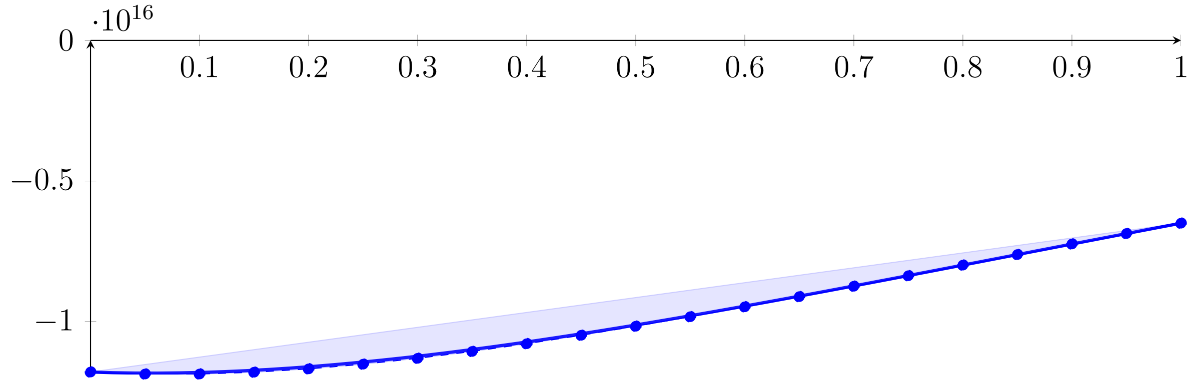
1.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[1, 1]$

Found root in interval $[1, 1]$ at recursion depth 10!

1.11 Recursion Branch 1 1 1 1 1 1 2 on the Second Half $[1.22951, 1.5584]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 9.85996{\cdot}10^{6}X^{20} - 4.26182{\cdot}10^{7}X^{19} + 3.00977{\cdot}10^{8}X^{18} - 1.16597{\cdot}10^{9}X^{17} + 6.77151{\cdot}10^{9}X^{16} - 5.2331{\cdot}10^{9}X^{15} + 2.04265{\cdot}10^{9}X^{14} + 1.17117{\cdot}10^{9}X^{13} + 5.63489{\cdot}10^{9}X^{12} + 7.39696{\cdot}10^{8}X^{11} + 5.71339{\cdot}10^{9}X^{10} - 7.85858{\cdot}10^{10}X^{9} + 1.1773{\cdot}10^{12}X^{8} - 1.36145{\cdot}10^{13}X^{7} + 1.19262{\cdot}10^{14}X^{6} - 7.65504{\cdot}10^{14}X^{5} + 3.41275{\cdot}10^{15}X^{4} - 9.59553{\cdot}10^{15}X^{3} + 1.35428{\cdot}10^{16}X^{2} - 1.40978{\cdot}10^{15}X - 1.17909{\cdot}10^{16}$$
$$= -1.17909{\cdot}10^{16}B_{0,20}(X) - 1.18614{\cdot}10^{16}B_{1,20}(X) - 1.18606{\cdot}10^{16}B_{2,20}(X) - 1.1797{\cdot}10^{16}B_{3,20}(X) - 1.16782{\cdot}10^{16}B_{4,20}(X) - 1.15113{\cdot}10^{16}B_{5,20}(X) - 1.13028{\cdot}10^{16}B_{6,20}(X) - 1.10585{\cdot}10^{16}B_{7,20}(X) - 1.07838{\cdot}10^{16}B_{8,20}(X) - 1.04836{\cdot}10^{16}B_{9,20}(X) - 1.01623{\cdot}10^{16}B_{10,20}(X) - 9.82386{\cdot}10^{15}B_{11,20}(X) - 9.47196{\cdot}10^{15}B_{12,20}(X) - 9.10984{\cdot}10^{15}B_{13,20}(X) - 8.74042{\cdot}10^{15}B_{14,20}(X) - 8.36634{\cdot}10^{15}B_{15,20}(X) - 7.98994{\cdot}10^{15}B_{16,20}(X) - 7.6133{\cdot}10^{15}B_{17,20}(X) - 7.23829{\cdot}10^{15}B_{18,20}(X) - 6.86652{\cdot}10^{15}B_{19,20}(X) - 6.49943{\cdot}10^{15}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

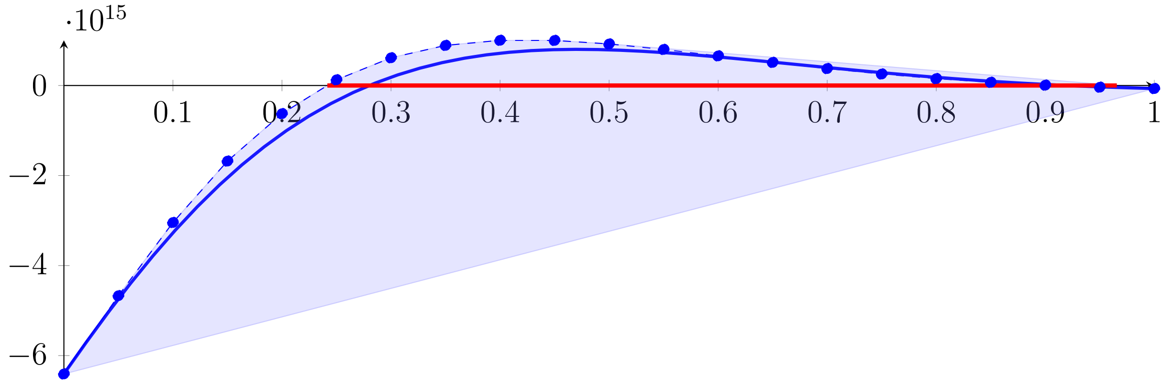
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

1.12 Recursion Branch 1 1 1 1 2 on the Second Half $[1.5625, 3.125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -854779X^{20} + 2.70678{\cdot}10^{6}X^{19} + 2.57285{\cdot}10^{7}X^{18} - 1.38387{\cdot}10^{9}X^{17} + 3.34218{\cdot}10^{10}X^{16} - 5.62474{\cdot}10^{11}X^{15} + 7.0799{\cdot}10^{12}X^{14} - 6.89484{\cdot}10^{13}X^{13} + 5.26324{\cdot}10^{14}X^{12} - 3.16741{\cdot}10^{15}X^{11} + 1.50317{\cdot}10^{16}X^{10} - 5.59783{\cdot}10^{16}X^{9} + 1.61826{\cdot}10^{17}X^{8} - 3.56531{\cdot}10^{17}X^{7} + 5.81008{\cdot}10^{17}X^{6} - 6.65758{\cdot}10^{17}X^{5} + 4.85849{\cdot}10^{17}X^{4} - 1.69752{\cdot}10^{17}X^{3} - 2.14228{\cdot}10^{16}X^{2} + 3.47712{\cdot}10^{16}X - 6.40794{\cdot}10^{15}$$
$$= -6.40794{\cdot}10^{15}B_{0,20}(X) - 4.66938{\cdot}10^{15}B_{1,20}(X) - 3.04357{\cdot}10^{15}B_{2,20}(X) - 1.67942{\cdot}10^{15}B_{3,20}(X) - 6.25553{\cdot}10^{14}B_{4,20}(X) + 1.26743{\cdot}10^{14}B_{5,20}(X) + 6.15563{\cdot}10^{14}B_{6,20}(X) + 8.9083{\cdot}10^{14}B_{7,20}(X) + 1.00381{\cdot}10^{15}B_{8,20}(X) + 1.00133{\cdot}10^{15}B_{9,20}(X) + 9.23073{\cdot}10^{14}B_{10,20}(X) + 8.00741{\cdot}10^{14}B_{11,20}(X) + 6.58338{\cdot}10^{14}B_{12,20}(X) + 5.13038{\cdot}10^{14}B_{13,20}(X) + 3.76314{\cdot}10^{14}B_{14,20}(X) + 2.55097{\cdot}10^{14}B_{15,20}(X) + 1.52873{\cdot}10^{14}B_{16,20}(X) + 7.06284{\cdot}10^{13}B_{17,20}(X) + 7.64979{\cdot}10^{12}B_{18,20}(X) - 3.78477{\cdot}10^{13}B_{19,20}(X) - 6.82353{\cdot}10^{13}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.241576, 0.965583\}$$

Intersection intervals with the x axis:

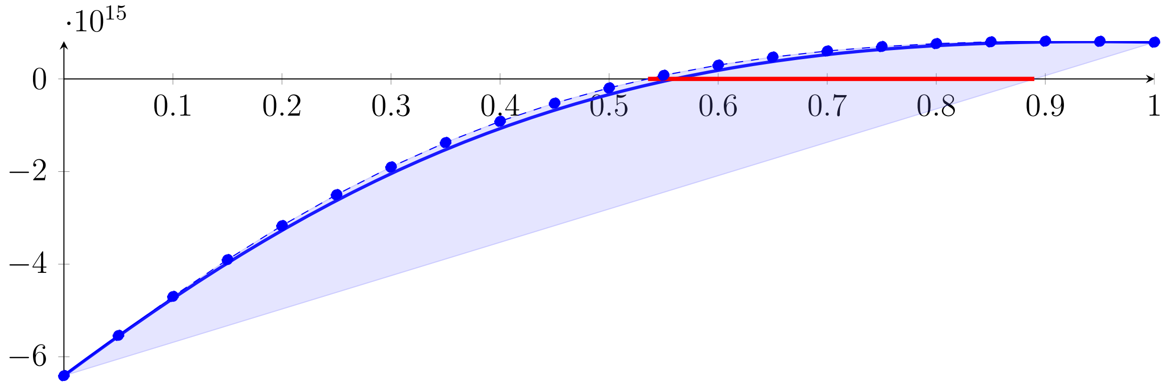
$$[0.241576, 0.965583]$$

Longest intersection interval: 0.724007
$\Longrightarrow$ Bisection: first half $[1.5625, 2.34375]$ und second half $[2.34375, 3.125]$

1.13 Recursion Branch 1 1 1 1 2 1 on the First Half $[1.5625, 2.34375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = {} & 628533X^{20} + 2.80214\cdot 10^{6}X^{19} + 4.20138\cdot 10^{7}X^{18} - 1.91759\cdot 10^{7}X^{17} + 5.57626\cdot 10^{8}X^{16} - 5.14314 \\ & \cdot 10^{8}X^{15} + 8.17565\cdot 10^{8}X^{14} - 8.21441\cdot 10^{9}X^{13} + 1.29423\cdot 10^{11}X^{12} - 1.5463\cdot 10^{12}X^{11} + 1.46797 \\ & \cdot 10^{13}X^{10} - 1.09332\cdot 10^{14}X^{9} + 6.32133\cdot 10^{14}X^{8} - 2.7854\cdot 10^{15}X^{7} + 9.07825\cdot 10^{15}X^{6} - 2.08049 \\ & \cdot 10^{16}X^{5} + 3.03655\cdot 10^{16}X^{4} - 2.1219\cdot 10^{16}X^{3} - 5.3557\cdot 10^{15}X^{2} + 1.73856\cdot 10^{16}X - 6.40794\cdot 10^{15} \\ = {} & -6.40794\cdot 10^{15}B_{0,20}(X) - 5.53866\cdot 10^{15}B_{1,20}(X) - 4.69757\cdot 10^{15}B_{2,20}(X) - 3.90328 \\ & \cdot 10^{15}B_{3,20}(X) - 3.16813\cdot 10^{15}B_{4,20}(X) - 2.49956\cdot 10^{15}B_{5,20}(X) - 1.90115\cdot 10^{15}B_{6,20}(X) \\ & - 1.3736\cdot 10^{15}B_{7,20}(X) - 9.15451\cdot 10^{14}B_{8,20}(X) - 5.23652\cdot 10^{14}B_{9,20}(X) - 1.94086 \\ & \cdot 10^{14}B_{10,20}(X) + 7.80618\cdot 10^{13}B_{11,20}(X) + 2.98005\cdot 10^{14}B_{12,20}(X) + 4.71115\cdot 10^{14}B_{13,20}(X) \\ & + 6.02746\cdot 10^{14}B_{14,20}(X) + 6.98102\cdot 10^{14}B_{15,20}(X) + 7.62138\cdot 10^{14}B_{16,20}(X) + 7.99497 \\ & \cdot 10^{14}B_{17,20}(X) + 8.14467\cdot 10^{14}B_{18,20}(X) + 8.10958\cdot 10^{14}B_{19,20}(X) + 7.92494\cdot 10^{14}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.535658, 0.889938\}$$

Intersection intervals with the x axis:

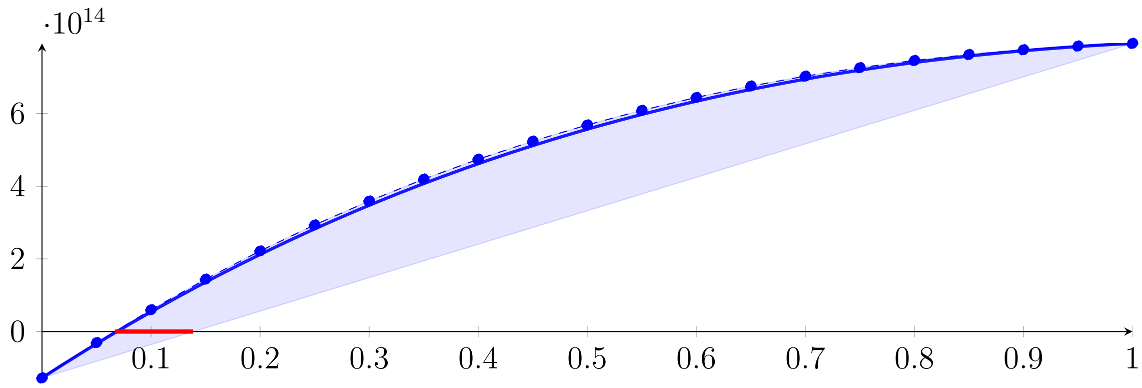
$$[0.535658, 0.889938]$$

Longest intersection interval: 0.35428
$\Longrightarrow$ Selective recursion: interval 1: $[1.98098, 2.25776]$,

1.14 Recursion Branch 1 1 1 1 2 1 1 in Interval 1: $[1.98098, 2.25776]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -494338X^{20} + 3.40087\cdot 10^{6}X^{19} - 1.36887\cdot 10^{7}X^{18} + 6.43046\cdot 10^{7}X^{17} - 3.14253\cdot 10^{8}X^{16} + 2.33211 \cdot 10^{8}X^{15} - 8.17115\cdot 10^{7}X^{14} - 1.41946\cdot 10^{7}X^{13} - 1.78856\cdot 10^{8}X^{12} - 1.28673\cdot 10^{7}X^{11} + 1.97397 \cdot 10^{8}X^{10} - 4.50346\cdot 10^{9}X^{9} + 6.45646\cdot 10^{10}X^{8} - 6.81614\cdot 10^{11}X^{7} + 4.9948\cdot 10^{12}X^{6} - 2.20869\cdot 10^{13}X^{5} + 2.89032\cdot 10^{13}X^{4} + 2.39776\cdot 10^{14}X^{3} - 1.27395\cdot 10^{15}X^{2} + 1.94369\cdot 10^{15}X - 1.27611\cdot 10^{14}$$
$$= -1.27611\cdot 10^{14}B_{0,20}(X) - 3.04264\cdot 10^{13}B_{1,20}(X) + 6.00529\cdot 10^{13}B_{2,20}(X) + 1.44037 \cdot 10^{14}B_{3,20}(X) + 2.21744\cdot 10^{14}B_{4,20}(X) + 2.93392\cdot 10^{14}B_{5,20}(X) + 3.59208\cdot 10^{14}B_{6,20}(X) + 4.19415\cdot 10^{14}B_{7,20}(X) + 4.74243\cdot 10^{14}B_{8,20}(X) + 5.23918\cdot 10^{14}B_{9,20}(X) + 5.68666 \cdot 10^{14}B_{10,20}(X) + 6.08713\cdot 10^{14}B_{11,20}(X) + 6.4428\cdot 10^{14}B_{12,20}(X) + 6.75587\cdot 10^{14}B_{13,20}(X) + 7.02852\cdot 10^{14}B_{14,20}(X) + 7.26284\cdot 10^{14}B_{15,20}(X) + 7.46094\cdot 10^{14}B_{16,20}(X) + 7.62484 \cdot 10^{14}B_{17,20}(X) + 7.75652\cdot 10^{14}B_{18,20}(X) + 7.85791\cdot 10^{14}B_{19,20}(X) + 7.9309\cdot 10^{14}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.066814, 0.138602\}$$

Intersection intervals with the x axis:

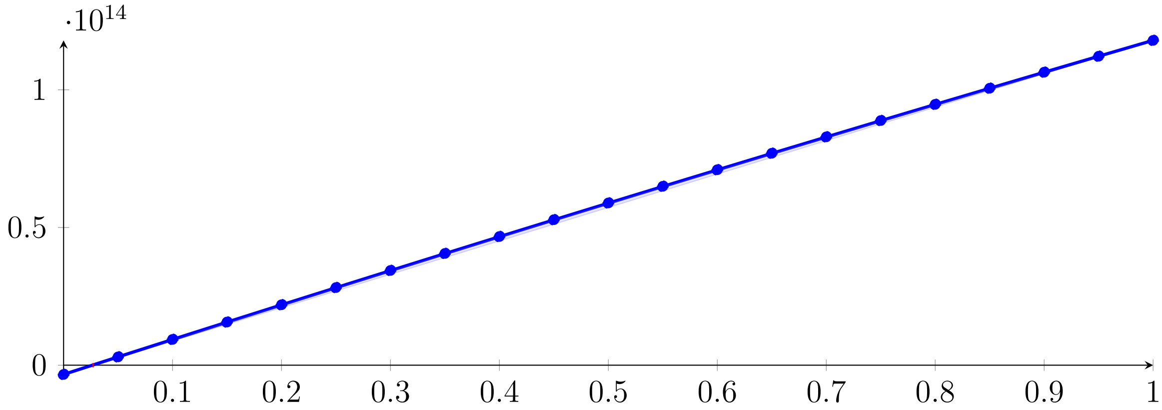
$$[0.066814, 0.138602]$$

Longest intersection interval: 0.0717877
$\Longrightarrow$ Selective recursion: interval 1: $[1.99948, 2.01935]$,

1.15 Recursion Branch 1 1 1 1 2 1 1 1 in Interval 1: $[1.99948, 2.01935]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -53647.6X^{20} + 382715X^{19} - 1.33077\cdot 10^{6}X^{18} + 6.69774\cdot 10^{6}X^{17} - 3.07973\cdot 10^{7}X^{16} + 2.32929\cdot 10^{7}X^{15} - 7.24971\cdot 10^{6}X^{14} - 2.27336\cdot 10^{6}X^{13} - 1.74058\cdot 10^{7}X^{12} + 93493.4X^{11} - 3.96107\cdot 10^{6}X^{10} - 67249.6X^{9} - 57818.3X^{8} - 8327.34X^{7} + 631591X^{6} - 3.84123\cdot 10^{7}X^{5} + 5.80354\cdot 10^{8}X^{4} + 9.12107\cdot 10^{10}X^{3} - 6.31394\cdot 10^{12}X^{2} + 1.27545\cdot 10^{14}X - 3.36024\cdot 10^{12}$$
$$= -3.36024\cdot 10^{12}B_{0,20}(X) + 3.017\cdot 10^{12}B_{1,20}(X) + 9.36101\cdot 10^{12}B_{2,20}(X) + 1.56719 \cdot 10^{13}B_{3,20}(X) + 2.19497\cdot 10^{13}B_{4,20}(X) + 2.81945\cdot 10^{13}B_{5,20}(X) + 3.44063\cdot 10^{13}B_{6,20}(X) + 4.05854\cdot 10^{13}B_{7,20}(X) + 4.67317\cdot 10^{13}B_{8,20}(X) + 5.28453\cdot 10^{13}B_{9,20}(X) + 5.89264 \cdot 10^{13}B_{10,20}(X) + 6.49749\cdot 10^{13}B_{11,20}(X) + 7.0991\cdot 10^{13}B_{12,20}(X) + 7.69748\cdot 10^{13}B_{13,20}(X) + 8.29263\cdot 10^{13}B_{14,20}(X) + 8.88457\cdot 10^{13}B_{15,20}(X) + 9.47329\cdot 10^{13}B_{16,20}(X) + 1.00588 \cdot 10^{14}B_{17,20}(X) + 1.06411\cdot 10^{14}B_{18,20}(X) + 1.12203\cdot 10^{14}B_{19,20}(X) + 1.17962\cdot 10^{14}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.0263455, 0.0276967\}$$

Intersection intervals with the x axis:

$$[0.0263455, 0.0276967]$$

Longest intersection interval: 0.00135117
$\Longrightarrow$ Selective recursion: interval 1: $[2, 2.00003]$,

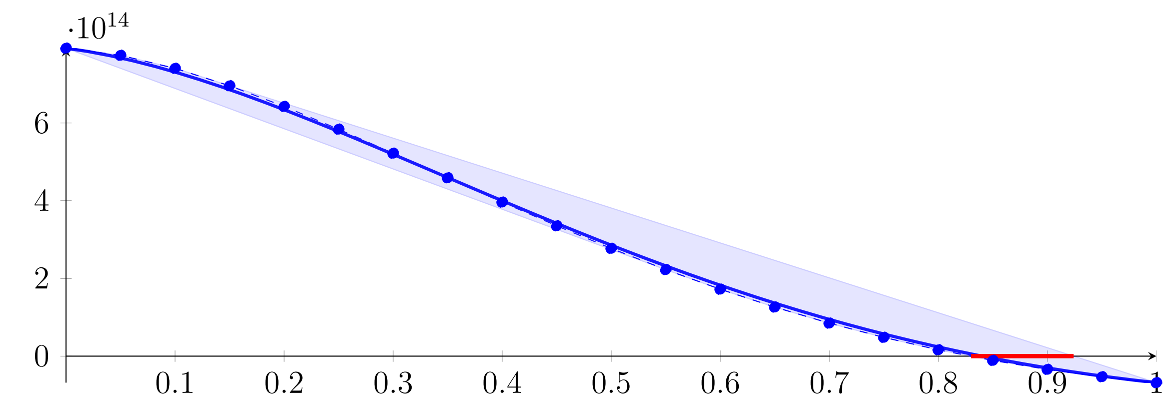
1.16 Recursion Branch 1 1 1 1 2 1 1 1 1 in Interval 1: $[2, 2.00003]$

Found root in interval $[2, 2.00003]$ at recursion depth 9!

1.17 Recursion Branch 1 1 1 1 2 2 on the Second Half $[2.34375, 3.125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -342205X^{20} + 740660X^{19} - 1.30129{\cdot}10^{7}X^{18} + 3.13739{\cdot}10^{7}X^{17} - 2.36954{\cdot}10^{8}X^{16} + 1.86963{\cdot}10^{8}X^{15} \\ & + 1.21978{\cdot}10^{8}X^{14} - 3.95236{\cdot}10^{9}X^{13} + 5.11961{\cdot}10^{10}X^{12} - 5.25185{\cdot}10^{11}X^{11} + 4.12541{\cdot}10^{12}X^{10} \\ & - 2.45648{\cdot}10^{13}X^{9} + 1.07492{\cdot}10^{14}X^{8} - 3.24451{\cdot}10^{14}X^{7} + 5.69883{\cdot}10^{14}X^{6} - 1.28897{\cdot}10^{14}X^{5} \\ & - 1.87079{\cdot}10^{15}X^{4} + 4.01756{\cdot}10^{15}X^{3} - 2.84135{\cdot}10^{15}X^{2} - 3.69266{\cdot}10^{14}X + 7.92494{\cdot}10^{14} \\ = & \; 7.92494{\cdot}10^{14}B_{0,20}(X) + 7.74031{\cdot}10^{14}B_{1,20}(X) + 7.40613{\cdot}10^{14}B_{2,20}(X) + 6.95765 \\ & {\cdot}10^{14}B_{3,20}(X) + 6.42625{\cdot}10^{14}B_{4,20}(X) + 5.83936{\cdot}10^{14}B_{5,20}(X) + 5.22054{\cdot}10^{14}B_{6,20}(X) \\ & + 4.58964{\cdot}10^{14}B_{7,20}(X) + 3.96302{\cdot}10^{14}B_{8,20}(X) + 3.35388{\cdot}10^{14}B_{9,20}(X) + 2.77253 \\ & {\cdot}10^{14}B_{10,20}(X) + 2.22674{\cdot}10^{14}B_{11,20}(X) + 1.72204{\cdot}10^{14}B_{12,20}(X) + 1.26205{\cdot}10^{14}B_{13,20}(X) \\ & + 8.48747{\cdot}10^{13}B_{14,20}(X) + 4.8274{\cdot}10^{13}B_{15,20}(X) + 1.63537{\cdot}10^{13}B_{16,20}(X) - 1.10251 \\ & {\cdot}10^{13}B_{17,20}(X) - 3.40702{\cdot}10^{13}B_{18,20}(X) - 5.30415{\cdot}10^{13}B_{19,20}(X) - 6.82353{\cdot}10^{13}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.829866, 0.924084\}$$

Intersection intervals with the x axis:

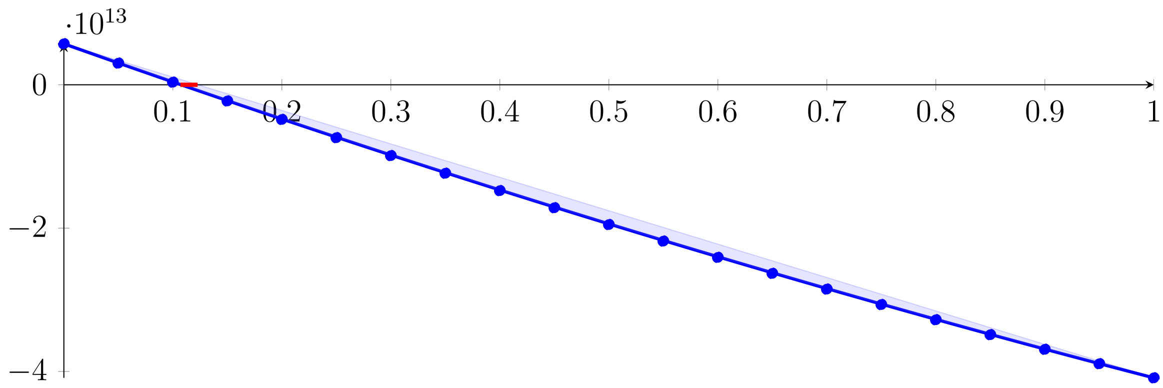
$$[0.829866, 0.924084]$$

Longest intersection interval: 0.0942182
$\Longrightarrow$ Selective recursion: interval 1: $[2.99208, 3.06569]$,

1.18 Recursion Branch 1 1 1 1 2 2 1 in Interval 1: $[2.99208, 3.06569]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = {} & 16409.9X^{20} - 135599X^{19} + 404723X^{18} - 2.39664\cdot 10^{6}X^{17} + 9.91527\cdot 10^{6}X^{16} - 7.11517 \\ & \cdot 10^{6}X^{15} + 2.39385\cdot 10^{6}X^{14} + 466104X^{13} + 4.90188\cdot 10^{6}X^{12} - 93001.3X^{11} + 998657X^{10} \\ & - 5740.82X^{9} + 103704X^{8} - 331201X^{7} - 6.44393\cdot 10^{7}X^{6} + 2.43964\cdot 10^{9}X^{5} - 3.28521 \\ & \cdot 10^{10}X^{4} - 1.17228\cdot 10^{11}X^{3} + 7.51574\cdot 10^{12}X^{2} - 5.39755\cdot 10^{13}X + 5.71896\cdot 10^{12} \\ = {} & 5.71896\cdot 10^{12}B_{0,20}(X) + 3.02018\cdot 10^{12}B_{1,20}(X) + 3.60966\cdot 10^{11}B_{2,20}(X) - 2.2588 \\ & \cdot 10^{12}B_{3,20}(X) - 4.83922\cdot 10^{12}B_{4,20}(X) - 7.38041\cdot 10^{12}B_{5,20}(X) - 9.88249\cdot 10^{12}B_{6,20}(X) \\ & - 1.23456\cdot 10^{13}B_{7,20}(X) - 1.47699\cdot 10^{13}B_{8,20}(X) - 1.71554\cdot 10^{13}B_{9,20}(X) - 1.95025 \\ & \cdot 10^{13}B_{10,20}(X) - 2.18111\cdot 10^{13}B_{11,20}(X) - 2.40814\cdot 10^{13}B_{12,20}(X) - 2.63137\cdot 10^{13}B_{13,20}(X) \\ & - 2.85081\cdot 10^{13}B_{14,20}(X) - 3.06648\cdot 10^{13}B_{15,20}(X) - 3.27839\cdot 10^{13}B_{16,20}(X) - 3.48656 \\ & \cdot 10^{13}B_{17,20}(X) - 3.69102\cdot 10^{13}B_{18,20}(X) - 3.89177\cdot 10^{13}B_{19,20}(X) - 4.08885\cdot 10^{13}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.106889, 0.122705\}$$

Intersection intervals with the x axis:

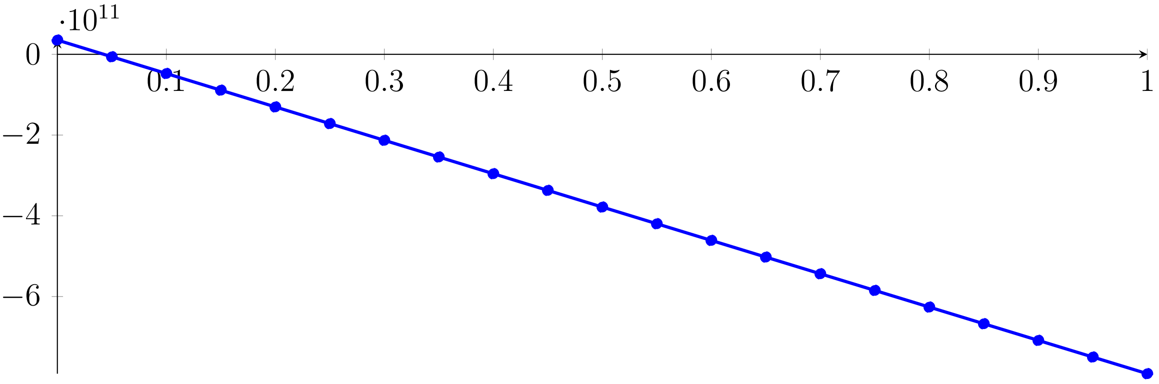
$$[0.106889, 0.122705]$$

Longest intersection interval: 0.0158155
$\Longrightarrow$ Selective recursion: interval 1: $[2.99995, 3.00111]$,

1.19 Recursion Branch 1 1 1 1 2 2 1 1 in Interval 1: $[2.99995, 3.00111]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 320.932X^{20} - 2473.41X^{19} + 8213.8X^{18} - 42410.5X^{17} + 199030X^{16} - 146117X^{15} + 50394.6X^{14} + 11372X^{13} + 109662X^{12} - 570.238X^{11} + 24339.2X^{10} + 639.435X^{9} + 543.007X^{8} - 102.909X^{7} + 78.0688X^{6} + 1.41943X^{5} - 1974.34X^{4} - 518217X^{3} + 1.86996 \cdot 10^{9}X^{2} - 8.28306 \cdot 10^{11}X + 3.52793 \cdot 10^{10}$$
$$= 3.52793 \cdot 10^{10}B_{0,20}(X) - 6.13598 \cdot 10^{9}B_{1,20}(X) - 4.75414 \cdot 10^{10}B_{2,20}(X) - 8.8937 \cdot 10^{10}B_{3,20}(X) - 1.30323 \cdot 10^{11}B_{4,20}(X) - 1.71699 \cdot 10^{11}B_{5,20}(X) - 2.13065 \cdot 10^{11}B_{6,20}(X) - 2.54421 \cdot 10^{11}B_{7,20}(X) - 2.95767 \cdot 10^{11}B_{8,20}(X) - 3.37104 \cdot 10^{11}B_{9,20}(X) - 3.78431 \cdot 10^{11}B_{10,20}(X) - 4.19748 \cdot 10^{11}B_{11,20}(X) - 4.61055 \cdot 10^{11}B_{12,20}(X) - 5.02352 \cdot 10^{11}B_{13,20}(X) - 5.43639 \cdot 10^{11}B_{14,20}(X) - 5.84917 \cdot 10^{11}B_{15,20}(X) - 6.26185 \cdot 10^{11}B_{16,20}(X) - 6.67442 \cdot 10^{11}B_{17,20}(X) - 7.08691 \cdot 10^{11}B_{18,20}(X) - 7.49929 \cdot 10^{11}B_{19,20}(X) - 7.91157 \cdot 10^{11}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.0425921, 0.0426885\}$$

Intersection intervals with the x axis:

$$[0.0425921, 0.0426885]$$

Longest intersection interval: $9.63456 \cdot 10^{-05}$
$\Longrightarrow$ Selective recursion: interval 1: $[3, 3]$,

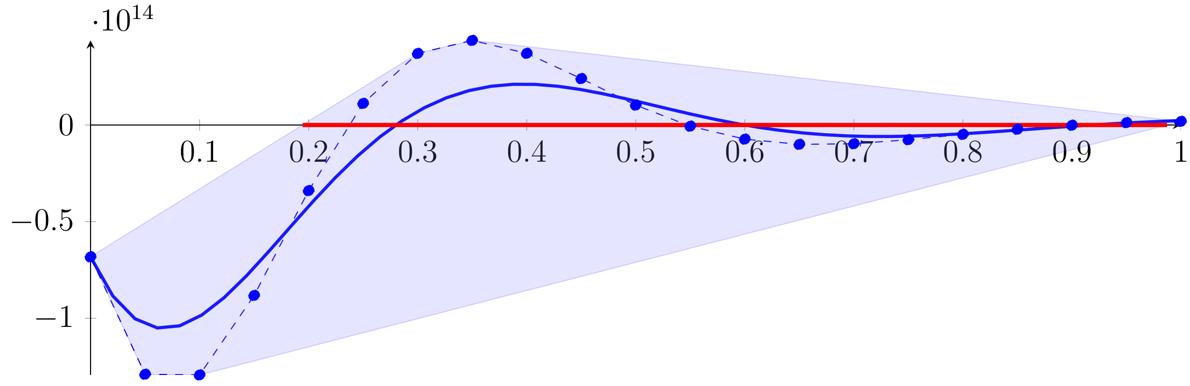
1.20 Recursion Branch 1 1 1 1 2 2 1 1 1 in Interval 1: $[3, 3]$

Found root in interval $[3, 3]$ at recursion depth 9!

1.21 Recursion Branch 1 1 1 2 on the Second Half $[3.125, 6.25]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 7.88859 \cdot 10^{9}X^{20} - 3.72342 \cdot 10^{11}X^{19} + 8.07932 \cdot 10^{12}X^{18} - 1.06797 \cdot 10^{14}X^{17} + 9.60483 \cdot 10^{14}X^{16} - 6.21458 \cdot 10^{15}X^{15} + 2.98115 \cdot 10^{16}X^{14} - 1.07566 \cdot 10^{17}X^{13} + 2.92576 \cdot 10^{17}X^{12} - 5.93362 \cdot 10^{17}X^{11} + 8.69791 \cdot 10^{17}X^{10} - 8.52613 \cdot 10^{17}X^{9} + 4.24784 \cdot 10^{17}X^{8} + 1.26126 \cdot 10^{17}X^{7} - 3.67434 \cdot 10^{17}X^{6} + 2.40127 \cdot 10^{17}X^{5} - 4.54599 \cdot 10^{16}X^{4} - 2.16249 \cdot 10^{16}X^{3} + 1.14835 \cdot 10^{16}X^{2} - 1.2155 \cdot 10^{15}X - 6.82353 \cdot 10^{13}$$
$$= -6.82353 \cdot 10^{13}B_{0,20}(X) - 1.2901 \cdot 10^{14}B_{1,20}(X) - 1.29346 \cdot 10^{14}B_{2,20}(X) - 8.82108 \cdot 10^{13}B_{3,20}(X) - 3.39572 \cdot 10^{13}B_{4,20}(X) + 1.11681 \cdot 10^{13}B_{5,20}(X) + 3.70318 \cdot 10^{13}B_{6,20}(X) + 4.37698 \cdot 10^{13}B_{7,20}(X) + 3.70894 \cdot 10^{13}B_{8,20}(X) + 2.40125 \cdot 10^{13}B_{9,20}(X) + 1.02825 \cdot 10^{13}B_{10,20}(X) - 6.08666 \cdot 10^{11}B_{11,20}(X) - 7.31328 \cdot 10^{12}B_{12,20}(X) - 1.00112 \cdot 10^{13}B_{13,20}(X) - 9.69955 \cdot 10^{12}B_{14,20}(X) - 7.61291 \cdot 10^{12}B_{15,20}(X) - 4.85196 \cdot 10^{12}B_{16,20}(X) - 2.20819 \cdot 10^{12}B_{17,20}(X) - 1.32423 \cdot 10^{11}B_{18,20}(X) + 1.21228 \cdot 10^{12}B_{19,20}(X) + 1.86285 \cdot 10^{12}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.194463, 0.987222\}$$

Intersection intervals with the x axis:

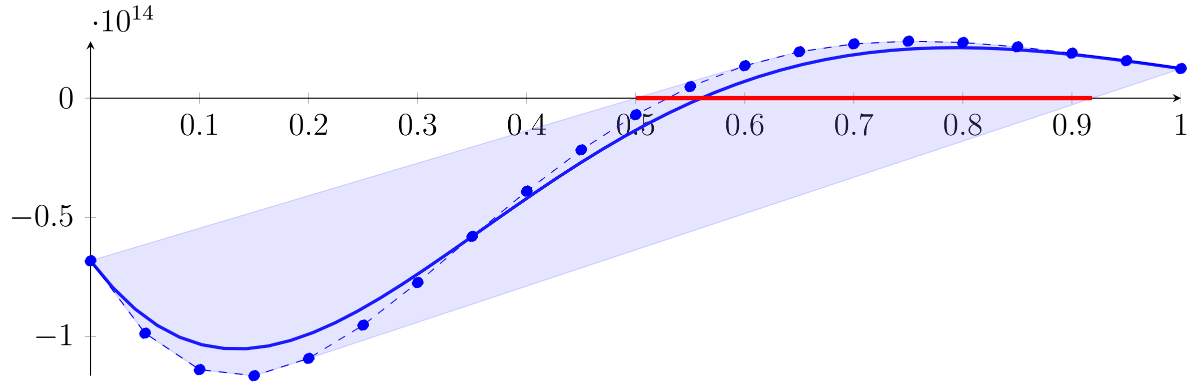
$$[0.194463, 0.987222]$$

Longest intersection interval: 0.792759
$\Longrightarrow$ Bisection: first half $[3.125, 4.6875]$ und second half $[4.6875, 6.25]$

1.22 Recursion Branch 1 1 1 2 1 on the First Half $[3.125, 4.6875]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = {} & 34696.8X^{20} - 584847X^{19} + 3.25592 \cdot 10^{7}X^{18} - 8.1589 \cdot 10^{8}X^{17} + 1.46784 \cdot 10^{10}X^{16} - 1.89675 \cdot 10^{11}X^{15} \\
& + 1.81956 \cdot 10^{12}X^{14} - 1.31307 \cdot 10^{13}X^{13} + 7.14297 \cdot 10^{13}X^{12} - 2.89728 \cdot 10^{14}X^{11} + 8.49406 \cdot 10^{14}X^{10} \\
& - 1.66526 \cdot 10^{15}X^{9} + 1.65931 \cdot 10^{15}X^{8} + 9.85362 \cdot 10^{14}X^{7} - 5.74116 \cdot 10^{15}X^{6} + 7.50397 \cdot 10^{15}X^{5} \\
& - 2.84125 \cdot 10^{15}X^{4} - 2.70311 \cdot 10^{15}X^{3} + 2.87089 \cdot 10^{15}X^{2} - 6.07752 \cdot 10^{14}X - 6.82353 \cdot 10^{13} \\
= {} & -6.82353 \cdot 10^{13}B_{0,20}(X) - 9.86229 \cdot 10^{13}B_{1,20}(X) - 1.13901 \cdot 10^{14}B_{2,20}(X) - 1.16439 \\
& \cdot 10^{14}B_{3,20}(X) - 1.09197 \cdot 10^{14}B_{4,20}(X) - 9.52335 \cdot 10^{13}B_{5,20}(X) - 7.73753 \cdot 10^{13}B_{6,20}(X) \\
& - 5.80151 \cdot 10^{13}B_{7,20}(X) - 3.90206 \cdot 10^{13}B_{8,20}(X) - 2.17241 \cdot 10^{13}B_{9,20}(X) - 6.96521 \\
& \cdot 10^{12}B_{10,20}(X) + 4.83558 \cdot 10^{12}B_{11,20}(X) + 1.35903 \cdot 10^{13}B_{12,20}(X) + 1.94553 \cdot 10^{13}B_{13,20}(X) \\
& + 2.27507 \cdot 10^{13}B_{14,20}(X) + 2.38903 \cdot 10^{13}B_{15,20}(X) + 2.33265 \cdot 10^{13}B_{16,20}(X) + 2.15075 \\
& \cdot 10^{13}B_{17,20}(X) + 1.88477 \cdot 10^{13}B_{18,20}(X) + 1.57094 \cdot 10^{13}B_{19,20}(X) + 1.23927 \cdot 10^{13}B_{20,20}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.500347, 0.918462\}$$

Intersection intervals with the x axis:

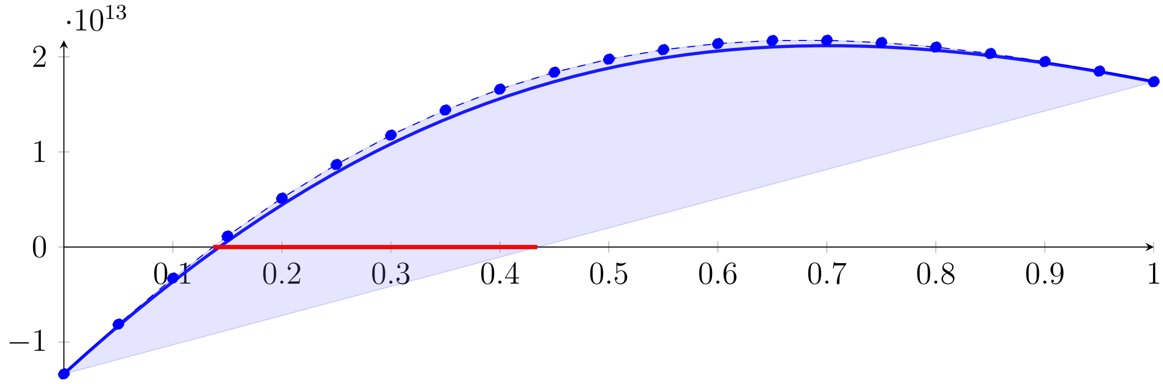
$$[0.500347, 0.918462]$$

Longest intersection interval: 0.418115
$\Longrightarrow$ Selective recursion: interval 1: $[3.90679, 4.5601]$,

1.23 Recursion Branch 1 1 1 2 1 1 in Interval 1: $[3.90679, 4.5601]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-18283.9X^{20} + 130461X^{19} - 475773X^{18} + 2.48277\cdot 10^{6}X^{17} - 1.11176\cdot 10^{7}X^{16} + 8.43745\\ &\cdot 10^{6}X^{15} + 912374X^{14} - 5.31926\cdot 10^{7}X^{13} + 5.12908\cdot 10^{8}X^{12} - 3.3219\cdot 10^{9}X^{11} + 1.02268\cdot 10^{10}X^{10}\\ &+ 2.96841\cdot 10^{10}X^{9} - 4.62951\cdot 10^{11}X^{8} + 1.92006\cdot 10^{12}X^{7} - 1.70977\cdot 10^{12}X^{6} - 1.3617\cdot 10^{13}X^{5}\\ &+ 4.77555\cdot 10^{13}X^{4} - 3.40527\cdot 10^{13}X^{3} - 7.3763\cdot 10^{13}X^{2} + 1.04615\cdot 10^{14}X - 1.33442\cdot 10^{13}\\ = &-1.33442\cdot 10^{13}B_{0,20}(X) - 8.11344\cdot 10^{12}B_{1,20}(X) - 3.27092\cdot 10^{12}B_{2,20}(X) + 1.15351\\ &\cdot 10^{12}B_{3,20}(X) + 5.13982\cdot 10^{12}B_{4,20}(X) + 8.67698\cdot 10^{12}B_{5,20}(X) + 1.1762\cdot 10^{13}B_{6,20}(X)\\ &+ 1.43991\cdot 10^{13}B_{7,20}(X) + 1.65984\cdot 10^{13}B_{8,20}(X) + 1.83757\cdot 10^{13}B_{9,20}(X) + 1.97507\\ &\cdot 10^{13}B_{10,20}(X) + 2.0747\cdot 10^{13}B_{11,20}(X) + 2.13908\cdot 10^{13}B_{12,20}(X) + 2.17102\cdot 10^{13}B_{13,20}(X)\\ &+ 2.17348\cdot 10^{13}B_{14,20}(X) + 2.14949\cdot 10^{13}B_{15,20}(X) + 2.10209\cdot 10^{13}B_{16,20}(X) + 2.03432\\ &\cdot 10^{13}B_{17,20}(X) + 1.94912\cdot 10^{13}B_{18,20}(X) + 1.84937\cdot 10^{13}B_{19,20}(X) + 1.7378\cdot 10^{13}B_{20,20}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.136964, 0.43435\}$$

Intersection intervals with the x axis:

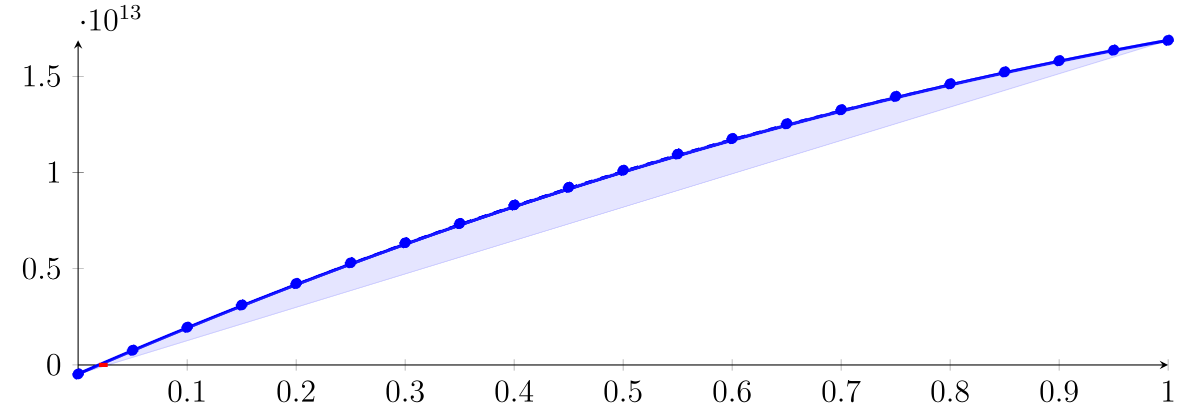
$$[0.136964, 0.43435]$$

Longest intersection interval: 0.297386
$\Longrightarrow$ Selective recursion: interval 1: $[3.99627, 4.19056]$,

1.24 Recursion Branch 1 1 1 2 1 1 1 in Interval 1: $[3.99627, 4.19056]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-8740X^{20} + 60830.8X^{19} - 216147X^{18} + 1.1391\cdot 10^{6}X^{17} - 5.30376\cdot 10^{6}X^{16} + 4.02612\\ &\cdot 10^{6}X^{15} - 1.32991\cdot 10^{6}X^{14} - 576782X^{13} - 3.04604\cdot 10^{6}X^{12} - 158939X^{11} - 676416X^{10}\\ &+ 678565X^{9} - 2.56399\cdot 10^{7}X^{8} + 2.95316\cdot 10^{8}X^{7} - 7.26215\cdot 10^{7}X^{6} - 3.33329\cdot 10^{10}X^{5}\\ &+ 2.98077\cdot 10^{11}X^{4} - 2.76401\cdot 10^{11}X^{3} - 7.31712\cdot 10^{12}X^{2} + 2.46709\cdot 10^{13}X - 4.7076\cdot 10^{11}\\ = &-4.7076\cdot 10^{11}B_{0,20}(X) + 7.62784\cdot 10^{11}B_{1,20}(X) + 1.95782\cdot 10^{12}B_{2,20}(X) + 3.1141\\ &\cdot 10^{12}B_{3,20}(X) + 4.23144\cdot 10^{12}B_{4,20}(X) + 5.30973\cdot 10^{12}B_{5,20}(X) + 6.3489\cdot 10^{12}B_{6,20}(X)\\ &+ 7.34894\cdot 10^{12}B_{7,20}(X) + 8.30989\cdot 10^{12}B_{8,20}(X) + 9.23185\cdot 10^{12}B_{9,20}(X) + 1.0115\\ &\cdot 10^{13}B_{10,20}(X) + 1.09594\cdot 10^{13}B_{11,20}(X) + 1.17654\cdot 10^{13}B_{12,20}(X) + 1.25333\cdot 10^{13}B_{13,20}(X)\\ &+ 1.32634\cdot 10^{13}B_{14,20}(X) + 1.3956\cdot 10^{13}B_{15,20}(X) + 1.46114\cdot 10^{13}B_{16,20}(X) + 1.52303\\ &\cdot 10^{13}B_{17,20}(X) + 1.58129\cdot 10^{13}B_{18,20}(X) + 1.63598\cdot 10^{13}B_{19,20}(X) + 1.68715\cdot 10^{13}B_{20,20}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.0190816, 0.0271452\}$$

Intersection intervals with the x axis:

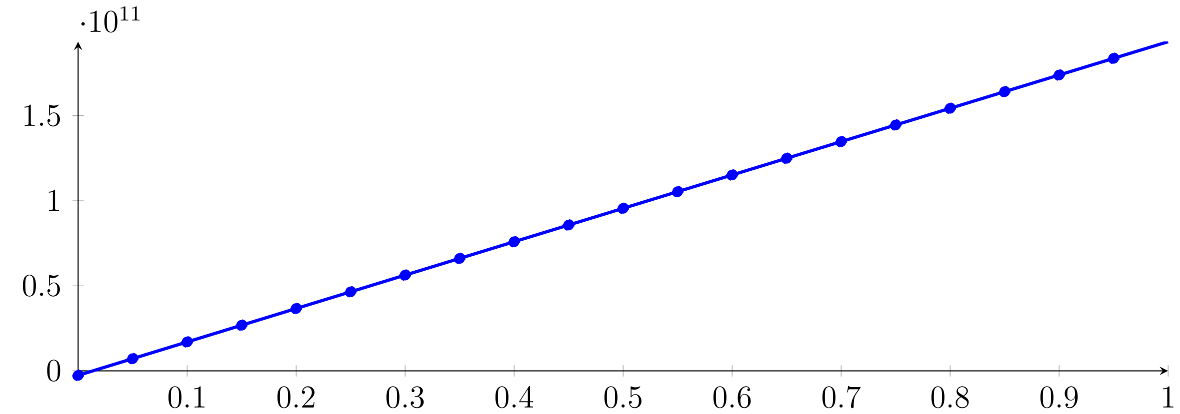
$$[0.0190816, 0.0271452]$$

Longest intersection interval: 0.00806358
$\Longrightarrow$ Selective recursion: interval 1: $[3.99998, 4.00155]$,

1.25 Recursion Branch 1 1 1 2 1 1 1 1 in Interval 1: $[3.99998, 4.00155]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-86.1632X^{20} + 669.497X^{19} - 2267.29X^{18} + 11696.8X^{17} - 51870.9X^{16} + 38130.7X^{15} - 14516.1X^{14} \\ &- 1406.42X^{13} - 29095.6X^{12} + 363.927X^{11} - 6926.66X^{10} + 51.2573X^{9} - 204.709X^{8} + 17.7429X^{7} \\ &- 18.6301X^{6} - 0.354858X^{5} + 1246.62X^{4} - 133053X^{3} - 4.76756 \cdot 10^{8}X^{2} + 1.96682 \cdot 10^{11}X - 2.6661 \cdot 10^{9} \\ = &-2.6661 \cdot 10^{9}B_{0,20}(X) + 7.16798 \cdot 10^{9}B_{1,20}(X) + 1.69996 \cdot 10^{10}B_{2,20}(X) + 2.68286 \\ &\cdot 10^{10}B_{3,20}(X) + 3.66552 \cdot 10^{10}B_{4,20}(X) + 4.64792 \cdot 10^{10}B_{5,20}(X) + 5.63007 \cdot 10^{10}B_{6,20}(X) \\ &+ 6.61198 \cdot 10^{10}B_{7,20}(X) + 7.59363 \cdot 10^{10}B_{8,20}(X) + 8.57503 \cdot 10^{10}B_{9,20}(X) + 9.55618 \\ &\cdot 10^{10}B_{10,20}(X) + 1.05371 \cdot 10^{11}B_{11,20}(X) + 1.15177 \cdot 10^{11}B_{12,20}(X) + 1.24981 \cdot 10^{11}B_{13,20}(X) \\ &+ 1.34783 \cdot 10^{11}B_{14,20}(X) + 1.44582 \cdot 10^{11}B_{15,20}(X) + 1.54378 \cdot 10^{11}B_{16,20}(X) + 1.64172 \\ &\cdot 10^{11}B_{17,20}(X) + 1.73963 \cdot 10^{11}B_{18,20}(X) + 1.83752 \cdot 10^{11}B_{19,20}(X) + 1.93539 \cdot 10^{11}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.0135554, 0.0135884\}$$

Intersection intervals with the x axis:

$$[0.0135554, 0.0135884]$$

Longest intersection interval: $3.29473 \cdot 10^{-05}$
$\Longrightarrow$ Selective recursion: interval 1: $[4, 4]$,

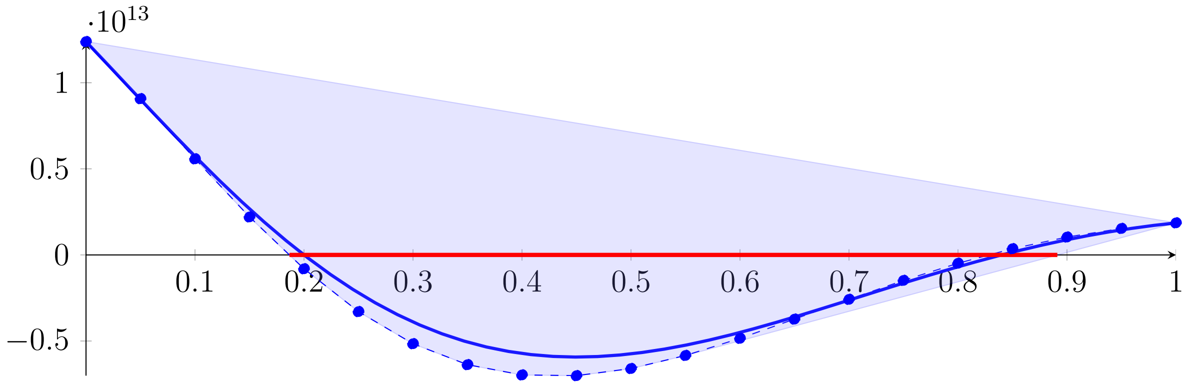
1.26 Recursion Branch 1 1 1 2 1 1 1 1 1 in Interval 1: $[4, 4]$

Found root in interval $[4, 4]$ at recursion depth 9!

1.27 Recursion Branch 1 1 1 2 2 on the Second Half $[4.6875, 6.25]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 13943.5X^{20} - 587258X^{19} + 1.89008\cdot10^{7}X^{18} - 3.73664\cdot10^{8}X^{17} + 4.87208\cdot10^{9}X^{16} - 4.34631\cdot10^{10}X^{15} + 2.65721\cdot10^{11}X^{14} - 1.05722\cdot10^{12}X^{13} + 2.18629\cdot10^{12}X^{12} + 1.53487\cdot10^{12}X^{11} - 2.39754\cdot10^{13}X^{10} + 6.26713\cdot10^{13}X^{9} - 3.75532\cdot10^{13}X^{8} - 1.53878\cdot10^{14}X^{7} + 3.47765\cdot10^{14}X^{6} - 1.50066\cdot10^{14}X^{5} - 3.00387\cdot10^{14}X^{4} + 3.42221\cdot10^{14}X^{3} - 3.38862\cdot10^{13}X^{2} - 6.63332\cdot10^{13}X + 1.23927\cdot10^{13}$$
$$= 1.23927\cdot10^{13}B_{0,20}(X) + 9.07608\cdot10^{12}B_{1,20}(X) + 5.58107\cdot10^{12}B_{2,20}(X) + 2.20791\cdot10^{12}B_{3,20}(X) - 8.05212\cdot10^{11}B_{4,20}(X) - 3.29178\cdot10^{12}B_{5,20}(X) - 5.15766\cdot10^{12}B_{6,20}(X) - 6.37482\cdot10^{12}B_{7,20}(X) - 6.97037\cdot10^{12}B_{8,20}(X) - 7.01303\cdot10^{12}B_{9,20}(X) - 6.5991\cdot10^{12}B_{10,20}(X) - 5.83916\cdot10^{12}B_{11,20}(X) - 4.8467\cdot10^{12}B_{12,20}(X) - 3.72904\cdot10^{12}B_{13,20}(X) - 2.58078\cdot10^{12}B_{14,20}(X) - 1.47983\cdot10^{12}B_{15,20}(X) - 4.85456\cdot10^{11}B_{16,20}(X) + 3.6178\cdot10^{11}B_{17,20}(X) + 1.03875\cdot10^{12}B_{18,20}(X) + 1.53757\cdot10^{12}B_{19,20}(X) + 1.86285\cdot10^{12}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.186638, 0.891161\}$$

Intersection intervals with the x axis:

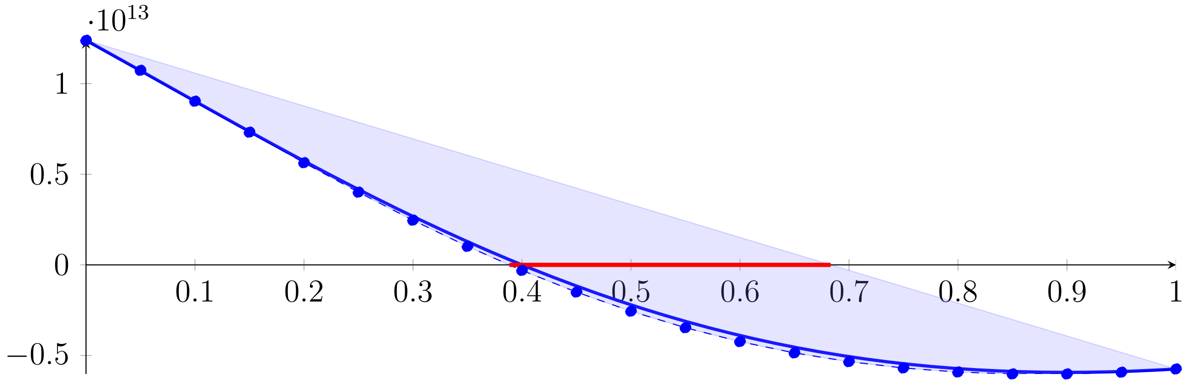
$$[0.186638, 0.891161]$$

Longest intersection interval: 0.704522
$\Longrightarrow$ Bisection: first half $[4.6875, 5.46875]$ und second half $[5.46875, 6.25]$

1.28 Recursion Branch 1 1 1 2 2 1 on the First Half $[4.6875, 5.46875]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 1200.11X^{20} - 25319.6X^{19} - 27809X^{18} - 307600X^{17} + 419906X^{16} - 1.36986\cdot10^{6}X^{15} + 1.58218\cdot10^{7}X^{14} - 1.29485\cdot10^{8}X^{13} + 5.32577\cdot10^{8}X^{12} + 7.48821\cdot10^{8}X^{11} - 2.34141\cdot10^{10}X^{10} + 1.22405\cdot10^{11}X^{9} - 1.46692\cdot10^{11}X^{8} - 1.20217\cdot10^{12}X^{7} + 5.43384\cdot10^{12}X^{6} - 4.68955\cdot10^{12}X^{5} - 1.87742\cdot10^{13}X^{4} + 4.27776\cdot10^{13}X^{3} - 8.47155\cdot10^{12}X^{2} - 3.31666\cdot10^{13}X + 1.23927\cdot10^{13}$$
$$= 1.23927\cdot10^{13}B_{0,20}(X) + 1.07344\cdot10^{13}B_{1,20}(X) + 9.03149\cdot10^{12}B_{2,20}(X) + 7.32151\cdot10^{12}B_{3,20}(X) + 5.63812\cdot10^{12}B_{4,20}(X) + 4.01079\cdot10^{12}B_{5,20}(X) + 2.46464\cdot10^{12}B_{6,20}(X) + 1.02044\cdot10^{12}B_{7,20}(X) - 3.05365\cdot10^{11}B_{8,20}(X) - 1.50048\cdot10^{12}B_{9,20}(X) - 2.5565\cdot10^{12}B_{10,20}(X) - 3.46866\cdot10^{12}B_{11,20}(X) - 4.23546\cdot10^{12}B_{12,20}(X) - 4.85834\cdot10^{12}B_{13,20}(X) - 5.34125\cdot10^{12}B_{14,20}(X) - 5.69032\cdot10^{12}B_{15,20}(X) - 5.91343\cdot10^{12}B_{16,20}(X) - 6.01989\cdot10^{12}B_{17,20}(X) - 6.02006\cdot10^{12}B_{18,20}(X) - 5.92504\cdot10^{12}B_{19,20}(X) - 5.74635\cdot10^{12}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.388484, 0.683206\}$$

Intersection intervals with the x axis:

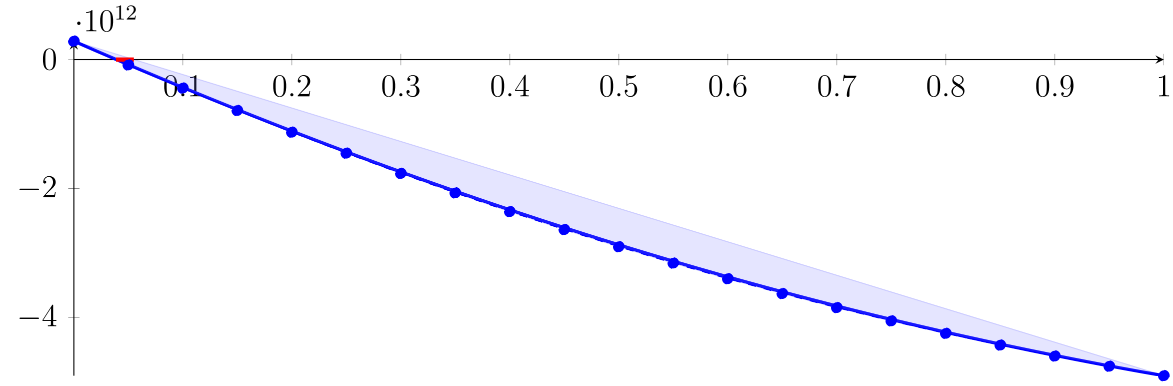
$$[0.388484, 0.683206]$$

Longest intersection interval: 0.294722
$\Longrightarrow$ Selective recursion: interval 1: $[4.991, 5.22125]$,

1.29 Recursion Branch 1 1 1 2 2 1 1 in Interval 1: $[4.991, 5.22125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = {} & 2589.39X^{20} - 20050.1X^{19} + 67168.1X^{18} - 350638X^{17} + 1.611\cdot10^{6}X^{16} - 1.17622 \\
& \cdot10^{6}X^{15} + 465468X^{14} + 13063.5X^{13} + 930528X^{12} - 20390.2X^{11} + 139593X^{10} \\
& + 716424X^{9} + 7.64629\cdot10^{6}X^{8} - 2.21945\cdot10^{8}X^{7} + 1.337\cdot10^{9}X^{6} + 8.86179\cdot10^{9}X^{5} \\
& - 1.37037\cdot10^{11}X^{4} + 3.04606\cdot10^{11}X^{3} + 2.02024\cdot10^{12}X^{2} - 7.38551\cdot10^{12}X + 2.85488\cdot10^{11} \\
= {} & 2.85488\cdot10^{11}B_{0,20}(X) - 8.37873\cdot10^{10}B_{1,20}(X) - 4.4243\cdot10^{11}B_{2,20}(X) - 7.90173 \\
& \cdot10^{11}B_{3,20}(X) - 1.12678\cdot10^{12}B_{4,20}(X) - 1.45203\cdot10^{12}B_{5,20}(X) - 1.76575\cdot10^{12}B_{6,20}(X) \\
& - 2.06778\cdot10^{12}B_{7,20}(X) - 2.35798\cdot10^{12}B_{8,20}(X) - 2.63625\cdot10^{12}B_{9,20}(X) - 2.90251 \\
& \cdot10^{12}B_{10,20}(X) - 3.1567\cdot10^{12}B_{11,20}(X) - 3.39879\cdot10^{12}B_{12,20}(X) - 3.62875\cdot10^{12}B_{13,20}(X) \\
& - 3.8466\cdot10^{12}B_{14,20}(X) - 4.05236\cdot10^{12}B_{15,20}(X) - 4.24609\cdot10^{12}B_{16,20}(X) - 4.42784 \\
& \cdot10^{12}B_{17,20}(X) - 4.59771\cdot10^{12}B_{18,20}(X) - 4.7558\cdot10^{12}B_{19,20}(X) - 4.90223\cdot10^{12}B_{20,20}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.0386552, 0.0550316\}$$

Intersection intervals with the x axis:

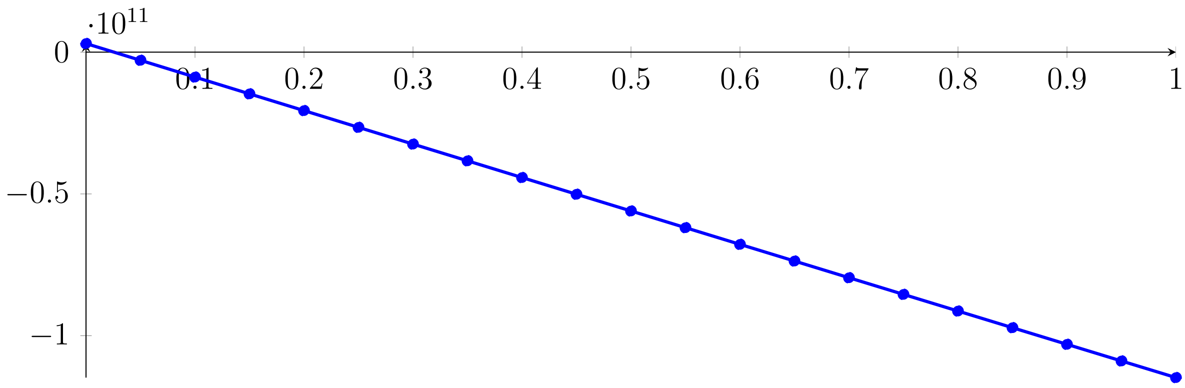
$$[0.0386552, 0.0550316]$$

Longest intersection interval: 0.0163764
$\Longrightarrow$ Selective recursion: interval 1: $[4.9999, 5.00367]$,

1.30 Recursion Branch 1 1 1 2 2 1 1 1 in Interval 1: $[4.9999, 5.00367]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = {} & 50.8198X^{20} - 382.316X^{19} + 1268.75X^{18} - 6880.37X^{17} + 29186X^{16} \\ & - 22805.6X^{15} + 7750.77X^{14} + 1258.42X^{13} + 16343.3X^{12} - 179.721X^{11} \\ & + 3814.67X^{10} + 88.0985X^{9} + 75.6846X^{8} - 4.58359X^{7} + 12.494X^{6} + 10.6458X^{5} \\ & - 9730.84X^{4} + 1.24534{\cdot}10^{6}X^{3} + 5.50946{\cdot}10^{8}X^{2} - 1.18368{\cdot}10^{11}X + 3.03598{\cdot}10^{9} \\ = {} & 3.03598{\cdot}10^{9}B_{0,20}(X) - 2.88244{\cdot}10^{9}B_{1,20}(X) - 8.79797{\cdot}10^{9}B_{2,20}(X) - 1.47106 \\ & {\cdot}10^{10}B_{3,20}(X) - 2.06203{\cdot}10^{10}B_{4,20}(X) - 2.65271{\cdot}10^{10}B_{5,20}(X) - 3.2431{\cdot}10^{10}B_{6,20}(X) \\ & - 3.8332{\cdot}10^{10}B_{7,20}(X) - 4.42301{\cdot}10^{10}B_{8,20}(X) - 5.01253{\cdot}10^{10}B_{9,20}(X) - 5.60176 \\ & {\cdot}10^{10}B_{10,20}(X) - 6.1907{\cdot}10^{10}B_{11,20}(X) - 6.77935{\cdot}10^{10}B_{12,20}(X) - 7.3677{\cdot}10^{10}B_{13,20}(X) \\ & - 7.95577{\cdot}10^{10}B_{14,20}(X) - 8.54354{\cdot}10^{10}B_{15,20}(X) - 9.13102{\cdot}10^{10}B_{16,20}(X) - 9.71821 \\ & {\cdot}10^{10}B_{17,20}(X) - 1.03051{\cdot}10^{11}B_{18,20}(X) - 1.08917{\cdot}10^{11}B_{19,20}(X) - 1.1478{\cdot}10^{11}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.0256485, 0.0257687\}$$

Intersection intervals with the x axis:

$$[0.0256485, 0.0257687]$$

Longest intersection interval: 0.00012021
$\Longrightarrow$ Selective recursion: interval 1: $[5, 5]$,

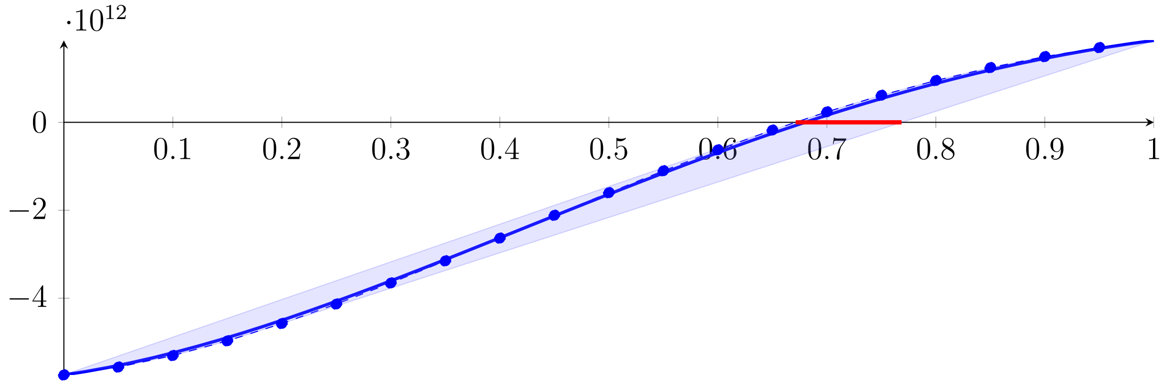
1.31 Recursion Branch 1 1 1 2 2 1 1 1 1 in Interval 1: $[5, 5]$

Found root in interval $[5, 5]$ at recursion depth 9!

1.32 Recursion Branch 1 1 1 2 2 2 on the Second Half $[5.46875, 6.25]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = {} & 2141.38X^{20} - 2148.75X^{19} + 90247X^{18} - 184573X^{17} + 1.58159{\cdot}10^{6}X^{16} - 1.80476{\cdot}10^{6}X^{15} \\ & + 4.2097{\cdot}10^{6}X^{14} - 5.47243{\cdot}10^{6}X^{13} - 1.50936{\cdot}10^{8}X^{12} + 1.47319{\cdot}10^{9}X^{11} - 4.05611{\cdot}10^{9}X^{10} \\ & - 1.87417{\cdot}10^{10}X^{9} + 1.64183{\cdot}10^{11}X^{8} - 2.82227{\cdot}10^{11}X^{7} - 1.06297{\cdot}10^{12}X^{6} + 4.60756{\cdot}10^{12}X^{5} \\ & - 2.11893{\cdot}10^{12}X^{4} - 1.31468{\cdot}10^{13}X^{3} + 1.58961{\cdot}10^{13}X^{2} + 3.57376{\cdot}10^{12}X - 5.74635{\cdot}10^{12} \\ = {} & -5.74635{\cdot}10^{12}B_{0,20}(X) - 5.56766{\cdot}10^{12}B_{1,20}(X) - 5.30531{\cdot}10^{12}B_{2,20}(X) - 4.97083 \\ & {\cdot}10^{12}B_{3,20}(X) - 4.57618{\cdot}10^{12}B_{4,20}(X) - 4.13349{\cdot}10^{12}B_{5,20}(X) - 3.65472{\cdot}10^{12}B_{6,20}(X) \\ & - 3.15149{\cdot}10^{12}B_{7,20}(X) - 2.63484{\cdot}10^{12}B_{8,20}(X) - 2.11506{\cdot}10^{12}B_{9,20}(X) - 1.60157 \\ & {\cdot}10^{12}B_{10,20}(X) - 1.1028{\cdot}10^{12}B_{11,20}(X) - 6.26115{\cdot}10^{11}B_{12,20}(X) - 1.77814{\cdot}10^{11}B_{13,20}(X) \\ & + 2.36929{\cdot}10^{11}B_{14,20}(X) + 6.14027{\cdot}10^{11}B_{15,20}(X) + 9.50455{\cdot}10^{11}B_{16,20}(X) + 1.2442 \\ & {\cdot}10^{12}B_{17,20}(X) + 1.49418{\cdot}10^{12}B_{18,20}(X) + 1.70021{\cdot}10^{12}B_{19,20}(X) + 1.86285{\cdot}10^{12}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.671437, 0.768555\}$$

Intersection intervals with the x axis:

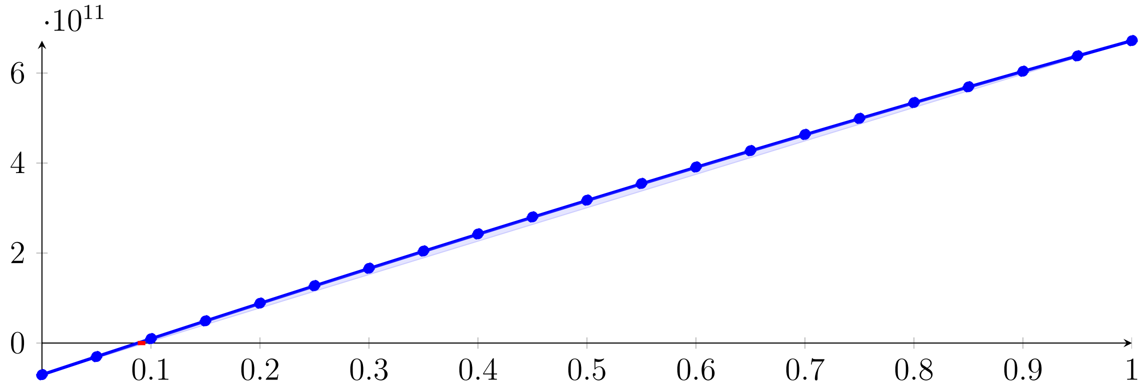
$$[0.671437, 0.768555]$$

Longest intersection interval: 0.0971186
$\Longrightarrow$ Selective recursion: interval 1: [5.99331, 6.06918],

1.33 Recursion Branch 1 1 1 2 2 2 1 in Interval 1: [5.99331, 6.06918]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-269.275X^{20} + 1999.71X^{19} - 6535.74X^{18} + 37676X^{17} - 158708X^{16} + 115882X^{15}\\ &- 35799.3X^{14} - 9115.13X^{13} - 82033.5X^{12} + 2280.95X^{11} - 18510.6X^{10}\\ &+ 210.155X^{9} - 338.298X^{8} + 18764.9X^{7} - 744285X^{6} - 1.10823\cdot 10^{6}X^{5} + 4.56688\\ &\cdot 10^{8}X^{4} - 4.96381\cdot 10^{9}X^{3} - 5.70191\cdot 10^{10}X^{2} + 8.03922\cdot 10^{11}X - 7.04107\cdot 10^{10}\\ = &-7.04107\cdot 10^{10}B_{0,20}(X) - 3.02146\cdot 10^{10}B_{1,20}(X) + 9.68141\cdot 10^{9}B_{2,20}(X) + 4.9273\\ &\cdot 10^{10}B_{3,20}(X) + 8.85558\cdot 10^{10}B_{4,20}(X) + 1.27526\cdot 10^{11}B_{5,20}(X) + 1.66179\cdot 10^{11}B_{6,20}(X)\\ &+ 2.04511\cdot 10^{11}B_{7,20}(X) + 2.42518\cdot 10^{11}B_{8,20}(X) + 2.80197\cdot 10^{11}B_{9,20}(X) + 3.17543\\ &\cdot 10^{11}B_{10,20}(X) + 3.54554\cdot 10^{11}B_{11,20}(X) + 3.91225\cdot 10^{11}B_{12,20}(X) + 4.27553\cdot 10^{11}B_{13,20}(X)\\ &+ 4.63535\cdot 10^{11}B_{14,20}(X) + 4.99168\cdot 10^{11}B_{15,20}(X) + 5.34448\cdot 10^{11}B_{16,20}(X) + 5.69372\\ &\cdot 10^{11}B_{17,20}(X) + 6.03938\cdot 10^{11}B_{18,20}(X) + 6.38143\cdot 10^{11}B_{19,20}(X) + 6.71983\cdot 10^{11}B_{20,20}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.0878667, 0.0948428\}$$

Intersection intervals with the x axis:

$$[0.0878667, 0.0948428]$$

Longest intersection interval: 0.00697607
$\Longrightarrow$ Selective recursion: interval 1: [5.99998, 6.00051],

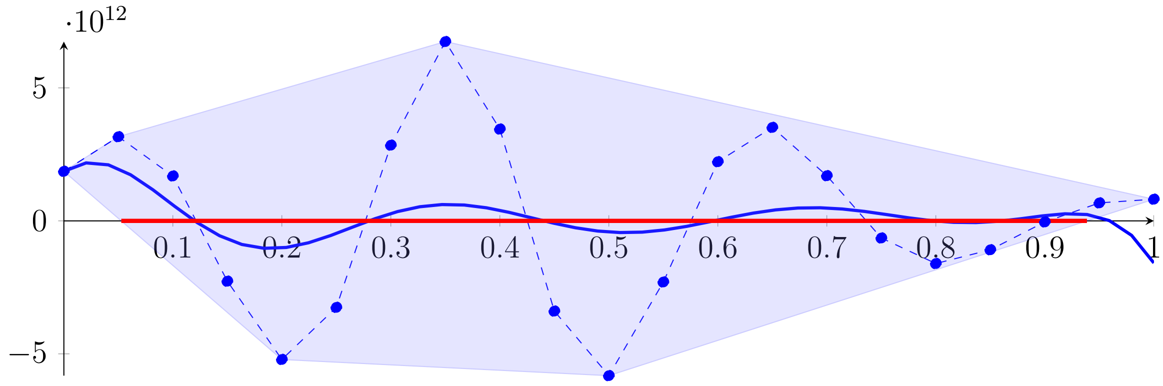
1.34 Recursion Branch 1 1 1 2 2 2 1 1 in Interval 1: $[5.99998, 6.00051]$

Found root in interval $[5.99998, 6.00051]$ at recursion depth 8!

1.35 Recursion Branch 1 1 2 on the Second Half $[6.25, 12.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 8.27181 \cdot 10^{15} X^{20} - 1.12497 \cdot 10^{17} X^{19} + 6.56318 \cdot 10^{17} X^{18} - 2.10324 \cdot 10^{18} X^{17} + 3.83361 \cdot 10^{18} X^{16} - 3.25611 \cdot 10^{18} X^{15} - 1.18134 \cdot 10^{18} X^{14} + 5.65844 \cdot 10^{18} X^{13} - 4.66119 \cdot 10^{18} X^{12} - 3.70393 \cdot 10^{17} X^{11} + 2.95436 \cdot 10^{18} X^{10} - 1.48062 \cdot 10^{18} X^{9} - 3.2208 \cdot 10^{17} X^{8} + 4.91145 \cdot 10^{17} X^{7} - 8.64752 \cdot 10^{16} X^{6} - 4.35417 \cdot 10^{16} X^{5} + 1.55034 \cdot 10^{16} X^{4} + 3.36768 \cdot 10^{14} X^{3} - 5.27545 \cdot 10^{14} X^{2} + 2.60227 \cdot 10^{13} X + 1.86285 \cdot 10^{12}$$
$$= 1.86285 \cdot 10^{12} B_{0,20}(X) + 3.16399 \cdot 10^{12} B_{1,20}(X) + 1.68857 \cdot 10^{12} B_{2,20}(X) - 2.268 \cdot 10^{12} B_{3,20}(X) - 5.21041 \cdot 10^{12} B_{4,20}(X) - 3.25192 \cdot 10^{12} B_{5,20}(X) + 2.84625 \cdot 10^{12} B_{6,20}(X) + 6.74009 \cdot 10^{12} B_{7,20}(X) + 3.45161 \cdot 10^{12} B_{8,20}(X) - 3.39194 \cdot 10^{12} B_{9,20}(X) - 5.81848 \cdot 10^{12} B_{10,20}(X) - 2.29738 \cdot 10^{12} B_{11,20}(X) + 2.22447 \cdot 10^{12} B_{12,20}(X) + 3.51385 \cdot 10^{12} B_{13,20}(X) + 1.69765 \cdot 10^{12} B_{14,20}(X) - 6.43381 \cdot 10^{11} B_{15,20}(X) - 1.60376 \cdot 10^{12} B_{16,20}(X) - 1.08654 \cdot 10^{12} B_{17,20}(X) - 4.06339 \cdot 10^{10} B_{18,20}(X) + 6.75764 \cdot 10^{11} B_{19,20}(X) + 8.1419 \cdot 10^{11} B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.052673, 0.938623\}$$

Intersection intervals with the x axis:

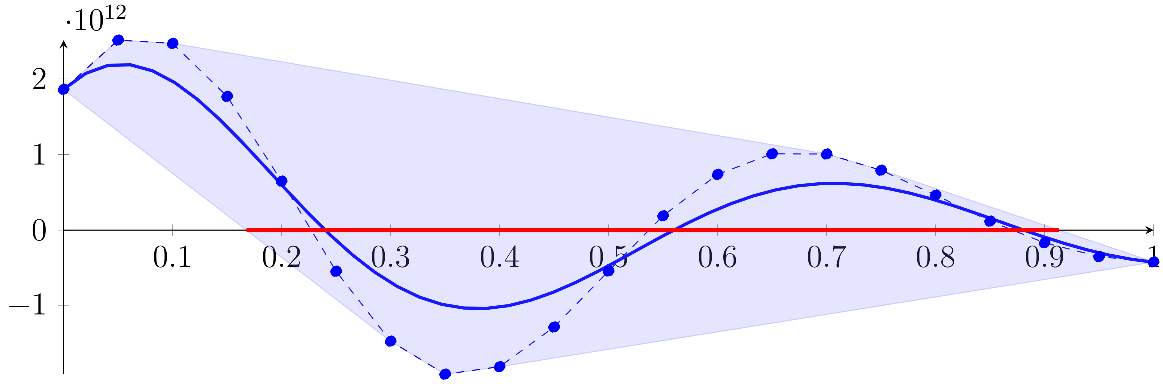
$$[0.052673, 0.938623]$$

Longest intersection interval: 0.88595
$\Longrightarrow$ Bisection: first half $[6.25, 9.375]$ und second half $[9.375, 12.5]$

1.36 Recursion Branch 1 1 2 1 on the First Half $[6.25, 9.375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 7.88861 \cdot 10^{9} X^{20} - 2.1457 \cdot 10^{11} X^{19} + 2.50366 \cdot 10^{12} X^{18} - 1.60464 \cdot 10^{13} X^{17} + 5.84963 \cdot 10^{13} X^{16} - 9.93687 \cdot 10^{13} X^{15} - 7.21032 \cdot 10^{13} X^{14} + 6.90728 \cdot 10^{14} X^{13} - 1.13799 \cdot 10^{15} X^{12} - 1.80856 \cdot 10^{14} X^{11} + 2.88511 \cdot 10^{15} X^{10} - 2.89183 \cdot 10^{15} X^{9} - 1.25813 \cdot 10^{15} X^{8} + 3.83707 \cdot 10^{15} X^{7} - 1.35117 \cdot 10^{15} X^{6} - 1.36068 \cdot 10^{15} X^{5} + 9.68965 \cdot 10^{14} X^{4} + 4.2096 \cdot 10^{13} X^{3} - 1.31886 \cdot 10^{14} X^{2} + 1.30114 \cdot 10^{13} X + 1.86285 \cdot 10^{12}$$
$$= 1.86285 \cdot 10^{12} B_{0,20}(X) + 2.51342 \cdot 10^{12} B_{1,20}(X) + 2.46985 \cdot 10^{12} B_{2,20}(X) + 1.76906 \cdot 10^{12} B_{3,20}(X) + 6.47986 \cdot 10^{11} B_{4,20}(X) - 5.44235 \cdot 10^{11} B_{5,20}(X) - 1.46885 \cdot 10^{12} B_{6,20}(X) - 1.90547 \cdot 10^{12} B_{7,20}(X) - 1.80595 \cdot 10^{12} B_{8,20}(X) - 1.28171 \cdot 10^{12} B_{9,20}(X) - 5.41242 \cdot 10^{11} B_{10,20}(X) + 1.90115 \cdot 10^{11} B_{11,20}(X) + 7.36986 \cdot 10^{11} B_{12,20}(X) + 1.00973 \cdot 10^{12} B_{13,20}(X) + 1.00677 \cdot 10^{12} B_{14,20}(X) + 7.92436 \cdot 10^{11} B_{15,20}(X) + 4.63782 \cdot 10^{11} B_{16,20}(X) + 1.1866 \cdot 10^{11} B_{17,20}(X) - 1.67068 \cdot 10^{11} B_{18,20}(X) - 3.50344 \cdot 10^{11} B_{19,20}(X) - 4.20945 \cdot 10^{11} B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.167739, 0.91327\}$$

Intersection intervals with the x axis:

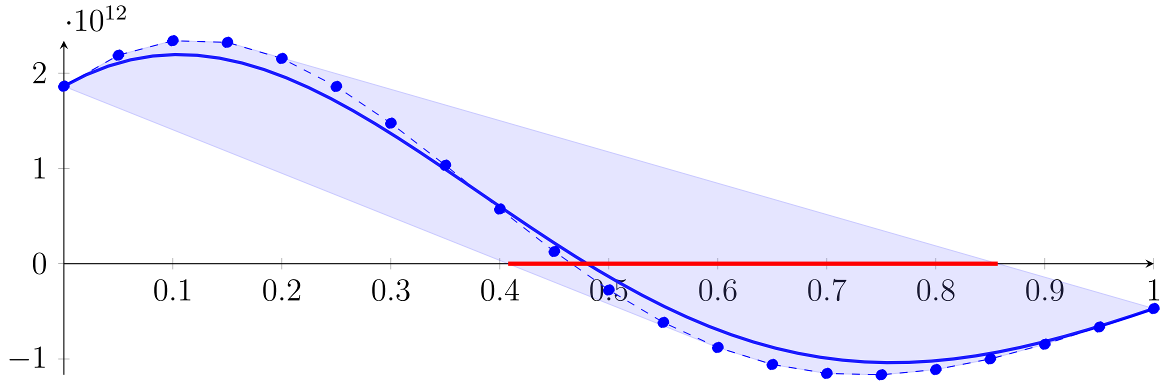
$$[0.167739, 0.91327]$$

Longest intersection interval: 0.745531
$\Longrightarrow$ Bisection: first half $[6.25, 7.8125]$ und second half $[7.8125, 9.375]$

1.37 Recursion Branch 1 1 2 1 1 on the First Half $[6.25, 7.8125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = {} & 7302.07X^{20} - 415766X^{19} + 9.52338 \cdot 10^{6} X^{18} - 1.22457 \cdot 10^{8} X^{17} + 8.92307 \cdot 10^{8} X^{16} - 3.03216 \\ & \cdot 10^{9} X^{15} - 4.40106 \cdot 10^{9} X^{14} + 8.43171 \cdot 10^{10} X^{13} - 2.77829 \cdot 10^{11} X^{12} - 8.83088 \cdot 10^{10} X^{11} + 2.81749 \\ & \cdot 10^{12} X^{10} - 5.64811 \cdot 10^{12} X^{9} - 4.91456 \cdot 10^{12} X^{8} + 2.99771 \cdot 10^{13} X^{7} - 2.11121 \cdot 10^{13} X^{6} - 4.25212 \\ & \cdot 10^{13} X^{5} + 6.05603 \cdot 10^{13} X^{4} + 5.262 \cdot 10^{12} X^{3} - 3.29716 \cdot 10^{13} X^{2} + 6.50568 \cdot 10^{12} X + 1.86285 \cdot 10^{12} \\ = {} & 1.86285 \cdot 10^{12} B_{0,20}(X) + 2.18813 \cdot 10^{12} B_{1,20}(X) + 2.33988 \cdot 10^{12} B_{2,20}(X) + 2.32271 \\ & \cdot 10^{12} B_{3,20}(X) + 2.15374 \cdot 10^{12} B_{4,20}(X) + 1.85984 \cdot 10^{12} B_{5,20}(X) + 1.47434 \cdot 10^{12} B_{6,20}(X) \\ & + 1.03362 \cdot 10^{12} B_{7,20}(X) + 5.7382 \cdot 10^{11} B_{8,20}(X) + 1.28041 \cdot 10^{11} B_{9,20}(X) - 2.75764 \\ & \cdot 10^{11} B_{10,20}(X) - 6.16213 \cdot 10^{11} B_{11,20}(X) - 8.79156 \cdot 10^{11} B_{12,20}(X) - 1.05766 \cdot 10^{12} B_{13,20}(X) \\ & - 1.15145 \cdot 10^{12} B_{14,20}(X) - 1.1659 \cdot 10^{12} B_{15,20}(X) - 1.11081 \cdot 10^{12} B_{16,20}(X) - 9.99056 \\ & \cdot 10^{11} B_{17,20}(X) - 8.45188 \cdot 10^{11} B_{18,20}(X) - 6.64233 \cdot 10^{11} B_{19,20}(X) - 4.70618 \cdot 10^{11} B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.407625, 0.856793\}$$

Intersection intervals with the x axis:

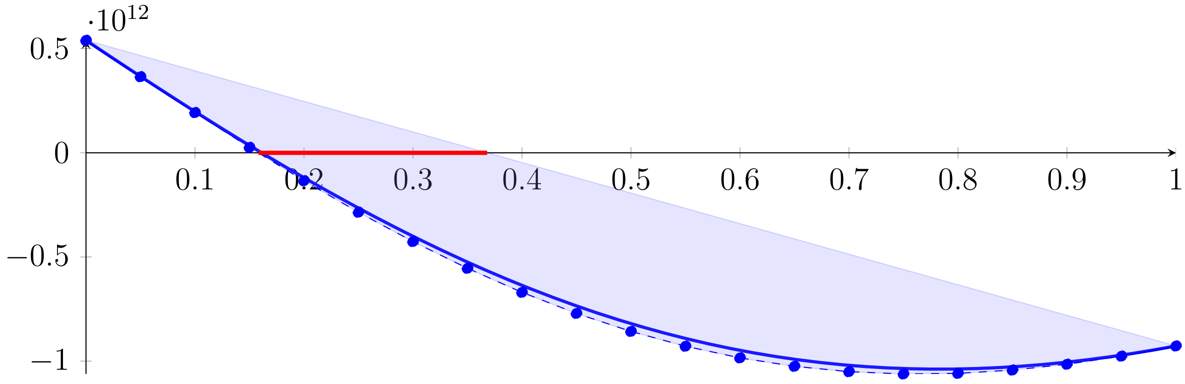
$$[0.407625, 0.856793]$$

Longest intersection interval: 0.449168
$\Longrightarrow$ Selective recursion: interval 1: $[6.88691, 7.58874]$,

1.38 Recursion Branch 1 1 2 1 1 1 in Interval 1: $[6.88691, 7.58874]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = {} & 756.997X^{20} - 5933.96X^{19} + 17488.3X^{18} - 107787X^{17} + 447115X^{16} - 319324X^{15} \\ & - 21440.5X^{14} + 1.00075\cdot 10^{6}X^{13} + 3.32296\cdot 10^{6}X^{12} - 8.10711\cdot 10^{7}X^{11} + 2.71652\cdot 10^{8}X^{10} \\ & + 1.78786\cdot 10^{9}X^{9} - 1.36195\cdot 10^{10}X^{8} + 1.50359\cdot 10^{9}X^{7} + 2.00212\cdot 10^{11}X^{6} - 3.72494\cdot 10^{11}X^{5} \\ & - 9.19659\cdot 10^{11}X^{4} + 2.63278\cdot 10^{12}X^{3} + 4.96735\cdot 10^{11}X^{2} - 3.49491\cdot 10^{12}X + 5.40127\cdot 10^{11} \\ = {} & 5.40127\cdot 10^{11}B_{0,20}(X) + 3.65382\cdot 10^{11}B_{1,20}(X) + 1.9325\cdot 10^{11}B_{2,20}(X) + 2.60428 \\ & \cdot 10^{10}B_{3,20}(X) - 1.34121\cdot 10^{11}B_{4,20}(X) - 2.85336\cdot 10^{11}B_{5,20}(X) - 4.25928\cdot 10^{11}B_{6,20}(X) \\ & - 5.54472\cdot 10^{11}B_{7,20}(X) - 6.69794\cdot 10^{11}B_{8,20}(X) - 7.70982\cdot 10^{11}B_{9,20}(X) - 8.57382 \\ & \cdot 10^{11}B_{10,20}(X) - 9.28588\cdot 10^{11}B_{11,20}(X) - 9.84442\cdot 10^{11}B_{12,20}(X) - 1.02501\cdot 10^{12}B_{13,20}(X) \\ & - 1.05059\cdot 10^{12}B_{14,20}(X) - 1.06166\cdot 10^{12}B_{15,20}(X) - 1.05887\cdot 10^{12}B_{16,20}(X) - 1.04304 \\ & \cdot 10^{12}B_{17,20}(X) - 1.01513\cdot 10^{12}B_{18,20}(X) - 9.7618\cdot 10^{11}B_{19,20}(X) - 9.27351\cdot 10^{11}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.15813, 0.368065\}$$

Intersection intervals with the x axis:

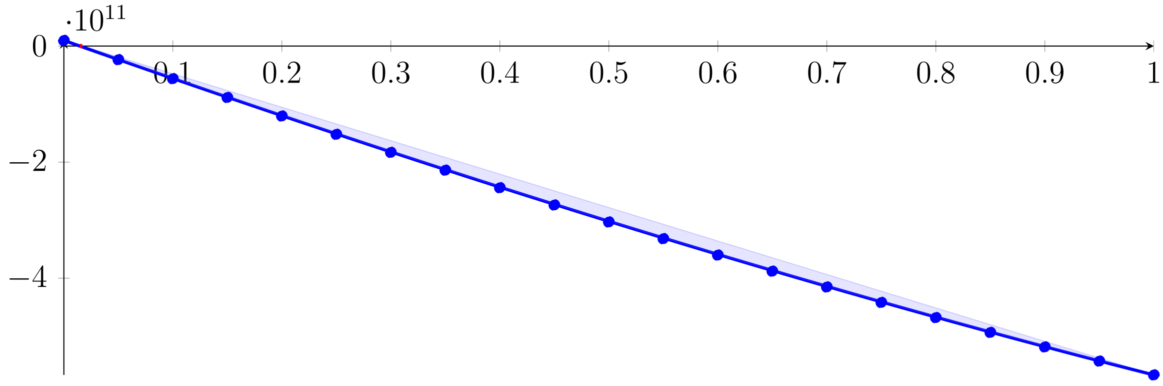
$$[0.15813, 0.368065]$$

Longest intersection interval: 0.209935
$\Longrightarrow$ Selective recursion: interval 1: $[6.99789, 7.14523]$,

1.39 Recursion Branch 1 1 2 1 1 1 1 in Interval 1: $[6.99789, 7.14523]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = {} & 263.295X^{20} - 1909.1X^{19} + 6637.3X^{18} - 33942.9X^{17} + 161695X^{16} - 116852X^{15} \\ & + 38960.3X^{14} + 14829.8X^{13} + 91507.3X^{12} + 387.634X^{11} + 22544.1X^{10} \\ & + 2186.77X^{9} - 40467.5X^{8} - 251676X^{7} + 1.65193\cdot 10^{7}X^{6} - 7.52822\cdot 10^{7}X^{5} \\ & - 2.21325\cdot 10^{9}X^{4} + 1.82617\cdot 10^{10}X^{3} + 7.029\cdot 10^{10}X^{2} - 6.62536\cdot 10^{11}X + 9.6987\cdot 10^{9} \\ = {} & 9.6987\cdot 10^{9}B_{0,20}(X) - 2.34281\cdot 10^{10}B_{1,20}(X) - 5.61849\cdot 10^{10}B_{2,20}(X) - 8.85558 \\ & \cdot 10^{10}B_{3,20}(X) - 1.20525\cdot 10^{11}B_{4,20}(X) - 1.52078\cdot 10^{11}B_{5,20}(X) - 1.83199\cdot 10^{11}B_{6,20}(X) \\ & - 2.13875\cdot 10^{11}B_{7,20}(X) - 2.44092\cdot 10^{11}B_{8,20}(X) - 2.73837\cdot 10^{11}B_{9,20}(X) - 3.03096 \\ & \cdot 10^{11}B_{10,20}(X) - 3.31858\cdot 10^{11}B_{11,20}(X) - 3.60112\cdot 10^{11}B_{12,20}(X) - 3.87844\cdot 10^{11}B_{13,20}(X) \\ & - 4.15046\cdot 10^{11}B_{14,20}(X) - 4.41706\cdot 10^{11}B_{15,20}(X) - 4.67815\cdot 10^{11}B_{16,20}(X) - 4.93363 \\ & \cdot 10^{11}B_{17,20}(X) - 5.18342\cdot 10^{11}B_{18,20}(X) - 5.42742\cdot 10^{11}B_{19,20}(X) - 5.66558\cdot 10^{11}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.0146388, 0.0168305\}$$

Intersection intervals with the x axis:

$$[0.0146388, 0.0168305]$$

Longest intersection interval: 0.00219177
$\Longrightarrow$ Selective recursion: interval 1: $[7.00005, 7.00037]$,

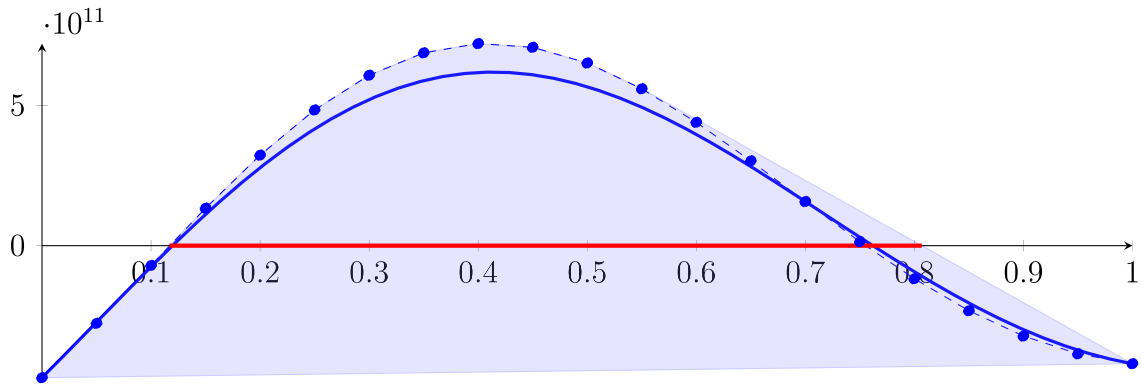
1.40 Recursion Branch 1 1 2 1 1 1 1 1 in Interval 1: $[7.00005, 7.00037]$

Found root in interval $[7.00005, 7.00037]$ at recursion depth 8!

1.41 Recursion Branch 1 1 2 1 2 on the Second Half $[7.8125, 9.375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = {} & 6877.98X^{20} - 256526X^{19} + 3.18676\cdot 10^{6}X^{18} - 1.18426\cdot 10^{7}X^{17} - 8.79174\cdot 10^{7}X^{16} + 9.23149 \\ & \cdot 10^{8}X^{15} - 1.26914\cdot 10^{9}X^{14} - 1.59203\cdot 10^{10}X^{13} + 6.26004\cdot 10^{10}X^{12} + 7.11942\cdot 10^{10}X^{11} - 7.3925 \\ & \cdot 10^{11}X^{10} + 5.09162\cdot 10^{11}X^{9} + 3.6295\cdot 10^{12}X^{8} - 5.56929\cdot 10^{12}X^{7} - 7.06545\cdot 10^{12}X^{6} + 1.64355 \\ & \cdot 10^{13}X^{5} + 2.9001\cdot 10^{12}X^{4} - 1.64458\cdot 10^{13}X^{3} + 2.40542\cdot 10^{12}X^{2} + 3.8723\cdot 10^{12}X - 4.70618\cdot 10^{11} \\ = {} & -4.70618\cdot 10^{11}B_{0,20}(X) - 2.77003\cdot 10^{11}B_{1,20}(X) - 7.07277\cdot 10^{10}B_{2,20}(X) + 1.33781 \\ & \cdot 10^{11}B_{3,20}(X) + 3.22697\cdot 10^{11}B_{4,20}(X) + 4.8385\cdot 10^{11}B_{5,20}(X) + 6.07608\cdot 10^{11}B_{6,20}(X) \\ & + 6.87499\cdot 10^{11}B_{7,20}(X) + 7.20537\cdot 10^{11}B_{8,20}(X) + 7.07242\cdot 10^{11}B_{9,20}(X) + 6.51366 \\ & \cdot 10^{11}B_{10,20}(X) + 5.59383\cdot 10^{11}B_{11,20}(X) + 4.398\cdot 10^{11}B_{12,20}(X) + 3.02359\cdot 10^{11}B_{13,20}(X) \\ & + 1.57223\cdot 10^{11}B_{14,20}(X) + 1.42063\cdot 10^{10}B_{15,20}(X) - 1.17894\cdot 10^{11}B_{16,20}(X) - 2.31815 \\ & \cdot 10^{11}B_{17,20}(X) - 3.22175\cdot 10^{11}B_{18,20}(X) - 3.85644\cdot 10^{11}B_{19,20}(X) - 4.20945\cdot 10^{11}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.116798, 0.806774\}$$

Intersection intervals with the x axis:

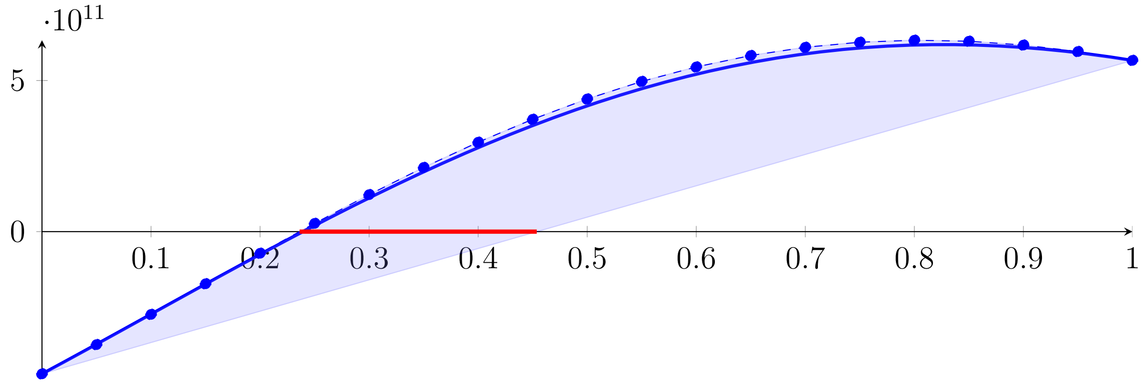
$$[0.116798, 0.806774]$$

Longest intersection interval: 0.689976
$\Longrightarrow$ Bisection: first half $[7.8125, 8.59375]$ und second half $[8.59375, 9.375]$

1.42 Recursion Branch 1 1 2 1 2 1 on the First Half $[7.8125, 8.59375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-354.8X^{20} + 3226.22X^{19} - 6973.15X^{18} + 50902.1X^{17} - 201307X^{16} + 165481X^{15} \\ &- 103434X^{14} - 1.94166\cdot10^{6}X^{13} + 1.52217\cdot10^{7}X^{12} + 3.47846\cdot10^{7}X^{11} - 7.21927\cdot10^{8}X^{10} \\ &+ 9.94463\cdot10^{8}X^{9} + 1.41777\cdot10^{10}X^{8} - 4.35101\cdot10^{10}X^{7} - 1.10398\cdot10^{11}X^{6} + 5.1361\cdot10^{11}X^{5} \\ &+ 1.81256\cdot10^{11}X^{4} - 2.05572\cdot10^{12}X^{3} + 6.01356\cdot10^{11}X^{2} + 1.93615\cdot10^{12}X - 4.70618\cdot10^{11} \\ = &-4.70618\cdot10^{11}B_{0,20}(X) - 3.7381\cdot10^{11}B_{1,20}(X) - 2.73838\cdot10^{11}B_{2,20}(X) - 1.72503 \\ &\cdot10^{11}B_{3,20}(X) - 7.15733\cdot10^{10}B_{4,20}(X) + 2.72575\cdot10^{10}B_{5,20}(X) + 1.22394\cdot10^{11}B_{6,20}(X) \\ &+ 2.12371\cdot10^{11}B_{7,20}(X) + 2.9587\cdot10^{11}B_{8,20}(X) + 3.71746\cdot10^{11}B_{9,20}(X) + 4.39036 \\ &\cdot10^{11}B_{10,20}(X) + 4.96971\cdot10^{11}B_{11,20}(X) + 5.44982\cdot10^{11}B_{12,20}(X) + 5.82699\cdot10^{11}B_{13,20}(X) \\ &+ 6.09952\cdot10^{11}B_{14,20}(X) + 6.2676\cdot10^{11}B_{15,20}(X) + 6.33323\cdot10^{11}B_{16,20}(X) + 6.30006 \\ &\cdot10^{11}B_{17,20}(X) + 6.1733\cdot10^{11}B_{18,20}(X) + 5.95947\cdot10^{11}B_{19,20}(X) + 5.66622\cdot10^{11}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.23621, 0.453721\}$$

Intersection intervals with the x axis:

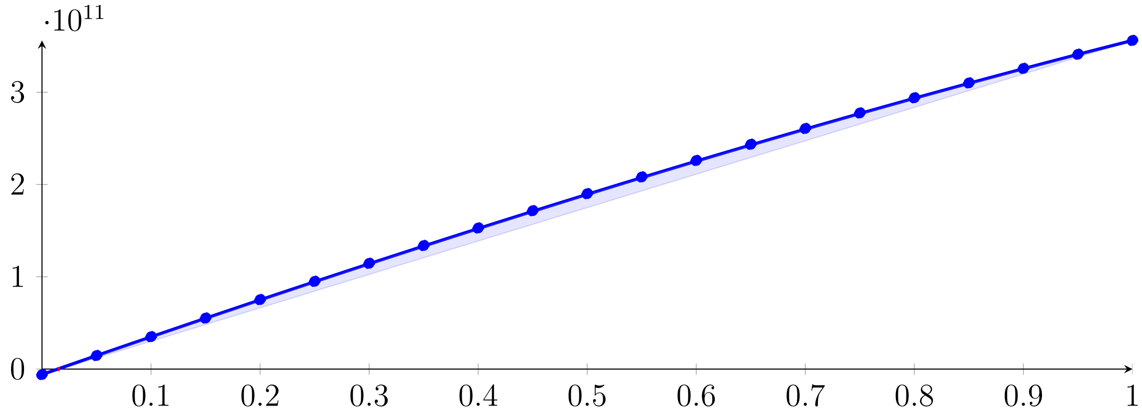
$$[0.23621, 0.453721]$$

Longest intersection interval: 0.217511
$\Longrightarrow$ Selective recursion: interval 1: $[7.99704, 8.16697]$,

1.43 Recursion Branch 1 1 2 1 2 1 1 in Interval 1: $[7.99704, 8.16697]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -163.525X^{20} + 1223.65X^{19} - 4252.75X^{18} + 21446.5X^{17} - 101186X^{16} + 74335.5X^{15} - 26048.4X^{14} - 7197.71X^{13} - 57020.6X^{12} - 1791.44X^{11} - 13320.5X^{10} - 1422.39X^{9} + 72789.9X^{8} - 364371X^{7} - 1.6896\cdot 10^{7}X^{6} + 1.54283\cdot 10^{8}X^{5} + 1.51882\cdot 10^{9}X^{4} - 1.67856\cdot 10^{10}X^{3} - 3.46686\cdot 10^{10}X^{2} + 4.11783\cdot 10^{11}X - 5.89908\cdot 10^{9}$$
$$= -5.89908\cdot 10^{9}B_{0,20}(X) + 1.46901\cdot 10^{10}B_{1,20}(X) + 3.50968\cdot 10^{10}B_{2,20}(X) + 5.53063\cdot 10^{10}B_{3,20}(X) + 7.53042\cdot 10^{10}B_{4,20}(X) + 9.50765\cdot 10^{10}B_{5,20}(X) + 1.14609\cdot 10^{11}B_{6,20}(X) + 1.33889\cdot 10^{11}B_{7,20}(X) + 1.52903\cdot 10^{11}B_{8,20}(X) + 1.71639\cdot 10^{11}B_{9,20}(X) + 1.90083\cdot 10^{11}B_{10,20}(X) + 2.08225\cdot 10^{11}B_{11,20}(X) + 2.26052\cdot 10^{11}B_{12,20}(X) + 2.43553\cdot 10^{11}B_{13,20}(X) + 2.60718\cdot 10^{11}B_{14,20}(X) + 2.77536\cdot 10^{11}B_{15,20}(X) + 2.93997\cdot 10^{11}B_{16,20}(X) + 3.10091\cdot 10^{11}B_{17,20}(X) + 3.2581\cdot 10^{11}B_{18,20}(X) + 3.41144\cdot 10^{11}B_{19,20}(X) + 3.56086\cdot 10^{11}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.0143257, 0.0162965\}$$

Intersection intervals with the x axis:

$$[0.0143257, 0.0162965]$$

Longest intersection interval: 0.00197078

$\Longrightarrow$ Selective recursion: interval 1: $[7.99947, 7.99981]$,

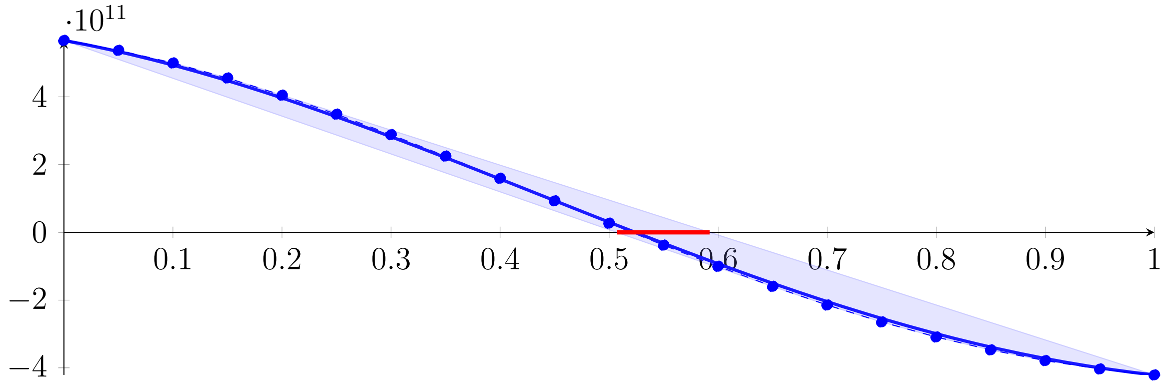
1.44 Recursion Branch 1 1 2 1 2 1 1 1 in Interval 1: $[7.99947, 7.99981]$

Found root in interval $[7.99947, 7.99981]$ at recursion depth 8!

1.45 Recursion Branch 1 1 2 1 2 2 on the Second Half $[8.59375, 9.375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -93.1062X^{20} - 758.462X^{19} - 5807.01X^{18} + 2574.74X^{17} - 87738.7X^{16} + 87645.3X^{15} + 103874X^{14} - 974015X^{13} - 7.14335\cdot 10^{6}X^{12} + 7.01872\cdot 10^{7}X^{11} + 1.08381\cdot 10^{8}X^{10} - 2.37106\cdot 10^{9}X^{9} + 1.37104\cdot 10^{9}X^{8} + 3.92216\cdot 10^{10}X^{7} - 5.93165\cdot 10^{10}X^{6} - 2.99569\cdot 10^{11}X^{5} + 5.54235\cdot 10^{11}X^{4} + 8.73982\cdot 10^{11}X^{3} - 1.50879\cdot 10^{12}X^{2} - 5.86495\cdot 10^{11}X + 5.66622\cdot 10^{11}$$
$$= 5.66622\cdot 10^{11}B_{0,20}(X) + 5.37297\cdot 10^{11}B_{1,20}(X) + 5.00031\cdot 10^{11}B_{2,20}(X) + 4.55591\cdot 10^{11}B_{3,20}(X) + 4.04858\cdot 10^{11}B_{4,20}(X) + 3.48807\cdot 10^{11}B_{5,20}(X) + 2.88489\cdot 10^{11}B_{6,20}(X) + 2.25007\cdot 10^{11}B_{7,20}(X) + 1.59494\cdot 10^{11}B_{8,20}(X) + 9.30894\cdot 10^{10}B_{9,20}(X) + 2.69192\cdot 10^{10}B_{10,20}(X) - 3.79257\cdot 10^{10}B_{11,20}(X) - 1.00409\cdot 10^{11}B_{12,20}(X) - 1.59568\cdot 10^{11}B_{13,20}(X) - 2.14527\cdot 10^{11}B_{14,20}(X) - 2.64511\cdot 10^{11}B_{15,20}(X) - 3.08858\cdot 10^{11}B_{16,20}(X) - 3.47027\cdot 10^{11}B_{17,20}(X) - 3.78602\cdot 10^{11}B_{18,20}(X) - 4.03295\cdot 10^{11}B_{19,20}(X) - 4.20945\cdot 10^{11}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.507173, 0.592208\}$$

Intersection intervals with the x axis:

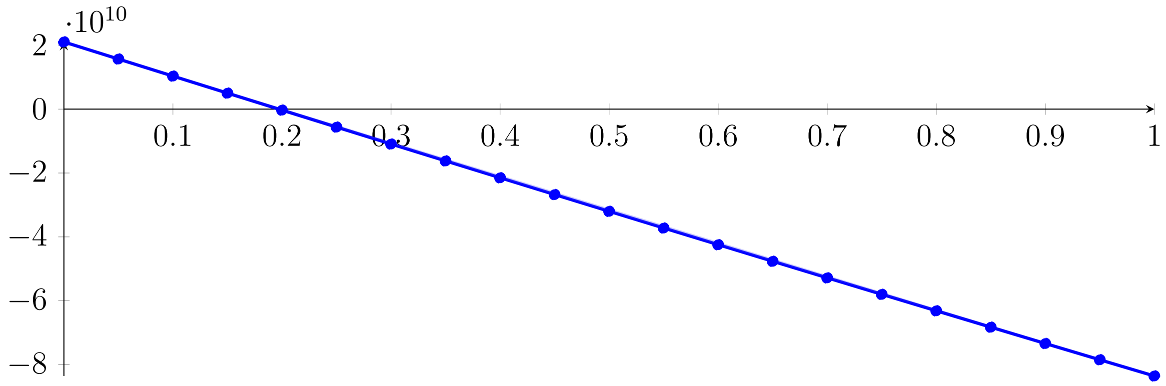
$$[0.507173, 0.592208]$$

Longest intersection interval: 0.0850344
$\Longrightarrow$ Selective recursion: interval 1: $[8.98998, 9.05641]$,

1.46 Recursion Branch 1 1 2 1 2 2 1 in Interval 1: $[8.98998, 9.05641]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = {} & 25.3484X^{20} - 228.663X^{19} + 550.98X^{18} - 3743.19X^{17} + 14954.7X^{16} - 10419X^{15} \\ & + 2592.54X^{14} + 386.204X^{13} + 5654X^{12} - 1146.88X^{11} + 821.783X^{10} \\ & - 234.502X^{9} + 18.2604X^{8} + 828.594X^{7} + 25059.2X^{6} - 1.22952 \cdot 10^{6}X^{5} - 1.35061 \\ & \cdot 10^{7}X^{4} + 7.15216 \cdot 10^{8}X^{3} + 1.83858 \cdot 10^{9}X^{2} - 1.07114 \cdot 10^{11}X + 2.10331 \cdot 10^{10} \\ = {} & 2.10331 \cdot 10^{10}B_{0,20}(X) + 1.56774 \cdot 10^{10}B_{1,20}(X) + 1.03314 \cdot 10^{10}B_{2,20}(X) + 4.99564 \\ & \cdot 10^{9}B_{3,20}(X) - 3.29179 \cdot 10^{8}B_{4,20}(X) - 5.64244 \cdot 10^{9}B_{5,20}(X) - 1.09435 \cdot 10^{10}B_{6,20}(X) \\ & - 1.62318 \cdot 10^{10}B_{7,20}(X) - 2.15068 \cdot 10^{10}B_{8,20}(X) - 2.67677 \cdot 10^{10}B_{9,20}(X) - 3.2014 \\ & \cdot 10^{10}B_{10,20}(X) - 3.7245 \cdot 10^{10}B_{11,20}(X) - 4.24603 \cdot 10^{10}B_{12,20}(X) - 4.76591 \cdot 10^{10}B_{13,20}(X) \\ & - 5.2841 \cdot 10^{10}B_{14,20}(X) - 5.80052 \cdot 10^{10}B_{15,20}(X) - 6.31513 \cdot 10^{10}B_{16,20}(X) - 6.82786 \\ & \cdot 10^{10}B_{17,20}(X) - 7.33866 \cdot 10^{10}B_{18,20}(X) - 7.84747 \cdot 10^{10}B_{19,20}(X) - 8.35422 \cdot 10^{10}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.196909, 0.201129\}$$

Intersection intervals with the x axis:

$$[0.196909, 0.201129]$$

Longest intersection interval: 0.00422004
$\Longrightarrow$ Selective recursion: interval 1: $[9.00306, 9.00334]$,

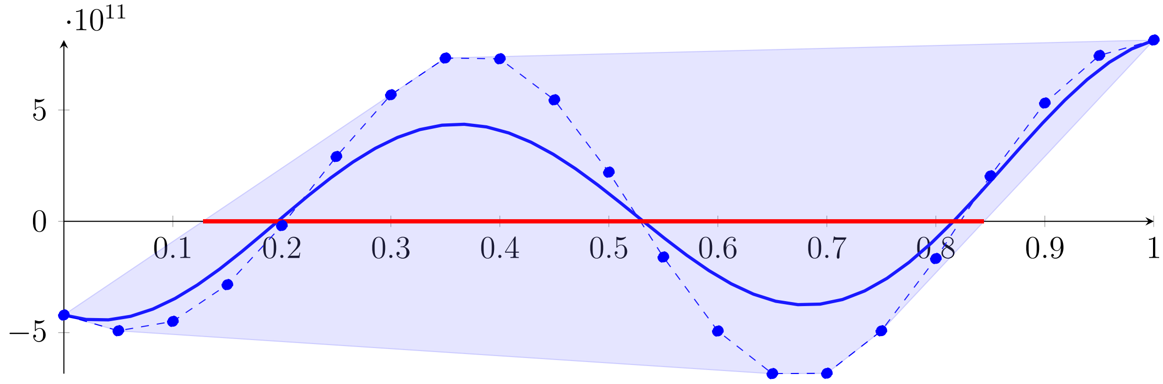
1.47 Recursion Branch 1 1 2 1 2 2 1 1 in Interval 1: $[9.00306, 9.00334]$

Found root in interval $[9.00306, 9.00334]$ at recursion depth 8!

1.48 Recursion Branch 1 1 2 2 on the Second Half $[9.375, 12.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = {} & 7.88861{\cdot}10^{9}X^{20} - 5.6798{\cdot}10^{10}X^{19} - 7.43423{\cdot}10^{10}X^{18} + 1.32089{\cdot}10^{12}X^{17} - 9.3169{\cdot}10^{11}X^{16} - 1.21266 \\ & {\cdot}10^{13}X^{15} + 1.72866{\cdot}10^{13}X^{14} + 5.61608{\cdot}10^{13}X^{13} - 1.04782{\cdot}10^{14}X^{12} - 1.38659{\cdot}10^{14}X^{11} + 3.15838 \\ & {\cdot}10^{14}X^{10} + 1.75102{\cdot}10^{14}X^{9} - 5.05882{\cdot}10^{14}X^{8} - 9.20246{\cdot}10^{13}X^{7} + 4.17973{\cdot}10^{14}X^{6} - 4.84112 \\ & {\cdot}10^{11}X^{5} - 1.59085{\cdot}10^{14}X^{4} + 1.16549{\cdot}10^{13}X^{3} + 2.14084{\cdot}10^{13}X^{2} - 1.41201{\cdot}10^{12}X - 4.20945{\cdot}10^{11} \\ = {} & -4.20945{\cdot}10^{11}B_{0,20}(X) - 4.91545{\cdot}10^{11}B_{1,20}(X) - 4.4947{\cdot}10^{11}B_{2,20}(X) - 2.84495 \\ & {\cdot}10^{11}B_{3,20}(X) - 1.92322{\cdot}10^{10}B_{4,20}(X) + 2.90841{\cdot}10^{11}B_{5,20}(X) + 5.68134{\cdot}10^{11}B_{6,20}(X) \\ & + 7.3329{\cdot}10^{11}B_{7,20}(X) + 7.29931{\cdot}10^{11}B_{8,20}(X) + 5.45616{\cdot}10^{11}B_{9,20}(X) + 2.20619 \\ & {\cdot}10^{11}B_{10,20}(X) - 1.60453{\cdot}10^{11}B_{11,20}(X) - 4.92917{\cdot}10^{11}B_{12,20}(X) - 6.84241{\cdot}10^{11}B_{13,20}(X) \\ & - 6.82665{\cdot}10^{11}B_{14,20}(X) - 4.91903{\cdot}10^{11}B_{15,20}(X) - 1.67279{\cdot}10^{11}B_{16,20}(X) + 2.0413 \\ & {\cdot}10^{11}B_{17,20}(X) + 5.31271{\cdot}10^{11}B_{18,20}(X) + 7.44977{\cdot}10^{11}B_{19,20}(X) + 8.1419{\cdot}10^{11}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.127644, 0.844155\}$$

Intersection intervals with the x axis:

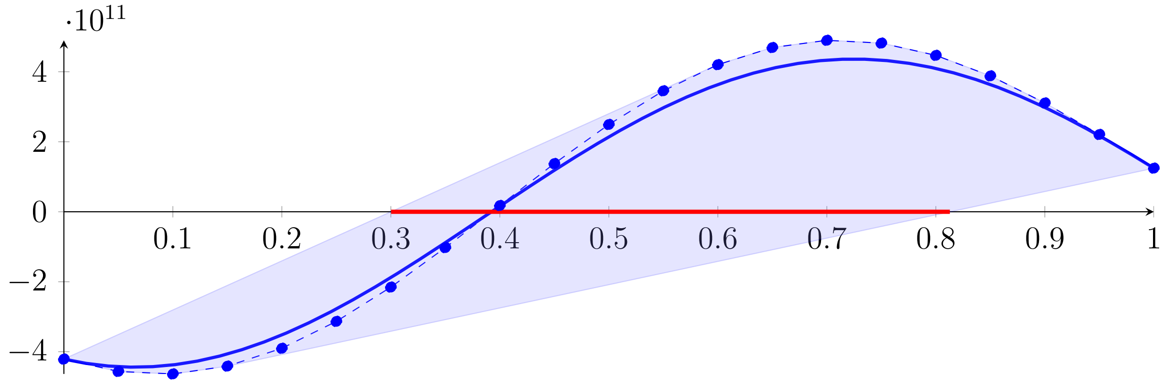
$$[0.127644, 0.844155]$$

Longest intersection interval: 0.716512
$\Longrightarrow$ Bisection: first half $[9.375, 10.9375]$ und second half $[10.9375, 12.5]$

1.49 Recursion Branch 1 1 2 2 1 on the First Half $[9.375, 10.9375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = {} & 7412.69X^{20} - 105681X^{19} - 281108X^{18} + 1.01062{\cdot}10^{7}X^{17} - 1.42567{\cdot}10^{7}X^{16} - 3.70075{\cdot}10^{8}X^{15} \\ & + 1.05512{\cdot}10^{9}X^{14} + 6.8556{\cdot}10^{9}X^{13} - 2.55814{\cdot}10^{10}X^{12} - 6.77046{\cdot}10^{10}X^{11} + 3.08436{\cdot}10^{11}X^{10} \\ & + 3.41997{\cdot}10^{11}X^{9} - 1.9761{\cdot}10^{12}X^{8} - 7.18943{\cdot}10^{11}X^{7} + 6.53083{\cdot}10^{12}X^{6} - 1.51285{\cdot}10^{10}X^{5} \\ & - 9.94282{\cdot}10^{12}X^{4} + 1.45686{\cdot}10^{12}X^{3} + 5.3521{\cdot}10^{12}X^{2} - 7.06004{\cdot}10^{11}X - 4.20945{\cdot}10^{11} \\ = {} & -4.20945{\cdot}10^{11}B_{0,20}(X) - 4.56245{\cdot}10^{11}B_{1,20}(X) - 4.63376{\cdot}10^{11}B_{2,20}(X) - 4.4106 \\ & {\cdot}10^{11}B_{3,20}(X) - 3.90072{\cdot}10^{11}B_{4,20}(X) - 3.13239{\cdot}10^{11}B_{5,20}(X) - 2.15273{\cdot}10^{11}B_{6,20}(X) \\ & - 1.02447{\cdot}10^{11}B_{7,20}(X) + 1.78698{\cdot}10^{10}B_{8,20}(X) + 1.37766{\cdot}10^{11}B_{9,20}(X) + 2.49392 \\ & {\cdot}10^{11}B_{10,20}(X) + 3.45561{\cdot}10^{11}B_{11,20}(X) + 4.2028{\cdot}10^{11}B_{12,20}(X) + 4.69189{\cdot}10^{11}B_{13,20}(X) \\ & + 4.89846{\cdot}10^{11}B_{14,20}(X) + 4.81857{\cdot}10^{11}B_{15,20}(X) + 4.46838{\cdot}10^{11}B_{16,20}(X) + 3.88213 \\ & {\cdot}10^{11}B_{17,20}(X) + 3.10886{\cdot}10^{11}B_{18,20}(X) + 2.20816{\cdot}10^{11}B_{19,20}(X) + 1.24532{\cdot}10^{11}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.300237, 0.812848\}$$

Intersection intervals with the x axis:

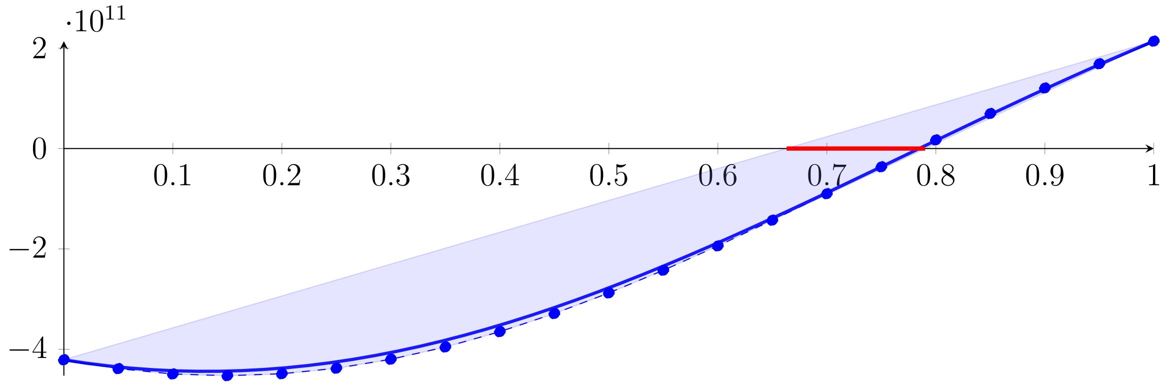
$$[0.300237, 0.812848]$$

Longest intersection interval: 0.512611
$\Longrightarrow$ Bisection: first half $[9.375, 10.1562]$ und second half $[10.1562, 10.9375]$

1.50 Recursion Branch 1 1 2 2 1 1 on the First Half $[9.375, 10.1562]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = {} & 322.224X^{20} - 905.543X^{19} + 11472.3X^{18} - 34069.5X^{17} + 223456X^{16} - 196568X^{15} \\ & + 139400X^{14} + 879760X^{13} - 6.03736\cdot 10^{6}X^{12} - 3.30226\cdot 10^{7}X^{11} + 3.0127\cdot 10^{8}X^{10} \\ & + 6.67972\cdot 10^{8}X^{9} - 7.71914\cdot 10^{9}X^{8} - 5.61674\cdot 10^{9}X^{7} + 1.02044\cdot 10^{11}X^{6} - 4.72766\cdot 10^{8}X^{5} \\ & - 6.21426\cdot 10^{11}X^{4} + 1.82108\cdot 10^{11}X^{3} + 1.33802\cdot 10^{12}X^{2} - 3.53002\cdot 10^{11}X - 4.20945\cdot 10^{11} \\ = {} & -4.20945\cdot 10^{11}B_{0,20}(X) - 4.38595\cdot 10^{11}B_{1,20}(X) - 4.49203\cdot 10^{11}B_{2,20}(X) - 4.52609 \\ & \cdot 10^{11}B_{3,20}(X) - 4.48781\cdot 10^{11}B_{4,20}(X) - 4.37817\cdot 10^{11}B_{5,20}(X) - 4.19938\cdot 10^{11}B_{6,20}(X) \\ & - 3.95489\cdot 10^{11}B_{7,20}(X) - 3.64924\cdot 10^{11}B_{8,20}(X) - 3.28803\cdot 10^{11}B_{9,20}(X) - 2.87777 \\ & \cdot 10^{11}B_{10,20}(X) - 2.42573\cdot 10^{11}B_{11,20}(X) - 1.93982\cdot 10^{11}B_{12,20}(X) - 1.42843\cdot 10^{11}B_{13,20}(X) \\ & - 9.00237\cdot 10^{10}B_{14,20}(X) - 3.64057\cdot 10^{10}B_{15,20}(X) + 1.71324\cdot 10^{10}B_{16,20}(X) + 6.9732 \\ & \cdot 10^{10}B_{17,20}(X) + 1.2057\cdot 10^{11}B_{18,20}(X) + 1.68871\cdot 10^{11}B_{19,20}(X) + 2.13926\cdot 10^{11}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.66304, 0.790222\}$$

Intersection intervals with the x axis:

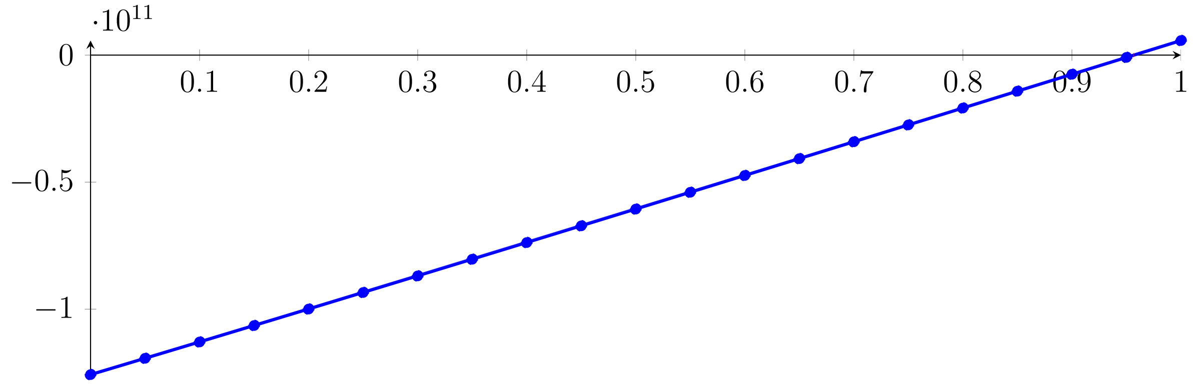
$$[0.66304, 0.790222]$$

Longest intersection interval: 0.127181
$\Longrightarrow$ Selective recursion: interval 1: $[9.893, 9.99236]$,

1.51 Recursion Branch 1 1 2 2 1 1 1 in Interval 1: $[9.893, 9.99236]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 65.8318X^{20} - 226.949X^{19} + 2405.66X^{18} - 6860.89X^{17} + 45977.9X^{16} - 36697.3X^{15} + 17582.6X^{14} + 6130.77X^{13} + 44103.7X^{12} + 6312.34X^{11} + 13760.3X^{10} + 1596.67X^{9} + 217.203X^{8} - 15104X^{7} + 30452.2X^{6} + 8.38818\cdot10^{6}X^{5} - 2.5213\cdot10^{7}X^{4} - 1.97492\cdot10^{9}X^{3} + 5.23883\cdot10^{9}X^{2} + 1.28253\cdot10^{11}X - 1.25767\cdot10^{11}$$
$$= -1.25767\cdot10^{11}B_{0,20}(X) - 1.19355\cdot10^{11}B_{1,20}(X) - 1.12914\cdot10^{11}B_{2,20}(X) - 1.06448\cdot10^{11}B_{3,20}(X) - 9.99582\cdot10^{10}B_{4,20}(X) - 9.34457\cdot10^{10}B_{5,20}(X) - 8.69125\cdot10^{10}B_{6,20}(X) - 8.03605\cdot10^{10}B_{7,20}(X) - 7.37914\cdot10^{10}B_{8,20}(X) - 6.7207\cdot10^{10}B_{9,20}(X) - 6.06089\cdot10^{10}B_{10,20}(X) - 5.3999\cdot10^{10}B_{11,20}(X) - 4.7379\cdot10^{10}B_{12,20}(X) - 4.07507\cdot10^{10}B_{13,20}(X) - 3.41158\cdot10^{10}B_{14,20}(X) - 2.74762\cdot10^{10}B_{15,20}(X) - 2.08335\cdot10^{10}B_{16,20}(X) - 1.41895\cdot10^{10}B_{17,20}(X) - 7.54595\cdot10^{9}B_{18,20}(X) - 9.04638\cdot10^{8}B_{19,20}(X) + 5.73271\cdot10^{9}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.956405, 0.956844\}$$

Intersection intervals with the x axis:

$$[0.956405, 0.956844]$$

Longest intersection interval: 0.000438317
$\Longrightarrow$ Selective recursion: interval 1: $[9.98803, 9.98807]$,

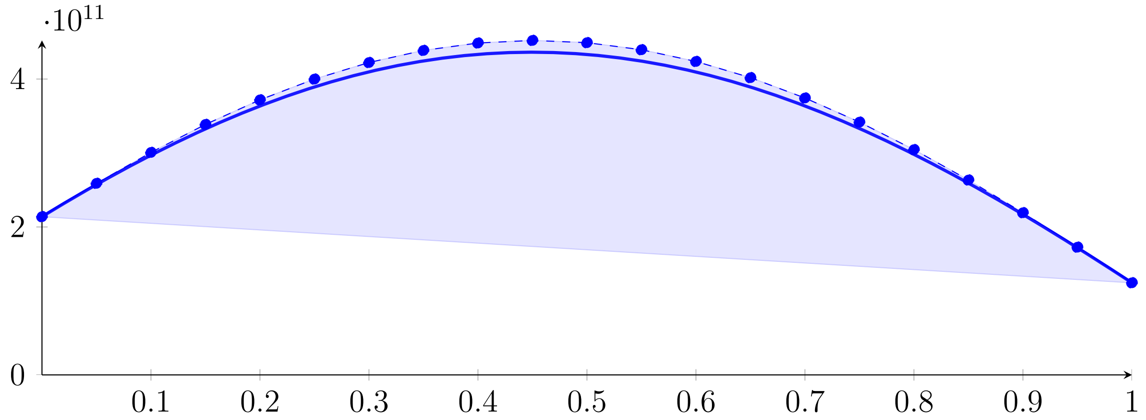
1.52 Recursion Branch 1 1 2 2 1 1 1 1 in Interval 1: $[9.98803, 9.98807]$

Found root in interval $[9.98803, 9.98807]$ at recursion depth 8!

1.53 Recursion Branch 1 1 2 2 1 2 on the Second Half $[10.1562, 10.9375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -437.747X^{20} + 2128.25X^{19} - 12933.7X^{18} + 52398.4X^{17} - 280438X^{16} + 205362X^{15} - 172248X^{14} + 564111X^{13} + 5.19508\cdot10^{6}X^{12} - 3.477\cdot10^{7}X^{11} - 2.05116\cdot10^{8}X^{10} + 1.15785\cdot10^{9}X^{9} + 4.49736\cdot10^{9}X^{8} - 2.14509\cdot10^{10}X^{7} - 5.35186\cdot10^{10}X^{6} + 2.02588\cdot10^{11}X^{5} + 3.0401\cdot10^{11}X^{4} - 8.10616\cdot10^{11}X^{3} - 6.16921\cdot10^{11}X^{2} + 9.01093\cdot10^{11}X + 2.13926\cdot10^{11}$$
$$= 2.13926\cdot10^{11}B_{0,20}(X) + 2.5898\cdot10^{11}B_{1,20}(X) + 3.00788\cdot10^{11}B_{2,20}(X) + 3.38638\cdot10^{11}B_{3,20}(X) + 3.71881\cdot10^{11}B_{4,20}(X) + 3.99946\cdot10^{11}B_{5,20}(X) + 4.22346\cdot10^{11}B_{6,20}(X) + 4.38696\cdot10^{11}B_{7,20}(X) + 4.48712\cdot10^{11}B_{8,20}(X) + 4.52225\cdot10^{11}B_{9,20}(X) + 4.4918\cdot10^{11}B_{10,20}(X) + 4.39639\cdot10^{11}B_{11,20}(X) + 4.23781\cdot10^{11}B_{12,20}(X) + 4.01896\cdot10^{11}B_{13,20}(X) + 3.74383\cdot10^{11}B_{14,20}(X) + 3.41739\cdot10^{11}B_{15,20}(X) + 3.0455\cdot10^{11}B_{16,20}(X) + 2.63481\cdot10^{11}B_{17,20}(X) + 2.19262\cdot10^{11}B_{18,20}(X) + 1.72674\cdot10^{11}B_{19,20}(X) + 1.24532\cdot10^{11}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

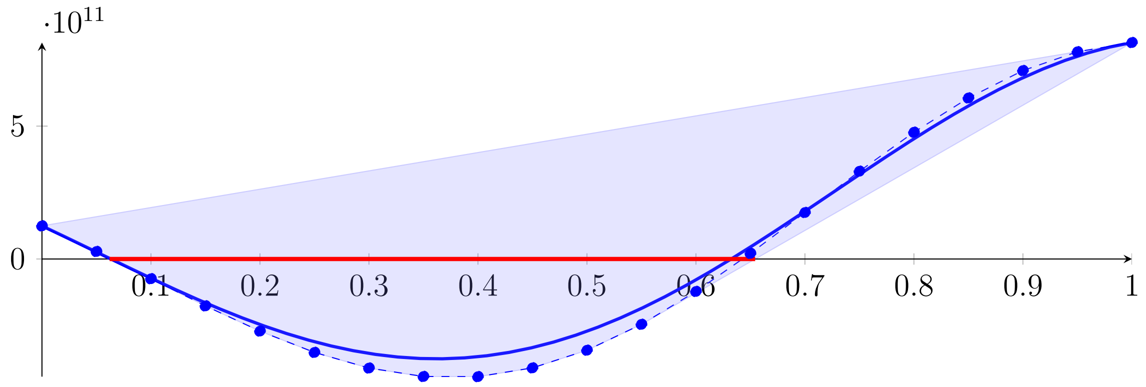
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

1.54 Recursion Branch 1 1 2 2 2 on the Second Half $[10.9375, 12.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = {} & 7874.96X^{20} + 41504.1X^{19} - 902121X^{18} - 5.01703\cdot 10^{6}X^{17} + 4.54393\cdot 10^{7}X^{16} + 2.38133\cdot 10^{8}X^{15} \\ & - 1.18503\cdot 10^{9}X^{14} - 5.9933\cdot 10^{9}X^{13} + 1.78815\cdot 10^{10}X^{12} + 8.56274\cdot 10^{10}X^{11} - 1.58071\cdot 10^{11}X^{10} \\ & - 7.03711\cdot 10^{11}X^{9} + 7.99866\cdot 10^{11}X^{8} + 3.21659\cdot 10^{12}X^{7} - 2.16687\cdot 10^{12}X^{6} - 7.4915\cdot 10^{12}X^{5} \\ & + 2.76126\cdot 10^{12}X^{4} + 7.44201\cdot 10^{12}X^{3} - 1.18084\cdot 10^{12}X^{2} - 1.92569\cdot 10^{12}X + 1.24532\cdot 10^{11} \\ = {} & 1.24532\cdot 10^{11}B_{0,20}(X) + 2.82469\cdot 10^{10}B_{1,20}(X) - 7.42527\cdot 10^{10}B_{2,20}(X) - 1.76439 \\ & \cdot 10^{11}B_{3,20}(X) - 2.71214\cdot 10^{11}B_{4,20}(X) - 3.51394\cdot 10^{11}B_{5,20}(X) - 4.10245\cdot 10^{11}B_{6,20}(X) \\ & - 4.42042\cdot 10^{11}B_{7,20}(X) - 4.42583\cdot 10^{11}B_{8,20}(X) - 4.09631\cdot 10^{11}B_{9,20}(X) - 3.43218 \\ & \cdot 10^{11}B_{10,20}(X) - 2.45778\cdot 10^{11}B_{11,20}(X) - 1.22096\cdot 10^{11}B_{12,20}(X) + 2.09582\cdot 10^{10}B_{13,20}(X) \\ & + 1.74882\cdot 10^{11}B_{14,20}(X) + 3.3015\cdot 10^{11}B_{15,20}(X) + 4.76935\cdot 10^{11}B_{16,20}(X) + 6.05883 \\ & \cdot 10^{11}B_{17,20}(X) + 7.08854\cdot 10^{11}B_{18,20}(X) + 7.79584\cdot 10^{11}B_{19,20}(X) + 8.1419\cdot 10^{11}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.062065, 0.654343\}$$

Intersection intervals with the x axis:

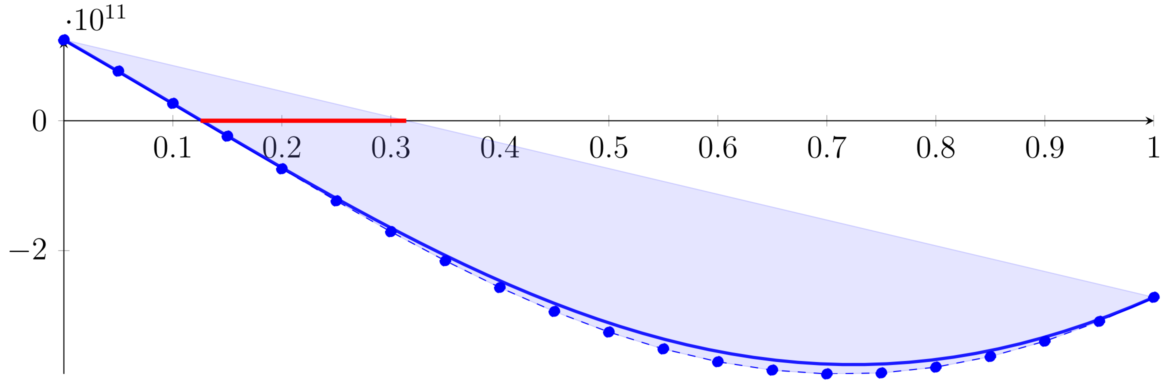
$$[0.062065, 0.654343]$$

Longest intersection interval: 0.592278
$\Longrightarrow$ Bisection: first half $[10.9375, 11.7188]$ und second half $[11.7188, 12.5]$

1.55 Recursion Branch 1 1 2 2 2 1 on the First Half [10.9375, 11.7188]

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = {} & 286.905X^{20} - 2051.04X^{19} + 6538.73X^{18} - 37870.8X^{17} + 168281X^{16} - 109479X^{15} \\
& - 37319.9X^{14} - 722808X^{13} + 4.44854\cdot 10^{6}X^{12} + 4.18049\cdot 10^{7}X^{11} - 1.54349\cdot 10^{8}X^{10} \\
& - 1.37444\cdot 10^{9}X^{9} + 3.12448\cdot 10^{9}X^{8} + 2.51296\cdot 10^{10}X^{7} - 3.38573\cdot 10^{10}X^{6} - 2.34109\cdot 10^{11}X^{5} \\
& + 1.72579\cdot 10^{11}X^{4} + 9.30252\cdot 10^{11}X^{3} - 2.9521\cdot 10^{11}X^{2} - 9.62846\cdot 10^{11}X + 1.24532\cdot 10^{11} \\
= {} & 1.24532\cdot 10^{11}B_{0,20}(X) + 7.63892\cdot 10^{10}B_{1,20}(X) + 2.66932\cdot 10^{10}B_{2,20}(X) - 2.37406 \\
& \cdot 10^{10}B_{3,20}(X) - 7.40605\cdot 10^{10}B_{4,20}(X) - 1.23394\cdot 10^{11}B_{5,20}(X) - 1.70865\cdot 10^{11}B_{6,20}(X) \\
& - 2.15609\cdot 10^{11}B_{7,20}(X) - 2.56789\cdot 10^{11}B_{8,20}(X) - 2.93615\cdot 10^{11}B_{9,20}(X) - 3.25357 \\
& \cdot 10^{11}B_{10,20}(X) - 3.51362\cdot 10^{11}B_{11,20}(X) - 3.71068\cdot 10^{11}B_{12,20}(X) - 3.84013\cdot 10^{11}B_{13,20}(X) \\
& - 3.89851\cdot 10^{11}B_{14,20}(X) - 3.88354\cdot 10^{11}B_{15,20}(X) - 3.79423\cdot 10^{11}B_{16,20}(X) - 3.63087 \\
& \cdot 10^{11}B_{17,20}(X) - 3.39506\cdot 10^{11}B_{18,20}(X) - 3.0897\cdot 10^{11}B_{19,20}(X) - 2.7189\cdot 10^{11}B_{20,20}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.125414, 0.314139\}$$

Intersection intervals with the x axis:

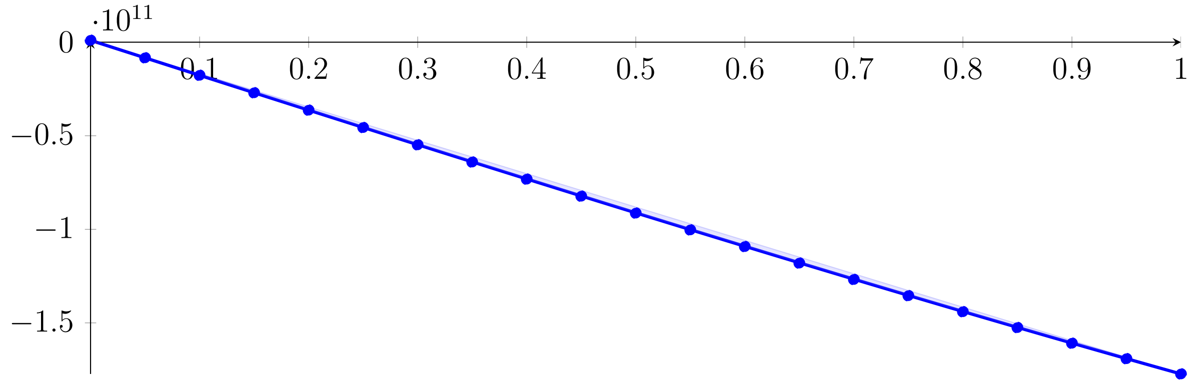
$$[0.125414, 0.314139]$$

Longest intersection interval: 0.188725
$\Longrightarrow$ Selective recursion: interval 1: $[11.0355, 11.1829]$,

1.56 Recursion Branch 1 1 2 2 2 1 1 in Interval 1: [11.0355, 11.1829]

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = {} & 78.8351X^{20} - 580.853X^{19} + 2004.97X^{18} - 10113.4X^{17} + 48525.5X^{16} - 36185.5X^{15} \\
& + 14125.5X^{14} + 1726.24X^{13} + 26419.2X^{12} + 621.175X^{11} + 6846.67X^{10} \\
& - 393.72X^{9} + 2534.59X^{8} + 234099X^{7} - 481407X^{6} - 6.00887\cdot 10^{7}X^{5} + 2.48273 \\
& \cdot 10^{7}X^{4} + 6.57993\cdot 10^{9}X^{3} + 2.36323\cdot 10^{9}X^{2} - 1.87202\cdot 10^{11}X + 1.00375\cdot 10^{9} \\
= {} & 1.00375\cdot 10^{9}B_{0,20}(X) - 8.35635\cdot 10^{9}B_{1,20}(X) - 1.7704\cdot 10^{10}B_{2,20}(X) - 2.70335 \\
& \cdot 10^{10}B_{3,20}(X) - 3.6339\cdot 10^{10}B_{4,20}(X) - 4.56147\cdot 10^{10}B_{5,20}(X) - 5.48548\cdot 10^{10}B_{6,20}(X) \\
& - 6.40537\cdot 10^{10}B_{7,20}(X) - 7.32055\cdot 10^{10}B_{8,20}(X) - 8.23044\cdot 10^{10}B_{9,20}(X) - 9.13449 \\
& \cdot 10^{10}B_{10,20}(X) - 1.00321\cdot 10^{11}B_{11,20}(X) - 1.09227\cdot 10^{11}B_{12,20}(X) - 1.18058\cdot 10^{11}B_{13,20}(X) \\
& - 1.26808\cdot 10^{11}B_{14,20}(X) - 1.3547\cdot 10^{11}B_{15,20}(X) - 1.44041\cdot 10^{11}B_{16,20}(X) - 1.52513 \\
& \cdot 10^{11}B_{17,20}(X) - 1.60883\cdot 10^{11}B_{18,20}(X) - 1.69144\cdot 10^{11}B_{19,20}(X) - 1.77291\cdot 10^{11}B_{20,20}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.00536186, 0.00562975\}$$

Intersection intervals with the x axis:

$$[0.00536186, 0.00562975]$$

Longest intersection interval: 0.000267881
$\Longrightarrow$ Selective recursion: interval 1: $[11.0363, 11.0363]$,

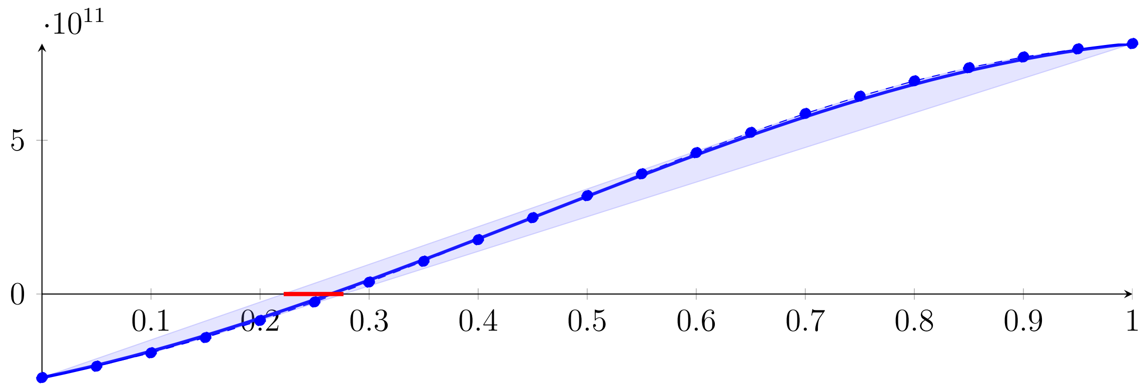
1.57 Recursion Branch 1 1 2 2 2 1 1 1 in Interval 1: $[11.0363, 11.0363]$

Found root in interval $[11.0363, 11.0363]$ at recursion depth 8!

1.58 Recursion Branch 1 1 2 2 2 2 on the Second Half $[11.7188, 12.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-239.286X^{20} + 2621.69X^{19} - 4218.74X^{18} + 36501.5X^{17} - 132212X^{16} + 92180.4X^{15} \\
&+ 68122.2X^{14} - 703261X^{13} - 7.4811\cdot 10^{6}X^{12} + 2.31877\cdot 10^{7}X^{11} + 3.38512\cdot 10^{8}X^{10} \\
&- 2.82813\cdot 10^{8}X^{9} - 8.23649\cdot 10^{9}X^{8} - 2.07318\cdot 10^{9}X^{7} + 1.03585\cdot 10^{11}X^{6} + 7.50723\cdot 10^{10}X^{5} \\
&- 5.978\cdot 10^{11}X^{4} - 4.69515\cdot 10^{11}X^{3} + 1.24337\cdot 10^{12}X^{2} + 7.41606\cdot 10^{11}X - 2.7189\cdot 10^{11} \\
= &-2.7189\cdot 10^{11}B_{0,20}(X) - 2.3481\cdot 10^{11}B_{1,20}(X) - 1.91185\cdot 10^{11}B_{2,20}(X) - 1.41429 \\
&\cdot 10^{11}B_{3,20}(X) - 8.60753\cdot 10^{10}B_{4,20}(X) - 2.57786\cdot 10^{10}B_{5,20}(X) + 3.86965\cdot 10^{10}B_{6,20}(X) \\
&+ 1.06484\cdot 10^{11}B_{7,20}(X) + 1.76631\cdot 10^{11}B_{8,20}(X) + 2.48109\cdot 10^{11}B_{9,20}(X) + 3.19837 \\
&\cdot 10^{11}B_{10,20}(X) + 3.90695\cdot 10^{11}B_{11,20}(X) + 4.59548\cdot 10^{11}B_{12,20}(X) + 5.25267\cdot 10^{11}B_{13,20}(X) \\
&+ 5.8675\cdot 10^{11}B_{14,20}(X) + 6.42947\cdot 10^{11}B_{15,20}(X) + 6.92882\cdot 10^{11}B_{16,20}(X) + 7.35673 \\
&\cdot 10^{11}B_{17,20}(X) + 7.70553\cdot 10^{11}B_{18,20}(X) + 7.96887\cdot 10^{11}B_{19,20}(X) + 8.1419\cdot 10^{11}B_{20,20}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.221656, 0.276489\}$$

Intersection intervals with the x axis:

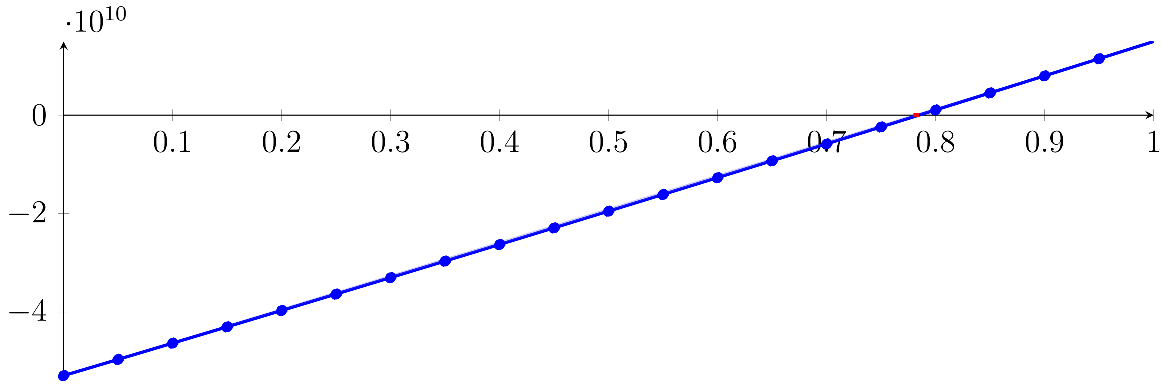
$$[0.221656, 0.276489]$$

Longest intersection interval: 0.0548323
$\Longrightarrow$ Selective recursion: interval 1: $[11.8919, 11.9348]$,

1.59 Recursion Branch 1 1 2 2 2 2 1 in Interval 1: $[11.8919, 11.9348]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 22.5781X^{20} - 56.8738X^{19} + 815.119X^{18} - 2269.91X^{17} + 16072.4X^{16} - 12946.2X^{15}\\ &\quad + 5959.26X^{14} + 4253.57X^{13} + 16461.3X^{12} + 3095.94X^{11} + 5164.69X^{10}\\ &\quad + 840.62X^{9} + 92.2632X^{8} - 33.1201X^{7} + 2453.25X^{6} + 101924X^{5} - 3.98149\\ &\quad \cdot 10^{6}X^{4} - 1.55052\cdot 10^{8}X^{3} + 2.30548\cdot 10^{9}X^{2} + 6.57329\cdot 10^{10}X - 5.29234\cdot 10^{10}\\ &= -5.29234\cdot 10^{10}B_{0,20}(X) - 4.96367\cdot 10^{10}B_{1,20}(X) - 4.63379\cdot 10^{10}B_{2,20}(X) - 4.30272\\ &\quad \cdot 10^{10}B_{3,20}(X) - 3.97045\cdot 10^{10}B_{4,20}(X) - 3.63702\cdot 10^{10}B_{5,20}(X) - 3.30242\cdot 10^{10}B_{6,20}(X)\\ &\quad - 2.96668\cdot 10^{10}B_{7,20}(X) - 2.62981\cdot 10^{10}B_{8,20}(X) - 2.29183\cdot 10^{10}B_{9,20}(X) - 1.95274\\ &\quad \cdot 10^{10}B_{10,20}(X) - 1.61256\cdot 10^{10}B_{11,20}(X) - 1.27131\cdot 10^{10}B_{12,20}(X) - 9.28999\cdot 10^{9}B_{13,20}(X)\\ &\quad - 5.85644\cdot 10^{9}B_{14,20}(X) - 2.41258\cdot 10^{9}B_{15,20}(X) + 1.04143\cdot 10^{9}B_{16,20}(X) + 4.50545\\ &\quad \cdot 10^{9}B_{17,20}(X) + 7.97934\cdot 10^{9}B_{18,20}(X) + 1.14629\cdot 10^{10}B_{19,20}(X) + 1.49561\cdot 10^{10}B_{20,20}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.779667, 0.784924\}$$

Intersection intervals with the x axis:

$$[0.779667, 0.784924]$$

Longest intersection interval: 0.00525758
$\Longrightarrow$ Selective recursion: interval 1: $[11.9253, 11.9255]$,

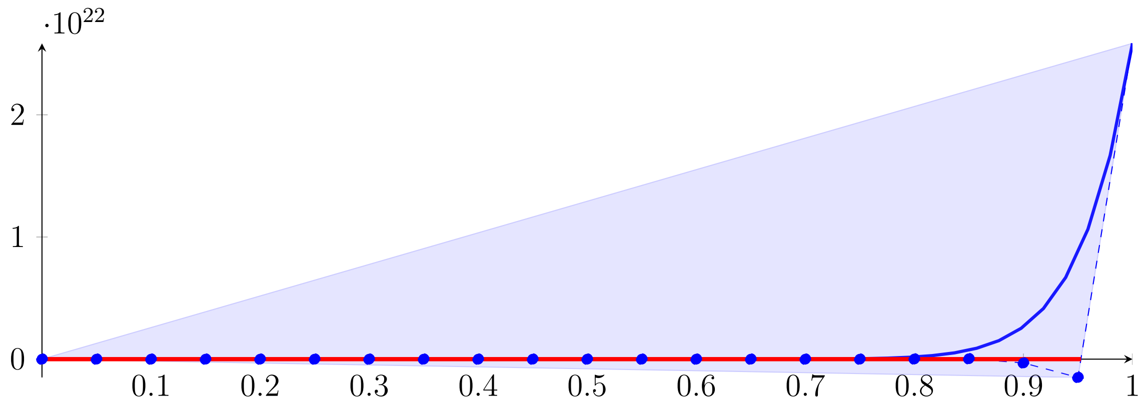
1.60 Recursion Branch 1 1 2 2 2 2 1 1 in Interval 1: $[11.9253, 11.9255]$

Found root in interval $[11.9253, 11.9255]$ at recursion depth 8!

1.61 Recursion Branch 1 2 on the Second Half $[12.5, 25]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 8.67362\cdot 10^{21}X^{20} + 2.77556\cdot 10^{22}X^{19} + 2.3731\cdot 10^{22}X^{18} - 1.26565\cdot 10^{22}X^{17} - 2.8638\cdot 10^{22}X^{16} - 6.33435 \cdot 10^{21}X^{15} + 1.06357\cdot 10^{22}X^{14} + 5.39429\cdot 10^{21}X^{13} - 1.50133\cdot 10^{21}X^{12} - 1.39249\cdot 10^{21}X^{11} + 1.05296 \cdot 10^{19}X^{10} + 1.67885\cdot 10^{20}X^{9} + 1.71006\cdot 10^{19}X^{8} - 9.83957\cdot 10^{18}X^{7} - 1.53217\cdot 10^{18}X^{6} + 2.57478 \cdot 10^{17}X^{5} + 4.72654\cdot 10^{16}X^{4} - 2.266\cdot 10^{15}X^{3} - 4.39258\cdot 10^{14}X^{2} + 5.53708\cdot 10^{12}X + 8.1419\cdot 10^{11}$$
$$= 8.1419\cdot 10^{11}B_{0,20}(X) + 1.09104\cdot 10^{12}B_{1,20}(X) - 9.43984\cdot 10^{11}B_{2,20}(X) - 7.27862 \cdot 10^{12}B_{3,20}(X) - 1.01451\cdot 10^{13}B_{4,20}(X) + 2.45871\cdot 10^{13}B_{5,20}(X) + 1.34488\cdot 10^{14}B_{6,20}(X) + 1.71188\cdot 10^{14}B_{7,20}(X) - 5.46645\cdot 10^{14}B_{8,20}(X) - 2.59384\cdot 10^{15}B_{9,20}(X) - 1.47677 \cdot 10^{15}B_{10,20}(X) + 2.00018\cdot 10^{16}B_{11,20}(X) + 5.97972\cdot 10^{16}B_{12,20}(X) - 8.43638\cdot 10^{16}B_{13,20}(X) - 9.00155\cdot 10^{17}B_{14,20}(X) - 6.30584\cdot 10^{17}B_{15,20}(X) + 1.35026\cdot 10^{19}B_{16,20}(X) + 3.45757 \cdot 10^{19}B_{17,20}(X) - 3.09468\cdot 10^{20}B_{18,20}(X) - 1.49659\cdot 10^{21}B_{19,20}(X) + 2.5852\cdot 10^{22}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{5.16831e-10, 0.952736\}$$

Intersection intervals with the x axis:

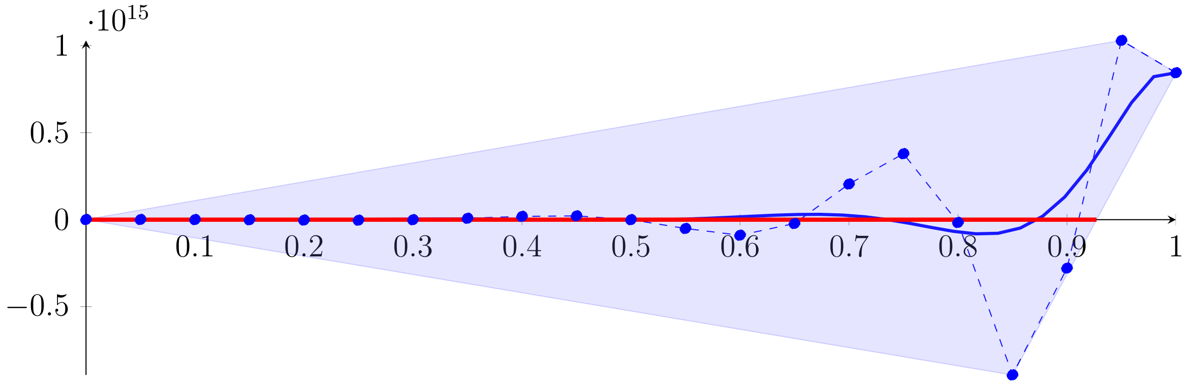
$$[5.16831e-10, 0.952736]$$

Longest intersection interval: 0.952736
$\Longrightarrow$ Bisection: first half $[12.5, 18.75]$ und second half $[18.75, 25]$

1.62 Recursion Branch 1 2 1 on the First Half $[12.5, 18.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 8.27181\cdot 10^{15}X^{20} + 5.29396\cdot 10^{16}X^{19} + 9.05266\cdot 10^{16}X^{18} - 9.65618\cdot 10^{16}X^{17} - 4.36981\cdot 10^{17}X^{16} - 1.93309\cdot 10^{17}X^{15} + 6.49154\cdot 10^{17}X^{14} + 6.58483\cdot 10^{17}X^{13} - 3.66535\cdot 10^{17}X^{12} - 6.79925\cdot 10^{17}X^{11} + 1.02828\cdot 10^{16}X^{10} + 3.279\cdot 10^{17}X^{9} + 6.67991\cdot 10^{16}X^{8} - 7.68717\cdot 10^{16}X^{7} - 2.39402\cdot 10^{16}X^{6} + 8.04618 \cdot 10^{15}X^{5} + 2.95408\cdot 10^{15}X^{4} - 2.8325\cdot 10^{14}X^{3} - 1.09814\cdot 10^{14}X^{2} + 2.76854\cdot 10^{12}X + 8.1419\cdot 10^{11}$$
$$= 8.1419\cdot 10^{11}B_{0,20}(X) + 9.52617\cdot 10^{11}B_{1,20}(X) + 5.13074\cdot 10^{11}B_{2,20}(X) - 7.52905 \cdot 10^{11}B_{3,20}(X) - 2.48407\cdot 10^{12}B_{4,20}(X) - 3.19047\cdot 10^{12}B_{5,20}(X) - 3.5214\cdot 10^{11}B_{6,20}(X) + 7.87292\cdot 10^{12}B_{7,20}(X) + 1.88702\cdot 10^{13}B_{8,20}(X) + 2.17404\cdot 10^{13}B_{9,20}(X) - 6.61543 \cdot 10^{10}B_{10,20}(X) - 5.06363\cdot 10^{13}B_{11,20}(X) - 8.94122\cdot 10^{13}B_{12,20}(X) - 2.20403\cdot 10^{13}B_{13,20}(X) + 2.04834\cdot 10^{14}B_{14,20}(X) + 3.789\cdot 10^{14}B_{15,20}(X) - 1.62511\cdot 10^{13}B_{16,20}(X) - 8.91971 \cdot 10^{14}B_{17,20}(X) - 2.7844\cdot 10^{14}B_{18,20}(X) + 1.02974\cdot 10^{15}B_{19,20}(X) + 8.43944\cdot 10^{14}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.000775172, 0.927075\}$$

Intersection intervals with the x axis:

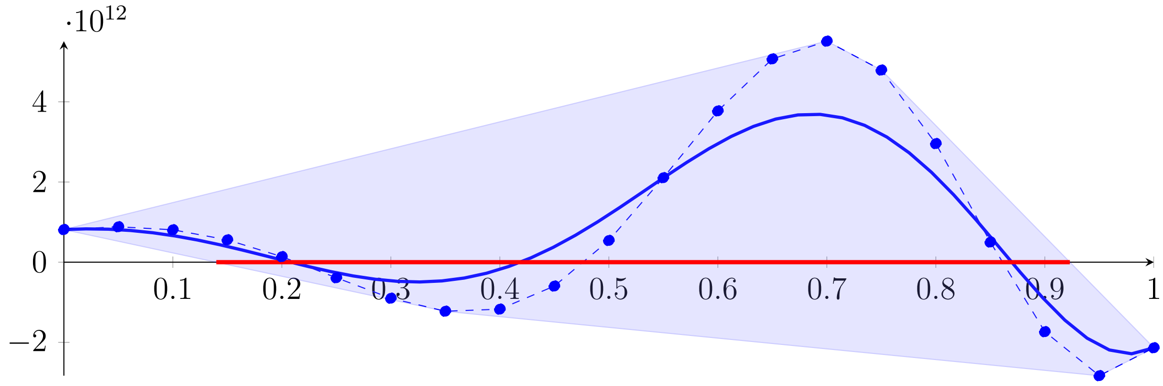
$$[0.000775172, 0.927075]$$

Longest intersection interval: 0.9263
$\Longrightarrow$ Bisection: first half $[12.5, 15.625]$ und second half $[15.625, 18.75]$

1.63 Recursion Branch 1 2 1 1 on the First Half $[12.5, 15.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 7.88861{\cdot}10^{9}X^{20} + 1.00974{\cdot}10^{11}X^{19} + 3.45332{\cdot}10^{11}X^{18} - 7.36708{\cdot}10^{11}X^{17} - 6.66779{\cdot}10^{12}X^{16} \\ &\quad - 5.89932{\cdot}10^{12}X^{15} + 3.96212{\cdot}10^{13}X^{14} + 8.03812{\cdot}10^{13}X^{13} - 8.94862{\cdot}10^{13}X^{12} - 3.31995{\cdot}10^{14}X^{11} \\ &\quad + 1.00418{\cdot}10^{13}X^{10} + 6.4043{\cdot}10^{14}X^{9} + 2.60934{\cdot}10^{14}X^{8} - 6.0056{\cdot}10^{14}X^{7} - 3.74065{\cdot}10^{14}X^{6} + 2.51443 \\ &\quad {\cdot}10^{14}X^{5} + 1.8463{\cdot}10^{14}X^{4} - 3.54063{\cdot}10^{13}X^{3} - 2.74536{\cdot}10^{13}X^{2} + 1.38427{\cdot}10^{12}X + 8.1419{\cdot}10^{11} \\ &= 8.1419{\cdot}10^{11}B_{0,20}(X) + 8.83404{\cdot}10^{11}B_{1,20}(X) + 8.08125{\cdot}10^{11}B_{2,20}(X) + 5.57295 \\ &\quad {\cdot}10^{11}B_{3,20}(X) + 1.37963{\cdot}10^{11}B_{4,20}(X) - 3.88495{\cdot}10^{11}B_{5,20}(X) - 8.99813{\cdot}10^{11}B_{6,20}(X) \\ &\quad - 1.22366{\cdot}10^{12}B_{7,20}(X) - 1.17156{\cdot}10^{12}B_{8,20}(X) - 5.95624{\cdot}10^{11}B_{9,20}(X) + 5.41725 \\ &\quad {\cdot}10^{11}B_{10,20}(X) + 2.10687{\cdot}10^{12}B_{11,20}(X) + 3.77349{\cdot}10^{12}B_{12,20}(X) + 5.07064{\cdot}10^{12}B_{13,20}(X) \\ &\quad + 5.51323{\cdot}10^{12}B_{14,20}(X) + 4.79225{\cdot}10^{12}B_{15,20}(X) + 2.95806{\cdot}10^{12}B_{16,20}(X) + 5.02527 \\ &\quad {\cdot}10^{11}B_{17,20}(X) - 1.7341{\cdot}10^{12}B_{18,20}(X) - 2.83115{\cdot}10^{12}B_{19,20}(X) - 2.1354{\cdot}10^{12}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.139837, 0.922939\}$$

Intersection intervals with the x axis:

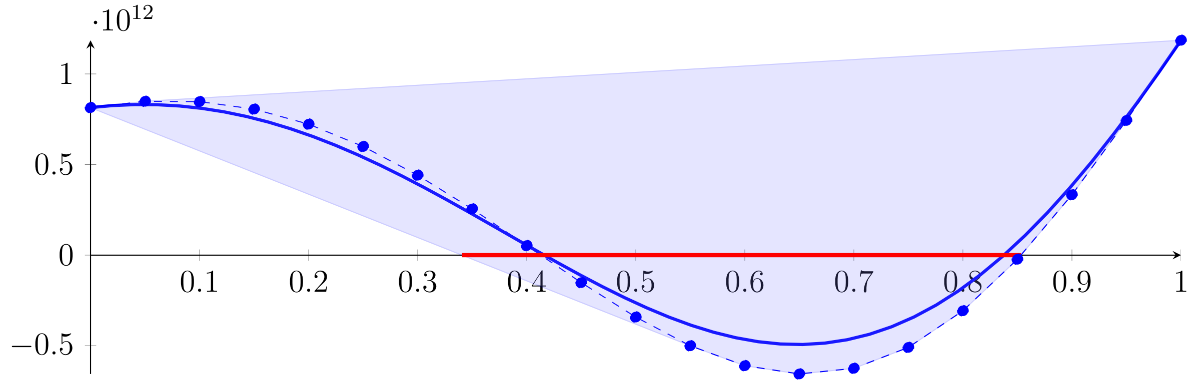
$$[0.139837, 0.922939]$$

Longest intersection interval: 0.783102
$\Longrightarrow$ Bisection: first half $[12.5, 14.0625]$ und second half $[14.0625, 15.625]$

1.64 Recursion Branch 1 2 1 1 1 on the First Half $[12.5, 14.0625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 7635.4X^{20} + 188595X^{19} + 1.31037\cdot 10^{6}X^{18} - 5.65847\cdot 10^{6}X^{17} - 1.0173\cdot 10^{8}X^{16} - 1.7999\cdot 10^{8}X^{15} + 2.41822\cdot 10^{9}X^{14} + 9.8121\cdot 10^{9}X^{13} - 2.18474\cdot 10^{10}X^{12} - 1.62107\cdot 10^{11}X^{11} + 9.80637\cdot 10^{9}X^{10} + 1.25084\cdot 10^{12}X^{9} + 1.01927\cdot 10^{12}X^{8} - 4.69187\cdot 10^{12}X^{7} - 5.84477\cdot 10^{12}X^{6} + 7.8576\cdot 10^{12}X^{5} + 1.15394\cdot 10^{13}X^{4} - 4.42579\cdot 10^{12}X^{3} - 6.8634\cdot 10^{12}X^{2} + 6.92135\cdot 10^{11}X + 8.1419\cdot 10^{11}$$
$$= 8.1419\cdot 10^{11}B_{0,20}(X) + 8.48797\cdot 10^{11}B_{1,20}(X) + 8.47281\cdot 10^{11}B_{2,20}(X) + 8.05759\cdot 10^{11}B_{3,20}(X) + 7.22731\cdot 10^{11}B_{4,20}(X) + 5.99585\cdot 10^{11}B_{5,20}(X) + 4.40954\cdot 10^{11}B_{6,20}(X) + 2.54859\cdot 10^{11}B_{7,20}(X) + 5.25918\cdot 10^{10}B_{8,20}(X) - 1.51705\cdot 10^{11}B_{9,20}(X) - 3.41772\cdot 10^{11}B_{10,20}(X) - 5.00267\cdot 10^{11}B_{11,20}(X) - 6.10048\cdot 10^{11}B_{12,20}(X) - 6.55614\cdot 10^{11}B_{13,20}(X) - 6.24598\cdot 10^{11}B_{14,20}(X) - 5.09162\cdot 10^{11}B_{15,20}(X) - 3.07139\cdot 10^{11}B_{16,20}(X) - 2.27736\cdot 10^{10}B_{17,20}(X) + 3.33058\cdot 10^{11}B_{18,20}(X) + 7.43235\cdot 10^{11}B_{19,20}(X) + 1.1854\cdot 10^{12}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.340676, 0.8532\}$$

Intersection intervals with the x axis:

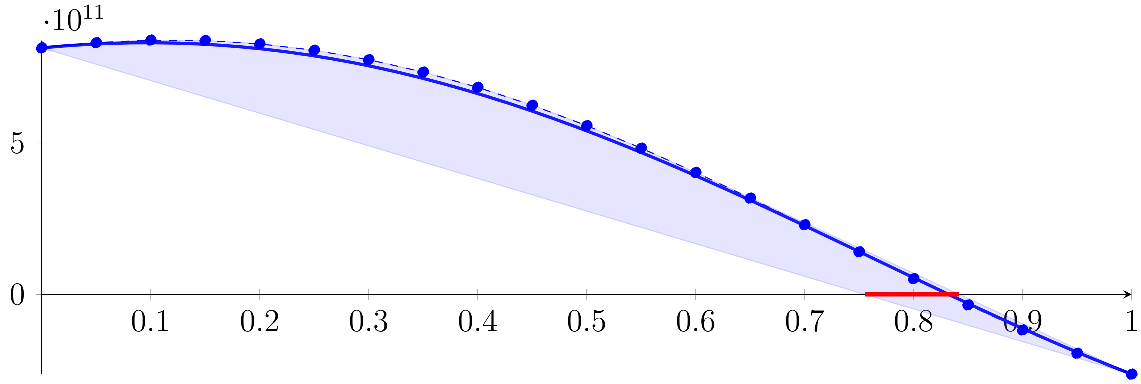
$$[0.340676, 0.8532]$$

Longest intersection interval: 0.512524
$\Longrightarrow$ Bisection: first half $[12.5, 13.2812]$ und second half $[13.2812, 14.0625]$

1.65 Recursion Branch 1 2 1 1 1 1 on the First Half $[12.5, 13.2812]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -615.164X^{20} + 1877.3X^{19} - 21086.7X^{18} + 65024.5X^{17} - 421013X^{16} + 344752X^{15} - 5223.52X^{14} + 1.14024\cdot 10^{6}X^{13} - 5.74098\cdot 10^{6}X^{12} - 7.91928\cdot 10^{7}X^{11} + 9.45612\cdot 10^{6}X^{10} + 2.44303\cdot 10^{9}X^{9} + 3.98153\cdot 10^{9}X^{8} - 3.66553\cdot 10^{10}X^{7} - 9.13245\cdot 10^{10}X^{6} + 2.4555\cdot 10^{11}X^{5} + 7.21212\cdot 10^{11}X^{4} - 5.53223\cdot 10^{11}X^{3} - 1.71585\cdot 10^{12}X^{2} + 3.46067\cdot 10^{11}X + 8.1419\cdot 10^{11}$$
$$= 8.1419\cdot 10^{11}B_{0,20}(X) + 8.31494\cdot 10^{11}B_{1,20}(X) + 8.39766\cdot 10^{11}B_{2,20}(X) + 8.38523\cdot 10^{11}B_{3,20}(X) + 8.27427\cdot 10^{11}B_{4,20}(X) + 8.06307\cdot 10^{11}B_{5,20}(X) + 7.75169\cdot 10^{11}B_{6,20}(X) + 7.34208\cdot 10^{11}B_{7,20}(X) + 6.83817\cdot 10^{11}B_{8,20}(X) + 6.24585\cdot 10^{11}B_{9,20}(X) + 5.57306\cdot 10^{11}B_{10,20}(X) + 4.82964\cdot 10^{11}B_{11,20}(X) + 4.02731\cdot 10^{11}B_{12,20}(X) + 3.17954\cdot 10^{11}B_{13,20}(X) + 2.30131\cdot 10^{11}B_{14,20}(X) + 1.40895\cdot 10^{11}B_{15,20}(X) + 5.19842\cdot 10^{10}B_{16,20}(X) - 3.47871\cdot 10^{10}B_{17,20}(X) - 1.17561\cdot 10^{11}B_{18,20}(X) - 1.94471\cdot 10^{11}B_{19,20}(X) - 2.63682\cdot 10^{11}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.755368, 0.841731\}$$

Intersection intervals with the x axis:

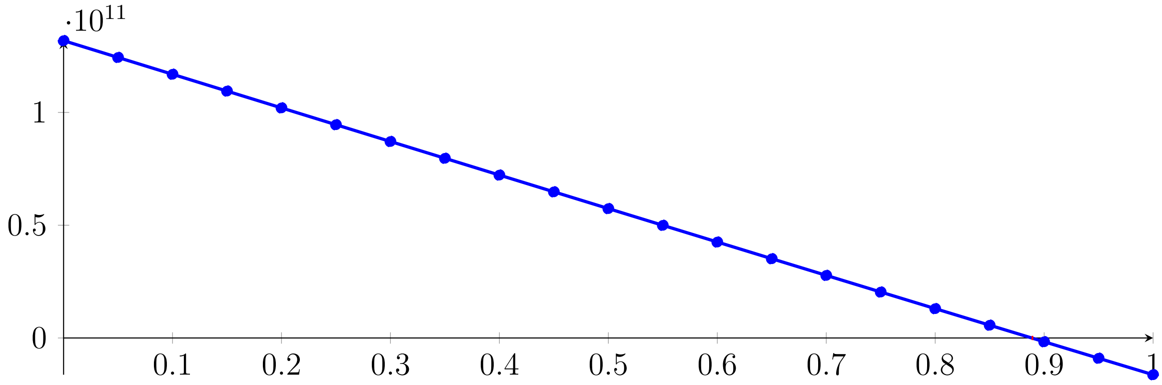
$$[0.755368, 0.841731]$$

Longest intersection interval: 0.0863626
$\Longrightarrow$ Selective recursion: interval 1: $[13.0901, 13.1576]$,

1.66 Recursion Branch 1 2 1 1 1 1 1 in Interval 1: $[13.0901, 13.1576]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-64.0016X^{20} + 206.039X^{19} - 2411.03X^{18} + 6920.09X^{17} - 45180.3X^{16} + 38282.1X^{15} \\ &- 19046.4X^{14} - 6136.68X^{13} - 44616.9X^{12} - 6694.21X^{11} - 14422.8X^{10} \\ &- 2019.54X^{9} - 238.347X^{8} + 1017.26X^{7} - 59008.2X^{6} - 2.0075\cdot 10^{6}X^{5} + 2.65348 \\ &\cdot 10^{7}X^{4} + 1.2327\cdot 10^{9}X^{3} - 3.3326\cdot 10^{8}X^{2} - 1.49035\cdot 10^{11}X + 1.31821\cdot 10^{11} \\ = &\, 1.31821\cdot 10^{11}B_{0,20}(X) + 1.24369\cdot 10^{11}B_{1,20}(X) + 1.16916\cdot 10^{11}B_{2,20}(X) + 1.09461 \\ &\cdot 10^{11}B_{3,20}(X) + 1.02008\cdot 10^{11}B_{4,20}(X) + 9.45554\cdot 10^{10}B_{5,20}(X) + 8.71057\cdot 10^{10}B_{6,20}(X) \\ &+ 7.96598\cdot 10^{10}B_{7,20}(X) + 7.22186\cdot 10^{10}B_{8,20}(X) + 6.47834\cdot 10^{10}B_{9,20}(X) + 5.73552 \\ &\cdot 10^{10}B_{10,20}(X) + 4.99352\cdot 10^{10}B_{11,20}(X) + 4.25245\cdot 10^{10}B_{12,20}(X) + 3.51242\cdot 10^{10}B_{13,20}(X) \\ &+ 2.77354\cdot 10^{10}B_{14,20}(X) + 2.03594\cdot 10^{10}B_{15,20}(X) + 1.29971\cdot 10^{10}B_{16,20}(X) + 5.64989 \\ &\cdot 10^{9}B_{17,20}(X) - 1.68122\cdot 10^{9}B_{18,20}(X) - 8.99506\cdot 10^{9}B_{19,20}(X) - 1.62905\cdot 10^{10}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.888534, 0.890012\}$$

Intersection intervals with the x axis:

$$[0.888534, 0.890012]$$

Longest intersection interval: 0.00147848
$\Longrightarrow$ Selective recursion: interval 1: $[13.1501, 13.1502]$,

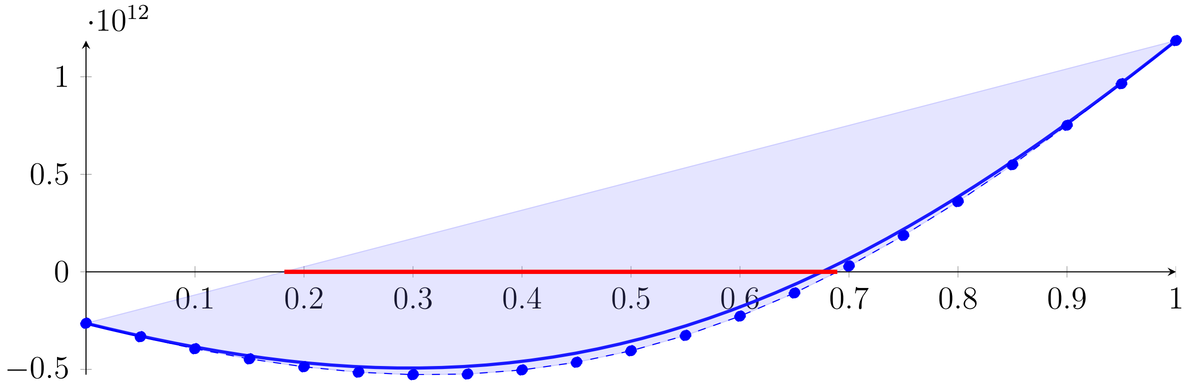
1.67 Recursion Branch 1 2 1 1 1 1 1 1 in Interval 1: $[13.1501, 13.1502]$

Found root in interval $[13.1501, 13.1502]$ at recursion depth 8!

1.68 Recursion Branch 1 2 1 1 1 2 on the Second Half $[13.2812, 14.0625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = {} & 419.785X^{20} - 1104.09X^{19} + 13183.9X^{18} - 49424.6X^{17} + 297884X^{16} - 280941X^{15} \\ & - 27766.5X^{14} + 1.81264\cdot10^{6}X^{13} + 1.84521\cdot10^{7}X^{12} - 1.05999\cdot10^{7}X^{11} - 7.5227\cdot10^{8}X^{10} \\ & - 1.88218\cdot10^{9}X^{9} + 1.2628\cdot10^{10}X^{8} + 5.6459\cdot10^{10}X^{7} - 6.82406\cdot10^{10}X^{6} - 5.77934\cdot10^{11}X^{5} \\ & - 1.43065\cdot10^{11}X^{4} + 2.09319\cdot10^{12}X^{3} + 1.46289\cdot10^{12}X^{2} - 1.38422\cdot10^{12}X - 2.63682\cdot10^{11} \\ = {} & -2.63682\cdot10^{11}B_{0,20}(X) - 3.32893\cdot10^{11}B_{1,20}(X) - 3.94405\cdot10^{11}B_{2,20}(X) - 4.46381 \\ & \cdot10^{11}B_{3,20}(X) - 4.87015\cdot10^{11}B_{4,20}(X) - 5.14566\cdot10^{11}B_{5,20}(X) - 5.27402\cdot10^{11}B_{6,20}(X) \\ & - 5.24034\cdot10^{11}B_{7,20}(X) - 5.03161\cdot10^{11}B_{8,20}(X) - 4.63705\cdot10^{11}B_{9,20}(X) - 4.04855 \\ & \cdot10^{11}B_{10,20}(X) - 3.26096\cdot10^{11}B_{11,20}(X) - 2.27246\cdot10^{11}B_{12,20}(X) - 1.08476\cdot10^{11}B_{13,20}(X) \\ & + 2.96648\cdot10^{10}B_{14,20}(X) + 1.86236\cdot10^{11}B_{15,20}(X) + 3.59903\cdot10^{11}B_{16,20}(X) + 5.48938 \\ & \cdot10^{11}B_{17,20}(X) + 7.51232\cdot10^{11}B_{18,20}(X) + 9.64317\cdot10^{11}B_{19,20}(X) + 1.1854\cdot10^{12}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.181965, 0.689263\}$$

Intersection intervals with the x axis:

$$[0.181965, 0.689263]$$

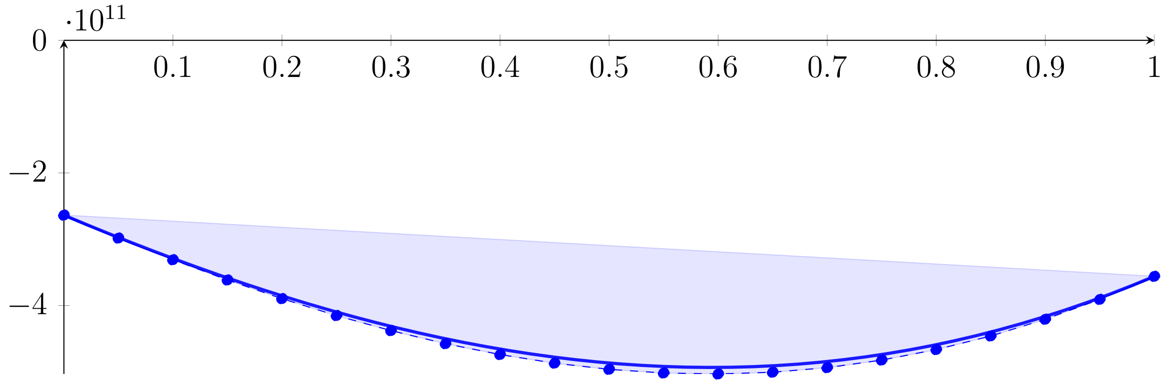
Longest intersection interval: 0.507298

$\Longrightarrow$ Bisection: first half $[13.2812, 13.6719]$ und second half $[13.6719, 14.0625]$

1.69 Recursion Branch 1 2 1 1 1 2 1 on the First Half $[13.2812, 13.6719]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = {} & 467.015X^{20} - 2659.99X^{19} + 13221.8X^{18} - 57116.3X^{17} + 301925X^{16} - 236897X^{15} \\ & + 87680.8X^{14} + 40657.3X^{13} + 216380X^{12} + 14444.3X^{11} - 675525X^{10} - 3.67149 \\ & \cdot10^{6}X^{9} + 4.93291\cdot10^{7}X^{8} + 4.41086\cdot10^{8}X^{7} - 1.06626\cdot10^{9}X^{6} - 1.80605\cdot10^{10}X^{5} \\ & - 8.94155\cdot10^{9}X^{4} + 2.61649\cdot10^{11}X^{3} + 3.65722\cdot10^{11}X^{2} - 6.9211\cdot10^{11}X - 2.63682\cdot10^{11} \\ = {} & -2.63682\cdot10^{11}B_{0,20}(X) - 2.98288\cdot10^{11}B_{1,20}(X) - 3.30968\cdot10^{11}B_{2,20}(X) - 3.61494 \\ & \cdot10^{11}B_{3,20}(X) - 3.89639\cdot10^{11}B_{4,20}(X) - 4.15176\cdot10^{11}B_{5,20}(X) - 4.37887\cdot10^{11}B_{6,20}(X) \\ & - 4.57555\cdot10^{11}B_{7,20}(X) - 4.73972\cdot10^{11}B_{8,20}(X) - 4.86939\cdot10^{11}B_{9,20}(X) - 4.96263 \\ & \cdot10^{11}B_{10,20}(X) - 5.01763\cdot10^{11}B_{11,20}(X) - 5.03271\cdot10^{11}B_{12,20}(X) - 5.00629\cdot10^{11}B_{13,20}(X) \\ & - 4.93695\cdot10^{11}B_{14,20}(X) - 4.82341\cdot10^{11}B_{15,20}(X) - 4.66456\cdot10^{11}B_{16,20}(X) - 4.45946 \\ & \cdot10^{11}B_{17,20}(X) - 4.20735\cdot10^{11}B_{18,20}(X) - 3.90766\cdot10^{11}B_{19,20}(X) - 3.56003\cdot10^{11}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

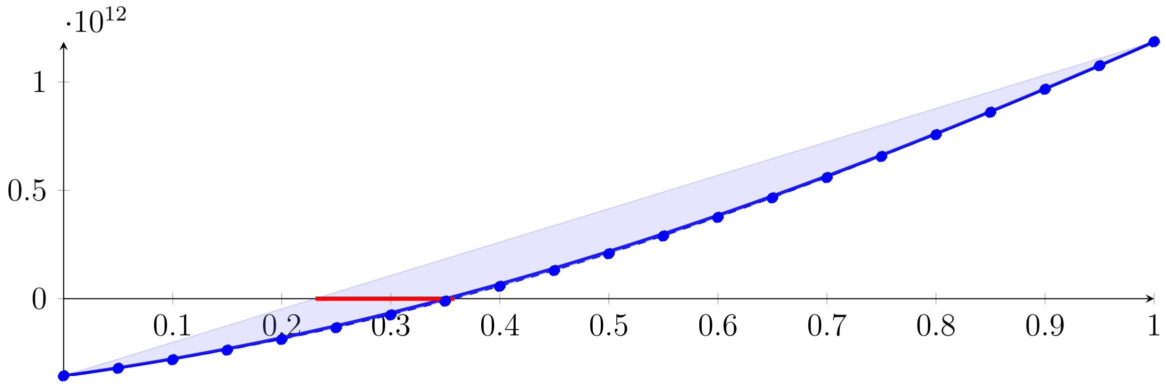
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

1.70 Recursion Branch 1 2 1 1 1 2 2 on the Second Half $[13.6719, 14.0625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -132.606X^{20} + 2258.87X^{19} - 1420.25X^{18} + 25358.7X^{17} - 57645X^{16} + 27031.7X^{15} \\ & + 4338.74X^{14} + 17742.9X^{13} + 39996.1X^{12} + 86829.9X^{11} - 428128X^{10} - 1.01922 \\ & \cdot 10^{7}X^{9} - 1.52294 \cdot 10^{7}X^{8} + 6.17345 \cdot 10^{8}X^{7} + 2.94149 \cdot 10^{9}X^{6} - 1.30798 \cdot 10^{10}X^{5} \\ & - 9.69636 \cdot 10^{10}X^{4} + 4.17565 \cdot 10^{10}X^{3} + 9.109 \cdot 10^{11}X^{2} + 6.95257 \cdot 10^{11}X - 3.56003 \cdot 10^{11} \\ = & -3.56003 \cdot 10^{11}B_{0,20}(X) - 3.2124 \cdot 10^{11}B_{1,20}(X) - 2.81683 \cdot 10^{11}B_{2,20}(X) - 2.37295 \\ & \cdot 10^{11}B_{3,20}(X) - 1.8806 \cdot 10^{11}B_{4,20}(X) - 1.33981 \cdot 10^{11}B_{5,20}(X) - 7.50855 \cdot 10^{10}B_{6,20}(X) \\ & - 1.14204 \cdot 10^{10}B_{7,20}(X) + 5.69428 \cdot 10^{10}B_{8,20}(X) + 1.2991 \cdot 10^{11}B_{9,20}(X) + 2.07362 \\ & \cdot 10^{11}B_{10,20}(X) + 2.89157 \cdot 10^{11}B_{11,20}(X) + 3.75129 \cdot 10^{11}B_{12,20}(X) + 4.65087 \cdot 10^{11}B_{13,20}(X) \\ & + 5.58815 \cdot 10^{11}B_{14,20}(X) + 6.56076 \cdot 10^{11}B_{15,20}(X) + 7.56607 \cdot 10^{11}B_{16,20}(X) + 8.60123 \\ & \cdot 10^{11}B_{17,20}(X) + 9.66316 \cdot 10^{11}B_{18,20}(X) + 1.07486 \cdot 10^{12}B_{19,20}(X) + 1.1854 \cdot 10^{12}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.23096, 0.358353\}$$

Intersection intervals with the x axis:

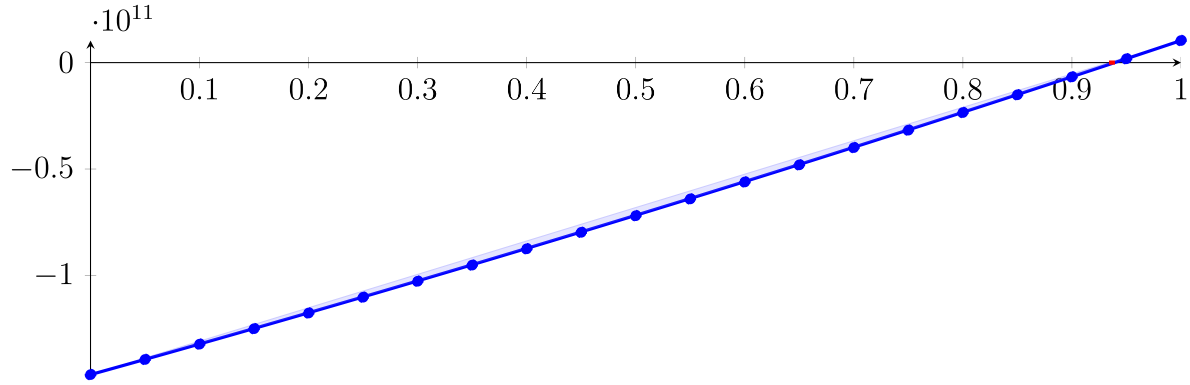
$$[0.23096, 0.358353]$$

Longest intersection interval: 0.127392
$\Longrightarrow$ Selective recursion: interval 1: $[13.7621, 13.8119]$,

1.71 Recursion Branch 1 2 1 1 1 2 2 1 in Interval 1: $[13.7621, 13.8119]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = {} & 79.8845X^{20} - 257.036X^{19} + 2797.81X^{18} - 8546.35X^{17} + 53357.5X^{16} - 43491X^{15} \\
& + 18896.2X^{14} + 10392.6X^{13} + 51632.8X^{12} + 9518.49X^{11} + 16142.5X^{10} \\
& + 2301.45X^{9} + 272.945X^{8} + 295.715X^{7} + 16804.9X^{6} - 279361X^{5} - 2.88269 \\
& \cdot 10^{7}X^{4} - 1.1167\cdot 10^{8}X^{3} + 1.47247\cdot 10^{10}X^{2} + 1.42393\cdot 10^{11}X - 1.46606\cdot 10^{11} \\
= {} & -1.46606\cdot 10^{11}B_{0,20}(X) - 1.39486\cdot 10^{11}B_{1,20}(X) - 1.32289\cdot 10^{11}B_{2,20}(X) - 1.25015 \\
& \cdot 10^{11}B_{3,20}(X) - 1.17663\cdot 10^{11}B_{4,20}(X) - 1.10234\cdot 10^{11}B_{5,20}(X) - 1.02728\cdot 10^{11}B_{6,20}(X) \\
& - 9.51446\cdot 10^{10}B_{7,20}(X) - 8.74848\cdot 10^{10}B_{8,20}(X) - 7.97482\cdot 10^{10}B_{9,20}(X) - 7.19351 \\
& \cdot 10^{10}B_{10,20}(X) - 6.40456\cdot 10^{10}B_{11,20}(X) - 5.60799\cdot 10^{10}B_{12,20}(X) - 4.8038\cdot 10^{10}B_{13,20}(X) \\
& - 3.99202\cdot 10^{10}B_{14,20}(X) - 3.17267\cdot 10^{10}B_{15,20}(X) - 2.34576\cdot 10^{10}B_{16,20}(X) - 1.51131 \\
& \cdot 10^{10}B_{17,20}(X) - 6.69342\cdot 10^{9}B_{18,20}(X) + 1.8013\cdot 10^{9}B_{19,20}(X) + 1.03708\cdot 10^{10}B_{20,20}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.933934, 0.939398\}$$

Intersection intervals with the x axis:

$$[0.933934, 0.939398]$$

Longest intersection interval: 0.00546356

$\Longrightarrow$ Selective recursion: interval 1: $[13.8086, 13.8088]$,

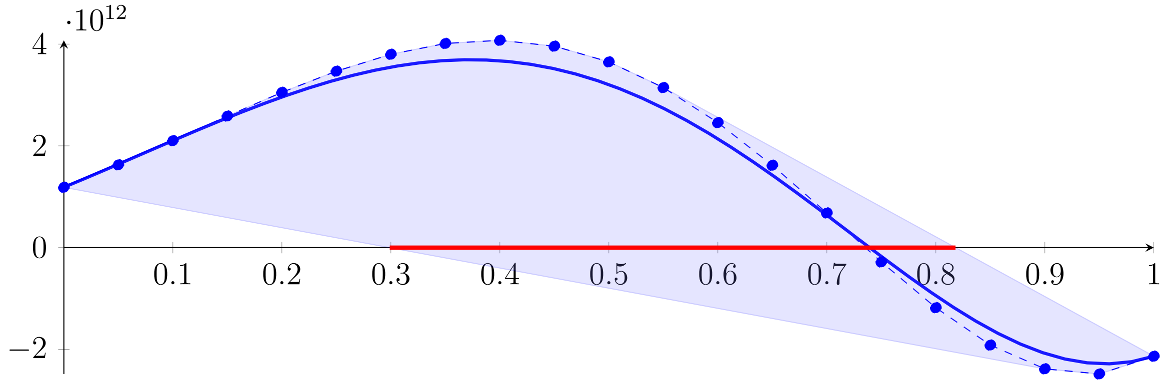
1.72 Recursion Branch 1 2 1 1 1 2 2 1 1 in Interval 1: $[13.8086, 13.8088]$

Found root in interval $[13.8086, 13.8088]$ at recursion depth 9!

1.73 Recursion Branch 1 2 1 1 2 on the Second Half $[14.0625, 15.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = {} & 3986.15X^{20} + 355982X^{19} + 6.295\cdot 10^{6}X^{18} + 6.00178\cdot 10^{7}X^{17} + 2.24902\cdot 10^{8}X^{16} - 6.32265 \\
& \cdot 10^{8}X^{15} - 9.75189\cdot 10^{9}X^{14} - 2.84929\cdot 10^{10}X^{13} + 5.90133\cdot 10^{10}X^{12} + 5.19357\cdot 10^{11}X^{11} + 6.2382 \\
& \cdot 10^{11}X^{10} - 2.63478\cdot 10^{12}X^{9} - 7.48493\cdot 10^{12}X^{8} + 1.62878\cdot 10^{12}X^{7} + 2.42459\cdot 10^{13}X^{6} + 1.56831 \\
& \cdot 10^{13}X^{5} - 2.53581\cdot 10^{13}X^{4} - 2.54855\cdot 10^{13}X^{3} + 6.07786\cdot 10^{12}X^{2} + 8.8433\cdot 10^{12}X + 1.1854\cdot 10^{12} \\
= {} & 1.1854\cdot 10^{12}B_{0,20}(X) + 1.62756\cdot 10^{12}B_{1,20}(X) + 2.10172\cdot 10^{12}B_{2,20}(X) + 2.58551 \\
& \cdot 10^{12}B_{3,20}(X) + 3.05134\cdot 10^{12}B_{4,20}(X) + 3.4674\cdot 10^{12}B_{5,20}(X) + 3.79929\cdot 10^{12}B_{6,20}(X) \\
& + 4.01233\cdot 10^{12}B_{7,20}(X) + 4.07439\cdot 10^{12}B_{8,20}(X) + 3.95934\cdot 10^{12}B_{9,20}(X) + 3.65071 \\
& \cdot 10^{12}B_{10,20}(X) + 3.14537\cdot 10^{12}B_{11,20}(X) + 2.4568\cdot 10^{12}B_{12,20}(X) + 1.61759\cdot 10^{12}B_{13,20}(X) \\
& + 6.80535\cdot 10^{11}B_{14,20}(X) - 2.82012\cdot 10^{11}B_{15,20}(X) - 1.18103\cdot 10^{12}B_{16,20}(X) - 1.91608 \\
& \cdot 10^{12}B_{17,20}(X) - 2.38295\cdot 10^{12}B_{18,20}(X) - 2.48328\cdot 10^{12}B_{19,20}(X) - 2.1354\cdot 10^{12}B_{20,20}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.298978, 0.818032\}$$

Intersection intervals with the x axis:

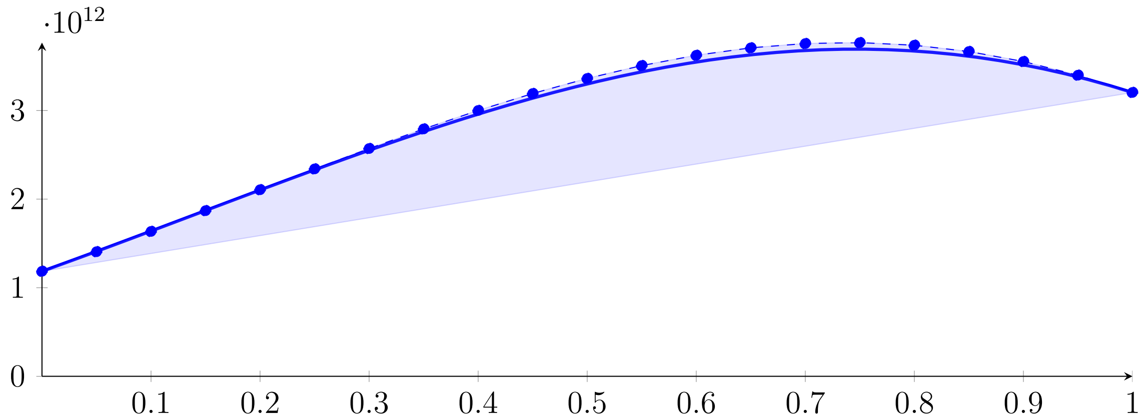
$$[0.298978, 0.818032]$$

Longest intersection interval: 0.519054
$\Longrightarrow$ Bisection: first half $[14.0625, 14.8438]$ und second half $[14.8438, 15.625]$

1.74 Recursion Branch 1 2 1 1 2 1 on the First Half $[14.0625, 14.8438]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -3164.52X^{20} + 19758.6X^{19} - 80800.6X^{18} + 419071X^{17} - 1.94135\cdot 10^6 X^{16} + 1.42971\cdot 10^6 X^{15} \\ & - 1.17599\cdot 10^6 X^{14} - 3.63583\cdot 10^6 X^{13} + 1.31048\cdot 10^7 X^{12} + 2.53539\cdot 10^8 X^{11} + 6.08859\cdot 10^8 X^{10} \\ & - 5.14608\cdot 10^9 X^9 - 2.9238\cdot 10^{10} X^8 + 1.27248\cdot 10^{10} X^7 + 3.78842\cdot 10^{11} X^6 + 4.90097\cdot 10^{11} X^5 \\ & - 1.58488\cdot 10^{12} X^4 - 3.18569\cdot 10^{12} X^3 + 1.51947\cdot 10^{12} X^2 + 4.42165\cdot 10^{12} X + 1.1854\cdot 10^{12} \\ = & \ 1.1854\cdot 10^{12} B_{0,20}(X) + 1.40648\cdot 10^{12} B_{1,20}(X) + 1.63556\cdot 10^{12} B_{2,20}(X) + 1.86984 \\ & \cdot 10^{12} B_{3,20}(X) + 2.10621\cdot 10^{12} B_{4,20}(X) + 2.34124\cdot 10^{12} B_{5,20}(X) + 2.57126\cdot 10^{12} B_{6,20}(X) \\ & + 2.7924\cdot 10^{12} B_{7,20}(X) + 3.00064\cdot 10^{12} B_{8,20}(X) + 3.1919\cdot 10^{12} B_{9,20}(X) + 3.3621 \\ & \cdot 10^{12} B_{10,20}(X) + 3.50725\cdot 10^{12} B_{11,20}(X) + 3.62358\cdot 10^{12} B_{12,20}(X) + 3.70757\cdot 10^{12} B_{13,20}(X) \\ & + 3.75612\cdot 10^{12} B_{14,20}(X) + 3.76662\cdot 10^{12} B_{15,20}(X) + 3.73705\cdot 10^{12} B_{16,20}(X) + 3.66611 \\ & \cdot 10^{12} B_{17,20}(X) + 3.55325\cdot 10^{12} B_{18,20}(X) + 3.39883\cdot 10^{12} B_{19,20}(X) + 3.2041\cdot 10^{12} B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

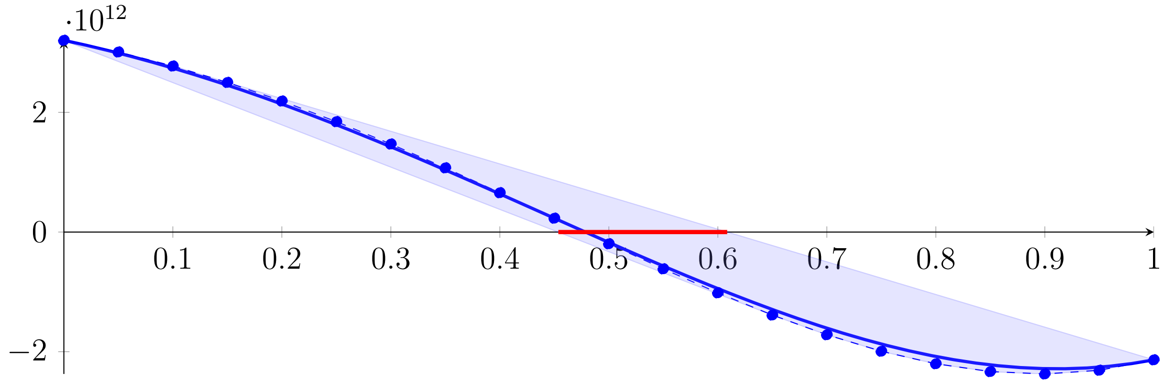
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

1.75 Recursion Branch 1 2 1 1 2 2 on the Second Half $[14.8438, 15.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -268.452X^{20} - 6664.56X^{19} - 27110.3X^{18} - 23656.7X^{17} - 301632X^{16} + 474424X^{15} - 325164X^{14} - 1.07899\cdot10^{7}X^{13} - 8.48682\cdot10^{7}X^{12} - 6.6458\cdot10^{7}X^{11} + 2.73851\cdot10^{9}X^{10} + 1.434\cdot10^{10}X^{9} - 5.49518\cdot10^{9}X^{8} - 2.46323\cdot10^{11}X^{7} - 5.32431\cdot10^{11}X^{6} + 1.02102\cdot10^{12}X^{5} + 4.51419\cdot10^{12}X^{4} + 1.44526\cdot10^{12}X^{3} - 7.65801\cdot10^{12}X^{2} - 3.89462\cdot10^{12}X + 3.2041\cdot10^{12}$$
$$= 3.2041\cdot10^{12}B_{0,20}(X) + 3.00937\cdot10^{12}B_{1,20}(X) + 2.77433\cdot10^{12}B_{2,20}(X) + 2.50026\cdot10^{12}B_{3,20}(X) + 2.18934\cdot10^{12}B_{4,20}(X) + 1.84479\cdot10^{12}B_{5,20}(X) + 1.47084\cdot10^{12}B_{6,20}(X) + 1.07283\cdot10^{12}B_{7,20}(X) + 6.57193\cdot10^{11}B_{8,20}(X) + 2.31445\cdot10^{11}B_{9,20}(X) - 1.9583\cdot10^{11}B_{10,20}(X) - 6.15059\cdot10^{11}B_{11,20}(X) - 1.01577\cdot10^{12}B_{12,20}(X) - 1.38672\cdot10^{12}B_{13,20}(X) - 1.71606\cdot10^{12}B_{14,20}(X) - 1.99154\cdot10^{12}B_{15,20}(X) - 2.20072\cdot10^{12}B_{16,20}(X) - 2.33127\cdot10^{12}B_{17,20}(X) - 2.37123\cdot10^{12}B_{18,20}(X) - 2.30934\cdot10^{12}B_{19,20}(X) - 2.1354\cdot10^{12}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.453659, 0.608561\}$$

Intersection intervals with the x axis:

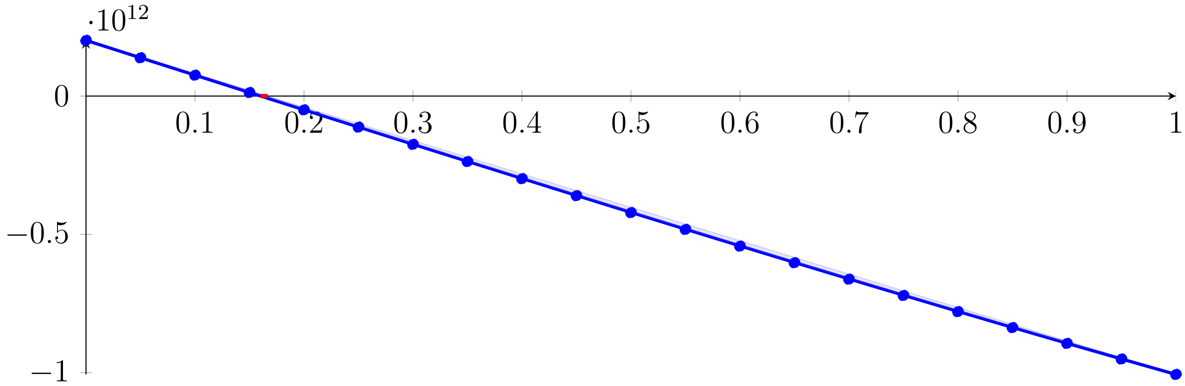
$$[0.453659, 0.608561]$$

Longest intersection interval: 0.154902
$\Longrightarrow$ Selective recursion: interval 1: $[15.1982, 15.3192]$,

1.76 Recursion Branch 1 2 1 1 2 2 1 in Interval 1: $[15.1982, 15.3192]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 367.168X^{20} - 3141.65X^{19} + 8979.92X^{18} - 50614.3X^{17} + 207614X^{16} - 144158X^{15} + 54176.2X^{14} - 967.581X^{13} + 95065.7X^{12} - 9395.47X^{11} + 15792.9X^{10} - 989.266X^{9} + 25560.7X^{8} - 281845X^{7} - 1.67275\cdot10^{7}X^{6} - 1.2774\cdot10^{8}X^{5} + 2.53607\cdot10^{9}X^{4} + 3.8598\cdot10^{10}X^{3} + 9.5922\cdot10^{9}X^{2} - 1.25854\cdot10^{12}X + 2.01351\cdot10^{11}$$
$$= 2.01351\cdot10^{11}B_{0,20}(X) + 1.38424\cdot10^{11}B_{1,20}(X) + 7.5547\cdot10^{10}B_{2,20}(X) + 1.27547\cdot10^{10}B_{3,20}(X) - 4.9919\cdot10^{10}B_{4,20}(X) - 1.12439\cdot10^{11}B_{5,20}(X) - 1.7477\cdot10^{11}B_{6,20}(X) - 2.36876\cdot10^{11}B_{7,20}(X) - 2.98721\cdot10^{11}B_{8,20}(X) - 3.60267\cdot10^{11}B_{9,20}(X) - 4.21478\cdot10^{11}B_{10,20}(X) - 4.82316\cdot10^{11}B_{11,20}(X) - 5.42742\cdot10^{11}B_{12,20}(X) - 6.02718\cdot10^{11}B_{13,20}(X) - 6.62205\cdot10^{11}B_{14,20}(X) - 7.21163\cdot10^{11}B_{15,20}(X) - 7.79552\cdot10^{11}B_{16,20}(X) - 8.37332\cdot10^{11}B_{17,20}(X) - 8.94463\cdot10^{11}B_{18,20}(X) - 9.50903\cdot10^{11}B_{19,20}(X) - 1.00661\cdot10^{12}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.160175, 0.166686\}$$

Intersection intervals with the x axis:

$$[0.160175, 0.166686]$$

Longest intersection interval: 0.00651098
$\Longrightarrow$ Selective recursion: interval 1: $[15.2176, 15.2183]$,

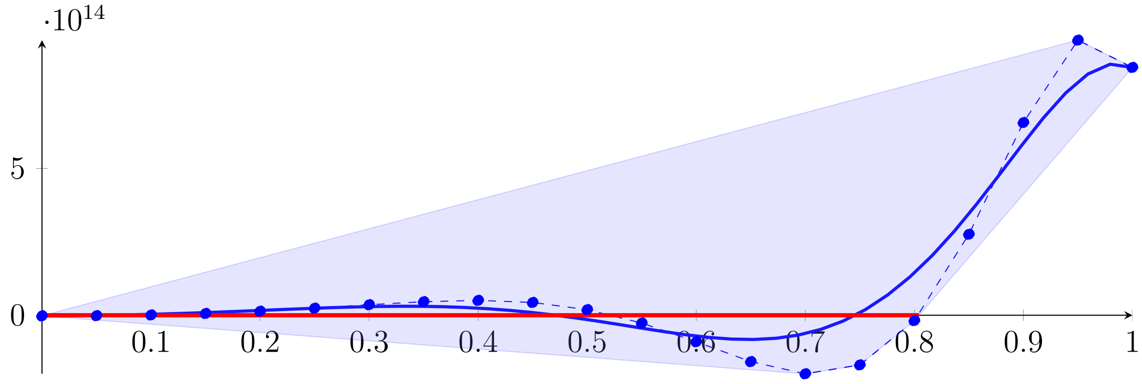
1.77 Recursion Branch 1 2 1 1 2 2 1 1 in Interval 1: $[15.2176, 15.2183]$

Found root in interval $[15.2176, 15.2183]$ at recursion depth 8!

1.78 Recursion Branch 1 2 1 2 on the Second Half $[15.625, 18.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = {} & 7.88858\cdot 10^{9} X^{20} + 2.58746\cdot 10^{11} X^{19} + 3.76268\cdot 10^{12} X^{18} + 3.17389\cdot 10^{13} X^{17} + 1.69708\cdot 10^{14} X^{16} \\
& + 5.82695\cdot 10^{14} X^{15} + 1.18664\cdot 10^{15} X^{14} + 8.38279\cdot 10^{14} X^{13} - 2.32497\cdot 10^{15} X^{12} - 7.06233\cdot 10^{15} X^{11} \\
& - 6.4407\cdot 10^{15} X^{10} + 3.31615\cdot 10^{15} X^{9} + 1.18856\cdot 10^{16} X^{8} + 7.3503\cdot 10^{15} X^{7} - 3.10022\cdot 10^{15} X^{6} - 5.3941 \\
& \cdot 10^{15} X^{5} - 1.29591\cdot 10^{15} X^{4} + 7.44661\cdot 10^{14} X^{3} + 3.40631\cdot 10^{14} X^{2} + 1.3915\cdot 10^{13} X - 2.1354\cdot 10^{12} \\
= {} & -2.1354\cdot 10^{12} B_{0,20}(X) - 1.43965\cdot 10^{12} B_{1,20}(X) + 1.04889\cdot 10^{12} B_{2,20}(X) + 5.98344 \\
& \cdot 10^{12} B_{3,20}(X) + 1.37497\cdot 10^{13} B_{4,20}(X) + 2.41181\cdot 10^{13} B_{5,20}(X) + 3.58157\cdot 10^{13} B_{6,20}(X) \\
& + 4.61131\cdot 10^{13} B_{7,20}(X) + 5.06156\cdot 10^{13} B_{8,20}(X) + 4.35612\cdot 10^{13} B_{9,20}(X) + 1.90286 \\
& \cdot 10^{13} B_{10,20}(X) - 2.65368\cdot 10^{13} B_{11,20}(X) - 9.02907\cdot 10^{13} B_{12,20}(X) - 1.57924\cdot 10^{14} B_{13,20}(X) \\
& - 1.99362\cdot 10^{14} B_{14,20}(X) - 1.69182\cdot 10^{14} B_{15,20}(X) - 1.82426\cdot 10^{13} B_{16,20}(X) + 2.75733 \\
& \cdot 10^{14} B_{17,20}(X) + 6.56245\cdot 10^{14} B_{18,20}(X) + 9.36841\cdot 10^{14} B_{19,20}(X) + 8.43944\cdot 10^{14} B_{20,20}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.00216047, 0.804232\}$$

Intersection intervals with the x axis:

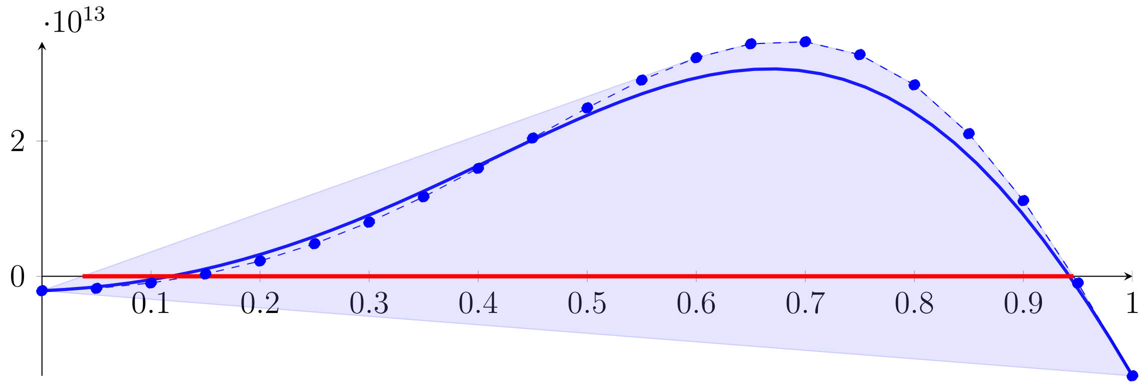
$$[0.00216047, 0.804232]$$

Longest intersection interval: 0.802071
$\Longrightarrow$ Bisection: first half $[15.625, 17.1875]$ und second half $[17.1875, 18.75]$

1.79 Recursion Branch 1 2 1 2 1 on the First Half $[15.625, 17.1875]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -12255.1X^{20} + 678571X^{19} + 1.39217\cdot10^{7}X^{18} + 2.45056\cdot10^{8}X^{17} + 2.57809\cdot10^{9}X^{16} + 1.77899 \\ & \cdot10^{10}X^{15} + 7.24246\cdot10^{10}X^{14} + 1.02329\cdot10^{11}X^{13} - 5.67624\cdot10^{11}X^{12} - 3.4484\cdot10^{12}X^{11} - 6.28975 \\ & \cdot10^{12}X^{10} + 6.47685\cdot10^{12}X^{9} + 4.6428\cdot10^{13}X^{8} + 5.74242\cdot10^{13}X^{7} - 4.8441\cdot10^{13}X^{6} - 1.68566\cdot10^{14}X^{5} \\ & - 8.09942\cdot10^{13}X^{4} + 9.30826\cdot10^{13}X^{3} + 8.51578\cdot10^{13}X^{2} + 6.95749\cdot10^{12}X - 2.1354\cdot10^{12} \\ = & -2.1354\cdot10^{12}B_{0,20}(X) - 1.78753\cdot10^{12}B_{1,20}(X) - 9.91453\cdot10^{11}B_{2,20}(X) + 3.34471 \\ & \cdot10^{11}B_{3,20}(X) + 2.25518\cdot10^{12}B_{4,20}(X) + 4.80802\cdot10^{12}B_{5,20}(X) + 7.99062\cdot10^{12}B_{6,20}(X) \\ & + 1.17483\cdot10^{13}B_{7,20}(X) + 1.59619\cdot10^{13}B_{8,20}(X) + 2.04381\cdot10^{13}B_{9,20}(X) + 2.49034 \\ & \cdot10^{13}B_{10,20}(X) + 2.90046\cdot10^{13}B_{11,20}(X) + 3.2318\cdot10^{13}B_{12,20}(X) + 3.43704\cdot10^{13}B_{13,20}(X) \\ & + 3.46731\cdot10^{13}B_{14,20}(X) + 3.27707\cdot10^{13}B_{15,20}(X) + 2.83048\cdot10^{13}B_{16,20}(X) + 2.10885 \\ & \cdot10^{13}B_{17,20}(X) + 1.11867\cdot10^{13}B_{18,20}(X) - 1.00816\cdot10^{12}B_{19,20}(X) - 1.47196\cdot10^{13}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.0371877, 0.945866\}$$

Intersection intervals with the x axis:

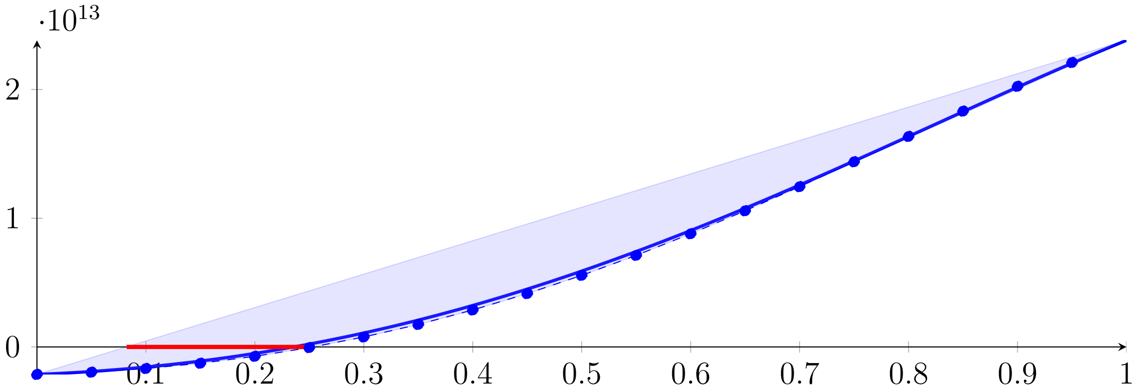
$$[0.0371877, 0.945866]$$

Longest intersection interval: 0.908679
$\Longrightarrow$ Bisection: first half $[15.625, 16.4062]$ und second half $[16.4062, 17.1875]$

1.80 Recursion Branch 1 2 1 2 1 1 on the First Half $[15.625, 16.4062]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -3857.86X^{20} + 47087.4X^{19} - 77088.1X^{18} + 646946X^{17} - 2.21154\cdot 10^{6}X^{16} + 1.86624\cdot 10^{6}X^{15} \\
& + 4.12701\cdot 10^{6}X^{14} + 1.25716\cdot 10^{7}X^{13} - 1.39021\cdot 10^{8}X^{12} - 1.68356\cdot 10^{9}X^{11} - 6.14235\cdot 10^{9}X^{10} \\
& + 1.26501\cdot 10^{10}X^{9} + 1.81359\cdot 10^{11}X^{8} + 4.48627\cdot 10^{11}X^{7} - 7.5689\cdot 10^{11}X^{6} - 5.26768\cdot 10^{12}X^{5} \\
& - 5.06214\cdot 10^{12}X^{4} + 1.16353\cdot 10^{13}X^{3} + 2.12894\cdot 10^{13}X^{2} + 3.47874\cdot 10^{12}X - 2.1354\cdot 10^{12} \\
= & -2.1354\cdot 10^{12}B_{0,20}(X) - 1.96146\cdot 10^{12}B_{1,20}(X) - 1.67548\cdot 10^{12}B_{2,20}(X) - 1.26723 \\
& \cdot 10^{12}B_{3,20}(X) - 7.27573\cdot 10^{11}B_{4,20}(X) - 4.87171\cdot 10^{10}B_{5,20}(X) + 7.75367\cdot 10^{11}B_{6,20}(X) \\
& + 1.74859\cdot 10^{12}B_{7,20}(X) + 2.87239\cdot 10^{12}B_{8,20}(X) + 4.14529\cdot 10^{12}B_{9,20}(X) + 5.56261 \\
& \cdot 10^{12}B_{10,20}(X) + 7.11607\cdot 10^{12}B_{11,20}(X) + 8.79353\cdot 10^{12}B_{12,20}(X) + 1.05787\cdot 10^{13}B_{13,20}(X) \\
& + 1.2451\cdot 10^{13}B_{14,20}(X) + 1.43854\cdot 10^{13}B_{15,20}(X) + 1.63524\cdot 10^{13}B_{16,20}(X) + 1.83185 \\
& \cdot 10^{13}B_{17,20}(X) + 2.02458\cdot 10^{13}B_{18,20}(X) + 2.20932\cdot 10^{13}B_{19,20}(X) + 2.38161\cdot 10^{13}B_{20,20}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.0822843, 0.252956\}$$

Intersection intervals with the x axis:

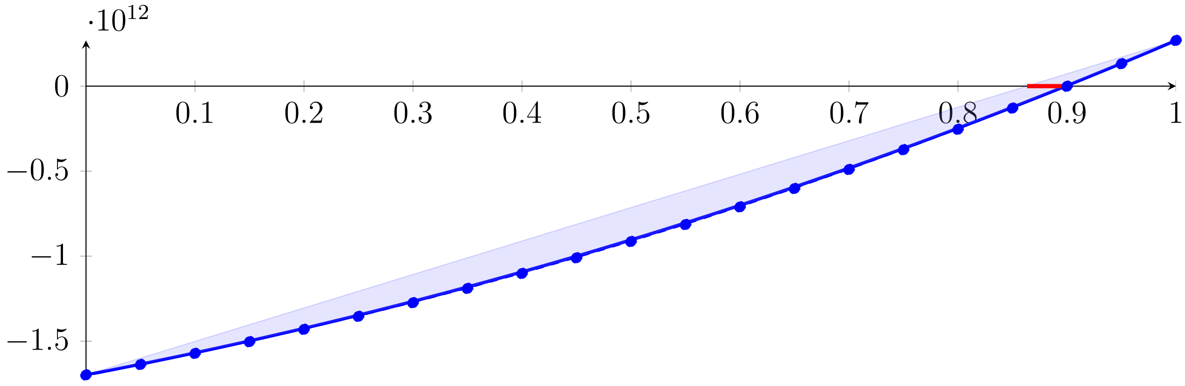
$$[0.0822843, 0.252956]$$

Longest intersection interval: 0.170672
$\implies$ Selective recursion: interval 1: $[15.6893, 15.8226]$,

1.81 Recursion Branch 1 2 1 2 1 1 1 in Interval 1: $[15.6893, 15.8226]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & \, 1004.35X^{20} - 3334.23X^{19} + 35109.3X^{18} - 108639X^{17} + 684378X^{16} - 525508X^{15} \\
& + 227583X^{14} + 131988X^{13} + 608752X^{12} + 108173X^{11} + 189537X^{10} + 27473.9X^{9} \\
& + 136119X^{8} + 2.40702\cdot 10^{6}X^{7} - 1.14562\cdot 10^{7}X^{6} - 8.06872\cdot 10^{8}X^{5} - 6.19132 \\
& \cdot 10^{9}X^{4} + 4.77501\cdot 10^{10}X^{3} + 6.96941\cdot 10^{11}X^{2} + 1.22989\cdot 10^{12}X - 1.69878\cdot 10^{12} \\
= & -1.69878\cdot 10^{12}B_{0,20}(X) - 1.63729\cdot 10^{12}B_{1,20}(X) - 1.57212\cdot 10^{12}B_{2,20}(X) - 1.50325 \\
& \cdot 10^{12}B_{3,20}(X) - 1.43063\cdot 10^{12}B_{4,20}(X) - 1.35422\cdot 10^{12}B_{5,20}(X) - 1.27397\cdot 10^{12}B_{6,20}(X) \\
& - 1.18987\cdot 10^{12}B_{7,20}(X) - 1.10187\cdot 10^{12}B_{8,20}(X) - 1.00993\cdot 10^{12}B_{9,20}(X) - 9.14028 \\
& \cdot 10^{11}B_{10,20}(X) - 8.14132\cdot 10^{11}B_{11,20}(X) - 7.10213\cdot 10^{11}B_{12,20}(X) - 6.02244\cdot 10^{11}B_{13,20}(X) \\
& - 4.902\cdot 10^{11}B_{14,20}(X) - 3.74059\cdot 10^{11}B_{15,20}(X) - 2.53798\cdot 10^{11}B_{16,20}(X) - 1.29399 \\
& \cdot 10^{11}B_{17,20}(X) - 8.44695\cdot 10^{8}B_{18,20}(X) + 1.3188\cdot 10^{11}B_{19,20}(X) + 2.68788\cdot 10^{11}B_{20,20}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.863391, 0.900318\}$$

Intersection intervals with the x axis:

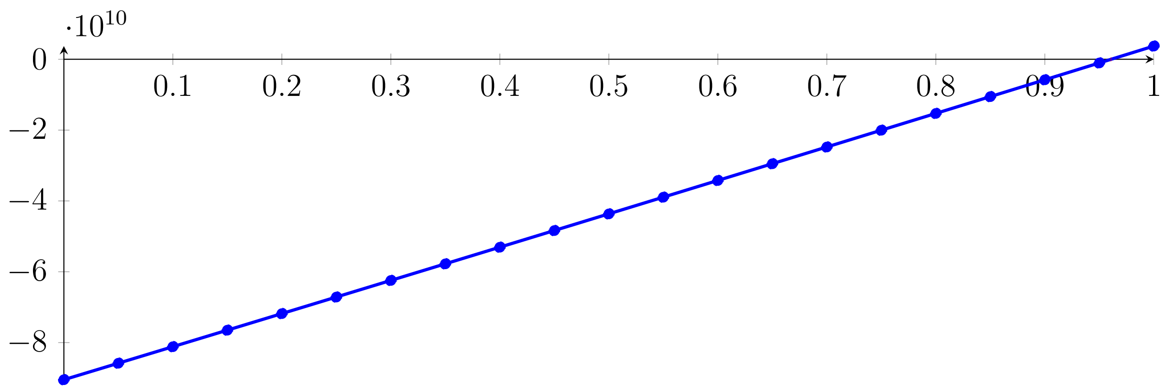
$$[0.863391, 0.900318]$$

Longest intersection interval: 0.0369277
$\Longrightarrow$ Selective recursion: interval 1: $[15.8044, 15.8093]$,

1.82 Recursion Branch 1 2 1 2 1 1 1 1 in Interval 1: $[15.8044, 15.8093]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = {} & 48.0636X^{20} - 160.153X^{19} + 1704.87X^{18} - 5195.49X^{17} + 33645.7X^{16} \\ & - 27791.8X^{15} + 13203.7X^{14} + 6031.41X^{13} + 31086.9X^{12} + 4907.89X^{11} \\ & + 10027.7X^{10} + 1401.89X^{9} + 163.383X^{8} - 5.91431X^{7} + 52.6373X^{6} - 57.0139X^{5} \\ & - 18116.9X^{4} + 1.02003\cdot 10^{6}X^{3} + 1.0741\cdot 10^{9}X^{2} + 9.31289\cdot 10^{10}X - 9.04784\cdot 10^{10} \\ = {} & -9.04784\cdot 10^{10}B_{0,20}(X) - 8.58219\cdot 10^{10}B_{1,20}(X) - 8.11598\cdot 10^{10}B_{2,20}(X) - 7.64921 \\ & \cdot 10^{10}B_{3,20}(X) - 7.18187\cdot 10^{10}B_{4,20}(X) - 6.71396\cdot 10^{10}B_{5,20}(X) - 6.24549\cdot 10^{10}B_{6,20}(X) \\ & - 5.77645\cdot 10^{10}B_{7,20}(X) - 5.30685\cdot 10^{10}B_{8,20}(X) - 4.83668\cdot 10^{10}B_{9,20}(X) - 4.36595 \\ & \cdot 10^{10}B_{10,20}(X) - 3.89464\cdot 10^{10}B_{11,20}(X) - 3.42278\cdot 10^{10}B_{12,20}(X) - 2.95034\cdot 10^{10}B_{13,20}(X) \\ & - 2.47734\cdot 10^{10}B_{14,20}(X) - 2.00378\cdot 10^{10}B_{15,20}(X) - 1.52964\cdot 10^{10}B_{16,20}(X) - 1.05494 \\ & \cdot 10^{10}B_{17,20}(X) - 5.79676\cdot 10^{9}B_{18,20}(X) - 1.03842\cdot 10^{9}B_{19,20}(X) + 3.72558\cdot 10^{9}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.960452, 0.960899\}$$

Intersection intervals with the x axis:

$$[0.960452, 0.960899]$$

Longest intersection interval: 0.000446654
$\Longrightarrow$ Selective recursion: interval 1: $[15.8091, 15.8091]$,

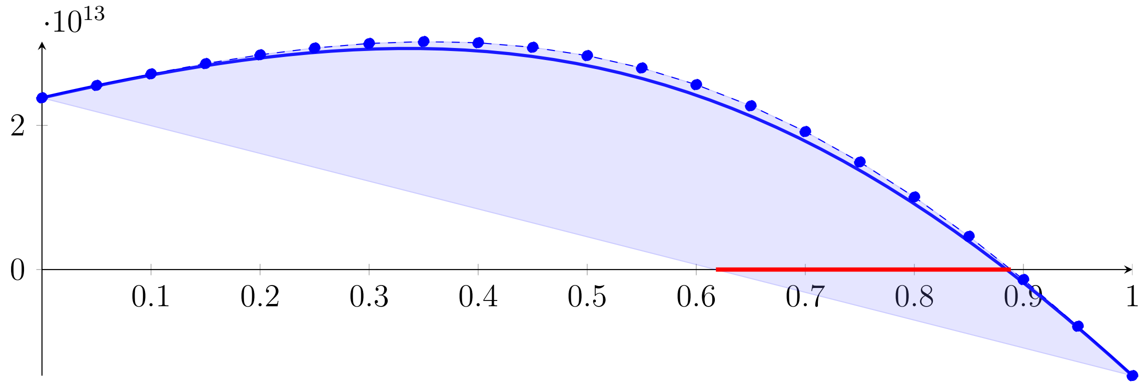
1.83 Recursion Branch 1 2 1 2 1 1 1 1 1 in Interval 1: $[15.8091, 15.8091]$

Found root in interval $[15.8091, 15.8091]$ at recursion depth 9!

1.84 Recursion Branch 1 2 1 2 1 2 on the Second Half $[16.4062, 17.1875]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -29580.3X^{20} + 141227X^{19} - 941631X^{18} + 3.42622\cdot10^{6}X^{17} - 1.97822\cdot10^{7}X^{16} + 1.69868\cdot10^{7}X^{15} \\ & + 1.26109\cdot10^{7}X^{14} + 1.56674\cdot10^{8}X^{13} + 7.41812\cdot10^{8}X^{12} + 1.73358\cdot10^{8}X^{11} - 2.38329\cdot10^{10}X^{10} \\ & - 1.50872\cdot10^{11}X^{9} - 2.94218\cdot10^{11}X^{8} + 9.93008\cdot10^{11}X^{7} + 6.36609\cdot10^{12}X^{6} + 8.95339\cdot10^{12}X^{5} \\ & - 1.46628\cdot10^{13}X^{4} - 5.05471\cdot10^{13}X^{3} - 2.36296\cdot10^{13}X^{2} + 3.44591\cdot10^{13}X + 2.38161\cdot10^{13} \\ = & \; 2.38161\cdot10^{13}B_{0,20}(X) + 2.55391\cdot10^{13}B_{1,20}(X) + 2.71376\cdot10^{13}B_{2,20}(X) + 2.85675 \\ & \cdot10^{13}B_{3,20}(X) + 2.97813\cdot10^{13}B_{4,20}(X) + 3.07293\cdot10^{13}B_{5,20}(X) + 3.13598\cdot10^{13}B_{6,20}(X) \\ & + 3.16206\cdot10^{13}B_{7,20}(X) + 3.14597\cdot10^{13}B_{8,20}(X) + 3.08267\cdot10^{13}B_{9,20}(X) + 2.96743 \\ & \cdot10^{13}B_{10,20}(X) + 2.79602\cdot10^{13}B_{11,20}(X) + 2.56485\cdot10^{13}B_{12,20}(X) + 2.27123\cdot10^{13}B_{13,20}(X) \\ & + 1.91355\cdot10^{13}B_{14,20}(X) + 1.49152\cdot10^{13}B_{15,20}(X) + 1.00642\cdot10^{13}B_{16,20}(X) + 4.61307 \\ & \cdot10^{12}B_{17,20}(X) - 1.38731\cdot10^{12}B_{18,20}(X) - 7.8639\cdot10^{12}B_{19,20}(X) - 1.47196\cdot10^{13}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.618027, 0.88844\}$$

Intersection intervals with the x axis:

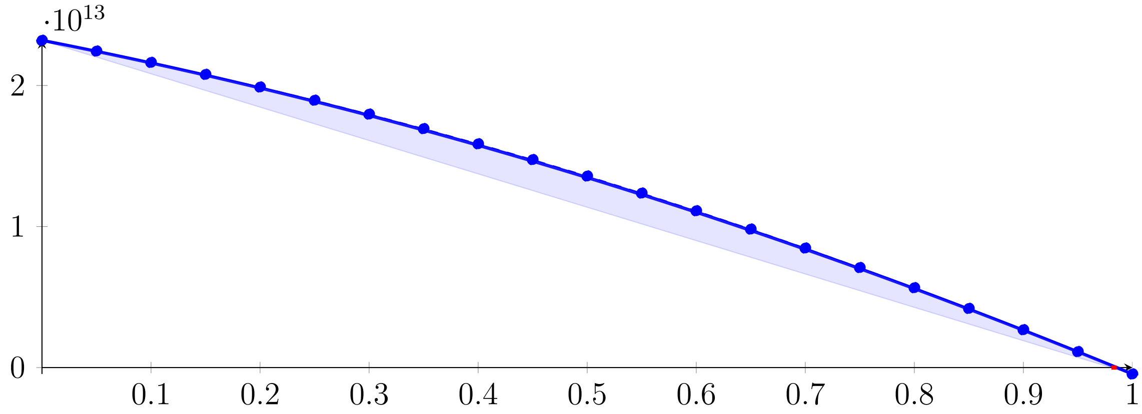
$$[0.618027, 0.88844]$$

Longest intersection interval: 0.270413
$\Longrightarrow$ Selective recursion: interval 1: $[16.8891, 17.1003]$,

1.85 Recursion Branch 1 2 1 2 1 2 1 in Interval 1: $[16.8891, 17.1003]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -14689.5X^{20} + 54430.9X^{19} - 485554X^{18} + 1.63139\cdot10^{6}X^{17} - 9.85253\cdot10^{6}X^{16} + 7.51145 \\ & \cdot10^{6}X^{15} - 3.36909\cdot10^{6}X^{14} - 1.73572\cdot10^{6}X^{13} - 8.67422\cdot10^{6}X^{12} - 1.24658\cdot10^{6}X^{11} - 2.53173 \\ & \cdot10^{6}X^{10} - 2.23334\cdot10^{6}X^{9} - 4.1761\cdot10^{7}X^{8} - 3.31058\cdot10^{8}X^{7} + 1.50558\cdot10^{9}X^{6} + 4.82221\cdot10^{10}X^{5} \\ & + 2.81768\cdot10^{11}X^{4} - 3.90688\cdot10^{11}X^{3} - 8.38526\cdot10^{12}X^{2} - 1.52147\cdot10^{13}X + 2.32037\cdot10^{13} \\ = & \; 2.32037\cdot10^{13}B_{0,20}(X) + 2.2443\cdot10^{13}B_{1,20}(X) + 2.16381\cdot10^{13}B_{2,20}(X) + 2.07888 \\ & \cdot10^{13}B_{3,20}(X) + 1.98947\cdot10^{13}B_{4,20}(X) + 1.89556\cdot10^{13}B_{5,20}(X) + 1.79714\cdot10^{13}B_{6,20}(X) \\ & + 1.69419\cdot10^{13}B_{7,20}(X) + 1.58672\cdot10^{13}B_{8,20}(X) + 1.47473\cdot10^{13}B_{9,20}(X) + 1.35823 \\ & \cdot10^{13}B_{10,20}(X) + 1.23725\cdot10^{13}B_{11,20}(X) + 1.11181\cdot10^{13}B_{12,20}(X) + 9.81946\cdot10^{12}B_{13,20}(X) \\ & + 8.47717\cdot10^{12}B_{14,20}(X) + 7.09173\cdot10^{12}B_{15,20}(X) + 5.66382\cdot10^{12}B_{16,20}(X) + 4.19419 \\ & \cdot10^{12}B_{17,20}(X) + 2.68372\cdot10^{12}B_{18,20}(X) + 1.13337\cdot10^{12}B_{19,20}(X) - 4.55762\cdot10^{11}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.980737, 0.98566\}$$

Intersection intervals with the x axis:

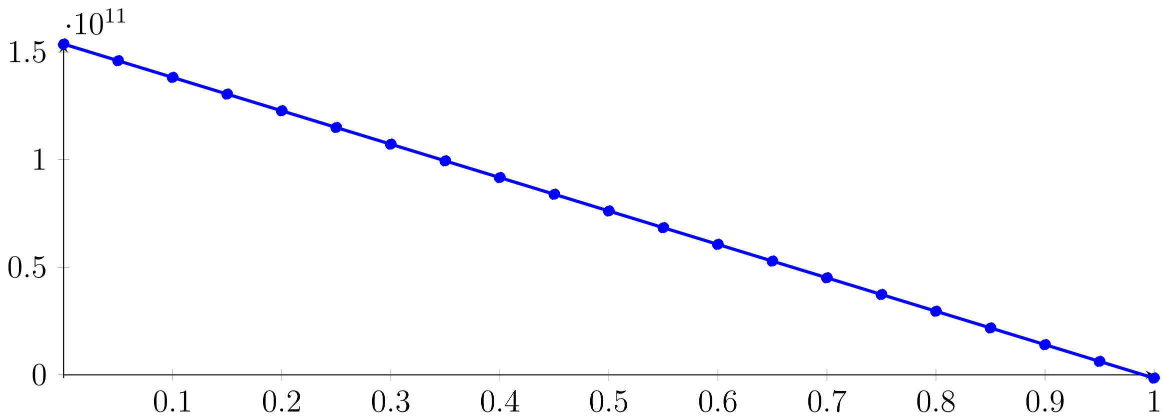
$$[0.980737, 0.98566]$$

Longest intersection interval: 0.00492344
$\Longrightarrow$ Selective recursion: interval 1: $[17.0963, 17.0973]$,

1.86 Recursion Branch 1 2 1 2 1 2 1 1 in Interval 1: $[17.0963, 17.0973]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-79.1494X^{20} + 259.296X^{19} - 2785.79X^{18} + 8457.56X^{17} - 56796.1X^{16} \\ &+ 44104.2X^{15} - 18243.3X^{14} - 10506.2X^{13} - 51048.5X^{12} - 10297.6X^{11} \\ &- 16052.3X^{10} - 2178.44X^{9} - 365.208X^{8} - 21.2915X^{7} - 98.1775X^{6} - 0.473145X^{5} \\ &+ 307.692X^{4} + 142490X^{3} - 1.80354 \cdot 10^{8}X^{2} - 1.55076 \cdot 10^{11}X + 1.53737 \cdot 10^{11} \\ = &\ 1.53737 \cdot 10^{11}B_{0,20}(X) + 1.45983 \cdot 10^{11}B_{1,20}(X) + 1.38228 \cdot 10^{11}B_{2,20}(X) + 1.30473 \\ &\cdot 10^{11}B_{3,20}(X) + 1.22716 \cdot 10^{11}B_{4,20}(X) + 1.14958 \cdot 10^{11}B_{5,20}(X) + 1.072 \cdot 10^{11}B_{6,20}(X) \\ &+ 9.94403 \cdot 10^{10}B_{7,20}(X) + 9.16799 \cdot 10^{10}B_{8,20}(X) + 8.39185 \cdot 10^{10}B_{9,20}(X) + 7.61562 \\ &\cdot 10^{10}B_{10,20}(X) + 6.83929 \cdot 10^{10}B_{11,20}(X) + 6.06287 \cdot 10^{10}B_{12,20}(X) + 5.28635 \cdot 10^{10}B_{13,20}(X) \\ &+ 4.50974 \cdot 10^{10}B_{14,20}(X) + 3.73304 \cdot 10^{10}B_{15,20}(X) + 2.95624 \cdot 10^{10}B_{16,20}(X) + 2.17934 \\ &\cdot 10^{10}B_{17,20}(X) + 1.40235 \cdot 10^{10}B_{18,20}(X) + 6.25264 \cdot 10^{9}B_{19,20}(X) - 1.51916 \cdot 10^{9}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.990215, 0.990226\}$$

Intersection intervals with the x axis:

$$[0.990215, 0.990226]$$

Longest intersection interval: $1.13355 \cdot 10^{-05}$
$\Longrightarrow$ Selective recursion: interval 1: $[17.0973, 17.0973]$,

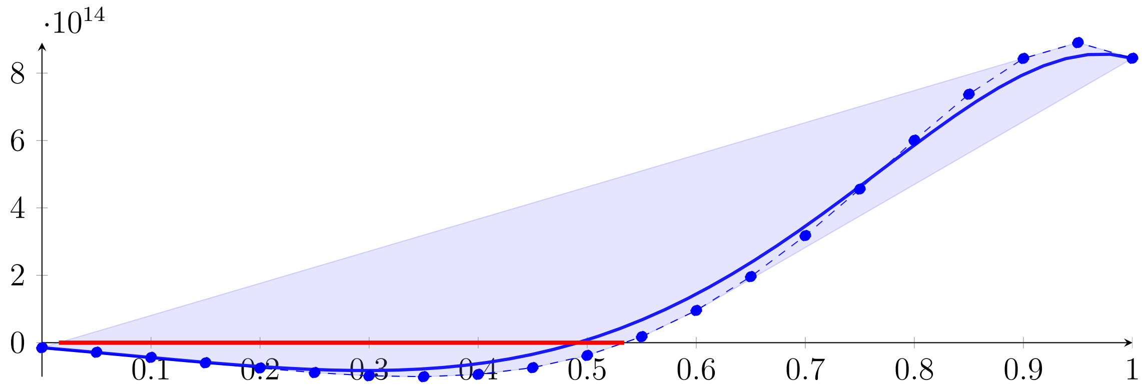
1.87 Recursion Branch 1 2 1 2 1 2 1 1 1 in Interval 1: $[17.0973, 17.0973]$

Found root in interval $[17.0973, 17.0973]$ at recursion depth 9!

1.88 Recursion Branch 1 2 1 2 2 on the Second Half $[17.1875, 18.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 51309.5X^{20} + 1.13609 \cdot 10^{6} X^{19} + 2.62513 \cdot 10^{7} X^{18} + 5.89696 \cdot 10^{8} X^{17} + 9.46063 \cdot 10^{9} X^{16} + 1.05848 \\ &\quad \cdot 10^{11} X^{15} + 8.64537 \cdot 10^{11} X^{14} + 5.14688 \cdot 10^{12} X^{13} + 2.19482 \cdot 10^{13} X^{12} + 6.31615 \cdot 10^{13} X^{11} + 9.96023 \\ &\quad \cdot 10^{13} X^{10} - 2.75387 \cdot 10^{13} X^{9} - 5.24772 \cdot 10^{14} X^{8} - 1.10687 \cdot 10^{15} X^{7} - 7.34813 \cdot 10^{14} X^{6} + 8.78049 \\ &\quad \cdot 10^{14} X^{5} + 1.86093 \cdot 10^{15} X^{4} + 8.85216 \cdot 10^{14} X^{3} - 2.8815 \cdot 10^{14} X^{2} - 2.74229 \cdot 10^{14} X - 1.47196 \cdot 10^{13} \\ &= -1.47196 \cdot 10^{13} B_{0,20}(X) - 2.84311 \cdot 10^{13} B_{1,20}(X) - 4.36591 \cdot 10^{13} B_{2,20}(X) - 5.96272 \\ &\quad \cdot 10^{13} B_{3,20}(X) - 7.51748 \cdot 10^{13} B_{4,20}(X) - 8.87006 \cdot 10^{13} B_{5,20}(X) - 9.81247 \cdot 10^{13} B_{6,20}(X) \\ &\quad - 1.00885 \cdot 10^{14} B_{7,20}(X) - 9.39824 \cdot 10^{13} B_{8,20}(X) - 7.41057 \cdot 10^{13} B_{9,20}(X) - 3.78514 \\ &\quad \cdot 10^{13} B_{10,20}(X) + 1.7921 \cdot 10^{13} B_{11,20}(X) + 9.55764 \cdot 10^{13} B_{12,20}(X) + 1.96007 \cdot 10^{14} B_{13,20}(X) \\ &\quad + 3.17738 \cdot 10^{14} B_{14,20}(X) + 4.5586 \cdot 10^{14} B_{15,20}(X) + 6.00841 \cdot 10^{14} B_{16,20}(X) + 7.37367 \\ &\quad \cdot 10^{14} B_{17,20}(X) + 8.43467 \cdot 10^{14} B_{18,20}(X) + 8.90392 \cdot 10^{14} B_{19,20}(X) + 8.43944 \cdot 10^{14} B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.0154368, 0.533934\}$$

Intersection intervals with the x axis:

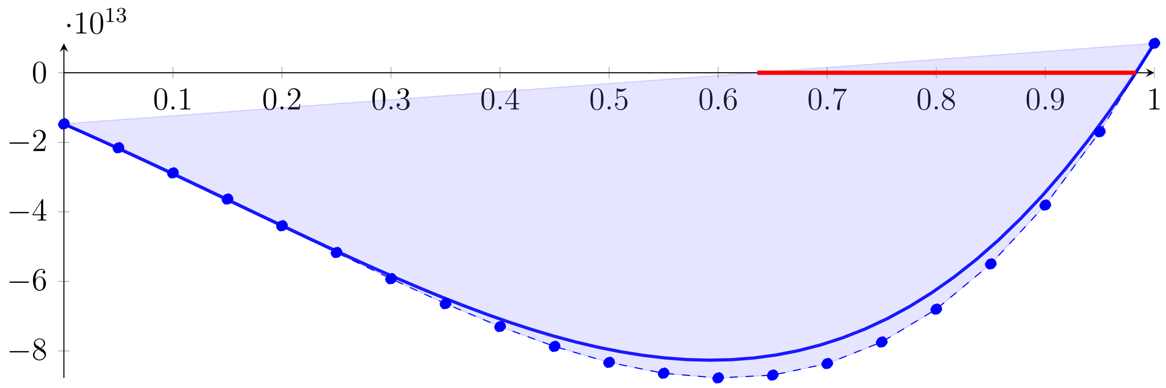
$$[0.0154368, 0.533934]$$

Longest intersection interval: 0.518497
$\Longrightarrow$ Bisection: first half $[17.1875, 17.9688]$ und second half $[17.9688, 18.75]$

1.89 Recursion Branch 1 2 1 2 2 1 on the First Half $[17.1875, 17.9688]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 77549.2X^{20} - 481986X^{19} + 1.96467{\cdot}10^{6}X^{18} - 9.60214{\cdot}10^{6}X^{17} + 4.74973{\cdot}10^{7}X^{16} - 3.11619{\cdot}10^{7}X^{15} + 6.44235{\cdot}10^{7}X^{14} + 6.32879{\cdot}10^{8}X^{13} + 5.38653{\cdot}10^{9}X^{12} + 3.08428{\cdot}10^{10}X^{11} + 9.72751{\cdot}10^{10}X^{10} - 5.37861{\cdot}10^{10}X^{9} - 2.04989{\cdot}10^{12}X^{8} - 8.64744{\cdot}10^{12}X^{7} - 1.14815{\cdot}10^{13}X^{6} + 2.7439{\cdot}10^{13}X^{5} + 1.16308{\cdot}10^{14}X^{4} + 1.10652{\cdot}10^{14}X^{3} - 7.20374{\cdot}10^{13}X^{2} - 1.37115{\cdot}10^{14}X - 1.47196{\cdot}10^{13}$$
$$= -1.47196{\cdot}10^{13}B_{0,20}(X) - 2.15754{\cdot}10^{13}B_{1,20}(X) - 2.88102{\cdot}10^{13}B_{2,20}(X) - 3.63272{\cdot}10^{13}B_{3,20}(X) - 4.40052{\cdot}10^{13}B_{4,20}(X) - 5.16973{\cdot}10^{13}B_{5,20}(X) - 5.92295{\cdot}10^{13}B_{6,20}(X) - 6.63994{\cdot}10^{13}B_{7,20}(X) - 7.29757{\cdot}10^{13}B_{8,20}(X) - 7.86984{\cdot}10^{13}B_{9,20}(X) - 8.328{\cdot}10^{13}B_{10,20}(X) - 8.6407{\cdot}10^{13}B_{11,20}(X) - 8.77436{\cdot}10^{13}B_{12,20}(X) - 8.69362{\cdot}10^{13}B_{13,20}(X) - 8.36193{\cdot}10^{13}B_{14,20}(X) - 7.74242{\cdot}10^{13}B_{15,20}(X) - 6.79884{\cdot}10^{13}B_{16,20}(X) - 5.49683{\cdot}10^{13}B_{17,20}(X) - 3.80537{\cdot}10^{13}B_{18,20}(X) - 1.69845{\cdot}10^{13}B_{19,20}(X) + 8.42928{\cdot}10^{12}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.635867, 0.983416\}$$

Intersection intervals with the x axis:

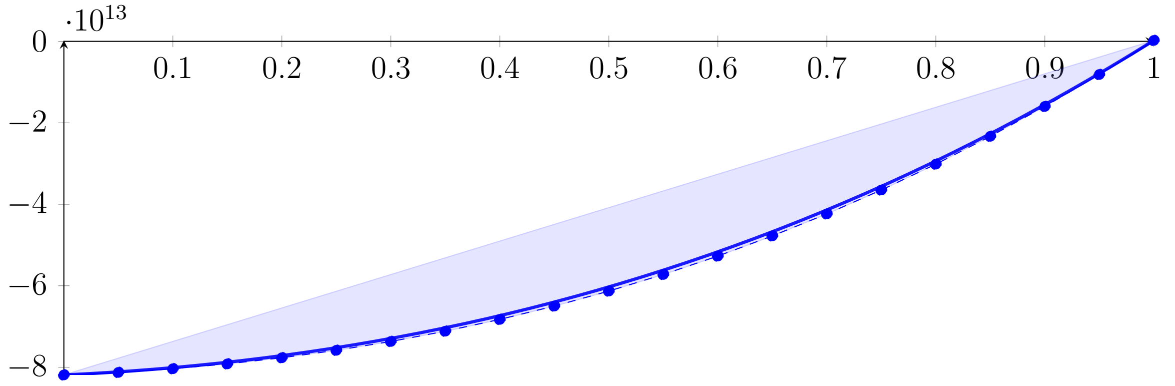
$$[0.635867, 0.983416]$$

Longest intersection interval: 0.347549
$\Longrightarrow$ Selective recursion: interval 1: $[17.6843, 17.9558]$,

1.90 Recursion Branch 1 2 1 2 2 1 1 in Interval 1: $[17.6843, 17.9558]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 64610.2X^{20} - 299553X^{19} + 2.10703{\cdot}10^{6}X^{18} - 7.18987{\cdot}10^{6}X^{17} + 4.23824{\cdot}10^{7}X^{16} - 3.34484{\cdot}10^{7}X^{15} + 1.60624{\cdot}10^{7}X^{14} + 3.68947{\cdot}10^{6}X^{13} + 3.6551{\cdot}10^{7}X^{12} + 5.15952{\cdot}10^{6}X^{11} + 2.53173{\cdot}10^{7}X^{10} + 1.22627{\cdot}10^{8}X^{9} + 2.66226{\cdot}10^{8}X^{8} - 8.98715{\cdot}10^{9}X^{7} - 1.21855{\cdot}10^{11}X^{6} - 5.91862{\cdot}10^{11}X^{5} + 4.93055{\cdot}10^{11}X^{4} + 1.66798{\cdot}10^{13}X^{3} + 5.32673{\cdot}10^{13}X^{2} + 1.24031{\cdot}10^{13}X - 8.18949{\cdot}10^{13}$$
$$= -8.18949{\cdot}10^{13}B_{0,20}(X) - 8.12747{\cdot}10^{13}B_{1,20}(X) - 8.03742{\cdot}10^{13}B_{2,20}(X) - 7.91787{\cdot}10^{13}B_{3,20}(X) - 7.76735{\cdot}10^{13}B_{4,20}(X) - 7.58438{\cdot}10^{13}B_{5,20}(X) - 7.36747{\cdot}10^{13}B_{6,20}(X) - 7.11515{\cdot}10^{13}B_{7,20}(X) - 6.82595{\cdot}10^{13}B_{8,20}(X) - 6.4984{\cdot}10^{13}B_{9,20}(X) - 6.13106{\cdot}10^{13}B_{10,20}(X) - 5.72251{\cdot}10^{13}B_{11,20}(X) - 5.27136{\cdot}10^{13}B_{12,20}(X) - 4.77627{\cdot}10^{13}B_{13,20}(X) - 4.23591{\cdot}10^{13}B_{14,20}(X) - 3.64903{\cdot}10^{13}B_{15,20}(X) - 3.01444{\cdot}10^{13}B_{16,20}(X) - 2.33099{\cdot}10^{13}B_{17,20}(X) - 1.59763{\cdot}10^{13}B_{18,20}(X) - 8.13394{\cdot}10^{12}B_{19,20}(X) + 2.26021{\cdot}10^{11}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.997248, 0.998648\}$$

Intersection intervals with the x axis:

$$[0.997248, 0.998648]$$

Longest intersection interval: 0.00140049
$\Longrightarrow$ Selective recursion: interval 1: $[17.955, 17.9554]$,

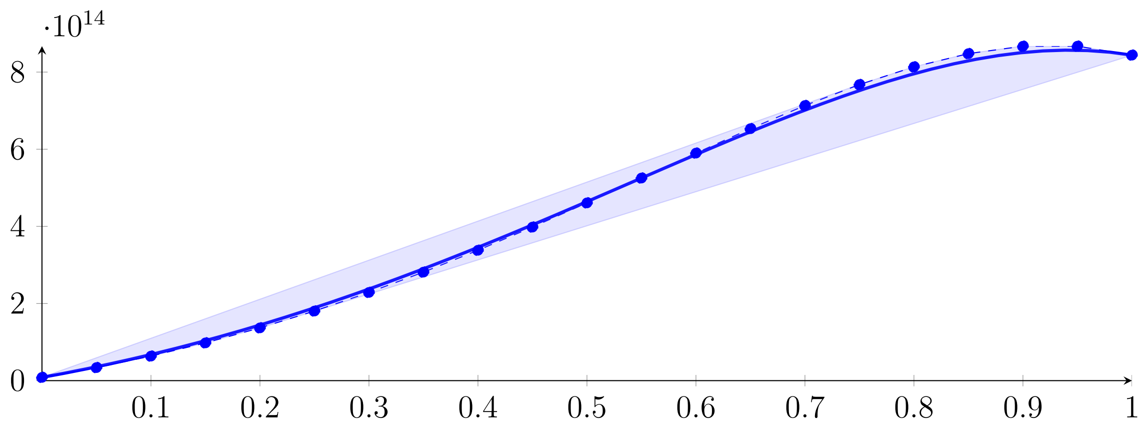
1.91 Recursion Branch 1 2 1 2 2 1 1 1 in Interval 1: $[17.955, 17.9554]$

Found root in interval $[17.955, 17.9554]$ at recursion depth 8!

1.92 Recursion Branch 1 2 1 2 2 2 on the Second Half $[17.9688, 18.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -406735X^{20} + 3.23454\cdot 10^{6}X^{19} - 9.27253\cdot 10^{6}X^{18} + 5.54286\cdot 10^{7}X^{17} - 2.32388\cdot 10^{8}X^{16} + 1.68297 \\ & \cdot 10^{8}X^{15} + 6.61085\cdot 10^{7}X^{14} + 1.79234\cdot 10^{9}X^{13} + 1.9977\cdot 10^{10}X^{12} + 1.68452\cdot 10^{11}X^{11} + 1.0336 \\ & \cdot 10^{12}X^{10} + 4.36677\cdot 10^{12}X^{9} + 1.0574\cdot 10^{13}X^{8} + 3.92297\cdot 10^{11}X^{7} - 9.30488\cdot 10^{13}X^{6} - 3.00689 \\ & \cdot 10^{14}X^{5} - 3.37885\cdot 10^{14}X^{4} + 2.16815\cdot 10^{14}X^{3} + 8.25489\cdot 10^{14}X^{2} + 5.08276\cdot 10^{14}X + 8.42928\cdot 10^{12} \\ = & \, 8.42928\cdot 10^{12}B_{0,20}(X) + 3.38431\cdot 10^{13}B_{1,20}(X) + 6.36016\cdot 10^{13}B_{2,20}(X) + 9.7895 \\ & \cdot 10^{13}B_{3,20}(X) + 1.36844\cdot 10^{14}B_{4,20}(X) + 1.80479\cdot 10^{14}B_{5,20}(X) + 2.28721\cdot 10^{14}B_{6,20}(X) \\ & + 2.81356\cdot 10^{14}B_{7,20}(X) + 3.38006\cdot 10^{14}B_{8,20}(X) + 3.98106\cdot 10^{14}B_{9,20}(X) + 4.60869 \\ & \cdot 10^{14}B_{10,20}(X) + 5.25254\cdot 10^{14}B_{11,20}(X) + 5.89938\cdot 10^{14}B_{12,20}(X) + 6.53281\cdot 10^{14}B_{13,20}(X) \\ & + 7.13299\cdot 10^{14}B_{14,20}(X) + 7.67635\cdot 10^{14}B_{15,20}(X) + 8.13539\cdot 10^{14}B_{16,20}(X) + 8.47861 \\ & \cdot 10^{14}B_{17,20}(X) + 8.67049\cdot 10^{14}B_{18,20}(X) + 8.67168\cdot 10^{14}B_{19,20}(X) + 8.43944\cdot 10^{14}B_{20,20}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

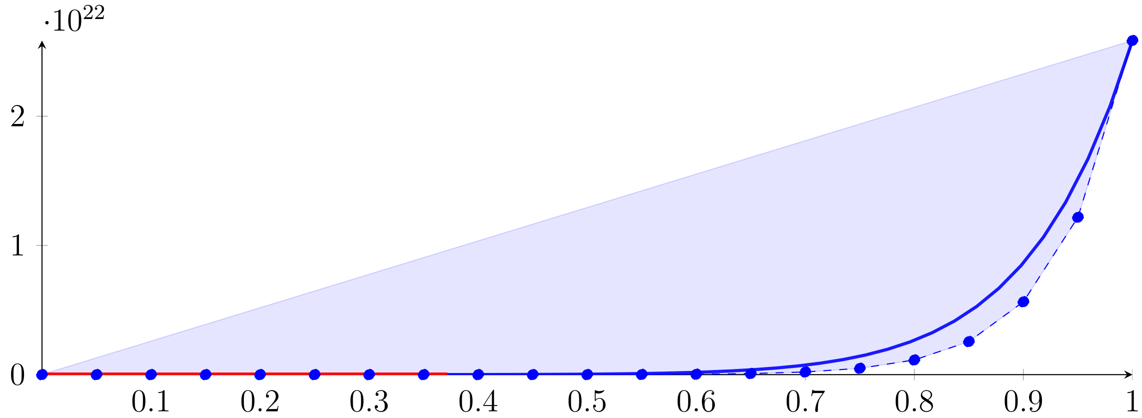
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

1.93 Recursion Branch 1 2 2 on the Second Half $[18.75, 25]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 8.27177 \cdot 10^{15} X^{20} + 2.18376 \cdot 10^{17} X^{19} + 2.66802 \cdot 10^{18} X^{18} + 2.00154 \cdot 10^{19} X^{17} + 1.03147 \cdot 10^{20} X^{16} \\ &\quad + 3.86992 \cdot 10^{20} X^{15} + 1.09286 \cdot 10^{21} X^{14} + 2.36814 \cdot 10^{21} X^{13} + 3.97654 \cdot 10^{21} X^{12} + 5.18646 \cdot 10^{21} X^{11} \\ &\quad + 5.22867 \cdot 10^{21} X^{10} + 4.02002 \cdot 10^{21} X^{9} + 2.29598 \cdot 10^{21} X^{8} + 9.25412 \cdot 10^{20} X^{7} + 2.3318 \cdot 10^{20} X^{6} + 2.12469 \\ &\quad \cdot 10^{19} X^{5} - 6.75399 \cdot 10^{18} X^{4} - 2.49502 \cdot 10^{18} X^{3} - 2.83854 \cdot 10^{17} X^{2} - 3.71586 \cdot 10^{15} X + 8.43944 \cdot 10^{14} \\ &= 8.43944 \cdot 10^{14} B_{0,20}(X) + 6.58151 \cdot 10^{14} B_{1,20}(X) - 1.02161 \cdot 10^{15} B_{2,20}(X) - 6.38396 \\ &\quad \cdot 10^{15} B_{3,20}(X) - 1.90115 \cdot 10^{16} B_{4,20}(X) - 4.25105 \cdot 10^{16} B_{5,20}(X) - 7.31244 \cdot 10^{16} B_{6,20}(X) \\ &\quad - 7.43935 \cdot 10^{16} B_{7,20}(X) + 9.63026 \cdot 10^{16} B_{8,20}(X) + 8.81646 \cdot 10^{17} B_{9,20}(X) + 3.50544 \\ &\quad \cdot 10^{18} B_{10,20}(X) + 1.11134 \cdot 10^{19} B_{11,20}(X) + 3.13849 \cdot 10^{19} B_{12,20}(X) + 8.23454 \cdot 10^{19} B_{13,20}(X) \\ &\quad + 2.04998 \cdot 10^{20} B_{14,20}(X) + 4.9022 \cdot 10^{20} B_{15,20}(X) + 1.13504 \cdot 10^{21} B_{16,20}(X) + 2.55855 \\ &\quad \cdot 10^{21} B_{17,20}(X) + 5.63734 \cdot 10^{21} B_{18,20}(X) + 1.21777 \cdot 10^{22} B_{19,20}(X) + 2.5852 \cdot 10^{22} B_{20,20}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.00342286, 0.371791\}$$

Intersection intervals with the x axis:

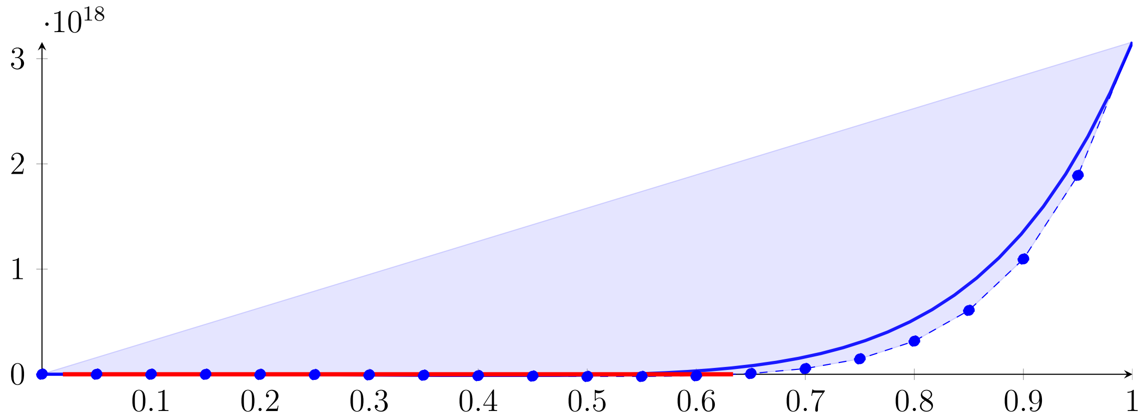
$$[0.00342286, 0.371791]$$

Longest intersection interval: 0.368368
$\Longrightarrow$ Selective recursion: interval 1: $[18.7714, 21.0737]$,

1.94 Recursion Branch 1 2 2 1 in Interval 1: $[18.7714, 21.0737]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 2.32734{\cdot}10^{7}X^{20} + 1.14125{\cdot}10^{9}X^{19} + 4.1768{\cdot}10^{10}X^{18} + 8.5231{\cdot}10^{11}X^{17} + 1.20004{\cdot}10^{13}X^{16} + 1.22531 {\cdot}10^{14}X^{15} + 9.42755{\cdot}10^{14}X^{14} + 5.56754{\cdot}10^{15}X^{13} + 2.54904{\cdot}10^{16}X^{12} + 9.07025{\cdot}10^{16}X^{11} + 2.49682 {\cdot}10^{17}X^{10} + 5.2485{\cdot}10^{17}X^{9} + 8.21375{\cdot}10^{17}X^{8} + 9.11198{\cdot}10^{17}X^{7} + 6.39938{\cdot}10^{17}X^{6} + 1.78176 {\cdot}10^{17}X^{5} - 1.16888{\cdot}10^{17}X^{4} - 1.29204{\cdot}10^{17}X^{3} - 4.20574{\cdot}10^{16}X^{2} - 2.11731{\cdot}10^{15}X + 8.27799{\cdot}10^{14}$$
$$= 8.27799{\cdot}10^{14}B_{0,20}(X) + 7.21933{\cdot}10^{14}B_{1,20}(X) + 3.94712{\cdot}10^{14}B_{2,20}(X) - 2.672 {\cdot}10^{14}B_{3,20}(X) - 1.40127{\cdot}10^{15}B_{4,20}(X) - 3.15758{\cdot}10^{15}B_{5,20}(X) - 5.67088{\cdot}10^{15}B_{6,20}(X) - 9.00424{\cdot}10^{15}B_{7,20}(X) - 1.30463{\cdot}10^{16}B_{8,20}(X) - 1.7334{\cdot}10^{16}B_{9,20}(X) - 2.07589 {\cdot}10^{16}B_{10,20}(X) - 2.1094{\cdot}10^{16}B_{11,20}(X) - 1.42505{\cdot}10^{16}B_{12,20}(X) + 6.87177{\cdot}10^{15}B_{13,20}(X) + 5.41324{\cdot}10^{16}B_{14,20}(X) + 1.46776{\cdot}10^{17}B_{15,20}(X) + 3.15335{\cdot}10^{17}B_{16,20}(X) + 6.07362 {\cdot}10^{17}B_{17,20}(X) + 1.09581{\cdot}10^{18}B_{18,20}(X) + 1.89111{\cdot}10^{18}B_{19,20}(X) + 3.15862{\cdot}10^{18}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.0191738, 0.633733\}$$

Intersection intervals with the x axis:

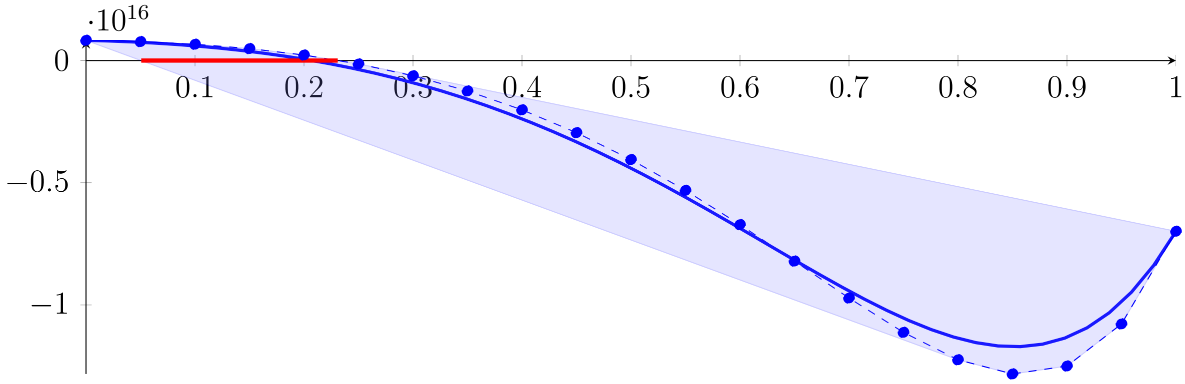
$$[0.0191738, 0.633733]$$

Longest intersection interval: 0.61456
$\Longrightarrow$ Bisection: first half $[18.7714, 19.9225]$ und second half $[19.9225, 21.0737]$

1.95 Recursion Branch 1 2 2 1 1 on the First Half $[18.7714, 19.9225]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 3.28066{\cdot}10^{6}X^{20} - 3.87246{\cdot}10^{7}X^{19} + 6.76014{\cdot}10^{7}X^{18} - 4.65807{\cdot}10^{8}X^{17} + 1.83861{\cdot}10^{9}X^{16} + 2.75976 {\cdot}10^{9}X^{15} + 5.7829{\cdot}10^{10}X^{14} + 6.79601{\cdot}10^{11}X^{13} + 6.22359{\cdot}10^{12}X^{12} + 4.42882{\cdot}10^{13}X^{11} + 2.4383 {\cdot}10^{14}X^{10} + 1.0251{\cdot}10^{15}X^{9} + 3.2085{\cdot}10^{15}X^{8} + 7.11873{\cdot}10^{15}X^{7} + 9.99903{\cdot}10^{15}X^{6} + 5.568{\cdot}10^{15}X^{5} - 7.30551{\cdot}10^{15}X^{4} - 1.61505{\cdot}10^{16}X^{3} - 1.05144{\cdot}10^{16}X^{2} - 1.05866{\cdot}10^{15}X + 8.27799{\cdot}10^{14}$$
$$= 8.27799{\cdot}10^{14}B_{0,20}(X) + 7.74866{\cdot}10^{14}B_{1,20}(X) + 6.66594{\cdot}10^{14}B_{2,20}(X) + 4.88817 {\cdot}10^{14}B_{3,20}(X) + 2.25859{\cdot}10^{14}B_{4,20}(X) - 1.39104{\cdot}10^{14}B_{5,20}(X) - 6.23427{\cdot}10^{14}B_{6,20}(X) - 1.24403{\cdot}10^{15}B_{7,20}(X) - 2.01596{\cdot}10^{15}B_{8,20}(X) - 2.95036{\cdot}10^{15}B_{9,20}(X) - 4.05157 {\cdot}10^{15}B_{10,20}(X) - 5.31329{\cdot}10^{15}B_{11,20}(X) - 6.71333{\cdot}10^{15}B_{12,20}(X) - 8.20664{\cdot}10^{15}B_{13,20}(X) - 9.7163{\cdot}10^{15}B_{14,20}(X) - 1.11216{\cdot}10^{16}B_{15,20}(X) - 1.22431{\cdot}10^{16}B_{16,20}(X) - 1.28227 {\cdot}10^{16}B_{17,20}(X) - 1.24993{\cdot}10^{16}B_{18,20}(X) - 1.07772{\cdot}10^{16}B_{19,20}(X) - 6.98679{\cdot}10^{15}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.0506651, 0.230943\}$$

Intersection intervals with the x axis:

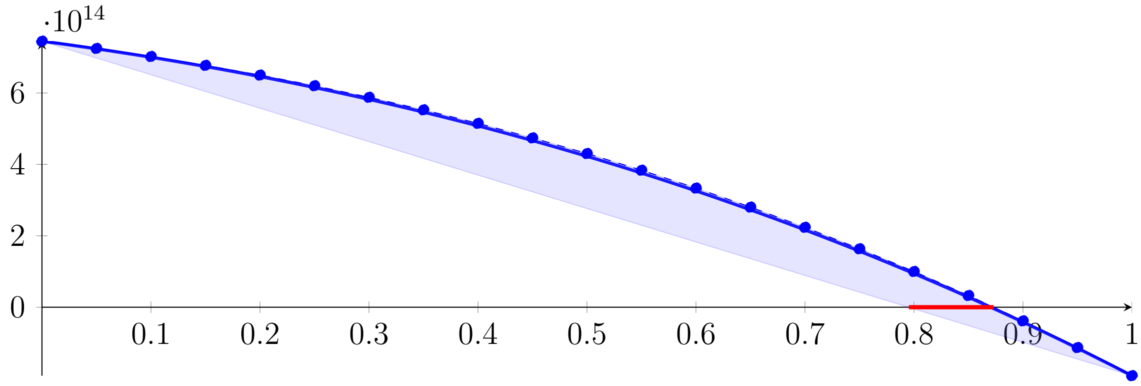
$$[0.0506651, 0.230943]$$

Longest intersection interval: 0.180278
$\Longrightarrow$ Selective recursion: interval 1: [18.8297, 19.0372],

1.96 Recursion Branch 1 2 2 1 1 1 in Interval 1: $[18.8297, 19.0372]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -474957X^{20} + 1.64363\cdot 10^{6}X^{19} - 1.55054\cdot 10^{7}X^{18} + 4.71109\cdot 10^{7}X^{17} - 3.1404\cdot 10^{8}X^{16} + 2.51837 \cdot 10^{8}X^{15} - 1.01604\cdot 10^{8}X^{14} - 5.44675\cdot 10^{7}X^{13} - 2.8096\cdot 10^{8}X^{12} - 5.21726\cdot 10^{7}X^{11} - 7.78516 \cdot 10^{7}X^{10} + 2.20594\cdot 10^{8}X^{9} + 4.13164\cdot 10^{9}X^{8} + 5.27135\cdot 10^{10}X^{7} + 4.38229\cdot 10^{11}X^{6} + 1.71673 \cdot 10^{12}X^{5} - 5.78409\cdot 10^{12}X^{4} - 1.02301\cdot 10^{14}X^{3} - 4.24886\cdot 10^{14}X^{2} - 4.05994\cdot 10^{14}X + 7.45025\cdot 10^{14}$$
$$= 7.45025\cdot 10^{14}B_{0,20}(X) + 7.24725\cdot 10^{14}B_{1,20}(X) + 7.0219\cdot 10^{14}B_{2,20}(X) + 6.77328 \cdot 10^{14}B_{3,20}(X) + 6.50049\cdot 10^{14}B_{4,20}(X) + 6.20261\cdot 10^{14}B_{5,20}(X) + 5.87871\cdot 10^{14}B_{6,20}(X) + 5.52786\cdot 10^{14}B_{7,20}(X) + 5.14911\cdot 10^{14}B_{8,20}(X) + 4.7415\cdot 10^{14}B_{9,20}(X) + 4.30409 \cdot 10^{14}B_{10,20}(X) + 3.83591\cdot 10^{14}B_{11,20}(X) + 3.33603\cdot 10^{14}B_{12,20}(X) + 2.80347\cdot 10^{14}B_{13,20}(X) + 2.2373\cdot 10^{14}B_{14,20}(X) + 1.63658\cdot 10^{14}B_{15,20}(X) + 1.00038\cdot 10^{14}B_{16,20}(X) + 3.27782 \cdot 10^{13}B_{17,20}(X) - 3.82111\cdot 10^{13}B_{18,20}(X) - 1.13018\cdot 10^{14}B_{19,20}(X) - 1.91727\cdot 10^{14}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.795328, 0.873087\}$$

Intersection intervals with the x axis:

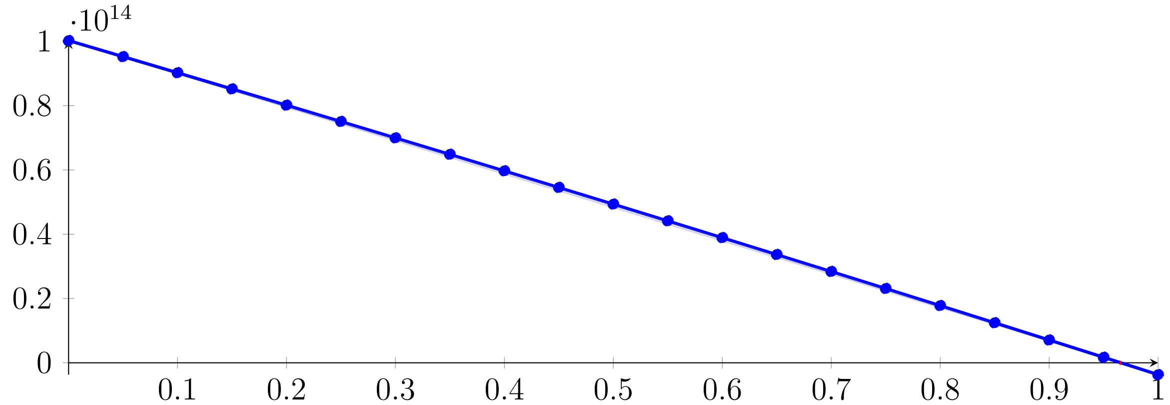
$$[0.795328, 0.873087]$$

Longest intersection interval: 0.0777588
$\Longrightarrow$ Selective recursion: interval 1: [18.9948, 19.0109],

1.97 Recursion Branch 1 2 2 1 1 1 1 in Interval 1: $[18.9948, 19.0109]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -55120X^{20} + 207260X^{19} - 1.93283\cdot10^{6}X^{18} + 5.76329\cdot10^{6}X^{17} - 3.74132\cdot10^{7}X^{16} + 2.97679\cdot10^{7}X^{15} - 1.42546\cdot10^{7}X^{14} - 6.52985\cdot10^{6}X^{13} - 3.53366\cdot10^{7}X^{12} - 5.12016\cdot10^{6}X^{11} - 1.08804\cdot10^{7}X^{10} - 1.65598\cdot10^{6}X^{9} - 49207X^{8} + 1211.25X^{7} + 120519X^{6} + 1.31815\cdot10^{7}X^{5} + 2.28662\cdot10^{8}X^{4} - 4.91868\cdot10^{10}X^{3} - 4.10719\cdot10^{12}X^{2} - 9.97831\cdot10^{13}X + 1.00256\cdot10^{14}$$
$$= 1.00256\cdot10^{14}B_{0,20}(X) + 9.52665\cdot10^{13}B_{1,20}(X) + 9.02557\cdot10^{13}B_{2,20}(X) + 8.52233\cdot10^{13}B_{3,20}(X) + 8.01692\cdot10^{13}B_{4,20}(X) + 7.50933\cdot10^{13}B_{5,20}(X) + 6.99956\cdot10^{13}B_{6,20}(X) + 6.48761\cdot10^{13}B_{7,20}(X) + 5.97347\cdot10^{13}B_{8,20}(X) + 5.45714\cdot10^{13}B_{9,20}(X) + 4.93862\cdot10^{13}B_{10,20}(X) + 4.41789\cdot10^{13}B_{11,20}(X) + 3.89496\cdot10^{13}B_{12,20}(X) + 3.36982\cdot10^{13}B_{13,20}(X) + 2.84247\cdot10^{13}B_{14,20}(X) + 2.3129\cdot10^{13}B_{15,20}(X) + 1.78111\cdot10^{13}B_{16,20}(X) + 1.24709\cdot10^{13}B_{17,20}(X) + 7.10846\cdot10^{12}B_{18,20}(X) + 1.72364\cdot10^{12}B_{19,20}(X) - 3.68356\cdot10^{12}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.96456, 0.965938\}$$

Intersection intervals with the x axis:

$$[0.96456, 0.965938]$$

Longest intersection interval: 0.00137795

$\Longrightarrow$ Selective recursion: interval 1: $[19.0103, 19.0104]$,

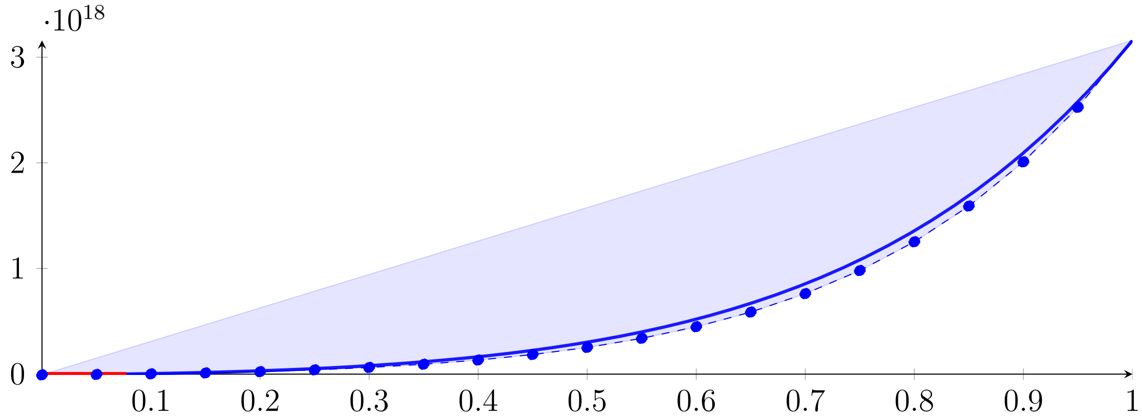
1.98 Recursion Branch 1 2 2 1 1 1 1 1 in Interval 1: $[19.0103, 19.0104]$

Found root in interval $[19.0103, 19.0104]$ at recursion depth 8!

1.99 Recursion Branch 1 2 2 1 2 on the Second Half $[19.9225, 21.0737]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.14933\cdot10^{8}X^{20} + 2.48514\cdot10^{9}X^{19} - 5.34024\cdot10^{9}X^{18} + 3.13797\cdot10^{10}X^{17} - 1.11649\cdot10^{11}X^{16} + 7.58578\cdot10^{10}X^{15} + 1.17134\cdot10^{11}X^{14} + 1.99981\cdot10^{12}X^{13} + 2.23381\cdot10^{13}X^{12} + 1.98917\cdot10^{14}X^{11} + 1.40653\cdot10^{15}X^{10} + 7.89048\cdot10^{15}X^{9} + 3.48689\cdot10^{16}X^{8} + 1.19884\cdot10^{17}X^{7} + 3.14551\cdot10^{17}X^{6} + 6.11731\cdot10^{17}X^{5} + 8.42882\cdot10^{17}X^{4} + 7.63381\cdot10^{17}X^{3} + 3.92979\cdot10^{17}X^{2} + 7.58081\cdot10^{16}X - 6.98679\cdot10^{15}$$
$$= -6.98679\cdot10^{15}B_{0,20}(X) - 3.19638\cdot10^{15}B_{1,20}(X) + 2.66233\cdot10^{15}B_{2,20}(X) + 1.1259\cdot10^{16}B_{3,20}(X) + 2.34372\cdot10^{16}B_{4,20}(X) + 4.0254\cdot10^{16}B_{5,20}(X) + 6.30274\cdot10^{16}B_{6,20}(X) + 9.33936\cdot10^{16}B_{7,20}(X) + 1.33376\cdot10^{17}B_{8,20}(X) + 1.85467\cdot10^{17}B_{9,20}(X) + 2.52726\cdot10^{17}B_{10,20}(X) + 3.38901\cdot10^{17}B_{11,20}(X) + 4.48561\cdot10^{17}B_{12,20}(X) + 5.87272\cdot10^{17}B_{13,20}(X) + 7.61788\cdot10^{17}B_{14,20}(X) + 9.8029\cdot10^{17}B_{15,20}(X) + 1.25267\cdot10^{18}B_{16,20}(X) + 1.59084\cdot10^{18}B_{17,20}(X) + 2.00916\cdot10^{18}B_{18,20}(X) + 2.52486\cdot10^{18}B_{19,20}(X) + 3.15862\cdot10^{18}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.00220709, 0.0772789\}$$

Intersection intervals with the x axis:

$$[0.00220709, 0.0772789]$$

Longest intersection interval: 0.0750718

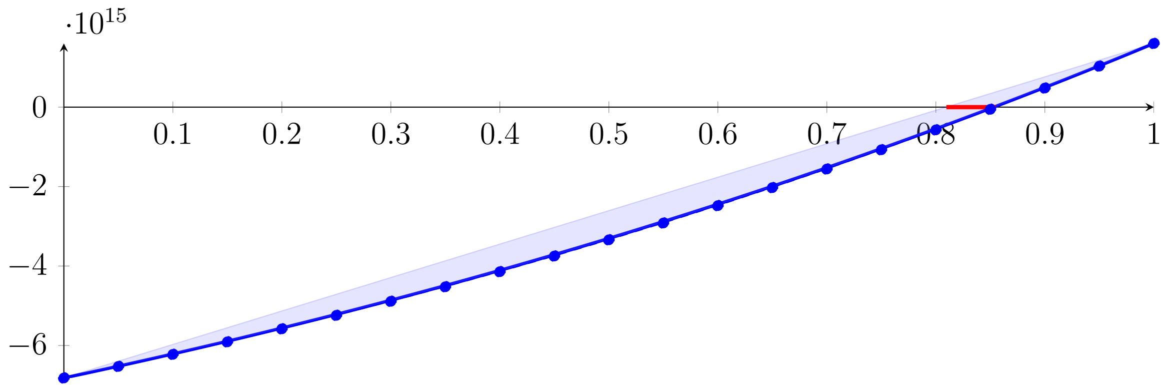
$\Longrightarrow$ Selective recursion: interval 1: $[19.9251, 20.0115]$,

1.100 Recursion Branch 1 2 2 1 2 1 in Interval 1: $[19.9251, 20.0115]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 3.68474 \cdot 10^6 X^{20} - 1.26801 \cdot 10^7 X^{19} + 1.21415 \cdot 10^8 X^{18} - 3.93167 \cdot 10^8 X^{17} + 2.48841 \cdot 10^9 X^{16} - 2.02574 \cdot 10^9 X^{15} + 8.49735 \cdot 10^8 X^{14} + 5.24423 \cdot 10^8 X^{13} + 2.41497 \cdot 10^9 X^{12} + 4.24099 \cdot 10^8 X^{11} + 7.54359 \cdot 10^8 X^{10} + 1.23115 \cdot 10^8 X^9 + 4.06883 \cdot 10^7 X^8 + 1.61963 \cdot 10^9 X^7 + 5.66421 \cdot 10^{10} X^6 + 1.46859 \cdot 10^{12} X^5 + 2.69867 \cdot 10^{13} X^4 + 3.26138 \cdot 10^{14} X^3 + 2.24336 \cdot 10^{15} X^2 + 5.82211 \cdot 10^{15} X - 6.81755 \cdot 10^{15}$$

$$= -6.81755 \cdot 10^{15} B_{0,20}(X) - 6.52644 \cdot 10^{15} B_{1,20}(X) - 6.22353 \cdot 10^{15} B_{2,20}(X) - 5.90852 \cdot 10^{15} B_{3,20}(X) - 5.58113 \cdot 10^{15} B_{4,20}(X) - 5.24106 \cdot 10^{15} B_{5,20}(X) - 4.888 \cdot 10^{15} B_{6,20}(X) - 4.52165 \cdot 10^{15} B_{7,20}(X) - 4.14169 \cdot 10^{15} B_{8,20}(X) - 3.74779 \cdot 10^{15} B_{9,20}(X) - 3.33964 \cdot 10^{15} B_{10,20}(X) - 2.9169 \cdot 10^{15} B_{11,20}(X) - 2.47923 \cdot 10^{15} B_{12,20}(X) - 2.02629 \cdot 10^{15} B_{13,20}(X) - 1.55771 \cdot 10^{15} B_{14,20}(X) - 1.07315 \cdot 10^{15} B_{15,20}(X) - 5.72224 \cdot 10^{14} B_{16,20}(X) - 5.45753 \cdot 10^{13} B_{17,20}(X) + 4.80183 \cdot 10^{14} B_{18,20}(X) + 1.03244 \cdot 10^{15} B_{19,20}(X) + 1.60258 \cdot 10^{15} B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.809673, 0.855103\}$$

Intersection intervals with the x axis:

$$[0.809673, 0.855103]$$

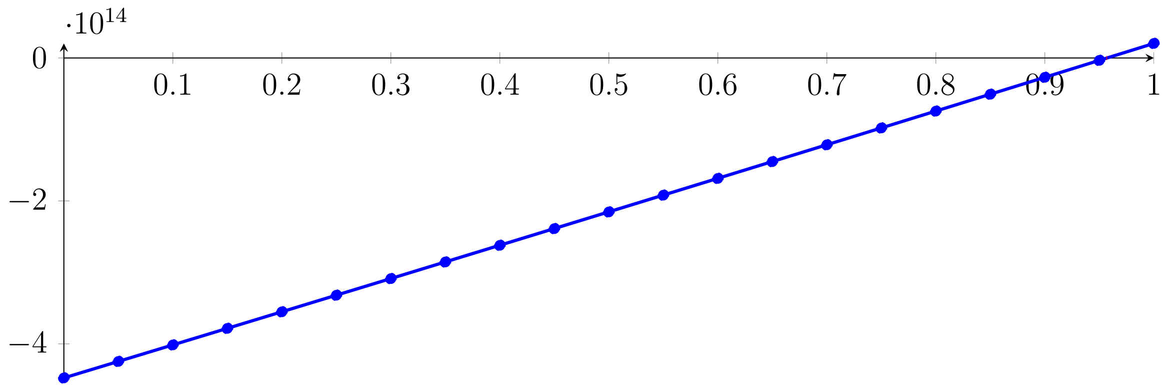
Longest intersection interval: 0.0454302

$\Longrightarrow$ Selective recursion: interval 1: $[19.9951, 19.999]$,

1.101 Recursion Branch 1 2 2 1 2 1 1 in Interval 1: $[19.9951, 19.999]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 235290X^{20} - 880615X^{19} + 8.4426\cdot10^{6}X^{18} - 2.57067\cdot10^{7}X^{17} + 1.64089\cdot10^{8}X^{16} - 1.31858\cdot10^{8}X^{15} + 6.49521\cdot10^{7}X^{14} + 2.90894\cdot10^{7}X^{13} + 1.5403\cdot10^{8}X^{12} + 2.80913\cdot10^{7}X^{11} + 4.82675\cdot10^{7}X^{10} + 7.43223\cdot10^{6}X^{9} + 913282X^{8} - 19380X^{7} + 281010X^{6} + 341088X^{5} + 1.42789\cdot10^{8}X^{4} + 3.97363\cdot10^{10}X^{3} + 6.50105\cdot10^{12}X^{2} + 4.61429\cdot10^{14}X - 4.47622\cdot10^{14}$$
$$= -4.47622\cdot10^{14}B_{0,20}(X) - 4.24551\cdot10^{14}B_{1,20}(X) - 4.01445\cdot10^{14}B_{2,20}(X) - 3.78305\cdot10^{14}B_{3,20}(X) - 3.55131\cdot10^{14}B_{4,20}(X) - 3.31922\cdot10^{14}B_{5,20}(X) - 3.08679\cdot10^{14}B_{6,20}(X) - 2.85402\cdot10^{14}B_{7,20}(X) - 2.6209\cdot10^{14}B_{8,20}(X) - 2.38744\cdot10^{14}B_{9,20}(X) - 2.15363\cdot10^{14}B_{10,20}(X) - 1.91948\cdot10^{14}B_{11,20}(X) - 1.68498\cdot10^{14}B_{12,20}(X) - 1.45014\cdot10^{14}B_{13,20}(X) - 1.21495\cdot10^{14}B_{14,20}(X) - 9.79413\cdot10^{13}B_{15,20}(X) - 7.43529\cdot10^{13}B_{16,20}(X) - 5.07298\cdot10^{13}B_{17,20}(X) - 2.70719\cdot10^{13}B_{18,20}(X) - 3.37918\cdot10^{12}B_{19,20}(X) + 2.03484\cdot10^{13}B_{20,20}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.956518, 0.957121\}$$

Intersection intervals with the x axis:

$$[0.956518, 0.957121]$$

Longest intersection interval: 0.000603
$\Longrightarrow$ Selective recursion: interval 1: $[19.9988, 19.9988]$,

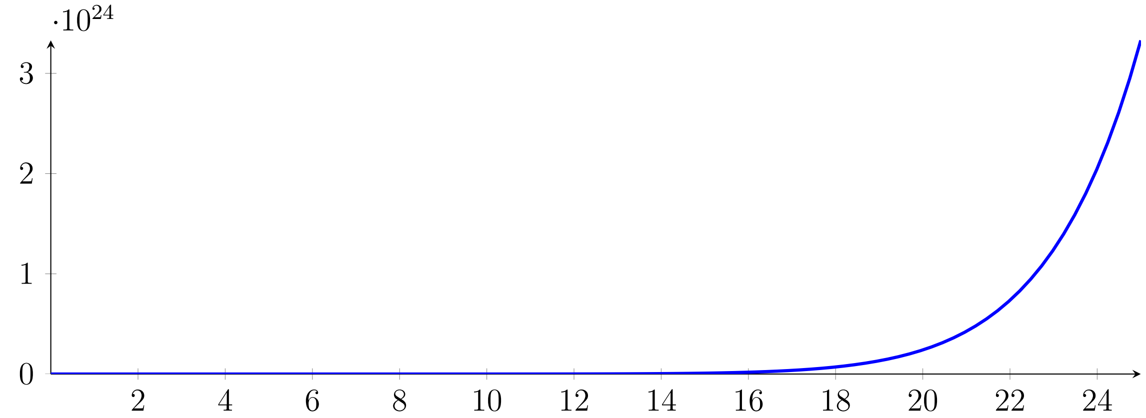
1.102 Recursion Branch 1 2 2 1 2 1 1 1 in Interval 1: $[19.9988, 19.9988]$

Found root in interval $[19.9988, 19.9988]$ at recursion depth 8!

1.103 Result: 20 Root Intervals

Input Polynomial on Interval $[0, 25]$

$$p = 1X^{20} - 210X^{19} + 20615X^{18} - 1.25685{\cdot}10^{6}X^{17} + 5.33279{\cdot}10^{7}X^{16} - 1.67228{\cdot}10^{9}X^{15} + 4.01718{\cdot}10^{10}X^{14} - 7.56111{\cdot}10^{11}X^{13} + 1.13103{\cdot}10^{13}X^{12} - 1.35585{\cdot}10^{14}X^{11} + 1.30754{\cdot}10^{15}X^{10} - 1.01423{\cdot}10^{16}X^{9} + 6.30308{\cdot}10^{16}X^{8} - 3.11334{\cdot}10^{17}X^{7} + 1.20665{\cdot}10^{18}X^{6} - 3.59998{\cdot}10^{18}X^{5} + 8.03781{\cdot}10^{18}X^{4} - 1.28709{\cdot}10^{19}X^{3} + 1.38038{\cdot}10^{19}X^{2} - 8.75295{\cdot}10^{18}X + 2.4329{\cdot}10^{18}$$



Result: Root Intervals

$[1, 1]$, $[2, 2.00003]$, $[3, 3]$, $[4, 4]$, $[5, 5]$, $[5.99998, 6.00051]$, $[7.00005, 7.00037]$, $[7.99947, 7.99981]$, $[9.00306, 9.00334]$, $[9.98803, 9.98807]$, $[11.0363, 11.0363]$, $[11.9253, 11.9255]$, $[13.1501, 13.1502]$, $[13.8086, 13.8088]$, $[15.2176, 15.2183]$, $[15.8091, 15.8091]$, $[17.0973, 17.0973]$, $[17.955, 17.9554]$, $[19.0103, 19.0104]$, $[19.9988, 19.9988]$

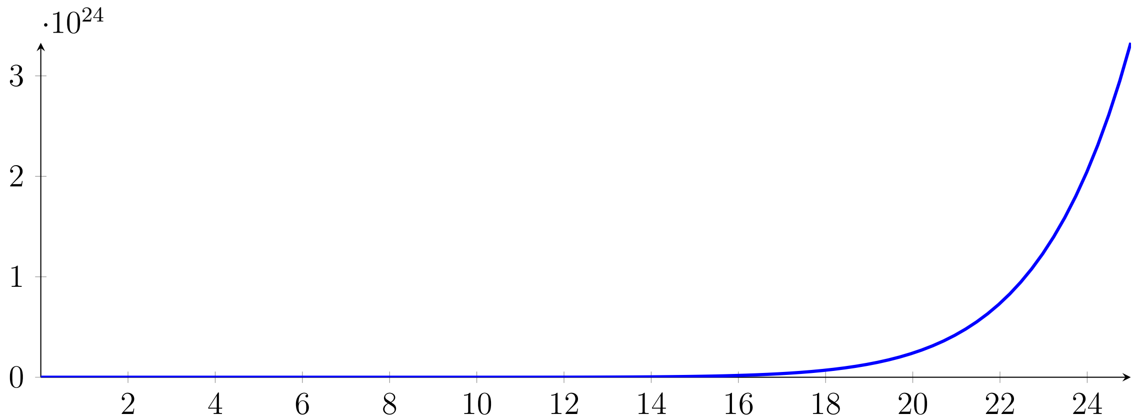
with precision $\varepsilon = 0.001$.

2 QuadClip Applied to the Wilkinson Polynomial

$$1X^{20} - 210X^{19} + 20615X^{18} - 1.25685\cdot10^{6}X^{17} + 5.33279\cdot10^{7}X^{16} - 1.67228\cdot10^{9}X^{15} + 4.01718\cdot10^{10}X^{14} - 7.56111\cdot10^{11}X^{13} + 1.13103\cdot10^{13}X^{12} - 1.35585\cdot10^{14}X^{11} + 1.30754\cdot10^{15}X^{10} - 1.01423\cdot10^{16}X^{9} + 6.30308\cdot10^{16}X^{8} - 3.11334\cdot10^{17}X^{7} + 1.20665\cdot10^{18}X^{6} - 3.59998\cdot10^{18}X^{5} + 8.03781\cdot10^{18}X^{4} - 1.28709\cdot10^{19}X^{3} + 1.38038\cdot10^{19}X^{2} - 8.75295\cdot10^{18}X + 2.4329\cdot10^{18}$$

Called QuadClip with input polynomial on interval $[0, 25]$:

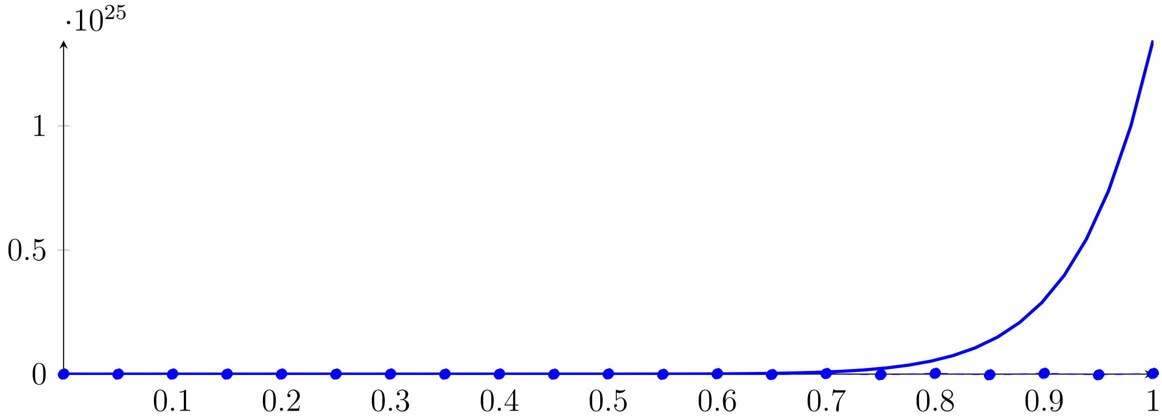
$$p = 1X^{20} - 210X^{19} + 20615X^{18} - 1.25685\cdot10^{6}X^{17} + 5.33279\cdot10^{7}X^{16} - 1.67228\cdot10^{9}X^{15} + 4.01718\cdot10^{10}X^{14} - 7.56111\cdot10^{11}X^{13} + 1.13103\cdot10^{13}X^{12} - 1.35585\cdot10^{14}X^{11} + 1.30754\cdot10^{15}X^{10} - 1.01423\cdot10^{16}X^{9} + 6.30308\cdot10^{16}X^{8} - 3.11334\cdot10^{17}X^{7} + 1.20665\cdot10^{18}X^{6} - 3.59998\cdot10^{18}X^{5} + 8.03781\cdot10^{18}X^{4} - 1.28709\cdot10^{19}X^{3} + 1.38038\cdot10^{19}X^{2} - 8.75295\cdot10^{18}X + 2.4329\cdot10^{18}$$



2.1 Recursion Branch 1 for Input Interval $[0, 25]$

Normalized monomial und Bézier representations and the Bézier polygon:

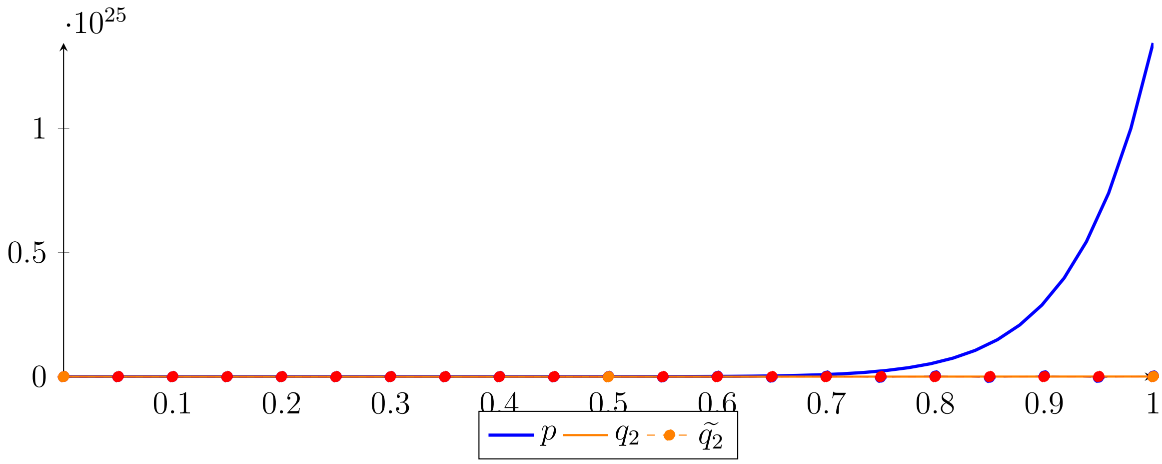
$$\begin{aligned} p &= 9.09495\cdot10^{27}X^{20} - 7.63976\cdot10^{28}X^{19} + 2.99988\cdot10^{29}X^{18} - 7.31583\cdot10^{29}X^{17} + 1.24164\cdot10^{30}X^{16} - 1.55743\cdot10^{30}X^{15} + 1.49652\cdot10^{30}X^{14} - 1.12669\cdot10^{30}X^{13} + 6.74145\cdot10^{29}X^{12} - 3.2326\cdot10^{29}X^{11} + 1.24696\cdot10^{29}X^{10} - 3.86898\cdot10^{28}X^{9} + 9.61774\cdot10^{27}X^{8} - 1.90023\cdot10^{27}X^{7} + 2.94592\cdot10^{26}X^{6} - 3.5156\cdot10^{25}X^{5} + 3.13977\cdot10^{24}X^{4} - 2.01108\cdot10^{23}X^{3} + 8.62735\cdot10^{21}X^{2} - 2.18824\cdot10^{20}X + 2.4329\cdot10^{18} \\ &= 2.4329\cdot10^{18}B_{0,20}(X) - 8.50828\cdot10^{18}B_{1,20}(X) + 2.59576\cdot10^{19}B_{2,20}(X) - 7.05801\cdot10^{19}B_{3,20}(X) + 1.73511\cdot10^{20}B_{4,20}(X) - 3.8964\cdot10^{20}B_{5,20}(X) + 8.05451\cdot10^{20}B_{6,20}(X) - 1.54188\cdot10^{21}B_{7,20}(X) + 2.74637\cdot10^{21}B_{8,20}(X) - 4.56922\cdot10^{21}B_{9,20}(X) + 7.12322\cdot10^{21}B_{10,20}(X) - 1.04331\cdot10^{22}B_{11,20}(X) + 1.43886\cdot10^{22}B_{12,20}(X) - 1.87204\cdot10^{22}B_{13,20}(X) + 2.30149\cdot10^{22}B_{14,20}(X) - 2.67735\cdot10^{22}B_{15,20}(X) + 2.95071\cdot10^{22}B_{16,20}(X) - 3.08413\cdot10^{22}B_{17,20}(X) + 3.06005\cdot10^{22}B_{18,20}(X) - 2.88452\cdot10^{22}B_{19,20}(X) + 2.5852\cdot10^{22}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = 1.51392{\cdot}10^{22}X^2 - 1.18412{\cdot}10^{22}X + 1.44787{\cdot}10^{21}$$
$$= 1.44787{\cdot}10^{21}B_{0,2} - 4.47275{\cdot}10^{21}B_{1,2} + 4.74584{\cdot}10^{21}B_{2,2}$$

$$\widetilde{q}_2 = 1.96809{\cdot}10^{24}X^{20} - 1.96783{\cdot}10^{25}X^{19} + 9.10597{\cdot}10^{25}X^{18} - 2.58749{\cdot}10^{26}X^{17} + 5.05148{\cdot}10^{26}X^{16} - 7.18268{\cdot}10^{26}X^{15} + 7.69415{\cdot}10^{26}X^{14} - 6.33548{\cdot}10^{26}X^{13} + 4.05559{\cdot}10^{26}X^{12} - 2.02812{\cdot}10^{26}X^{11} + 7.91923{\cdot}10^{25}X^{10} - 2.4012{\cdot}10^{25}X^{9} + 5.59295{\cdot}10^{24}X^{8} - 9.8408{\cdot}10^{23}X^{7} + 1.27701{\cdot}10^{23}X^{6} - 1.18258{\cdot}10^{22}X^{5} + 7.45345{\cdot}10^{20}X^{4} - 2.94849{\cdot}10^{19}X^{3} + 1.51399{\cdot}10^{22}X^{2} - 1.18412{\cdot}10^{22}X + 1.44787{\cdot}10^{21}$$
$$= 1.44787{\cdot}10^{21}B_{0,20} + 8.55808{\cdot}10^{20}B_{1,20} + 3.43429{\cdot}10^{20}B_{2,20} - 8.92922{\cdot}10^{19}B_{3,20} - 4.42228{\cdot}10^{20}B_{4,20} - 7.15859{\cdot}10^{20}B_{5,20} - 9.08743{\cdot}10^{20}B_{6,20} - 1.02438{\cdot}10^{21}B_{7,20} - 1.05579{\cdot}10^{21}B_{8,20} - 1.0146{\cdot}10^{21}B_{9,20} - 8.84506{\cdot}10^{20}B_{10,20} - 6.84807{\cdot}10^{20}B_{11,20} - 3.96186{\cdot}10^{20}B_{12,20} - 3.49934{\cdot}10^{19}B_{13,20} + 4.10439{\cdot}10^{20}B_{14,20} + 9.33122{\cdot}10^{20}B_{15,20} + 1.53656{\cdot}10^{21}B_{16,20} + 2.21929{\cdot}10^{21}B_{17,20} + 2.98181{\cdot}10^{21}B_{18,20} + 3.82398{\cdot}10^{21}B_{19,20} + 4.74584{\cdot}10^{21}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 3.30606{\cdot}10^{22}$.

Bounding polynomials M and m:

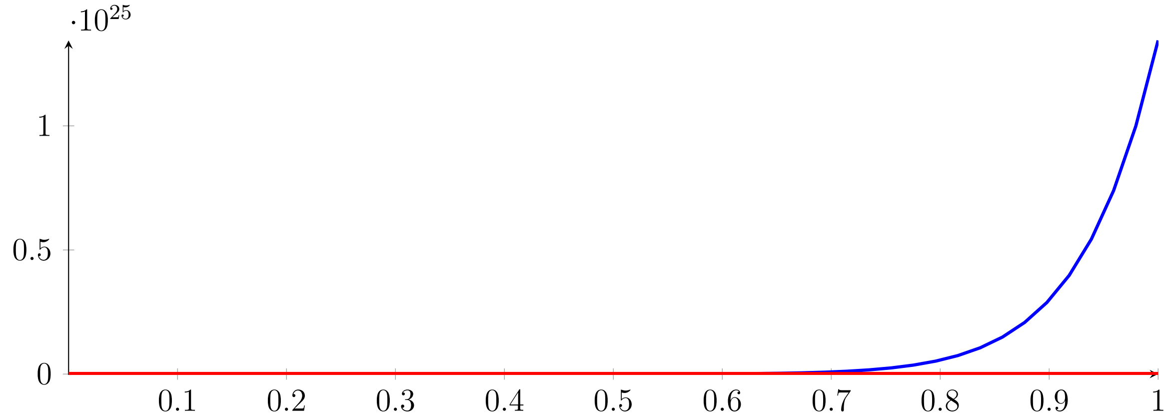
$$M = 1.51392{\cdot}10^{22}X^2 - 1.18412{\cdot}10^{22}X + 3.45085{\cdot}10^{22}$$

$$m = 1.51392{\cdot}10^{22}X^2 - 1.18412{\cdot}10^{22}X - 3.16127{\cdot}10^{22}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{-1.10594, 1.8881\}$$

Intersection intervals:



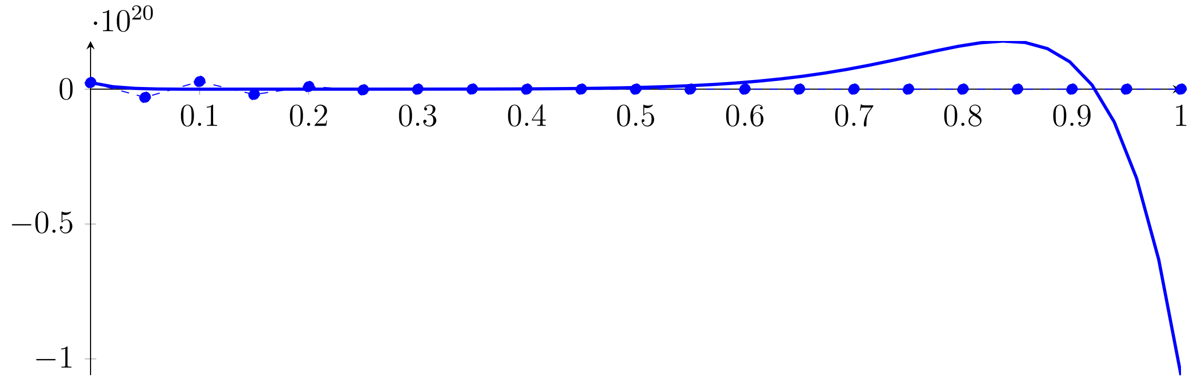
$[0, 1]$

Longest intersection interval: 1
$\Longrightarrow$ Bisection: first half $[0, 12.5]$ und second half $[12.5, 25]$

2.2 Recursion Branch 1 1 on the First Half $[0, 12.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

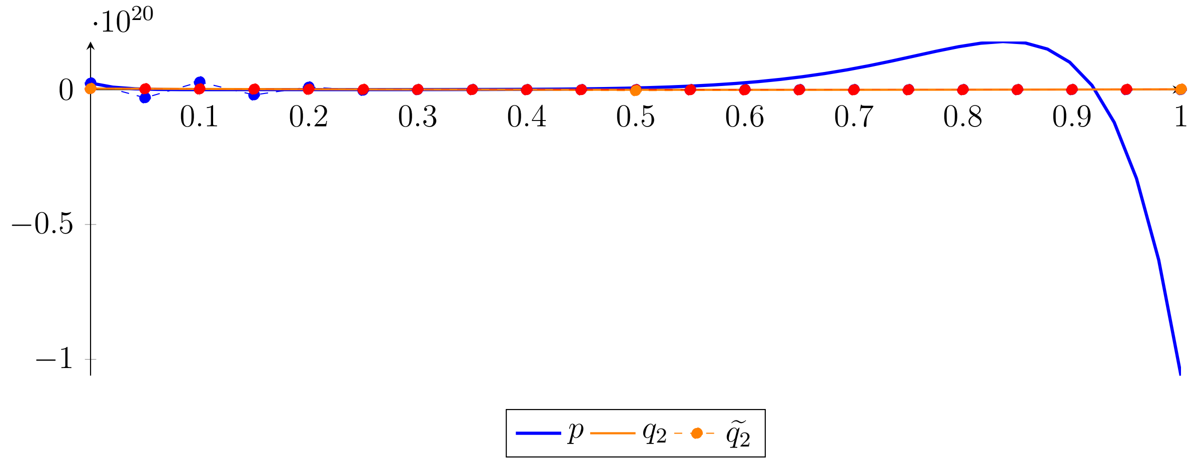
$$\begin{aligned}p &= 8.67362\cdot 10^{21}X^{20} - 1.45717\cdot 10^{23}X^{19} + 1.14436\cdot 10^{24}X^{18} - 5.58154\cdot 10^{24}X^{17} + 1.89459\cdot 10^{25}X^{16} \\ &\quad - 4.75291\cdot 10^{25}X^{15} + 9.134\cdot 10^{25}X^{14} - 1.37536\cdot 10^{26}X^{13} + 1.64586\cdot 10^{26}X^{12} - 1.57842\cdot 10^{26}X^{11} \\ &\quad + 1.21774\cdot 10^{26}X^{10} - 7.5566\cdot 10^{25}X^{9} + 3.75693\cdot 10^{25}X^{8} - 1.48455\cdot 10^{25}X^{7} + 4.603\cdot 10^{24}X^{6} - 1.09863 \\ &\quad \cdot 10^{24}X^{5} + 1.96236\cdot 10^{23}X^{4} - 2.51385\cdot 10^{22}X^{3} + 2.15684\cdot 10^{21}X^{2} - 1.09412\cdot 10^{20}X + 2.4329\cdot 10^{18} \\ &= 2.4329\cdot 10^{18}B_{0,20}(X) - 3.03769\cdot 10^{18}B_{1,20}(X) + 2.84349\cdot 10^{18}B_{2,20}(X) - 1.9749 \\ &\quad \cdot 10^{18}B_{3,20}(X) + 9.58506\cdot 10^{17}B_{4,20}(X) - 2.63073\cdot 10^{17}B_{5,20}(X) - 9.0343\cdot 10^{15}B_{6,20}(X) \\ &\quad + 3.44399\cdot 10^{16}B_{7,20}(X) - 5.41351\cdot 10^{15}B_{8,20}(X) - 4.28958\cdot 10^{15}B_{9,20}(X) + 1.09675 \\ &\quad \cdot 10^{15}B_{10,20}(X) + 6.89924\cdot 10^{14}B_{11,20}(X) - 1.57583\cdot 10^{14}B_{12,20}(X) - 1.3719\cdot 10^{14}B_{13,20}(X) \\ &\quad + 1.13888\cdot 10^{13}B_{14,20}(X) + 2.83586\cdot 10^{13}B_{15,20}(X) + 3.54186\cdot 10^{12}B_{16,20}(X) - 4.9643 \\ &\quad \cdot 10^{12}B_{17,20}(X) - 2.0514\cdot 10^{12}B_{18,20}(X) + 5.37337\cdot 10^{11}B_{19,20}(X) + 8.1419\cdot 10^{11}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 1.26565\cdot 10^{18}X^2 - 1.5358\cdot 10^{18}X + 3.92519\cdot 10^{17} \\ &= 3.92519\cdot 10^{17}B_{0,2} - 3.75383\cdot 10^{17}B_{1,2} + 1.22361\cdot 10^{17}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 1.43874{\cdot}10^{20}X^{20} - 1.43821{\cdot}10^{21}X^{19} + 6.65403{\cdot}10^{21}X^{18} - 1.8905{\cdot}10^{22}X^{17} + 3.69021{\cdot}10^{22}X^{16} - 5.24588 \\ &\quad \cdot 10^{22}X^{15} + 5.61715\cdot 10^{22}X^{14} - 4.62204\cdot 10^{22}X^{13} + 2.95552\cdot 10^{22}X^{12} - 1.47569\cdot 10^{22}X^{11} + 5.75046 \\ &\quad \cdot 10^{21}X^{10} - 1.73943\cdot 10^{21}X^{9} + 4.04107\cdot 10^{20}X^{8} - 7.09124\cdot 10^{19}X^{7} + 9.17622\cdot 10^{18}X^{6} - 8.47315 \\ &\quad \cdot 10^{17}X^{5} + 5.32762\cdot 10^{16}X^{4} - 2.10352\cdot 10^{15}X^{3} + 1.26569\cdot 10^{18}X^{2} - 1.5358\cdot 10^{18}X + 3.92519\cdot 10^{17} \\ &= 3.92519\cdot 10^{17}B_{0,20} + 3.15729\cdot 10^{17}B_{1,20} + 2.456\cdot 10^{17}B_{2,20} + 1.82131\cdot 10^{17}B_{3,20} + 1.25331 \\ &\quad \cdot 10^{17}B_{4,20} + 7.51648\cdot 10^{16}B_{5,20} + 3.17373\cdot 10^{16}B_{6,20} - 5.20553\cdot 10^{15}B_{7,20} - 3.51554\cdot 10^{16}B_{8,20} \\ &\quad - 5.8963\cdot 10^{16}B_{9,20} - 7.54331\cdot 10^{16}B_{10,20} - 8.59766\cdot 10^{16}B_{11,20} - 8.91853\cdot 10^{16}B_{12,20} \\ &\quad - 8.62544\cdot 10^{16}B_{13,20} - 7.63256\cdot 10^{16}B_{14,20} - 5.9915\cdot 10^{16}B_{15,20} - 3.67639\cdot 10^{16}B_{16,20} \\ &\quad - 6.97975\cdot 10^{15}B_{17,20} + 2.94735\cdot 10^{16}B_{18,20} + 7.25864\cdot 10^{16}B_{19,20} + 1.22361\cdot 10^{17}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.35342\cdot 10^{18}$.

Bounding polynomials M and m:

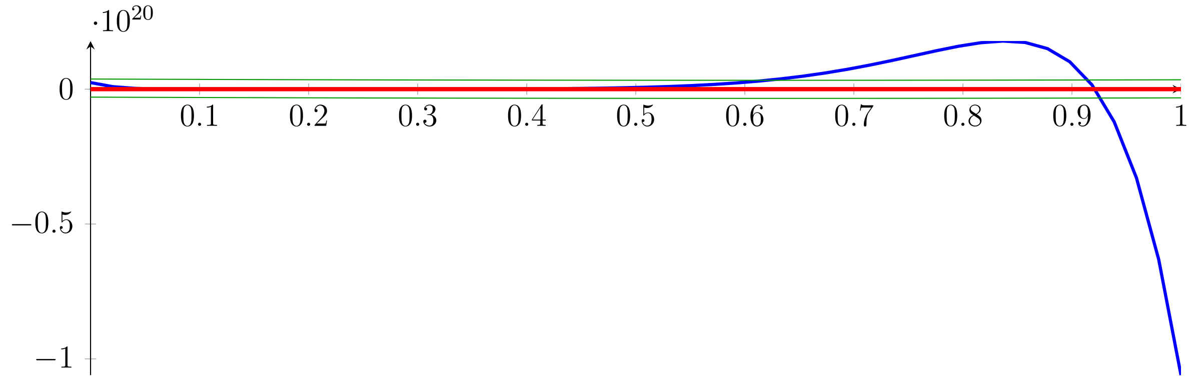
$$M = 1.26565\cdot 10^{18}X^2 - 1.5358\cdot 10^{18}X + 3.74594\cdot 10^{18}$$

$$m = 1.26565\cdot 10^{18}X^2 - 1.5358\cdot 10^{18}X - 2.9609\cdot 10^{18}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{-1.03874, 2.25219\}$$

Intersection intervals:



$$[0, 1]$$

Longest intersection interval: 1

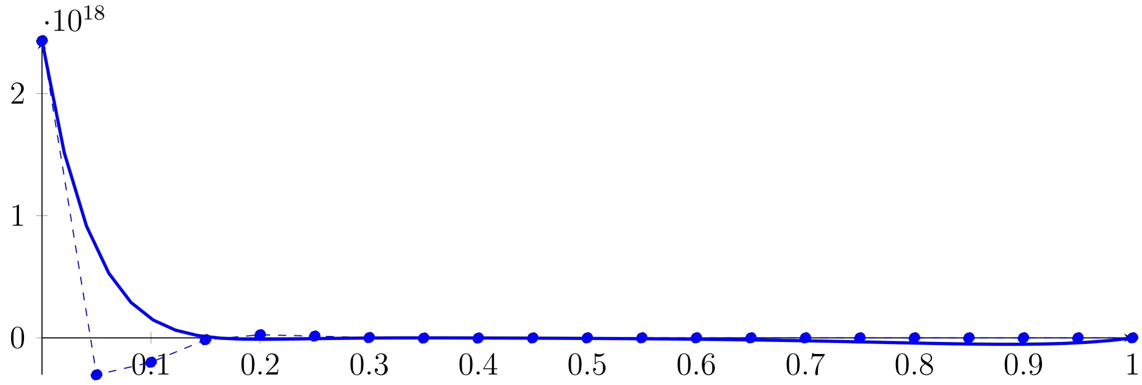
$\Longrightarrow$ Bisection: first half $[0, 6.25]$ und second half $[6.25, 12.5]$

2.3 Recursion Branch 1 1 1 on the First Half $[0, 6.25]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 8.27181{\cdot}10^{15}X^{20} - 2.77933{\cdot}10^{17}X^{19} + 4.3654{\cdot}10^{18}X^{18} - 4.25837{\cdot}10^{19}X^{17} + 2.89091{\cdot}10^{20}X^{16} - 1.45047{\cdot}10^{21}X^{15} + 5.57495{\cdot}10^{21}X^{14} - 1.6789{\cdot}10^{22}X^{13} + 4.01822{\cdot}10^{22}X^{12} - 7.70713{\cdot}10^{22}X^{11} + 1.1892{\cdot}10^{23}X^{10} - 1.4759{\cdot}10^{23}X^{9} + 1.46755{\cdot}10^{23}X^{8} - 1.15981{\cdot}10^{23}X^{7} + 7.19218{\cdot}10^{22}X^{6} - 3.43321{\cdot}10^{22}X^{5} + 1.22647{\cdot}10^{22}X^{4} - 3.14232{\cdot}10^{21}X^{3} + 5.39209{\cdot}10^{20}X^{2} - 5.47059{\cdot}10^{19}X + 2.4329{\cdot}10^{18}$$

$$= 2.4329{\cdot}10^{18}B_{0,20}(X) - 3.02394{\cdot}10^{17}B_{1,20}(X) - 1.99746{\cdot}10^{17}B_{2,20}(X) - 1.55733{\cdot}10^{16}B_{3,20}(X) + 2.51263{\cdot}10^{16}B_{4,20}(X) + 1.43711{\cdot}10^{16}B_{5,20}(X) + 2.36483{\cdot}10^{15}B_{6,20}(X) - 1.91069{\cdot}10^{15}B_{7,20}(X) - 1.81457{\cdot}10^{15}B_{8,20}(X) - 7.4091{\cdot}10^{14}B_{9,20}(X) - 3.15634{\cdot}10^{13}B_{10,20}(X) + 1.92739{\cdot}10^{14}B_{11,20}(X) + 1.62719{\cdot}10^{14}B_{12,20}(X) + 7.31276{\cdot}10^{13}B_{13,20}(X) + 9.11723{\cdot}10^{12}B_{14,20}(X) - 1.65546{\cdot}10^{13}B_{15,20}(X) - 1.79828{\cdot}10^{13}B_{16,20}(X) - 1.06656{\cdot}10^{13}B_{17,20}(X) - 3.51597{\cdot}10^{12}B_{18,20}(X) + 5.61716{\cdot}10^{11}B_{19,20}(X) + 1.86285{\cdot}10^{12}B_{20,20}(X)$$

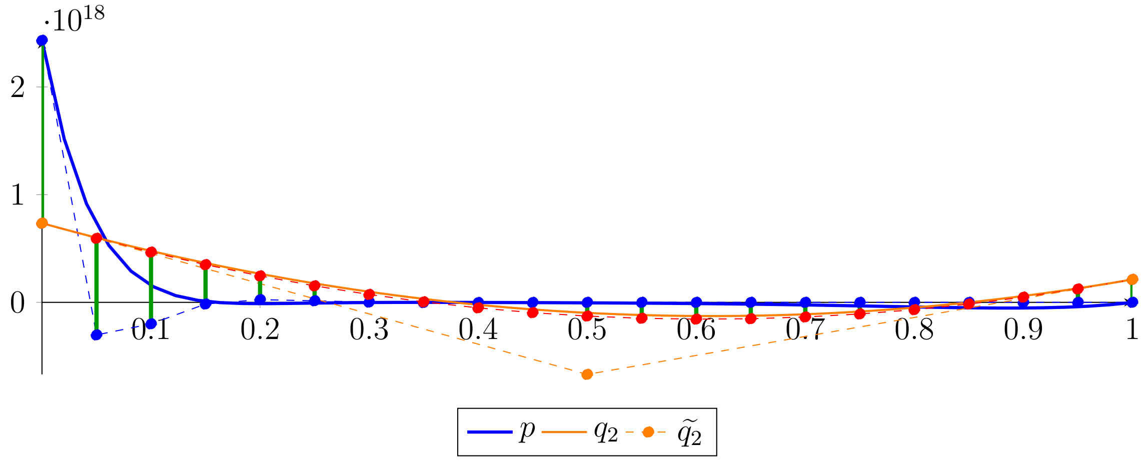


Degree reduction and raising:

$$q_2 = 2.28545{\cdot}10^{18}X^2 - 2.8081{\cdot}10^{18}X + 7.3523{\cdot}10^{17}$$
$$= 7.3523{\cdot}10^{17}B_{0,2} - 6.68821{\cdot}10^{17}B_{1,2} + 2.12582{\cdot}10^{17}B_{2,2}$$

$$\widetilde{q}_2 = 2.56822{\cdot}10^{20}X^{20} - 2.56716{\cdot}10^{21}X^{19} + 1.18767{\cdot}10^{22}X^{18} - 3.37418{\cdot}10^{22}X^{17} + 6.58596{\cdot}10^{22}X^{16} - 9.36183{\cdot}10^{22}X^{15} + 1.00237{\cdot}10^{23}X^{14} - 8.24729{\cdot}10^{22}X^{13} + 5.27315{\cdot}10^{22}X^{12} - 2.63258{\cdot}10^{22}X^{11} + 1.02574{\cdot}10^{22}X^{10} - 3.10231{\cdot}10^{21}X^{9} + 7.20643{\cdot}10^{20}X^{8} - 1.26441{\cdot}10^{20}X^{7} + 1.63583{\cdot}10^{19}X^{6} - 1.50988{\cdot}10^{18}X^{5} + 9.48628{\cdot}10^{16}X^{4} - 3.74057{\cdot}10^{15}X^{3} + 2.28554{\cdot}10^{18}X^{2} - 2.8081{\cdot}10^{18}X + 7.3523{\cdot}10^{17}$$

$$= 7.3523{\cdot}10^{17}B_{0,20} + 5.94825{\cdot}10^{17}B_{1,20} + 4.66449{\cdot}10^{17}B_{2,20} + 3.50099{\cdot}10^{17}B_{3,20} + 2.45791{\cdot}10^{17}B_{4,20} + 1.53464{\cdot}10^{17}B_{5,20} + 7.33023{\cdot}10^{16}B_{6,20} + 4.85473{\cdot}10^{15}B_{7,20} - 5.09725{\cdot}10^{16}B_{8,20} - 9.56974{\cdot}10^{16}B_{9,20} - 1.27187{\cdot}10^{17}B_{10,20} - 1.47958{\cdot}10^{17}B_{11,20} - 1.55499{\cdot}10^{17}B_{12,20} - 1.51943{\cdot}10^{17}B_{13,20} - 1.35756{\cdot}10^{17}B_{14,20} - 1.07862{\cdot}10^{17}B_{15,20} - 6.77974{\cdot}10^{16}B_{16,20} - 1.57547{\cdot}10^{16}B_{17,20} + 4.83308{\cdot}10^{16}B_{18,20} + 1.24442{\cdot}10^{17}B_{19,20} + 2.12582{\cdot}10^{17}B_{20,20}$$



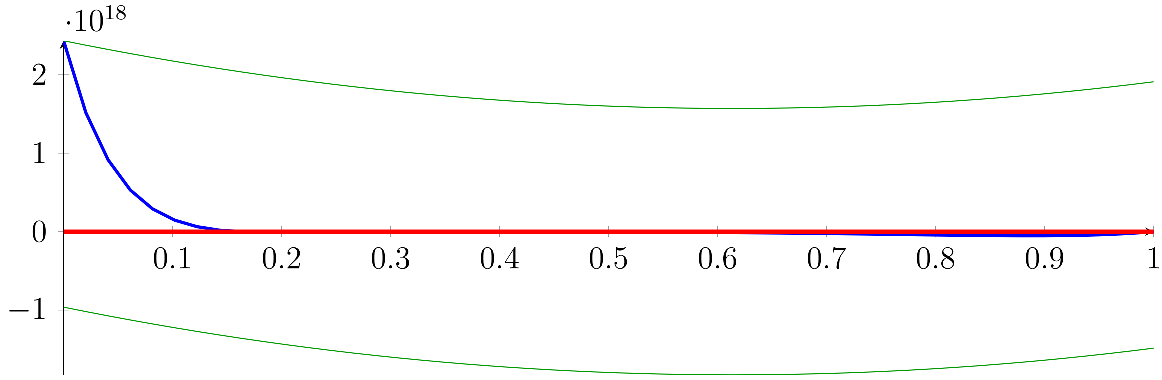
The maximum difference of the Bézier coefficients is $\delta = 1.69767 \cdot 10^{18}$.
Bounding polynomials M and m:

$$M = 2.28545 \cdot 10^{18} X^2 - 2.8081 \cdot 10^{18} X + 2.4329 \cdot 10^{18}$$
$$m = 2.28545 \cdot 10^{18} X^2 - 2.8081 \cdot 10^{18} X - 9.62441 \cdot 10^{17}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{-0.279264, 1.50795\}$$

Intersection intervals:



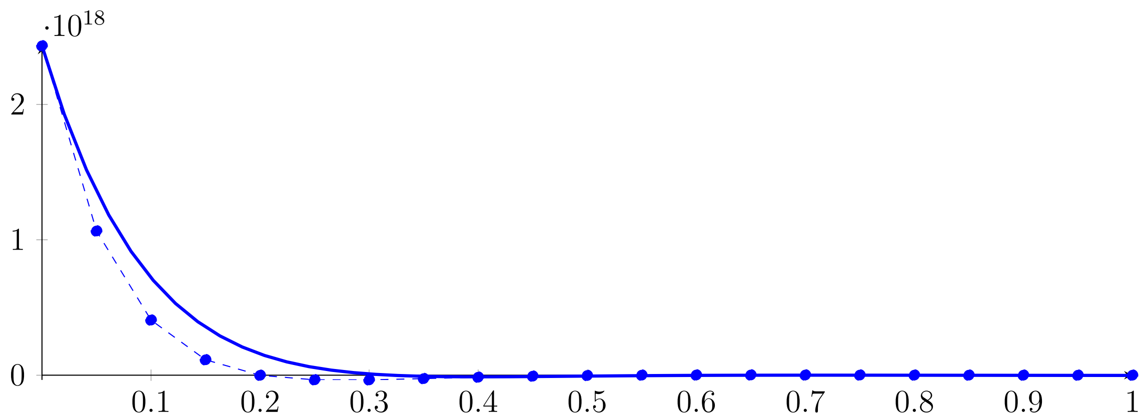
$[0, 1]$

Longest intersection interval: 1
$\Longrightarrow$ Bisection: first half $[0, 3.125]$ und second half $[3.125, 6.25]$
Bisection point is very near to a root?!?

2.4 Recursion Branch 1 1 1 1 on the First Half $[0, 3.125]$

Normalized monomial und Bézier representations and the Bézier polygon:

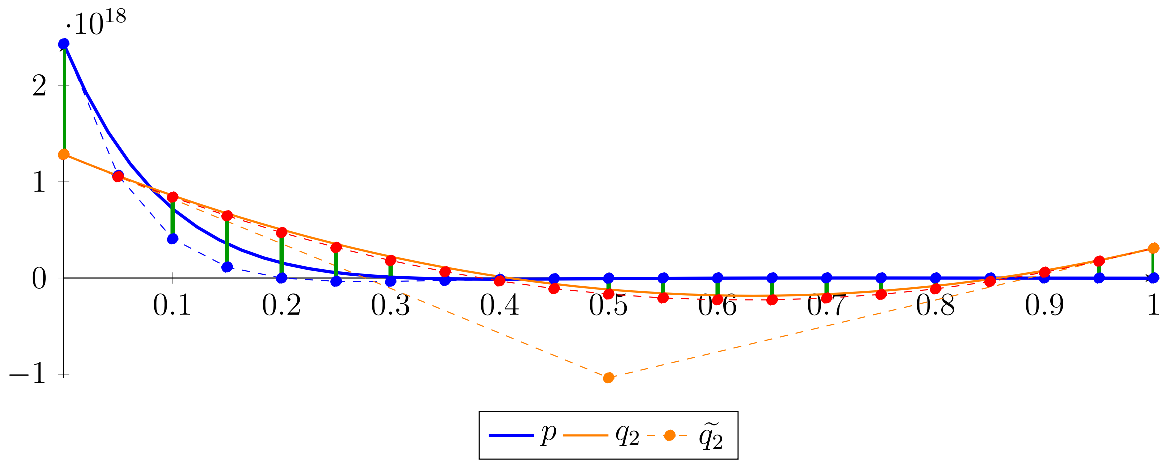
$$\begin{aligned} p &= 7.89961 \cdot 10^{9} X^{20} - 5.30084 \cdot 10^{11} X^{19} + 1.66534 \cdot 10^{13} X^{18} - 3.24889 \cdot 10^{14} X^{17} + 4.41119 \cdot 10^{15} X^{16} \\ &\quad - 4.42649 \cdot 10^{16} X^{15} + 3.40268 \cdot 10^{17} X^{14} - 2.04944 \cdot 10^{18} X^{13} + 9.8101 \cdot 10^{18} X^{12} - 3.76324 \cdot 10^{19} X^{11} \\ &\quad + 1.16132 \cdot 10^{20} X^{10} - 2.88261 \cdot 10^{20} X^{9} + 5.73262 \cdot 10^{20} X^{8} - 9.061 \cdot 10^{20} X^{7} + 1.12378 \cdot 10^{21} X^{6} - 1.07288 \\ &\quad \cdot 10^{21} X^{5} + 7.66545 \cdot 10^{20} X^{4} - 3.9279 \cdot 10^{20} X^{3} + 1.34802 \cdot 10^{20} X^{2} - 2.7353 \cdot 10^{19} X + 2.4329 \cdot 10^{18} \\ &= 2.4329 \cdot 10^{18} B_{0,20}(X) + 1.06525 \cdot 10^{18} B_{1,20}(X) + 4.07092 \cdot 10^{17} B_{2,20}(X) + 1.13863 \\ &\quad \cdot 10^{17} B_{3,20}(X) - 7.70051 \cdot 10^{14} B_{4,20}(X) - 3.41333 \cdot 10^{16} B_{5,20}(X) - 3.47444 \cdot 10^{16} B_{6,20}(X) \\ &\quad - 2.52167 \cdot 10^{16} B_{7,20}(X) - 1.49942 \cdot 10^{16} B_{8,20}(X) - 7.22308 \cdot 10^{15} B_{9,20}(X) - 2.31656 \\ &\quad \cdot 10^{15} B_{10,20}(X) + 2.94801 \cdot 10^{14} B_{11,20}(X) + 1.37334 \cdot 10^{15} B_{12,20}(X) + 1.56871 \cdot 10^{15} B_{13,20}(X) \\ &\quad + 1.33924 \cdot 10^{15} B_{14,20}(X) + 9.67327 \cdot 10^{14} B_{15,20}(X) + 6.03998 \cdot 10^{14} B_{16,20}(X) + 3.14379 \\ &\quad \cdot 10^{14} B_{17,20}(X) + 1.13755 \cdot 10^{14} B_{18,20}(X) - 7.46015 \cdot 10^{12} B_{19,20}(X) - 6.82353 \cdot 10^{13} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = 3.66492{\cdot}10^{18}X^2 - 4.63944{\cdot}10^{18}X + 1.28409{\cdot}10^{18}$$
$$= 1.28409{\cdot}10^{18}B_{0,2} - 1.03563{\cdot}10^{18}B_{1,2} + 3.0957{\cdot}10^{17}B_{2,2}$$

$$\widetilde{q}_2 = 3.99888{\cdot}10^{20}X^{20} - 3.99679{\cdot}10^{21}X^{19} + 1.84886{\cdot}10^{22}X^{18} - 5.25193{\cdot}10^{22}X^{17} + 1.02496{\cdot}10^{23}X^{16} - 1.45674{\cdot}10^{23}X^{15} + 1.55946{\cdot}10^{23}X^{14} - 1.28281{\cdot}10^{23}X^{13} + 8.19998{\cdot}10^{22}X^{12} - 4.09259{\cdot}10^{22}X^{11} + 1.59409{\cdot}10^{22}X^{10} - 4.81969{\cdot}10^{21}X^{9} + 1.11922{\cdot}10^{21}X^{8} - 1.96304{\cdot}10^{20}X^{7} + 2.5383{\cdot}10^{19}X^{6} - 2.3404{\cdot}10^{18}X^{5} + 1.4674{\cdot}10^{17}X^{4} - 5.76583{\cdot}10^{15}X^{3} + 3.66505{\cdot}10^{18}X^{2} - 4.63944{\cdot}10^{18}X + 1.28409{\cdot}10^{18}$$
$$= 1.28409{\cdot}10^{18}B_{0,20} + 1.05212{\cdot}10^{18}B_{1,20} + 8.39436{\cdot}10^{17}B_{2,20} + 6.46038{\cdot}10^{17}B_{3,20} + 4.7195{\cdot}10^{17}B_{4,20} + 3.17077{\cdot}10^{17}B_{5,20} + 1.81706{\cdot}10^{17}B_{6,20} + 6.51344{\cdot}10^{16}B_{7,20} - 3.12282{\cdot}10^{16}B_{8,20} - 1.09743{\cdot}10^{17}B_{9,20} - 1.67091{\cdot}10^{17}B_{10,20} - 2.0719{\cdot}10^{17}B_{11,20} - 2.26127{\cdot}10^{17}B_{12,20} - 2.27227{\cdot}10^{17}B_{13,20} - 2.08101{\cdot}10^{17}B_{14,20} - 1.70186{\cdot}10^{17}B_{15,20} - 1.12761{\cdot}10^{17}B_{16,20} - 3.61261{\cdot}10^{16}B_{17,20} + 5.98198{\cdot}10^{16}B_{18,20} + 1.7505{\cdot}10^{17}B_{19,20} + 3.0957{\cdot}10^{17}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.14881{\cdot}10^{18}$.

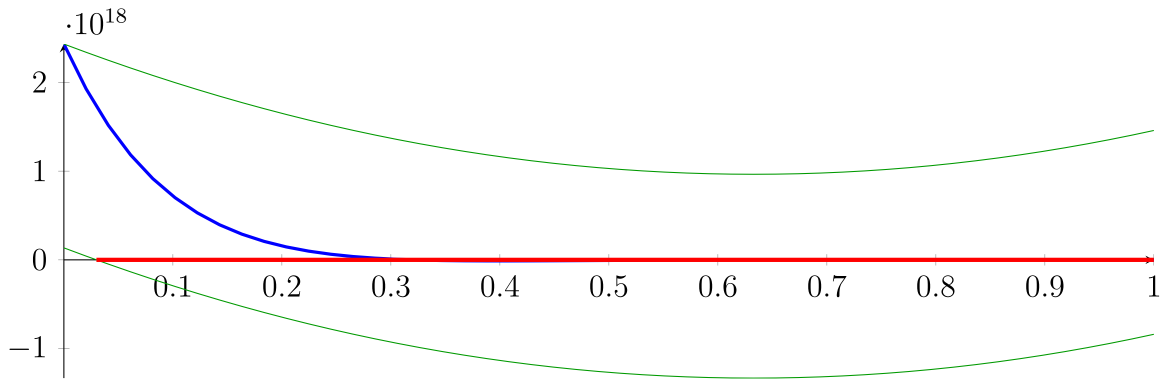
Bounding polynomials M and m:

$$M = 3.66492{\cdot}10^{18}X^2 - 4.63944{\cdot}10^{18}X + 2.4329{\cdot}10^{18}$$
$$m = 3.66492{\cdot}10^{18}X^2 - 4.63944{\cdot}10^{18}X + 1.35279{\cdot}10^{17}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.029863, 1.23604\}$$

Intersection intervals:



$[0.029863, 1]$

Longest intersection interval: 0.970137

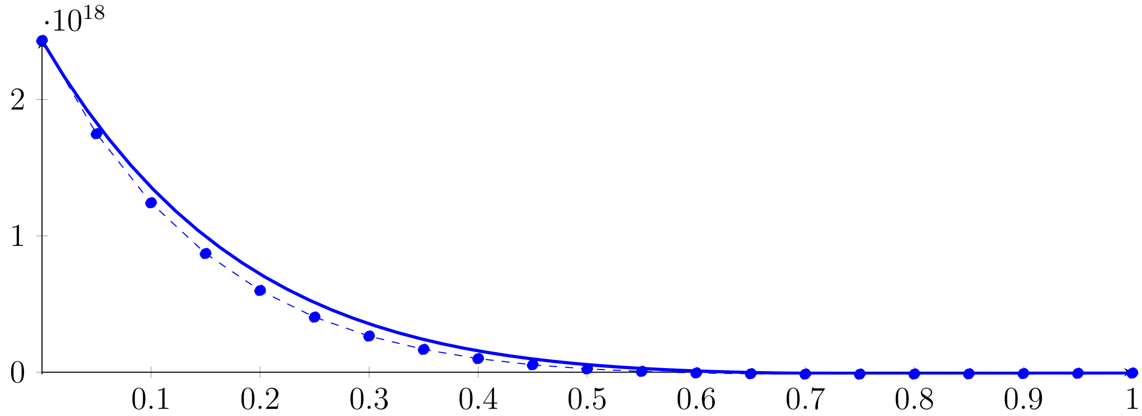
$\Longrightarrow$ Bisection: first half $[0, 1.5625]$ und second half $[1.5625, 3.125]$

Bisection point is very near to a root?!?

2.5 Recursion Branch 1 1 1 1 1 on the First Half $[0, 1.5625]$

Normalized monomial und Bézier representations and the Bézier polygon:

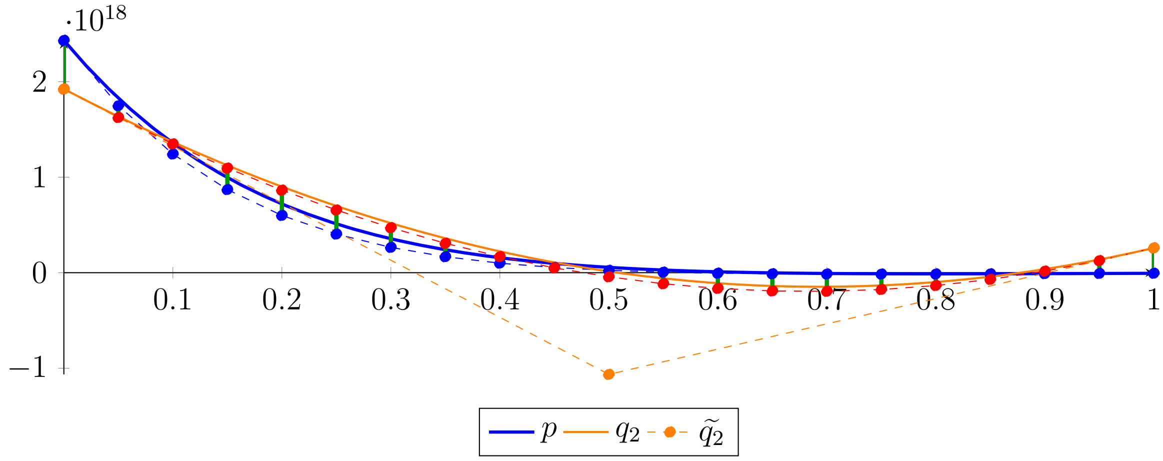
$$p = -8.46356\cdot 10^{7}X^{20} - 1.83419\cdot 10^{8}X^{19} - 5.89672\cdot 10^{9}X^{18} - 1.44753\cdot 10^{8}X^{17} - 9.10891\cdot 10^{9}X^{16} - 1.29397\cdot 10^{12}X^{15} + 2.06942\cdot 10^{13}X^{14} - 2.50213\cdot 10^{14}X^{13} + 2.39489\cdot 10^{15}X^{12} - 1.83753\cdot 10^{16}X^{11} + 1.13411\cdot 10^{17}X^{10} - 5.63011\cdot 10^{17}X^{9} + 2.2393\cdot 10^{18}X^{8} - 7.07891\cdot 10^{18}X^{7} + 1.7559\cdot 10^{19}X^{6} - 3.35274\cdot 10^{19}X^{5} + 4.79091\cdot 10^{19}X^{4} - 4.90987\cdot 10^{19}X^{3} + 3.37006\cdot 10^{19}X^{2} - 1.36765\cdot 10^{19}X + 2.4329\cdot 10^{18}$$
$$= 2.4329\cdot 10^{18}B_{0,20}(X) + 1.74908\cdot 10^{18}B_{1,20}(X) + 1.24263\cdot 10^{18}B_{2,20}(X) + 8.70475\cdot 10^{17}B_{3,20}(X) + 5.99447\cdot 10^{17}B_{4,20}(X) + 4.04086\cdot 10^{17}B_{5,20}(X) + 2.64953\cdot 10^{17}B_{6,20}(X) + 1.67278\cdot 10^{17}B_{7,20}(X) + 9.9902\cdot 10^{16}B_{8,20}(X) + 5.44408\cdot 10^{16}B_{9,20}(X) + 2.46418\cdot 10^{16}B_{10,20}(X) + 5.87625\cdot 10^{15}B_{11,20}(X) - 5.2528\cdot 10^{15}B_{12,20}(X) - 1.12129\cdot 10^{16}B_{13,20}(X) - 1.37757\cdot 10^{16}B_{14,20}(X) - 1.41949\cdot 10^{16}B_{15,20}(X) - 1.33428\cdot 10^{16}B_{16,20}(X) - 1.1813\cdot 10^{16}B_{17,20}(X) - 9.99781\cdot 10^{15}B_{18,20}(X) - 8.1465\cdot 10^{15}B_{19,20}(X) - 6.40794\cdot 10^{15}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 4.31191\cdot 10^{18}X^{2} - 5.97639\cdot 10^{18}X + 1.92335\cdot 10^{18}$$
$$= 1.92335\cdot 10^{18}B_{0,2} - 1.06485\cdot 10^{18}B_{1,2} + 2.58866\cdot 10^{17}B_{2,2}$$

$$\widetilde{q}_2 = 4.22647\cdot 10^{20}X^{20} - 4.22238\cdot 10^{21}X^{19} + 1.95228\cdot 10^{22}X^{18} - 5.54285\cdot 10^{22}X^{17} + 1.08113\cdot 10^{23}X^{16} - 1.53561\cdot 10^{23}X^{15} + 1.64271\cdot 10^{23}X^{14} - 1.35017\cdot 10^{23}X^{13} + 8.62206\cdot 10^{22}X^{12} - 4.29838\cdot 10^{22}X^{11} + 1.67217\cdot 10^{22}X^{10} - 5.0494\cdot 10^{21}X^{9} + 1.17114\cdot 10^{21}X^{8} - 2.05136\cdot 10^{20}X^{7} + 2.64681\cdot 10^{19}X^{6} - 2.43005\cdot 10^{18}X^{5} + 1.51077\cdot 10^{17}X^{4} - 5.84913\cdot 10^{15}X^{3} + 4.31203\cdot 10^{18}X^{2} - 5.97639\cdot 10^{18}X + 1.92335\cdot 10^{18}$$
$$= 1.92335\cdot 10^{18}B_{0,20} + 1.62453\cdot 10^{18}B_{1,20} + 1.3484\cdot 10^{18}B_{2,20} + 1.09497\cdot 10^{18}B_{3,20} + 8.64249\cdot 10^{17}B_{4,20} + 6.56147\cdot 10^{17}B_{5,20} + 4.70961\cdot 10^{17}B_{6,20} + 3.07958\cdot 10^{17}B_{7,20} + 1.68614\cdot 10^{17}B_{8,20} + 5.04439\cdot 10^{16}B_{9,20} - 4.30478\cdot 10^{16}B_{10,20} - 1.15999\cdot 10^{17}B_{11,20} - 1.64274\cdot 10^{17}B_{12,20} - 1.91396\cdot 10^{17}B_{13,20} - 1.94829\cdot 10^{17}B_{14,20} - 1.76097\cdot 10^{17}B_{15,20} - 1.34438\cdot 10^{17}B_{16,20} - 7.01689\cdot 10^{16}B_{17,20} + 1.68181\cdot 10^{16}B_{18,20} + 1.26495\cdot 10^{17}B_{19,20} + 2.58866\cdot 10^{17}B_{20,20}$$



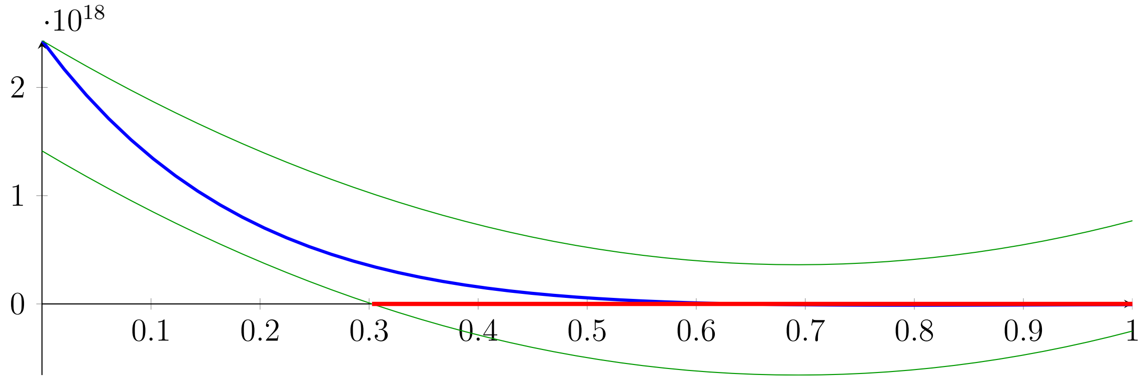
The maximum difference of the Bézier coefficients is $\delta = 5.09555 \cdot 10^{17}$.
Bounding polynomials M and m:

$$M = 4.31191 \cdot 10^{18} X^2 - 5.97639 \cdot 10^{18} X + 2.4329 \cdot 10^{18}$$
$$m = 4.31191 \cdot 10^{18} X^2 - 5.97639 \cdot 10^{18} X + 1.41379 \cdot 10^{18}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.302649, 1.08337\}$$

Intersection intervals:



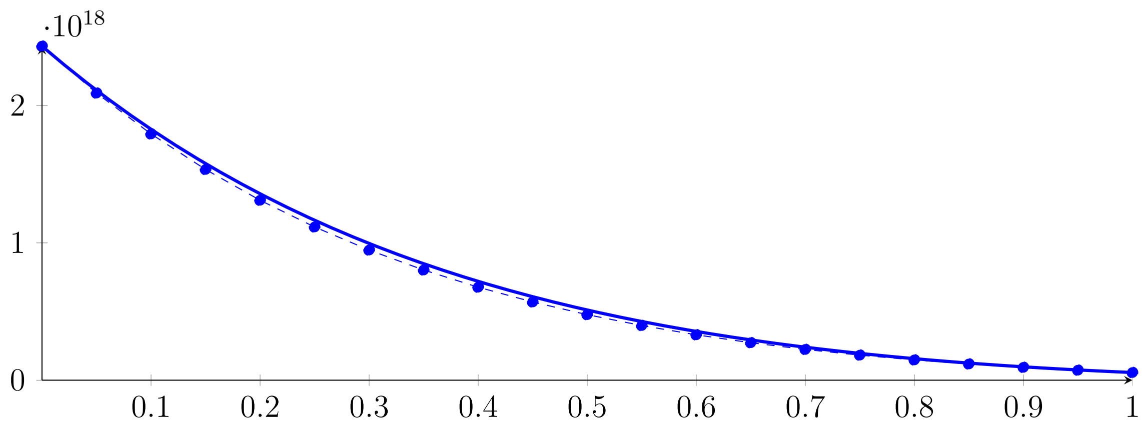
$[0.302649, 1]$

Longest intersection interval: 0.697351
$\Longrightarrow$ Bisection: first half $[0, 0.78125]$ und second half $[0.78125, 1.5625]$

2.6 Recursion Branch 1 1 1 1 1 1 on the First Half $[0, 0.78125]$

Normalized monomial und Bézier representations and the Bézier polygon:

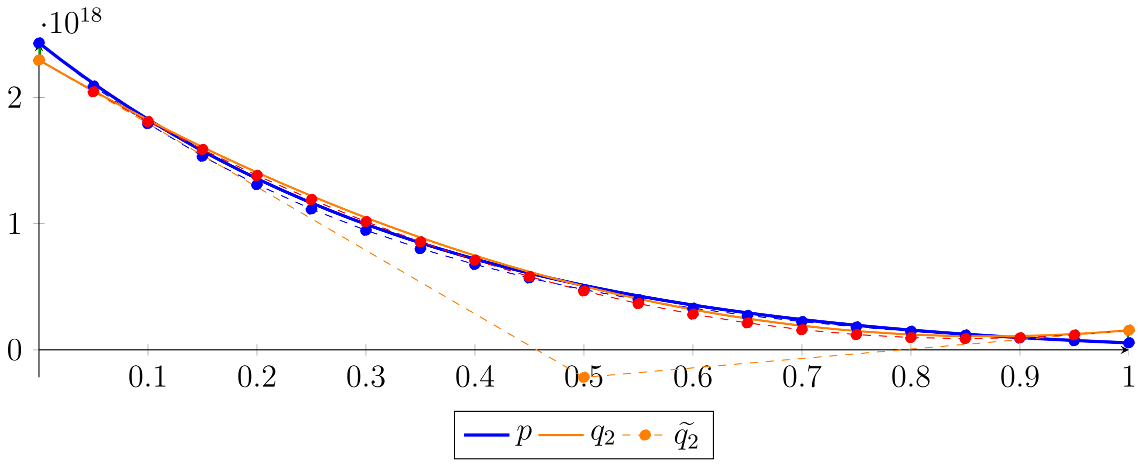
$$\begin{aligned} p = & -5.99458 \cdot 10^{8} X^{20} + 1.22241 \cdot 10^{9} X^{19} - 2.5064 \cdot 10^{10} X^{18} + 5.69111 \cdot 10^{10} X^{17} - 4.32167 \cdot 10^{11} X^{16} \\ & + 3.59181 \cdot 10^{11} X^{15} - 1.88449 \cdot 10^{11} X^{14} - 1.23516 \cdot 10^{11} X^{13} + 9.39071 \cdot 10^{10} X^{12} - 9.07218 \cdot 10^{12} X^{11} \\ & + 1.10587 \cdot 10^{14} X^{10} - 1.09966 \cdot 10^{15} X^{9} + 8.74728 \cdot 10^{15} X^{8} - 5.5304 \cdot 10^{16} X^{7} + 2.7436 \cdot 10^{17} X^{6} - 1.04773 \\ & \cdot 10^{18} X^{5} + 2.99432 \cdot 10^{18} X^{4} - 6.13734 \cdot 10^{18} X^{3} + 8.42515 \cdot 10^{18} X^{2} - 6.83824 \cdot 10^{18} X + 2.4329 \cdot 10^{18} \\ = & \, 2.4329 \cdot 10^{18} B_{0,20}(X) + 2.09099 \cdot 10^{18} B_{1,20}(X) + 1.79342 \cdot 10^{18} B_{2,20}(X) + 1.53481 \\ & \cdot 10^{18} B_{3,20}(X) + 1.31039 \cdot 10^{18} B_{4,20}(X) + 1.11596 \cdot 10^{18} B_{5,20}(X) + 9.47772 \cdot 10^{17} B_{6,20}(X) \\ & + 8.02552 \cdot 10^{17} B_{7,20}(X) + 6.77393 \cdot 10^{17} B_{8,20}(X) + 5.69738 \cdot 10^{17} B_{9,20}(X) + 4.77334 \\ & \cdot 10^{17} B_{10,20}(X) + 3.98201 \cdot 10^{17} B_{11,20}(X) + 3.30596 \cdot 10^{17} B_{12,20}(X) + 2.72992 \cdot 10^{17} B_{13,20}(X) \\ & + 2.24047 \cdot 10^{17} B_{14,20}(X) + 1.82588 \cdot 10^{17} B_{15,20}(X) + 1.47587 \cdot 10^{17} B_{16,20}(X) + 1.18148 \\ & \cdot 10^{17} B_{17,20}(X) + 9.34869 \cdot 10^{16} B_{18,20}(X) + 7.29227 \cdot 10^{16} B_{19,20}(X) + 5.58617 \cdot 10^{16} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = 2.88766{\cdot}10^{18}X^2 - 5.029{\cdot}10^{18}X + 2.29717{\cdot}10^{18}$$
$$= 2.29717{\cdot}10^{18}B_{0,2} - 2.17329{\cdot}10^{17}B_{1,2} + 1.55831{\cdot}10^{17}B_{2,2}$$

$$\widetilde{q_2} = 1.53374{\cdot}10^{20}X^{20} - 1.52613{\cdot}10^{21}X^{19} + 7.02511{\cdot}10^{21}X^{18} - 1.98489{\cdot}10^{22}X^{17} + 3.85094{\cdot}10^{22}X^{16} - 5.43764{\cdot}10^{22}X^{15} + 5.77864{\cdot}10^{22}X^{14} - 4.71419{\cdot}10^{22}X^{13} + 2.98498{\cdot}10^{22}X^{12} - 1.47408{\cdot}10^{22}X^{11} + 5.67716{\cdot}10^{21}X^{10} - 1.69749{\cdot}10^{21}X^{9} + 3.901{\cdot}10^{20}X^{8} - 6.76184{\cdot}10^{19}X^{7} + 8.5589{\cdot}10^{18}X^{6} - 7.52323{\cdot}10^{17}X^{5} + 4.2384{\cdot}10^{16}X^{4} - 1.33437{\cdot}10^{15}X^{3} + 2.88768{\cdot}10^{18}X^{2} - 5.029{\cdot}10^{18}X + 2.29717{\cdot}10^{18}$$
$$= 2.29717{\cdot}10^{18}B_{0,20} + 2.04572{\cdot}10^{18}B_{1,20} + 1.80947{\cdot}10^{18}B_{2,20} + 1.58841{\cdot}10^{18}B_{3,20} + 1.38256{\cdot}10^{18}B_{4,20} + 1.19189{\cdot}10^{18}B_{5,20} + 1.01648{\cdot}10^{18}B_{6,20} + 8.56104{\cdot}10^{17}B_{7,20} + 7.11254{\cdot}10^{17}B_{8,20} + 5.8106{\cdot}10^{17}B_{9,20} + 4.66779{\cdot}10^{17}B_{10,20} + 3.66922{\cdot}10^{17}B_{11,20} + 2.82996{\cdot}10^{17}B_{12,20} + 2.13687{\cdot}10^{17}B_{13,20} + 1.5995{\cdot}10^{17}B_{14,20} + 1.21215{\cdot}10^{17}B_{15,20} + 9.77616{\cdot}10^{16}B_{16,20} + 8.9476{\cdot}10^{16}B_{17,20} + 9.63971{\cdot}10^{16}B_{18,20} + 1.18515{\cdot}10^{17}B_{19,20} + 1.55831{\cdot}10^{17}B_{20,20}$$



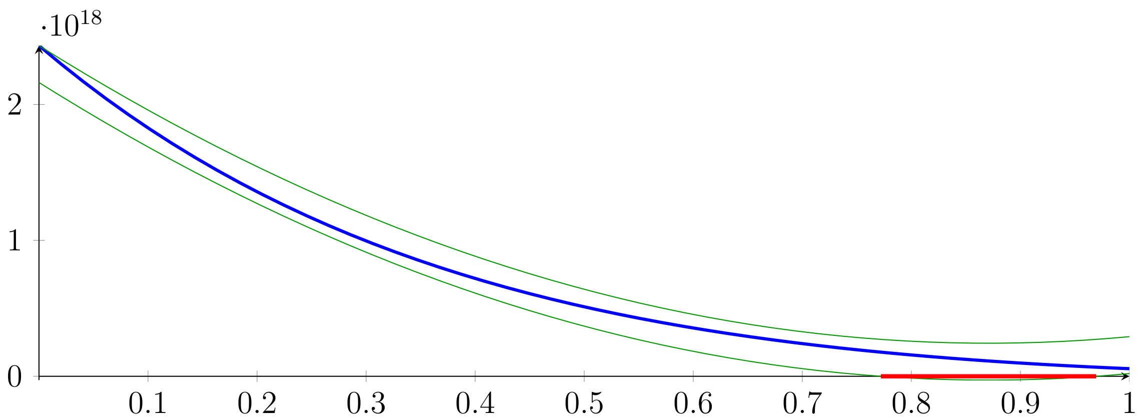
The maximum difference of the Bézier coefficients is $\delta = 1.35733{\cdot}10^{17}$.
Bounding polynomials M and m:

$$M = 2.88766{\cdot}10^{18}X^2 - 5.029{\cdot}10^{18}X + 2.4329{\cdot}10^{18}$$
$$m = 2.88766{\cdot}10^{18}X^2 - 5.029{\cdot}10^{18}X + 2.16144{\cdot}10^{18}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.772086, 0.969463\}$$

Intersection intervals:



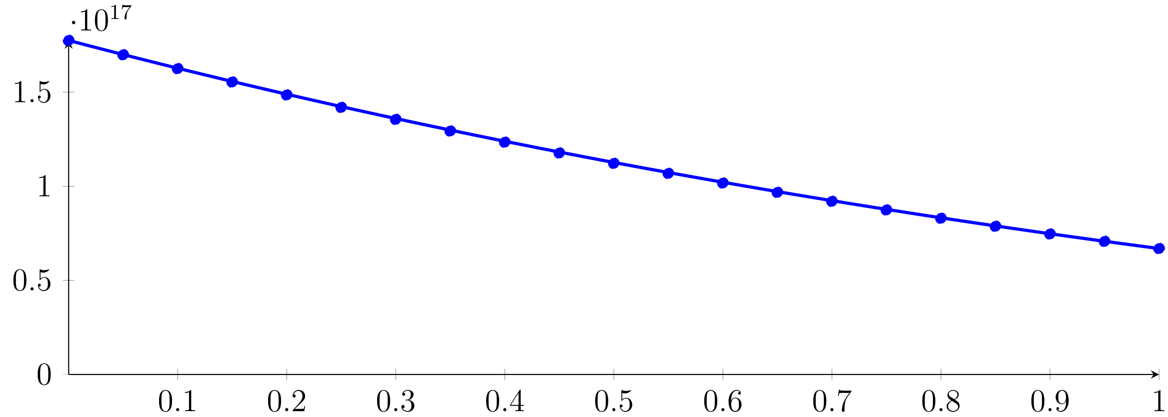
$$[0.772086, 0.969463]$$

Longest intersection interval: 0.197377
$\Longrightarrow$ Selective recursion: interval 1: $[0.603192, 0.757393]$,

2.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.603192, 0.757393]$

Normalized monomial und Bézier representations and the Bézier polygon:

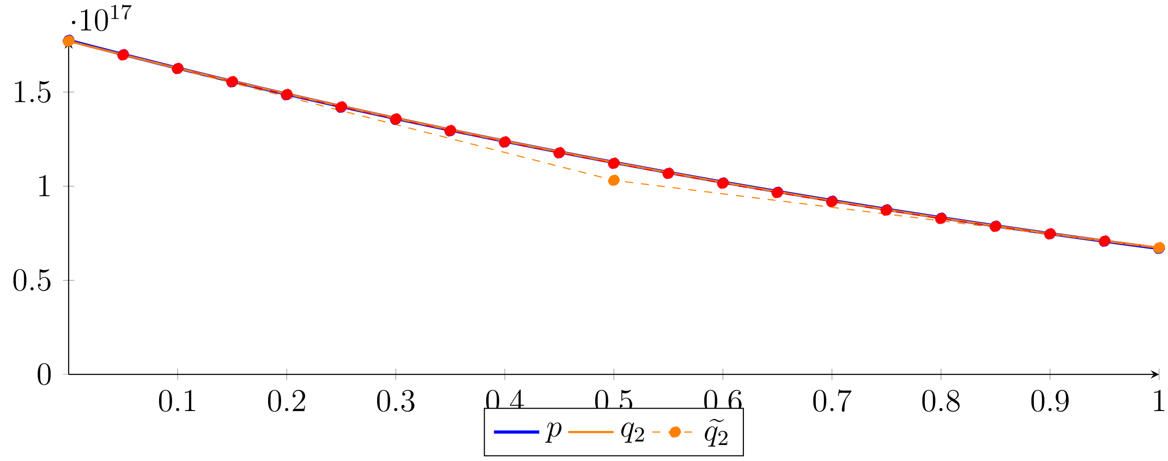
$$p = -1.08865\cdot10^{8}X^{20} + 4.97088\cdot10^{8}X^{19} - 3.54145\cdot10^{9}X^{18} + 1.26406\cdot10^{10}X^{17} - 7.69042\cdot10^{10}X^{16} + 5.84141\cdot10^{10}X^{15} - 2.37496\cdot10^{10}X^{14} - 1.24528\cdot10^{10}X^{13} - 6.65767\cdot10^{10}X^{12} - 1.0083\cdot10^{10}X^{11} - 2.05981\cdot10^{10}X^{10} - 3.50432\cdot10^{9}X^{9} + 7.88471\cdot10^{9}X^{8} - 2.29816\cdot10^{11}X^{7} + 5.09584\cdot10^{12}X^{6} - 8.56759\cdot10^{13}X^{5} + 1.05639\cdot10^{15}X^{4} - 9.08165\cdot10^{15}X^{3} + 5.01317\cdot10^{16}X^{2} - 1.52601\cdot10^{17}X + 1.77497\cdot10^{17}$$
$$= 1.77497\cdot10^{17}B_{0,20}(X) + 1.69867\cdot10^{17}B_{1,20}(X) + 1.62501\cdot10^{17}B_{2,20}(X) + 1.55391\cdot10^{17}B_{3,20}(X) + 1.48528\cdot10^{17}B_{4,20}(X) + 1.41907\cdot10^{17}B_{5,20}(X) + 1.35519\cdot10^{17}B_{6,20}(X) + 1.29356\cdot10^{17}B_{7,20}(X) + 1.23413\cdot10^{17}B_{8,20}(X) + 1.17683\cdot10^{17}B_{9,20}(X) + 1.12158\cdot10^{17}B_{10,20}(X) + 1.06833\cdot10^{17}B_{11,20}(X) + 1.01702\cdot10^{17}B_{12,20}(X) + 9.67576\cdot10^{16}B_{13,20}(X) + 9.19948\cdot10^{16}B_{14,20}(X) + 8.74079\cdot10^{16}B_{15,20}(X) + 8.29912\cdot10^{16}B_{16,20}(X) + 7.87394\cdot10^{16}B_{17,20}(X) + 7.46473\cdot10^{16}B_{18,20}(X) + 7.07098\cdot10^{16}B_{19,20}(X) + 6.6922\cdot10^{16}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 3.81759\cdot10^{16}X^{2} - 1.48031\cdot10^{17}X + 1.77125\cdot10^{17}$$
$$= 1.77125\cdot10^{17}B_{0,2} + 1.03109\cdot10^{17}B_{1,2} + 6.72695\cdot10^{16}B_{2,2}$$

$$\widetilde{q}_2 = -2.04042\cdot10^{19}X^{20} + 2.05104\cdot10^{20}X^{19} - 9.54889\cdot10^{20}X^{18} + 2.73172\cdot10^{21}X^{17} - 5.37215\cdot10^{21}X^{16} + 7.69806\cdot10^{21}X^{15} - 8.31293\cdot10^{21}X^{14} + 6.90142\cdot10^{21}X^{13} - 4.45407\cdot10^{21}X^{12} + 2.24475\cdot10^{21}X^{11} - 8.82501\cdot10^{20}X^{10} + 2.68972\cdot10^{20}X^{9} - 6.28712\cdot10^{19}X^{8} + 1.1113\cdot10^{19}X^{7} - 1.46277\cdot10^{18}X^{6} + 1.40955\cdot10^{17}X^{5} - 9.69355\cdot10^{15}X^{4} + 4.41737\cdot10^{14}X^{3} + 3.81644\cdot10^{16}X^{2} - 1.48031\cdot10^{17}X + 1.77125\cdot10^{17}$$
$$= 1.77125\cdot10^{17}B_{0,20} + 1.69723\cdot10^{17}B_{1,20} + 1.62523\cdot10^{17}B_{2,20} + 1.55523\cdot10^{17}B_{3,20} + 1.48723\cdot10^{17}B_{4,20} + 1.42129\cdot10^{17}B_{5,20} + 1.35723\cdot10^{17}B_{6,20} + 1.29546\cdot10^{17}B_{7,20} + 1.23519\cdot10^{17}B_{8,20} + 1.17768\cdot10^{17}B_{9,20} + 1.12121\cdot10^{17}B_{10,20} + 1.0678\cdot10^{17}B_{11,20} + 1.01548\cdot10^{17}B_{12,20} + 9.65874\cdot10^{16}B_{13,20} + 9.17811\cdot10^{16}B_{14,20} + 8.72012\cdot10^{16}B_{15,20} + 8.28102\cdot10^{16}B_{16,20} + 7.86244\cdot10^{16}B_{17,20} + 7.46383\cdot10^{16}B_{18,20} + 7.08535\cdot10^{16}B_{19,20} + 6.72695\cdot10^{16}B_{20,20}$$



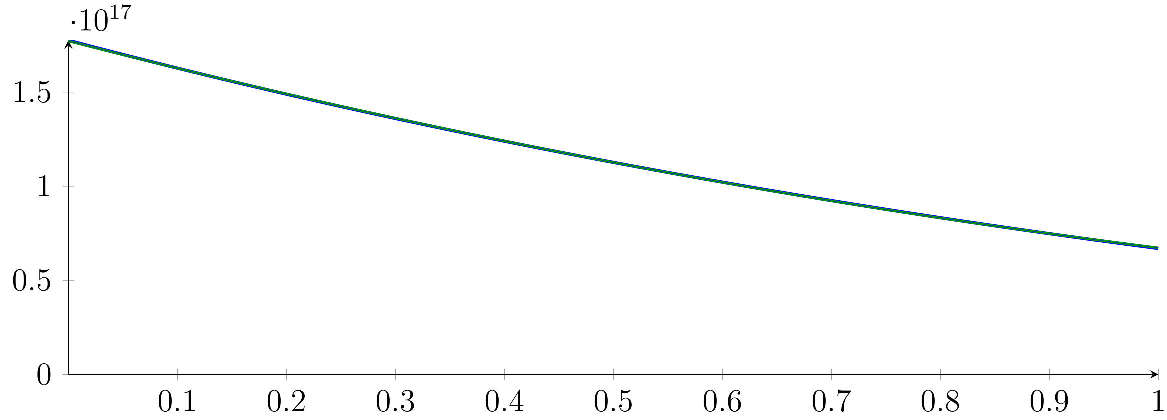
The maximum difference of the Bézier coefficients is $\delta = 3.72135{\cdot}10^{14}$.
Bounding polynomials M and m:

$$M = 3.81759{\cdot}10^{16}X^2 - 1.48031{\cdot}10^{17}X + 1.77497{\cdot}10^{17}$$
$$m = 3.81759{\cdot}10^{16}X^2 - 1.48031{\cdot}10^{17}X + 1.76753{\cdot}10^{17}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{\}$$

Intersection intervals:

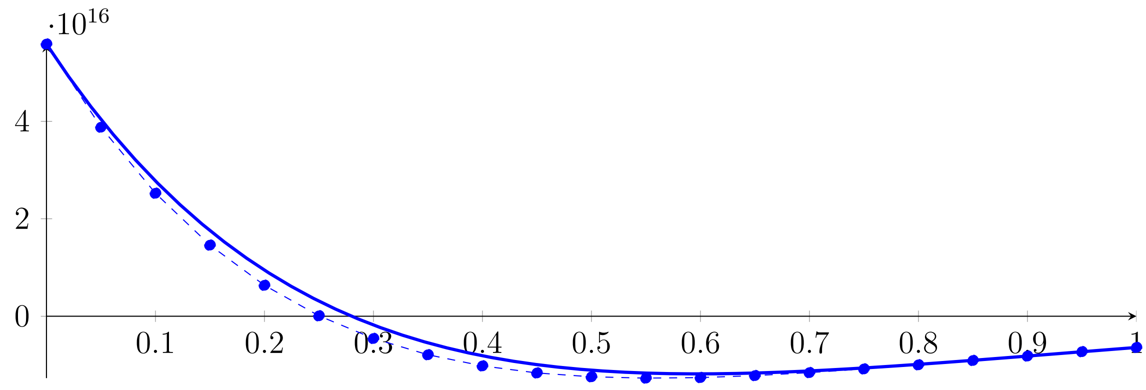


No intersection intervals with the x axis.

2.8 Recursion Branch 1 1 1 1 1 2 on the Second Half $[0.78125, 1.5625]$

Normalized monomial und Bézier representations and the Bézier polygon:

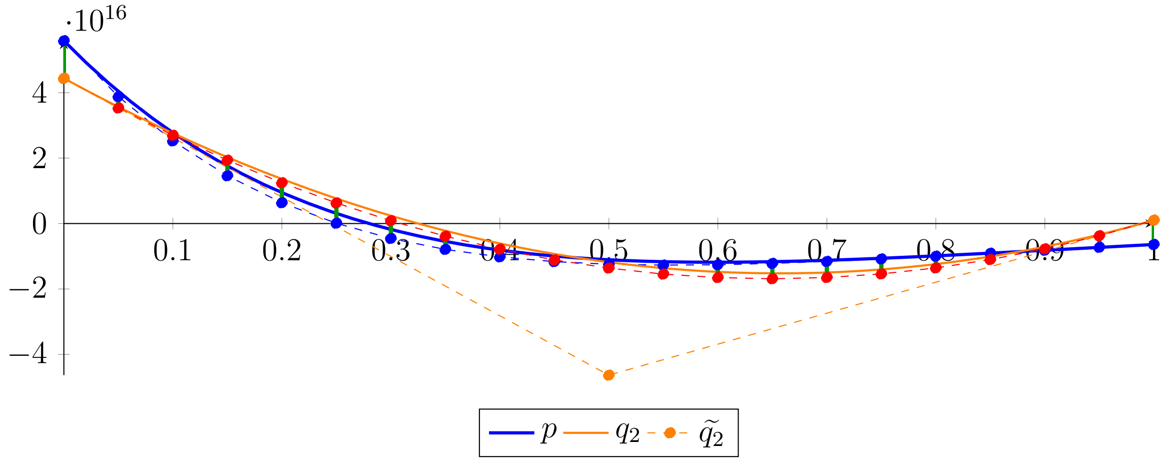
$$\begin{aligned}p &= 1.04049{\cdot}10^{7}X^{20} - 6.97254{\cdot}10^{7}X^{19} + 2.1021{\cdot}10^{8}X^{18} - 1.44877{\cdot}10^{9}X^{17} + 6.15974{\cdot}10^{9}X^{16} - 4.81145\\&\quad{\cdot}10^{9}X^{15} + 1.28745{\cdot}10^{9}X^{14} - 1.6789{\cdot}10^{10}X^{13} + 2.87929{\cdot}10^{11}X^{12} - 3.92923{\cdot}10^{12}X^{11} + 4.3068\\&\quad{\cdot}10^{13}X^{10} - 3.76433{\cdot}10^{14}X^{9} + 2.60774{\cdot}10^{15}X^{8} - 1.41682{\cdot}10^{16}X^{7} + 5.93915{\cdot}10^{16}X^{6} - 1.8747{\cdot}10^{17}X^{5}\\&\quad + 4.29741{\cdot}10^{17}X^{4} - 6.76415{\cdot}10^{17}X^{3} + 6.65601{\cdot}10^{17}X^{2} - 3.41221{\cdot}10^{17}X + 5.58617{\cdot}10^{16}\\&= 5.58617{\cdot}10^{16}B_{0,20}(X) + 3.88007{\cdot}10^{16}B_{1,20}(X) + 2.52428{\cdot}10^{16}B_{2,20}(X) + 1.45947\\&\quad{\cdot}10^{16}B_{3,20}(X) + 6.35188{\cdot}10^{15}B_{4,20}(X) + 8.61285{\cdot}10^{13}B_{5,20}(X) - 4.56449{\cdot}10^{15}B_{6,20}(X)\\&\quad - 7.90513{\cdot}10^{15}B_{7,20}(X) - 1.01922{\cdot}10^{16}B_{8,20}(X) - 1.16401{\cdot}10^{16}B_{9,20}(X) - 1.24276\\&\quad{\cdot}10^{16}B_{10,20}(X) - 1.27029{\cdot}10^{16}B_{11,20}(X) - 1.25884{\cdot}10^{16}B_{12,20}(X) - 1.21842{\cdot}10^{16}B_{13,20}(X)\\&\quad - 1.15719{\cdot}10^{16}B_{14,20}(X) - 1.08174{\cdot}10^{16}B_{15,20}(X) - 9.97347{\cdot}10^{15}B_{16,20}(X) - 9.08173\\&\quad{\cdot}10^{15}B_{17,20}(X) - 8.17469{\cdot}10^{15}B_{18,20}(X) - 7.27722{\cdot}10^{15}B_{19,20}(X) - 6.40794{\cdot}10^{15}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$q_2 = 1.38042{\cdot}10^{17}X^2 - 1.8141{\cdot}10^{17}X + 4.43781{\cdot}10^{16}$$
$$= 4.43781{\cdot}10^{16}B_{0,2} - 4.63271{\cdot}10^{16}B_{1,2} + 1.01021{\cdot}10^{15}B_{2,2}$$

$$\widetilde{q_2} = 1.614{\cdot}10^{19}X^{20} - 1.61381{\cdot}10^{20}X^{19} + 7.46879{\cdot}10^{20}X^{18} - 2.12276{\cdot}10^{21}X^{17} + 4.1452{\cdot}10^{21}X^{16} - 5.89507{\cdot}10^{21}X^{15} + 6.31465{\cdot}10^{21}X^{14} - 5.19757{\cdot}10^{21}X^{13} + 3.32419{\cdot}10^{21}X^{12} - 1.65983{\cdot}10^{21}X^{11} + 6.46707{\cdot}10^{20}X^{10} - 1.9555{\cdot}10^{20}X^{9} + 4.5407{\cdot}10^{19}X^{8} - 7.96422{\cdot}10^{18}X^{7} + 1.03072{\cdot}10^{18}X^{6} - 9.53514{\cdot}10^{16}X^{5} + 6.029{\cdot}10^{15}X^{4} - 2.40634{\cdot}10^{14}X^{3} + 1.38048{\cdot}10^{17}X^{2} - 1.8141{\cdot}10^{17}X + 4.43781{\cdot}10^{16}$$
$$= 4.43781{\cdot}10^{16}B_{0,20} + 3.53076{\cdot}10^{16}B_{1,20} + 2.69636{\cdot}10^{16}B_{2,20} + 1.9346{\cdot}10^{16}B_{3,20} + 1.24558{\cdot}10^{16}B_{4,20} + 6.28914{\cdot}10^{15}B_{5,20} + 8.57628{\cdot}10^{14}B_{6,20} - 3.8672{\cdot}10^{15}B_{7,20} - 7.82815{\cdot}10^{15}B_{8,20} - 1.11209{\cdot}10^{16}B_{9,20} - 1.36112{\cdot}10^{16}B_{10,20} - 1.54573{\cdot}10^{16}B_{11,20} - 1.65016{\cdot}10^{16}B_{12,20} - 1.68777{\cdot}10^{16}B_{13,20} - 1.64895{\cdot}10^{16}B_{14,20} - 1.53949{\cdot}10^{16}B_{15,20} - 1.35649{\cdot}10^{16}B_{16,20} - 1.10115{\cdot}10^{16}B_{17,20} - 7.73069{\cdot}10^{15}B_{18,20} - 3.72352{\cdot}10^{15}B_{19,20} + 1.01021{\cdot}10^{15}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.14836{\cdot}10^{16}$.

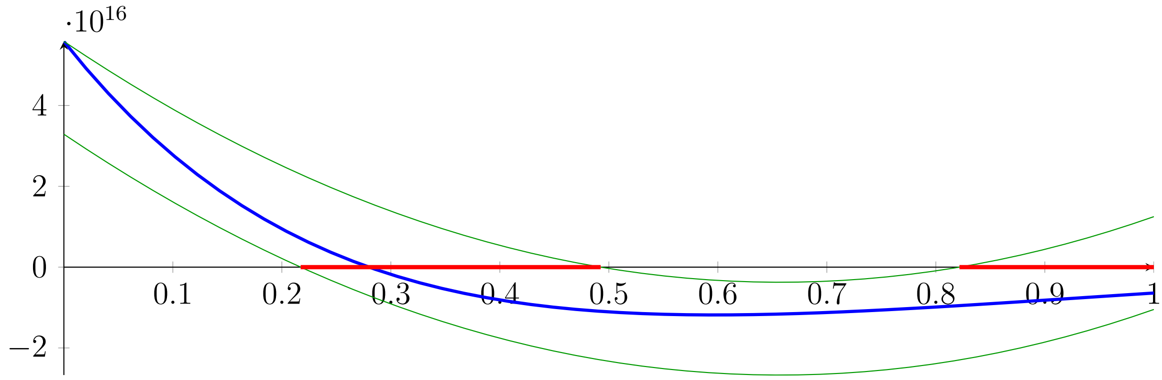
Bounding polynomials M and m:

$$M = 1.38042{\cdot}10^{17}X^2 - 1.8141{\cdot}10^{17}X + 5.58617{\cdot}10^{16}$$
$$m = 1.38042{\cdot}10^{17}X^2 - 1.8141{\cdot}10^{17}X + 3.28945{\cdot}10^{16}$$

Root of M and m:

$$N(M) = \{0.492503, 0.82166\} \qquad N(m) = \{0.217237, 1.09693\}$$

Intersection intervals:



$$[0.217237, 0.492503],\ [0.82166, 1]$$

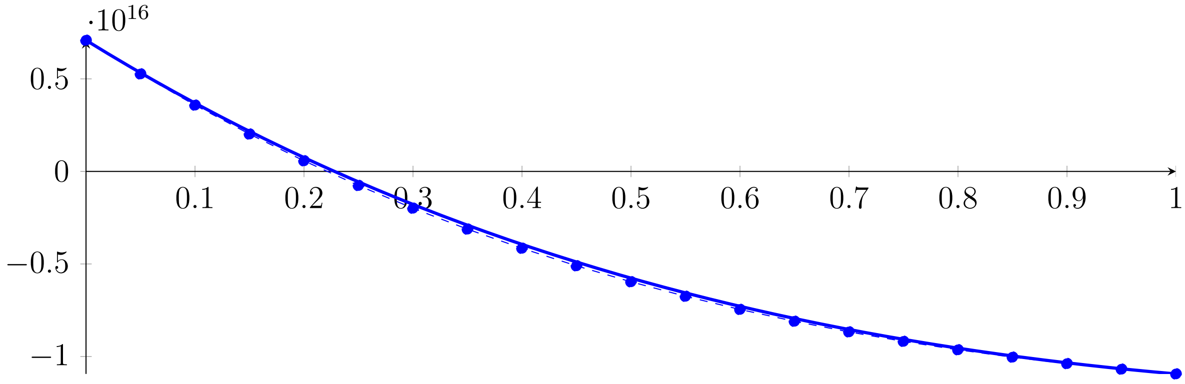
Longest intersection interval: 0.275266

$\Longrightarrow$ Selective recursion: interval 1: $[0.950966, 1.16602]$, interval 2: $[1.42317, 1.5625]$,

2.9 Recursion Branch 1 1 1 1 1 2 1 in Interval 1: $[0.950966, 1.16602]$

Normalized monomial und Bézier representations and the Bézier polygon:

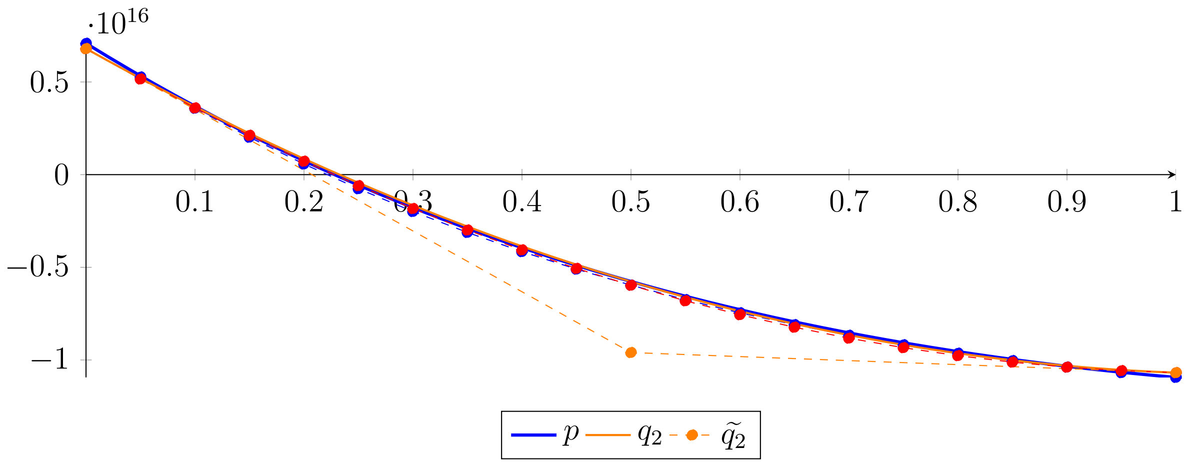
$$p = 4.86008 \cdot 10^{6} X^{20} - 4.21994 \cdot 10^{7} X^{19} + 1.05285 \cdot 10^{8} X^{18} - 7.03463 \cdot 10^{8} X^{17} + 2.78855 \cdot 10^{9} X^{16} - 1.96295 \cdot 10^{9} X^{15} + 5.11051 \cdot 10^{8} X^{14} - 4.59694 \cdot 10^{7} X^{13} + 9.61529 \cdot 10^{8} X^{12} - 2.64873 \cdot 10^{8} X^{11} + 1.7016 \cdot 10^{8} X^{10} - 2.72649 \cdot 10^{9} X^{9} + 6.45089 \cdot 10^{10} X^{8} - 1.22449 \cdot 10^{12} X^{7} + 1.78307 \cdot 10^{13} X^{6} - 1.93918 \cdot 10^{14} X^{5} + 1.51259 \cdot 10^{15} X^{4} - 7.93252 \cdot 10^{15} X^{3} + 2.49352 \cdot 10^{16} X^{2} - 3.63629 \cdot 10^{16} X + 7.08479 \cdot 10^{15}$$
$$= 7.08479 \cdot 10^{15} B_{0,20}(X) + 5.26664 \cdot 10^{15} B_{1,20}(X) + 3.57974 \cdot 10^{15} B_{2,20}(X) + 2.01711 \cdot 10^{15} B_{3,20}(X) + 5.72121 \cdot 10^{14} B_{4,20}(X) - 7.61584 \cdot 10^{14} B_{5,20}(X) - 1.99006 \cdot 10^{15} B_{6,20}(X) - 3.11909 \cdot 10^{15} B_{7,20}(X) - 4.1542 \cdot 10^{15} B_{8,20}(X) - 5.10064 \cdot 10^{15} B_{9,20}(X) - 5.96344 \cdot 10^{15} B_{10,20}(X) - 6.74738 \cdot 10^{15} B_{11,20}(X) - 7.45702 \cdot 10^{15} B_{12,20}(X) - 8.09672 \cdot 10^{15} B_{13,20}(X) - 8.67061 \cdot 10^{15} B_{14,20}(X) - 9.18264 \cdot 10^{15} B_{15,20}(X) - 9.63656 \cdot 10^{15} B_{16,20}(X) - 1.00359 \cdot 10^{16} B_{17,20}(X) - 1.03842 \cdot 10^{16} B_{18,20}(X) - 1.06845 \cdot 10^{16} B_{19,20}(X) - 1.09401 \cdot 10^{16} B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 1.5313 \cdot 10^{16} X^{2} - 3.27975 \cdot 10^{16} X + 6.79901 \cdot 10^{15}$$
$$= 6.79901 \cdot 10^{15} B_{0,2} - 9.59976 \cdot 10^{15} B_{1,2} - 1.06856 \cdot 10^{16} B_{2,2}$$

$$\widetilde{q}_2 = 2.11713 \cdot 10^{18} X^{20} - 2.12001 \cdot 10^{19} X^{19} + 9.82997 \cdot 10^{19} X^{18} - 2.79999 \cdot 10^{20} X^{17} + 5.4808 \cdot 10^{20} X^{16} - 7.81336 \cdot 10^{20} X^{15} + 8.38806 \cdot 10^{20} X^{14} - 6.91628 \cdot 10^{20} X^{13} + 4.42786 \cdot 10^{20} X^{12} - 2.21089 \cdot 10^{20} X^{11} + 8.60346 \cdot 10^{19} X^{10} - 2.59489 \cdot 10^{19} X^{9} + 6.00391 \cdot 10^{18} X^{8} - 1.04955 \cdot 10^{18} X^{7} + 1.3587 \cdot 10^{17} X^{6} - 1.27062 \cdot 10^{16} X^{5} + 8.3018 \cdot 10^{14} X^{4} - 3.51997 \cdot 10^{13} X^{3} + 1.53138 \cdot 10^{16} X^{2} - 3.27975 \cdot 10^{16} X + 6.79901 \cdot 10^{15}$$
$$= 6.79901 \cdot 10^{15} B_{0,20} + 5.15913 \cdot 10^{15} B_{1,20} + 3.59986 \cdot 10^{15} B_{2,20} + 2.12115 \cdot 10^{15} B_{3,20} + 7.23144 \cdot 10^{14} B_{4,20} - 5.94657 \cdot 10^{14} B_{5,20} - 1.83073 \cdot 10^{15} B_{6,20} - 2.98885 \cdot 10^{15} B_{7,20} - 4.06136 \cdot 10^{15} B_{8,20} - 5.06108 \cdot 10^{15} B_{9,20} - 5.97012 \cdot 10^{15} B_{10,20} - 6.80933 \cdot 10^{15} B_{11,20} - 7.55828 \cdot 10^{15} B_{12,20} - 8.23421 \cdot 10^{15} B_{13,20} - 8.82454 \cdot 10^{15} B_{14,20} - 9.33698 \cdot 10^{15} B_{15,20} - 9.7676 \cdot 10^{15} B_{16,20} - 1.01181 \cdot 10^{16} B_{17,20} - 1.03878 \cdot 10^{16} B_{18,20} - 1.0577 \cdot 10^{16} B_{19,20} - 1.06856 \cdot 10^{16} B_{20,20}$$



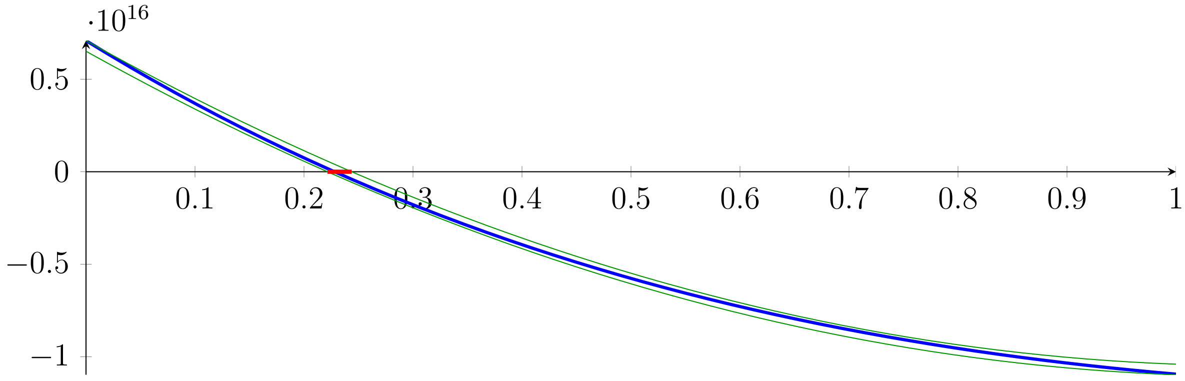
The maximum difference of the Bézier coefficients is $\delta = 2.85775 \cdot 10^{14}$.
Bounding polynomials M and m:

$$M = 1.5313 \cdot 10^{16} X^2 - 3.27975 \cdot 10^{16} X + 7.08479 \cdot 10^{15}$$
$$m = 1.5313 \cdot 10^{16} X^2 - 3.27975 \cdot 10^{16} X + 6.51324 \cdot 10^{15}$$

Root of M and m:

$$N(M) = \{0.243758, 1.89806\} \qquad N(m) = \{0.221495, 1.92032\}$$

Intersection intervals:



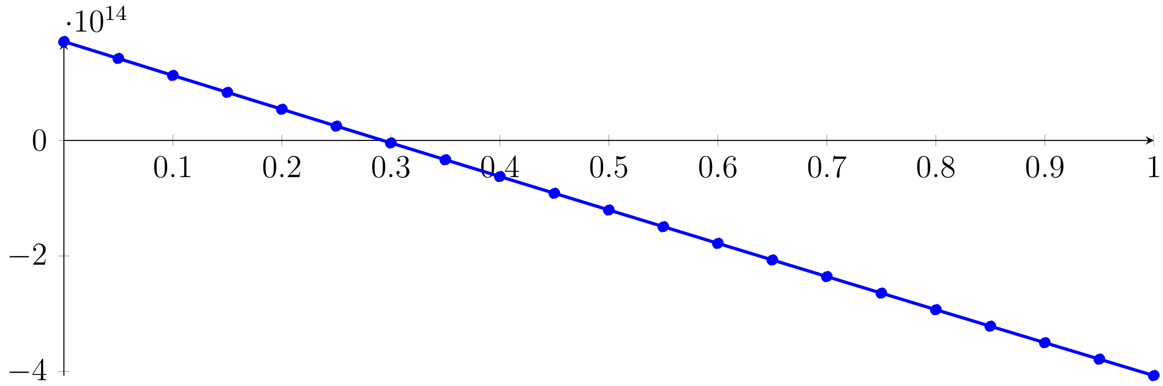
$$[0.221495, 0.243758]$$

Longest intersection interval: 0.0222626
$\Longrightarrow$ Selective recursion: interval 1: [0.998599, 1.00339],

2.10 Recursion Branch 1 1 1 1 1 2 1 1 in Interval 1: [0.998599, 1.00339]

Normalized monomial und Bézier representations and the Bézier polygon:

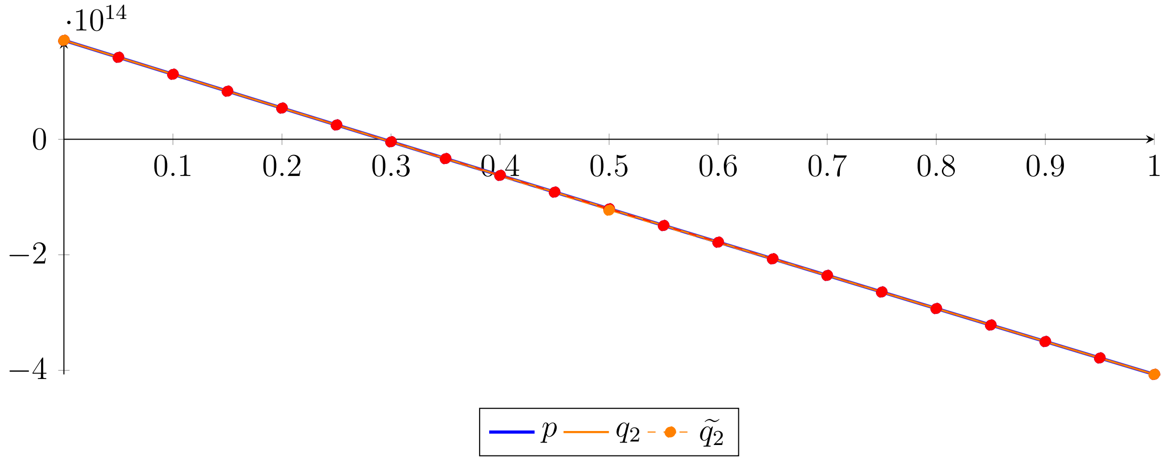
$$\begin{aligned} p = {} & 85020.7X^{20} - 1.04705 \cdot 10^6 X^{19} + 1.67945 \cdot 10^6 X^{18} - 1.42172 \cdot 10^7 X^{17} + 4.70434 \cdot 10^7 X^{16} \\ & - 3.10535 \cdot 10^7 X^{15} + 3.83482 \cdot 10^6 X^{14} - 4.95644 \cdot 10^6 X^{13} + 3.73973 \cdot 10^6 X^{12} - 9.56322 \cdot 10^6 X^{11} \\ & - 3.47572 \cdot 10^6 X^{10} - 2.17298 \cdot 10^6 X^9 + 86604.4X^8 - 14535X^7 - 55717.5X^6 - 937023X^5 \\ & + 3.2191 \cdot 10^8 X^4 - 7.37474 \cdot 10^{10} X^3 + 9.95654 \cdot 10^{12} X^2 - 5.88196 \cdot 10^{14} X + 1.71257 \cdot 10^{14} \\ = {} & 1.71257 \cdot 10^{14} B_{0,20}(X) + 1.41847 \cdot 10^{14} B_{1,20}(X) + 1.1249 \cdot 10^{14} B_{2,20}(X) + 8.31849 \\ & \cdot 10^{13} B_{3,20}(X) + 5.39321 \cdot 10^{13} B_{4,20}(X) + 2.47315 \cdot 10^{13} B_{5,20}(X) - 4.4169 \cdot 10^{12} B_{6,20}(X) \\ & - 3.35133 \cdot 10^{13} B_{7,20}(X) - 6.25576 \cdot 10^{13} B_{8,20}(X) - 9.155 \cdot 10^{13} B_{9,20}(X) - 1.2049 \\ & \cdot 10^{14} B_{10,20}(X) - 1.49379 \cdot 10^{14} B_{11,20}(X) - 1.78216 \cdot 10^{14} B_{12,20}(X) - 2.07001 \cdot 10^{14} B_{13,20}(X) \\ & - 2.35735 \cdot 10^{14} B_{14,20}(X) - 2.64417 \cdot 10^{14} B_{15,20}(X) - 2.93047 \cdot 10^{14} B_{16,20}(X) - 3.21627 \\ & \cdot 10^{14} B_{17,20}(X) - 3.50154 \cdot 10^{14} B_{18,20}(X) - 3.78631 \cdot 10^{14} B_{19,20}(X) - 4.07056 \cdot 10^{14} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 9.84647{\cdot}10^{12}X^2 - 5.88152{\cdot}10^{14}X + 1.71254{\cdot}10^{14} \\ &= 1.71254{\cdot}10^{14}B_{0,2} - 1.22823{\cdot}10^{14}B_{1,2} - 4.07052{\cdot}10^{14}B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 8.72356{\cdot}10^{14}X^{20} - 9.4143{\cdot}10^{15}X^{19} + 4.8034{\cdot}10^{16}X^{18} - 1.52018{\cdot}10^{17}X^{17} + 3.29969{\cdot}10^{17}X^{16} - 5.14887 \\ &\quad {\cdot}10^{17}X^{15} + 5.91016{\cdot}10^{17}X^{14} - 5.0363{\cdot}10^{17}X^{13} + 3.18223{\cdot}10^{17}X^{12} - 1.47354{\cdot}10^{17}X^{11} + 4.86751 \\ &\quad {\cdot}10^{16}X^{10} - 1.0828{\cdot}10^{16}X^{9} + 1.40214{\cdot}10^{15}X^{8} - 5.43259{\cdot}10^{13}X^{7} - 4.89487{\cdot}10^{12}X^{6} - 2.06568 \\ &\quad {\cdot}10^{12}X^{5} + 8.01143{\cdot}10^{11}X^{4} - 8.66498{\cdot}10^{10}X^{3} + 9.85016{\cdot}10^{12}X^{2} - 5.88152{\cdot}10^{14}X + 1.71254{\cdot}10^{14} \\ &= 1.71254{\cdot}10^{14}B_{0,20} + 1.41846{\cdot}10^{14}B_{1,20} + 1.1249{\cdot}10^{14}B_{2,20} + 8.31862{\cdot}10^{13}B_{3,20} + 5.3934 \\ &\quad {\cdot}10^{13}B_{4,20} + 2.47338{\cdot}10^{13}B_{5,20} - 4.41446{\cdot}10^{12}B_{6,20} - 3.35123{\cdot}10^{13}B_{7,20} - 6.25539{\cdot}10^{13}B_{8,20} \\ &\quad - 9.15511{\cdot}10^{13}B_{9,20} - 1.20489{\cdot}10^{14}B_{10,20} - 1.49376{\cdot}10^{14}B_{11,20} - 1.78215{\cdot}10^{14}B_{12,20} \\ &\quad - 2.07004{\cdot}10^{14}B_{13,20} - 2.35736{\cdot}10^{14}B_{14,20} - 2.6442{\cdot}10^{14}B_{15,20} - 2.93049{\cdot}10^{14}B_{16,20} \\ &\quad - 3.21628{\cdot}10^{14}B_{17,20} - 3.50154{\cdot}10^{14}B_{18,20} - 3.78629{\cdot}10^{14}B_{19,20} - 4.07052{\cdot}10^{14}B_{20,20} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.6924{\cdot}10^{9}$.

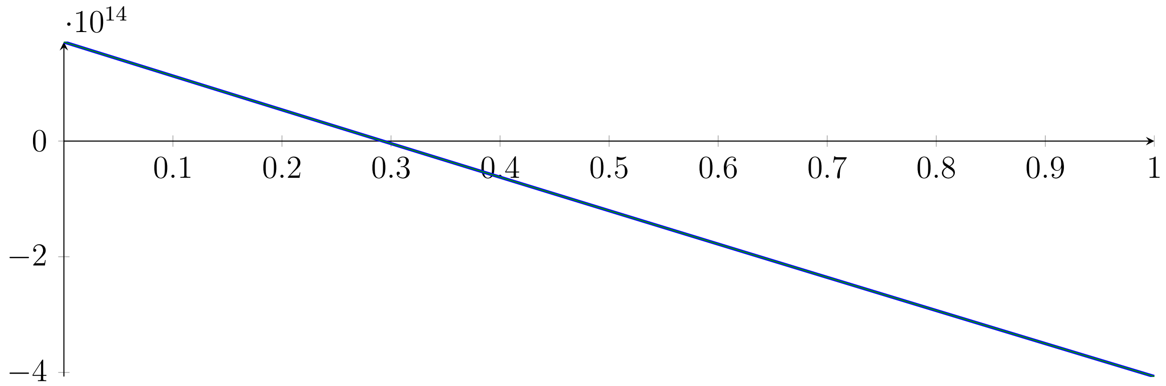
Bounding polynomials M and m:

$$M = 9.84647{\cdot}10^{12}X^2 - 5.88152{\cdot}10^{14}X + 1.71257{\cdot}10^{14}$$
$$m = 9.84647{\cdot}10^{12}X^2 - 5.88152{\cdot}10^{14}X + 1.7125{\cdot}10^{14}$$

Root of M and m:

$$N(M) = \{0.292612, 59.4397\} \qquad N(m) = \{0.292599, 59.4397\}$$

Intersection intervals:



$$[0.292599, 0.292612]$$

Longest intersection interval: $1.26802{\cdot}10^{-05}$

$\Longrightarrow$ Selective recursion: interval 1: $[1, 1]$,

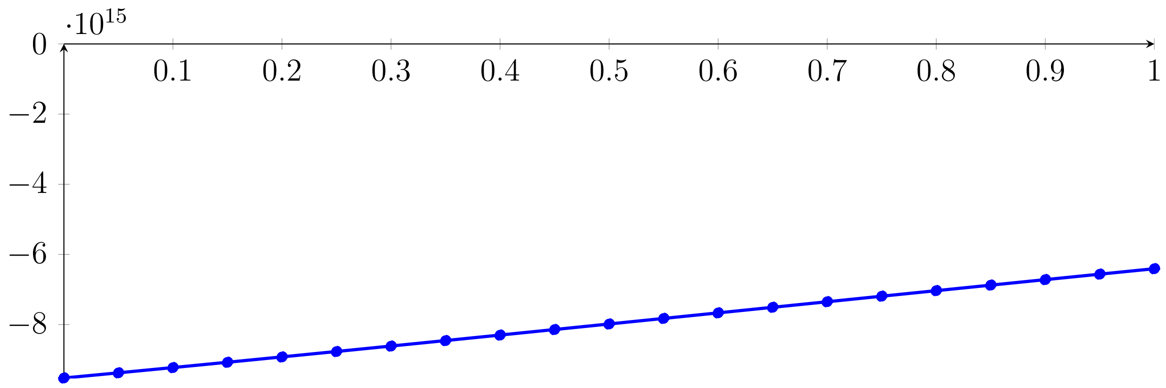
2.11 Recursion Branch 1 1 1 1 1 2 1 1 1 in Interval 1: $[1, 1]$

Found root in interval $[1, 1]$ at recursion depth 9!

2.12 Recursion Branch 1 1 1 1 1 2 2 in Interval 2: $[1.42317, 1.5625]$

Normalized monomial und Bézier representations and the Bézier polygon:

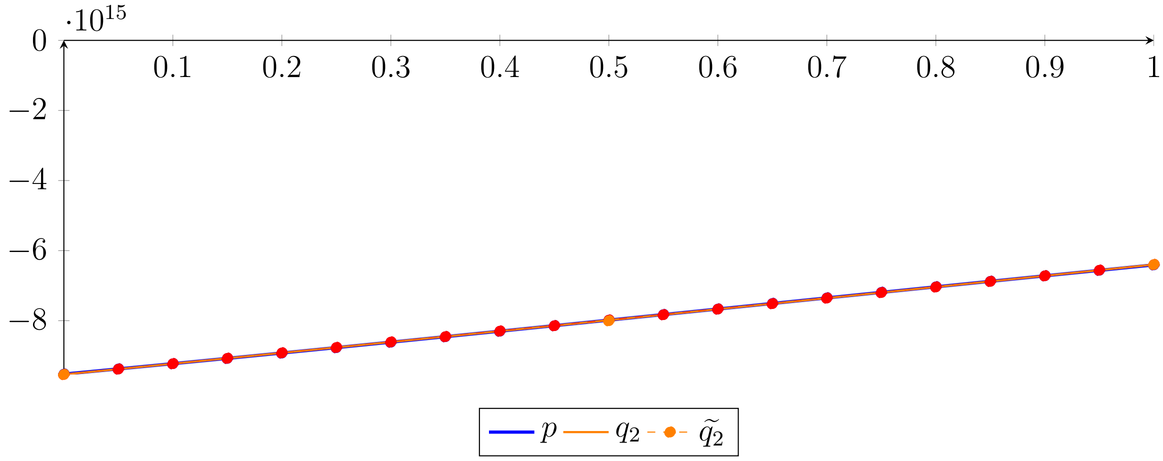
$$p = 8.0089{\cdot}10^{6}X^{20} - 3.91398{\cdot}10^{7}X^{19} + 2.60412{\cdot}10^{8}X^{18} - 9.70186{\cdot}10^{8}X^{17} + 5.26764{\cdot}10^{9}X^{16} - 4.18782{\cdot}10^{9}X^{15} + 1.83815{\cdot}10^{9}X^{14} + 6.75974{\cdot}10^{8}X^{13} + 4.32732{\cdot}10^{9}X^{12} + 5.83157{\cdot}10^{8}X^{11} + 1.26225{\cdot}10^{9}X^{10} + 1.28993{\cdot}10^{8}X^{9} + 8.74736{\cdot}10^{8}X^{8} - 2.19338{\cdot}10^{10}X^{7} + 4.23841{\cdot}10^{11}X^{6} - 5.88013{\cdot}10^{12}X^{5} + 5.44709{\cdot}10^{13}X^{4} - 2.87193{\cdot}10^{14}X^{3} + 4.17296{\cdot}10^{14}X^{2} + 2.93665{\cdot}10^{15}X - 9.52368{\cdot}10^{15}$$
$$= -9.52368{\cdot}10^{15}B_{0,20}(X) - 9.37685{\cdot}10^{15}B_{1,20}(X) - 9.22782{\cdot}10^{15}B_{2,20}(X) - 9.07685{\cdot}10^{15}B_{3,20}(X) - 8.92417{\cdot}10^{15}B_{4,20}(X) - 8.77002{\cdot}10^{15}B_{5,20}(X) - 8.61462{\cdot}10^{15}B_{6,20}(X) - 8.45817{\cdot}10^{15}B_{7,20}(X) - 8.30087{\cdot}10^{15}B_{8,20}(X) - 8.14292{\cdot}10^{15}B_{9,20}(X) - 7.98449{\cdot}10^{15}B_{10,20}(X) - 7.82576{\cdot}10^{15}B_{11,20}(X) - 7.66689{\cdot}10^{15}B_{12,20}(X) - 7.50803{\cdot}10^{15}B_{13,20}(X) - 7.34934{\cdot}10^{15}B_{14,20}(X) - 7.19095{\cdot}10^{15}B_{15,20}(X) - 7.033{\cdot}10^{15}B_{16,20}(X) - 6.87561{\cdot}10^{15}B_{17,20}(X) - 6.71889{\cdot}10^{15}B_{18,20}(X) - 6.56297{\cdot}10^{15}B_{19,20}(X) - 6.40794{\cdot}10^{15}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 7.01056{\cdot}10^{13}X^{2} + 3.065{\cdot}10^{15}X - 9.53396{\cdot}10^{15}$$
$$= -9.53396{\cdot}10^{15}B_{0,2} - 8.00146{\cdot}10^{15}B_{1,2} - 6.39885{\cdot}10^{15}B_{2,2}$$

$$\widetilde{q}_2 = 1.58249{\cdot}10^{18}X^{20} - 1.58955{\cdot}10^{19}X^{19} + 7.39473{\cdot}10^{19}X^{18} - 2.11378{\cdot}10^{20}X^{17} + 4.15339{\cdot}10^{20}X^{16} - 5.94593{\cdot}10^{20}X^{15} + 6.41365{\cdot}10^{20}X^{14} - 5.3174{\cdot}10^{20}X^{13} + 3.42607{\cdot}10^{20}X^{12} - 1.72324{\cdot}10^{20}X^{11} + 6.75953{\cdot}10^{19}X^{10} - 2.05533{\cdot}10^{19}X^{9} + 4.79295{\cdot}10^{18}X^{8} - 8.45077{\cdot}10^{17}X^{7} + 1.1086{\cdot}10^{17}X^{6} - 1.06261{\cdot}10^{16}X^{5} + 7.24918{\cdot}10^{14}X^{4} - 3.26978{\cdot}10^{13}X^{3} + 7.09507{\cdot}10^{13}X^{2} + 3.06499{\cdot}10^{15}X - 9.53396{\cdot}10^{15}$$
$$= -9.53396{\cdot}10^{15}B_{0,20} - 9.38071{\cdot}10^{15}B_{1,20} - 9.22708{\cdot}10^{15}B_{2,20} - 9.07312{\cdot}10^{15}B_{3,20} - 8.91868{\cdot}10^{15}B_{4,20} - 8.7642{\cdot}10^{15}B_{5,20} - 8.60844{\cdot}10^{15}B_{6,20} - 8.45441{\cdot}10^{15}B_{7,20} - 8.2961{\cdot}10^{15}B_{8,20} - 8.14331{\cdot}10^{15}B_{9,20} - 7.98259{\cdot}10^{15}B_{10,20} - 7.82962{\cdot}10^{15}B_{11,20} - 7.66913{\cdot}10^{15}B_{12,20} - 7.51377{\cdot}10^{15}B_{13,20} - 7.35441{\cdot}10^{15}B_{14,20} - 7.19666{\cdot}10^{15}B_{15,20} - 7.03762{\cdot}10^{15}B_{16,20} - 6.87854{\cdot}10^{15}B_{17,20} - 6.719{\cdot}10^{15}B_{18,20} - 6.55911{\cdot}10^{15}B_{19,20} - 6.39885{\cdot}10^{15}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.02729 \cdot 10^{13}$.

Bounding polynomials M and m:

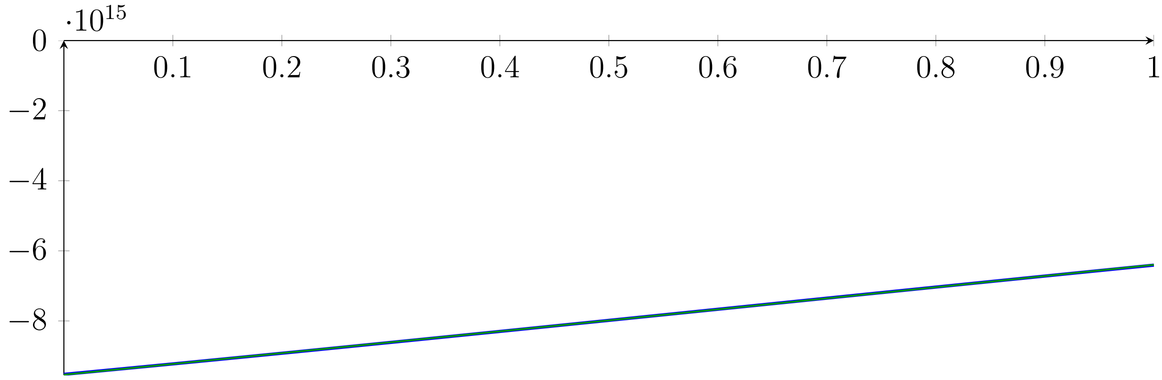
$$M = 7.01056 \cdot 10^{13} X^2 + 3.065 \cdot 10^{15} X - 9.52368 \cdot 10^{15}$$

$$m = 7.01056 \cdot 10^{13} X^2 + 3.065 \cdot 10^{15} X - 9.54423 \cdot 10^{15}$$

Root of M and m:

$$N(M) = \{-46.6329, 2.91313\} \qquad N(m) = \{-46.6388, 2.91904\}$$

Intersection intervals:

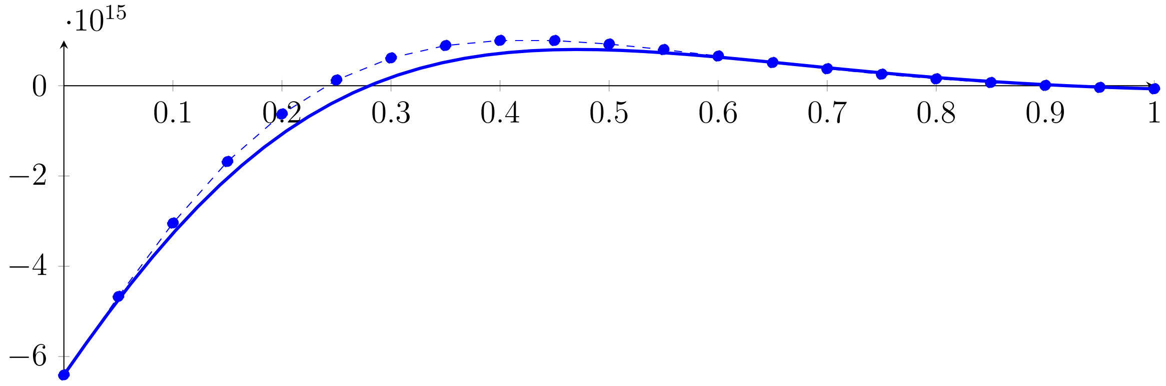


No intersection intervals with the x axis.

2.13 Recursion Branch 1 1 1 1 2 on the Second Half $[1.5625, 3.125]$

Normalized monomial und Bézier representations and the Bézier polygon:

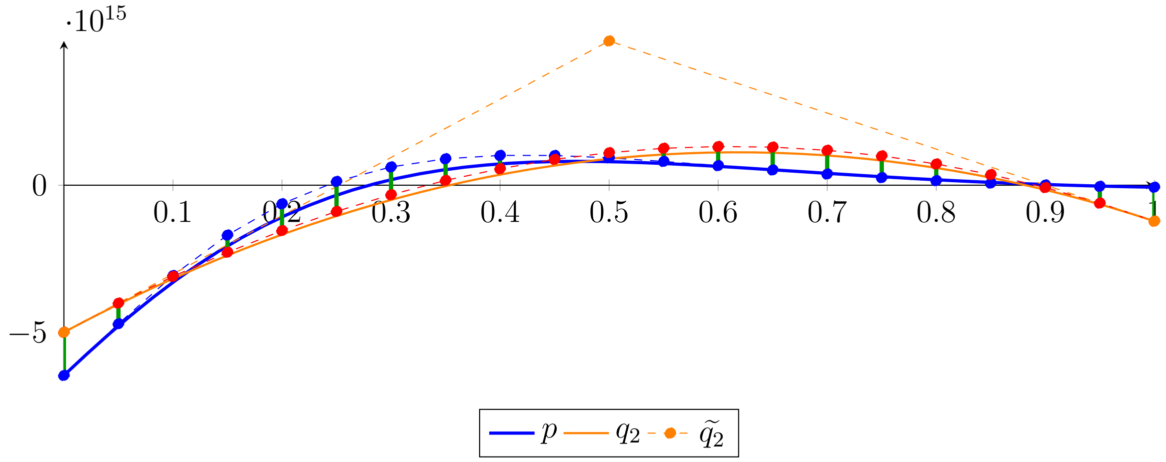
$$\begin{aligned} p = {} & -854779 X^{20} + 2.70678 \cdot 10^{6} X^{19} + 2.57285 \cdot 10^{7} X^{18} - 1.38387 \cdot 10^{9} X^{17} + 3.34218 \cdot 10^{10} X^{16} - 5.62474 \\ & \cdot 10^{11} X^{15} + 7.0799 \cdot 10^{12} X^{14} - 6.89484 \cdot 10^{13} X^{13} + 5.26324 \cdot 10^{14} X^{12} - 3.16741 \cdot 10^{15} X^{11} + 1.50317 \\ & \cdot 10^{16} X^{10} - 5.59783 \cdot 10^{16} X^{9} + 1.61826 \cdot 10^{17} X^{8} - 3.56531 \cdot 10^{17} X^{7} + 5.81008 \cdot 10^{17} X^{6} - 6.65758 \\ & \cdot 10^{17} X^{5} + 4.85849 \cdot 10^{17} X^{4} - 1.69752 \cdot 10^{17} X^{3} - 2.14228 \cdot 10^{16} X^{2} + 3.47712 \cdot 10^{16} X - 6.40794 \cdot 10^{15} \\ = {} & -6.40794 \cdot 10^{15} B_{0,20}(X) - 4.66938 \cdot 10^{15} B_{1,20}(X) - 3.04357 \cdot 10^{15} B_{2,20}(X) - 1.67942 \\ & \cdot 10^{15} B_{3,20}(X) - 6.25553 \cdot 10^{14} B_{4,20}(X) + 1.26743 \cdot 10^{14} B_{5,20}(X) + 6.15563 \cdot 10^{14} B_{6,20}(X) \\ & + 8.9083 \cdot 10^{14} B_{7,20}(X) + 1.00381 \cdot 10^{15} B_{8,20}(X) + 1.00133 \cdot 10^{15} B_{9,20}(X) + 9.23073 \\ & \cdot 10^{14} B_{10,20}(X) + 8.00741 \cdot 10^{14} B_{11,20}(X) + 6.58338 \cdot 10^{14} B_{12,20}(X) + 5.13038 \cdot 10^{14} B_{13,20}(X) \\ & + 3.76314 \cdot 10^{14} B_{14,20}(X) + 2.55097 \cdot 10^{14} B_{15,20}(X) + 1.52873 \cdot 10^{14} B_{16,20}(X) + 7.06284 \\ & \cdot 10^{13} B_{17,20}(X) + 7.64979 \cdot 10^{12} B_{18,20}(X) - 3.78477 \cdot 10^{13} B_{19,20}(X) - 6.82353 \cdot 10^{13} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = -1.58819\cdot 10^{16}X^2 + 1.96241\cdot 10^{16}X - 4.95309\cdot 10^{15}$$
$$= -4.95309\cdot 10^{15}B_{0,2} + 4.85894\cdot 10^{15}B_{1,2} - 1.21098\cdot 10^{15}B_{2,2}$$

$$\widetilde{q}_2 = -1.81894\cdot 10^{18}X^{20} + 1.81838\cdot 10^{19}X^{19} - 8.41361\cdot 10^{19}X^{18} + 2.39064\cdot 10^{20}X^{17} - 4.66691\cdot 10^{20}X^{16} + 6.635\cdot 10^{20}X^{15} - 7.10525\cdot 10^{20}X^{14} + 5.84697\cdot 10^{20}X^{13} - 3.73899\cdot 10^{20}X^{12} + 1.86691\cdot 10^{20}X^{11} - 7.27479\cdot 10^{19}X^{10} + 2.20036\cdot 10^{19}X^{9} - 5.11132\cdot 10^{18}X^{8} + 8.96838\cdot 10^{17}X^{7} - 1.16058\cdot 10^{17}X^{6} + 1.07216\cdot 10^{16}X^{5} - 6.75075\cdot 10^{14}X^{4} + 2.67259\cdot 10^{13}X^{3} - 1.58825\cdot 10^{16}X^{2} + 1.96241\cdot 10^{16}X - 4.95309\cdot 10^{15}$$
$$= -4.95309\cdot 10^{15}B_{0,20} - 3.97189\cdot 10^{15}B_{1,20} - 3.07428\cdot 10^{15}B_{2,20} - 2.26024\cdot 10^{15}B_{3,20} - 1.52988\cdot 10^{15}B_{4,20} - 8.8277\cdot 10^{14}B_{5,20} - 3.20223\cdot 10^{14}B_{6,20} + 1.60968\cdot 10^{14}B_{7,20} + 5.54373\cdot 10^{14}B_{8,20} + 8.70755\cdot 10^{14}B_{9,20} + 1.095\cdot 10^{15}B_{10,20} + 1.24493\cdot 10^{15}B_{11,20} + 1.30278\cdot 10^{15}B_{12,20} + 1.28362\cdot 10^{15}B_{13,20} + 1.17662\cdot 10^{15}B_{14,20} + 9.883\cdot 10^{14}B_{15,20} + 7.15387\cdot 10^{14}B_{16,20} + 3.59245\cdot 10^{14}B_{17,20} - 8.05864\cdot 10^{13}B_{18,20} - 6.03986\cdot 10^{14}B_{19,20} - 1.21098\cdot 10^{15}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.45485\cdot 10^{15}$.

Bounding polynomials M and m:

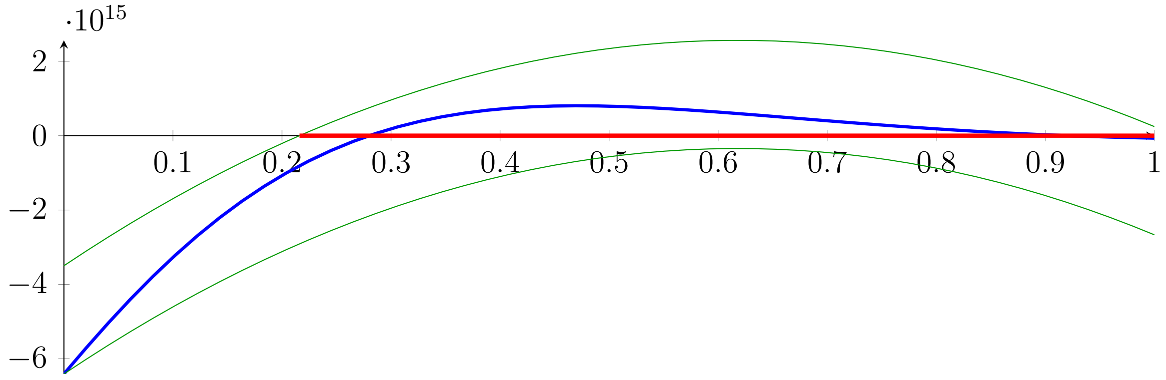
$$M = -1.58819\cdot 10^{16}X^2 + 1.96241\cdot 10^{16}X - 3.49824\cdot 10^{15}$$

$$m = -1.58819\cdot 10^{16}X^2 + 1.96241\cdot 10^{16}X - 6.40794\cdot 10^{15}$$

Root of M and m:

$$N(M) = \{0.216034, 1.01959\} \qquad N(m) = \{\}$$

Intersection intervals:



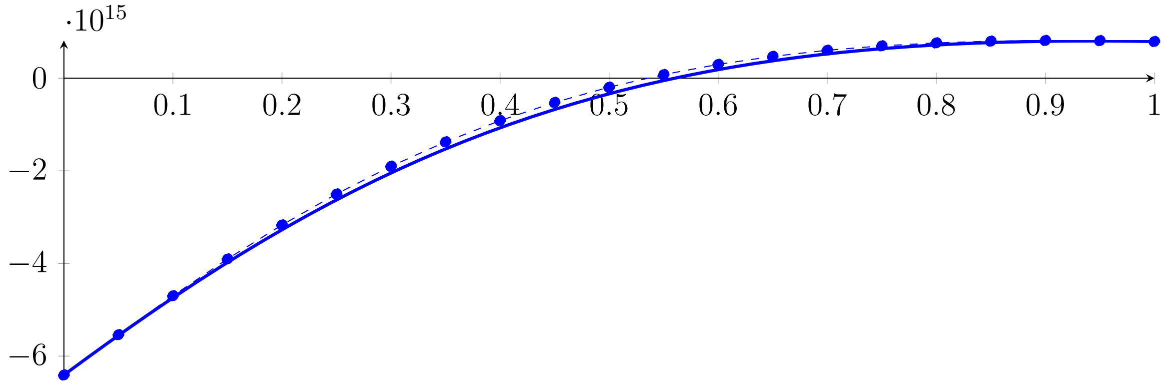
$[0.216034, 1]$

Longest intersection interval: 0.783966
$\Longrightarrow$ Bisection: first half $[1.5625, 2.34375]$ und second half $[2.34375, 3.125]$

2.14 Recursion Branch 1 1 1 1 2 1 on the First Half $[1.5625, 2.34375]$

Normalized monomial und Bézier representations and the Bézier polygon:

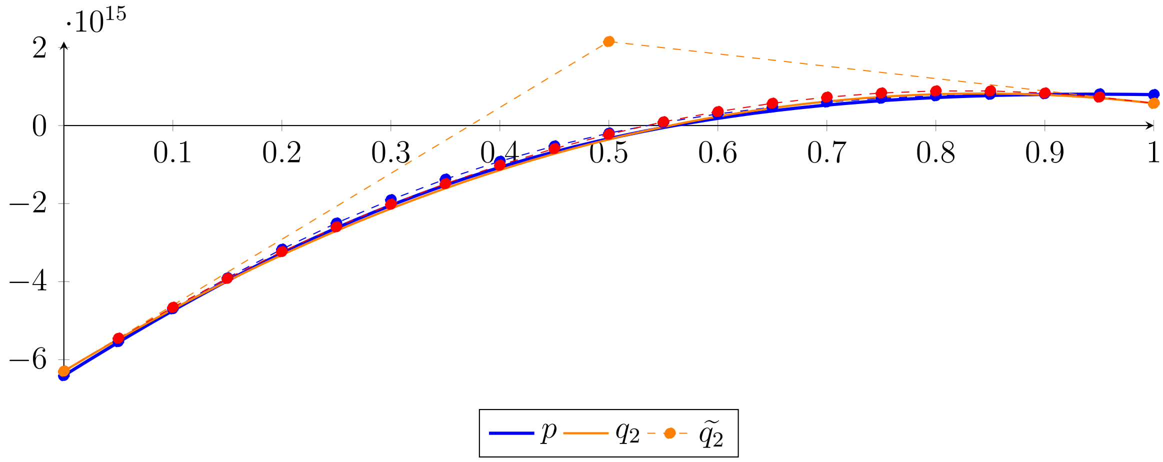
$$
\begin{aligned}
p &= 628533X^{20} + 2.80214\cdot 10^{6}X^{19} + 4.20138\cdot 10^{7}X^{18} - 1.91759\cdot 10^{7}X^{17} + 5.57626\cdot 10^{8}X^{16} - 5.14314 \\
&\quad \cdot 10^{8}X^{15} + 8.17565\cdot 10^{8}X^{14} - 8.21441\cdot 10^{9}X^{13} + 1.29423\cdot 10^{11}X^{12} - 1.5463\cdot 10^{12}X^{11} + 1.46797 \\
&\quad \cdot 10^{13}X^{10} - 1.09332\cdot 10^{14}X^{9} + 6.32133\cdot 10^{14}X^{8} - 2.7854\cdot 10^{15}X^{7} + 9.07825\cdot 10^{15}X^{6} - 2.08049 \\
&\quad \cdot 10^{16}X^{5} + 3.03655\cdot 10^{16}X^{4} - 2.1219\cdot 10^{16}X^{3} - 5.3557\cdot 10^{15}X^{2} + 1.73856\cdot 10^{16}X - 6.40794\cdot 10^{15} \\
&= -6.40794\cdot 10^{15}B_{0,20}(X) - 5.53866\cdot 10^{15}B_{1,20}(X) - 4.69757\cdot 10^{15}B_{2,20}(X) - 3.90328 \\
&\quad \cdot 10^{15}B_{3,20}(X) - 3.16813\cdot 10^{15}B_{4,20}(X) - 2.49956\cdot 10^{15}B_{5,20}(X) - 1.90115\cdot 10^{15}B_{6,20}(X) \\
&\quad - 1.3736\cdot 10^{15}B_{7,20}(X) - 9.15451\cdot 10^{14}B_{8,20}(X) - 5.23652\cdot 10^{14}B_{9,20}(X) - 1.94086 \\
&\quad \cdot 10^{14}B_{10,20}(X) + 7.80618\cdot 10^{13}B_{11,20}(X) + 2.98005\cdot 10^{14}B_{12,20}(X) + 4.71115\cdot 10^{14}B_{13,20}(X) \\
&\quad + 6.02746\cdot 10^{14}B_{14,20}(X) + 6.98102\cdot 10^{14}B_{15,20}(X) + 7.62138\cdot 10^{14}B_{16,20}(X) + 7.99497 \\
&\quad \cdot 10^{14}B_{17,20}(X) + 8.14467\cdot 10^{14}B_{18,20}(X) + 8.10958\cdot 10^{14}B_{19,20}(X) + 7.92494\cdot 10^{14}B_{20,20}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 &= -1.00292\cdot 10^{16}X^{2} + 1.68945\cdot 10^{16}X - 6.29445\cdot 10^{15} \\
&= -6.29445\cdot 10^{15}B_{0,2} + 2.15279\cdot 10^{15}B_{1,2} + 5.70871\cdot 10^{14}B_{2,2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_2 = & -8.09263{\cdot}10^{17}X^{20} + 8.07858{\cdot}10^{18}X^{19} - 3.73235{\cdot}10^{19}X^{18} + 1.05882{\cdot}10^{20}X^{17} - 2.06343{\cdot}10^{20}X^{16} \\
& + 2.92784{\cdot}10^{20}X^{15} - 3.12805{\cdot}10^{20}X^{14} + 2.56677{\cdot}10^{20}X^{13} - 1.63561{\cdot}10^{20}X^{12} + 8.1321{\cdot}10^{19}X^{11} \\
& - 3.15347{\cdot}10^{19}X^{10} + 9.489{\cdot}10^{18}X^{9} - 2.19288{\cdot}10^{18}X^{8} + 3.82613{\cdot}10^{17}X^{7} - 4.91122{\cdot}10^{16}X^{6} + 4.47131 \\
& {\cdot}10^{15}X^{5} - 2.74008{\cdot}10^{14}X^{4} + 1.03552{\cdot}10^{13}X^{3} - 1.00294{\cdot}10^{16}X^{2} + 1.68945{\cdot}10^{16}X - 6.29445{\cdot}10^{15} \\
= & -6.29445{\cdot}10^{15}B_{0,20} - 5.44973{\cdot}10^{15}B_{1,20} - 4.65779{\cdot}10^{15}B_{2,20} - 3.91863{\cdot}10^{15}B_{3,20} - 3.23229 \\
& {\cdot}10^{15}B_{4,20} - 2.59859{\cdot}10^{15}B_{5,20} - 2.0181{\cdot}10^{15}B_{6,20} - 1.48943{\cdot}10^{15}B_{7,20} - 1.01537{\cdot}10^{15}B_{8,20} \\
& - 5.91185{\cdot}10^{14}B_{9,20} - 2.23588{\cdot}10^{14}B_{10,20} + 9.53314{\cdot}10^{13}B_{11,20} + 3.5767{\cdot}10^{14}B_{12,20} \\
& + 5.70203{\cdot}10^{14}B_{13,20} + 7.28019{\cdot}10^{14}B_{14,20} + 8.34077{\cdot}10^{14}B_{15,20} + 8.869{\cdot}10^{14}B_{16,20} \\
& + 8.871{\cdot}10^{14}B_{17,20} + 8.3447{\cdot}10^{14}B_{18,20} + 7.29064{\cdot}10^{14}B_{19,20} + 5.70871{\cdot}10^{14}B_{20,20}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 2.21623{\cdot}10^{14}$.
Bounding polynomials M and m:

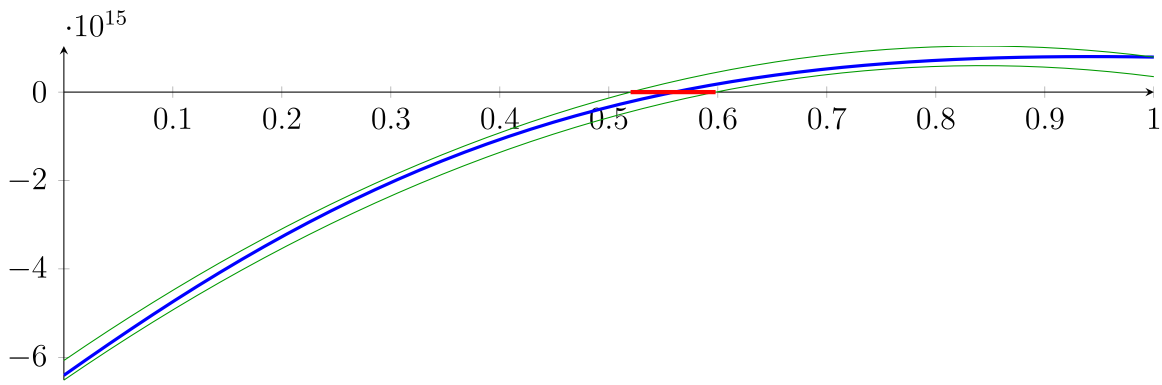
$$M = -1.00292{\cdot}10^{16}X^2 + 1.68945{\cdot}10^{16}X - 6.07283{\cdot}10^{15}$$

$$m = -1.00292{\cdot}10^{16}X^2 + 1.68945{\cdot}10^{16}X - 6.51608{\cdot}10^{15}$$

Root of M and m:

$$N(M) = \{0.519935, 1.1646\} \qquad N(m) = \{0.597926, 1.08661\}$$

Intersection intervals:



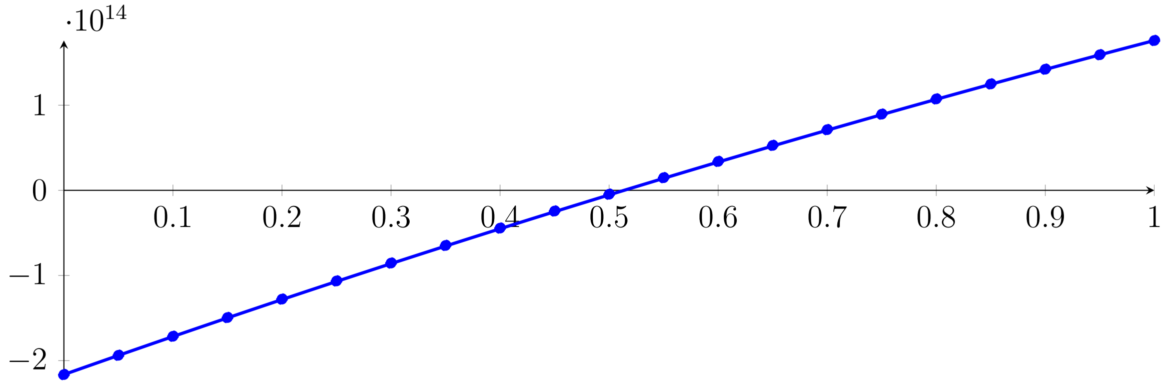
$$[0.519935, 0.597926]$$

Longest intersection interval: 0.0779914
$\Longrightarrow$ Selective recursion: interval 1: $[1.9687, 2.02963]$,

2.15 Recursion Branch 1 1 1 1 2 1 1 in Interval 1: $[1.9687, 2.02963]$

Normalized monomial und Bézier representations and the Bézier polygon:

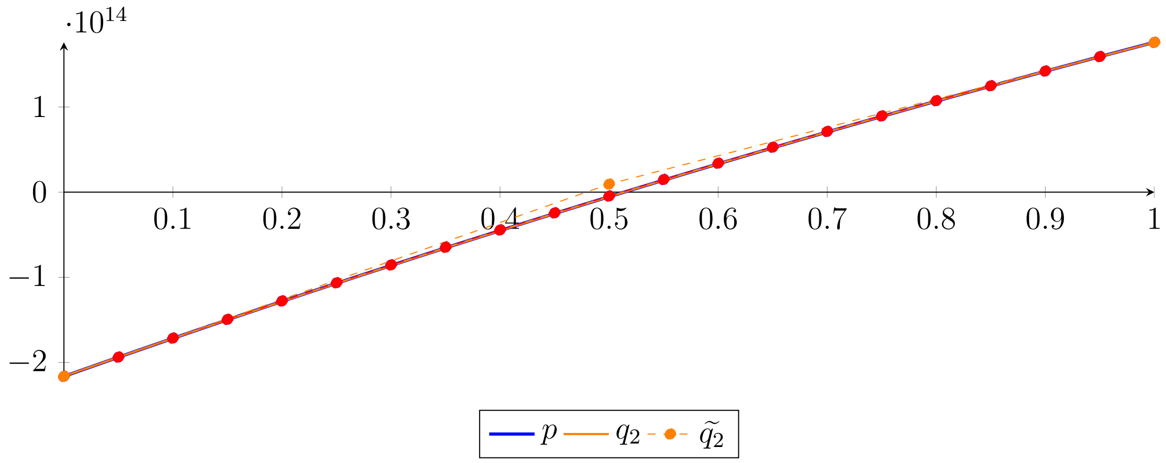
$$p = 24794.8X^{20} + 249674X^{19} + 1.70065\cdot10^{6}X^{18} - 423261X^{17} + 2.40907\cdot10^{7}X^{16} - 2.32371\cdot10^{7}X^{15} + 1.49529\cdot10^{7}X^{14} + 9.97828\cdot10^{6}X^{13} + 4.02828\cdot10^{7}X^{12} + 1.19199\cdot10^{7}X^{11} + 1.5202\cdot10^{7}X^{10} + 3.18599\cdot10^{6}X^{9} + 602294X^{8} - 1.76576\cdot10^{7}X^{7} + 5.93107\cdot10^{8}X^{6} - 1.21216\cdot10^{10}X^{5} + 7.9743\cdot10^{10}X^{4} + 2.49857\cdot10^{12}X^{3} - 6.32678\cdot10^{13}X^{2} + 4.53087\cdot10^{14}X - 2.16403\cdot10^{14}$$
$$= -2.16403\cdot10^{14}B_{0,20}(X) - 1.93749\cdot10^{14}B_{1,20}(X) - 1.71427\cdot10^{14}B_{2,20}(X) - 1.49437\cdot10^{14}B_{3,20}(X) - 1.27775\cdot10^{14}B_{4,20}(X) - 1.06439\cdot10^{14}B_{5,20}(X) - 8.54277\cdot10^{13}B_{6,20}(X) - 6.4738\cdot10^{13}B_{7,20}(X) - 4.4368\cdot10^{13}B_{8,20}(X) - 2.43154\cdot10^{13}B_{9,20}(X) - 4.57771\cdot10^{12}B_{10,20}(X) + 1.48472\cdot10^{13}B_{11,20}(X) + 3.39617\cdot10^{13}B_{12,20}(X) + 5.27681\cdot10^{13}B_{13,20}(X) + 7.12687\cdot10^{13}B_{14,20}(X) + 8.94659\cdot10^{13}B_{15,20}(X) + 1.07362\cdot10^{14}B_{16,20}(X) + 1.24959\cdot10^{14}B_{17,20}(X) + 1.4226\cdot10^{14}B_{18,20}(X) + 1.59268\cdot10^{14}B_{19,20}(X) + 1.75983\cdot10^{14}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = -5.94039\cdot10^{13}X^{2} + 4.51527\cdot10^{14}X - 2.16273\cdot10^{14}$$
$$= -2.16273\cdot10^{14}B_{0,2} + 9.49117\cdot10^{12}B_{1,2} + 1.75851\cdot10^{14}B_{2,2}$$

$$\widetilde{q}_2 = 9.27351\cdot10^{15}X^{20} - 9.30527\cdot10^{16}X^{19} + 4.319\cdot10^{17}X^{18} - 1.23065\cdot10^{18}X^{17} + 2.40972\cdot10^{18}X^{16} - 3.43989\cdot10^{18}X^{15} + 3.7067\cdot10^{18}X^{14} - 3.07989\cdot10^{18}X^{13} + 1.99783\cdot10^{18}X^{12} - 1.01719\cdot10^{18}X^{11} + 4.06133\cdot10^{17}X^{10} - 1.26276\cdot10^{17}X^{9} + 3.01897\cdot10^{16}X^{8} - 5.45383\cdot10^{15}X^{7} + 7.28115\cdot10^{14}X^{6} - 6.96404\cdot10^{13}X^{5} + 4.54637\cdot10^{12}X^{4} - 1.87407\cdot10^{11}X^{3} - 5.93995\cdot10^{13}X^{2} + 4.51527\cdot10^{14}X - 2.16273\cdot10^{14}$$
$$= -2.16273\cdot10^{14}B_{0,20} - 1.93696\cdot10^{14}B_{1,20} - 1.71432\cdot10^{14}B_{2,20} - 1.49481\cdot10^{14}B_{3,20} - 1.27843\cdot10^{14}B_{4,20} - 1.06518\cdot10^{14}B_{5,20} - 8.55011\cdot10^{13}B_{6,20} - 6.48093\cdot10^{13}B_{7,20} - 4.44074\cdot10^{13}B_{8,20} - 2.43511\cdot10^{13}B_{9,20} - 4.56472\cdot10^{12}B_{10,20} + 1.48599\cdot10^{13}B_{11,20} + 3.40166\cdot10^{13}B_{12,20} + 5.28289\cdot10^{13}B_{13,20} + 7.13477\cdot10^{13}B_{14,20} + 8.95437\cdot10^{13}B_{15,20} + 1.07431\cdot10^{14}B_{16,20} + 1.25005\cdot10^{14}B_{17,20} + 1.42266\cdot10^{14}B_{18,20} + 1.59215\cdot10^{14}B_{19,20} + 1.75851\cdot10^{14}B_{20,20}$$



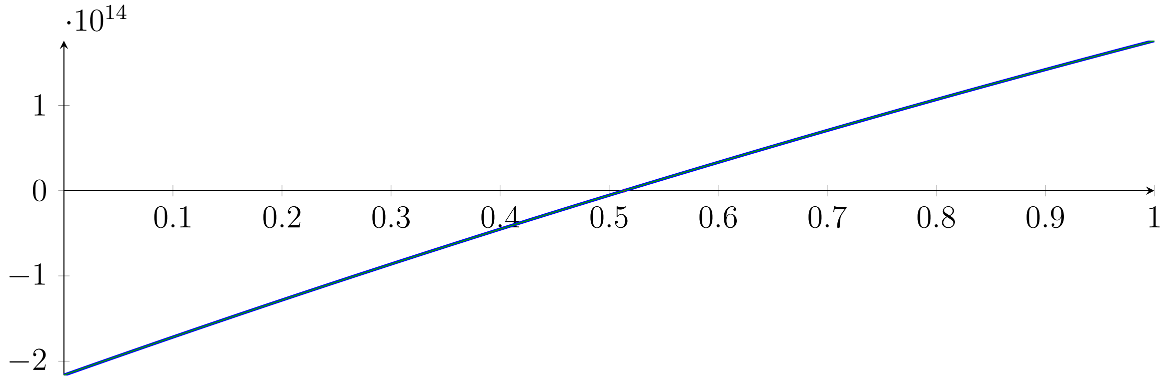
The maximum difference of the Bézier coefficients is $\delta = 1.32013{\cdot}10^{11}$.
Bounding polynomials M and m:

$$M = -5.94039{\cdot}10^{13}X^2 + 4.51527{\cdot}10^{14}X - 2.1614{\cdot}10^{14}$$
$$m = -5.94039{\cdot}10^{13}X^2 + 4.51527{\cdot}10^{14}X - 2.16405{\cdot}10^{14}$$

Root of M and m:

$$N(M) = \{0.513359, 7.08762\} \qquad N(m) = \{0.514035, 7.08694\}$$

Intersection intervals:



$$[0.513359, 0.514035]$$

Longest intersection interval: 0.000676132
$\Longrightarrow$ Selective recursion: interval 1: $[1.99998, 2.00002]$,

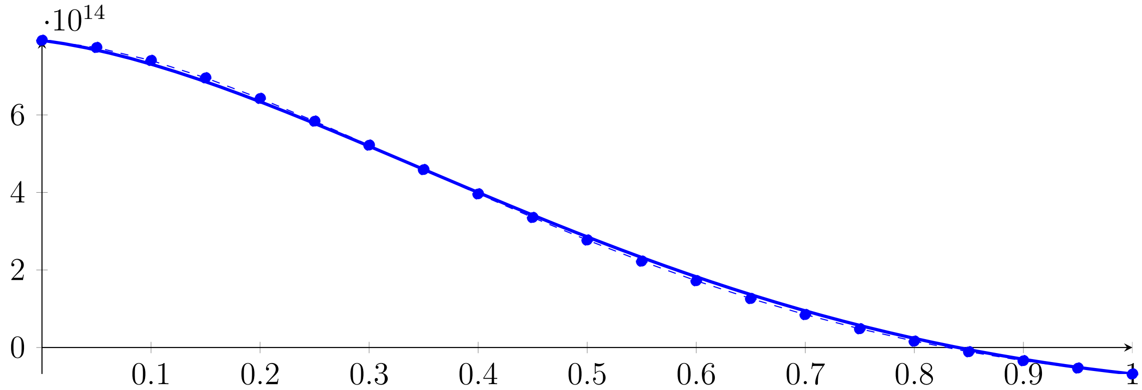
2.16 Recursion Branch 1 1 1 1 2 1 1 1 in Interval 1: $[1.99998, 2.00002]$

Found root in interval $[1.99998, 2.00002]$ at recursion depth 8!

2.17 Recursion Branch 1 1 1 1 2 2 on the Second Half $[2.34375, 3.125]$

Normalized monomial und Bézier representations and the Bézier polygon:

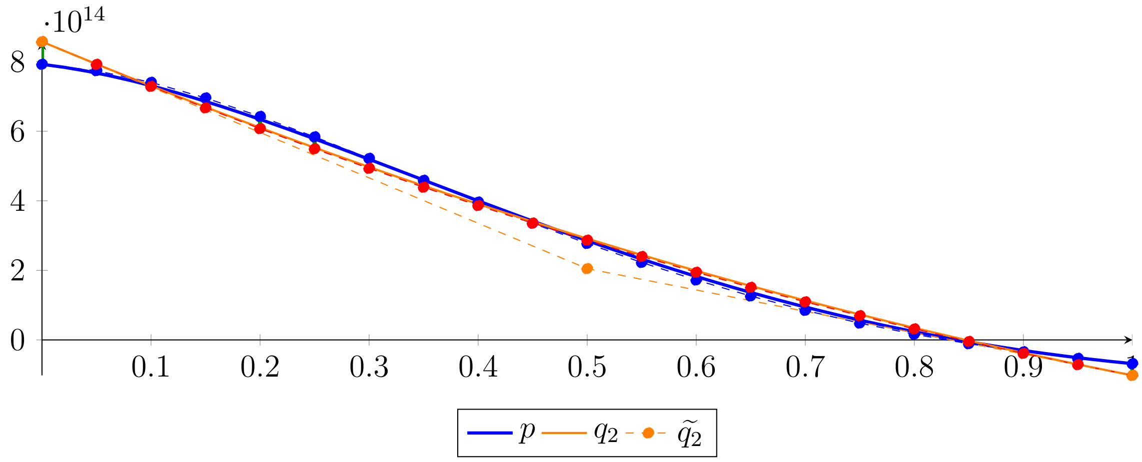
$$\begin{aligned} p = {} & -342205X^{20} + 740660X^{19} - 1.30129{\cdot}10^{7}X^{18} + 3.13739{\cdot}10^{7}X^{17} - 2.36954{\cdot}10^{8}X^{16} + 1.86963{\cdot}10^{8}X^{15} \\ & + 1.21978{\cdot}10^{8}X^{14} - 3.95236{\cdot}10^{9}X^{13} + 5.11961{\cdot}10^{10}X^{12} - 5.25185{\cdot}10^{11}X^{11} + 4.12541{\cdot}10^{12}X^{10} \\ & - 2.45648{\cdot}10^{13}X^{9} + 1.07492{\cdot}10^{14}X^{8} - 3.24451{\cdot}10^{14}X^{7} + 5.69883{\cdot}10^{14}X^{6} - 1.28897{\cdot}10^{14}X^{5} \\ & - 1.87079{\cdot}10^{15}X^{4} + 4.01756{\cdot}10^{15}X^{3} - 2.84135{\cdot}10^{15}X^{2} - 3.69266{\cdot}10^{14}X + 7.92494{\cdot}10^{14} \\ = {} & 7.92494{\cdot}10^{14}B_{0,20}(X) + 7.74031{\cdot}10^{14}B_{1,20}(X) + 7.40613{\cdot}10^{14}B_{2,20}(X) + 6.95765 \\ & {\cdot}10^{14}B_{3,20}(X) + 6.42625{\cdot}10^{14}B_{4,20}(X) + 5.83936{\cdot}10^{14}B_{5,20}(X) + 5.22054{\cdot}10^{14}B_{6,20}(X) \\ & + 4.58964{\cdot}10^{14}B_{7,20}(X) + 3.96302{\cdot}10^{14}B_{8,20}(X) + 3.35388{\cdot}10^{14}B_{9,20}(X) + 2.77253 \\ & {\cdot}10^{14}B_{10,20}(X) + 2.22674{\cdot}10^{14}B_{11,20}(X) + 1.72204{\cdot}10^{14}B_{12,20}(X) + 1.26205{\cdot}10^{14}B_{13,20}(X) \\ & + 8.48747{\cdot}10^{13}B_{14,20}(X) + 4.8274{\cdot}10^{13}B_{15,20}(X) + 1.63537{\cdot}10^{13}B_{16,20}(X) - 1.10251 \\ & {\cdot}10^{13}B_{17,20}(X) - 3.40702{\cdot}10^{13}B_{18,20}(X) - 5.30415{\cdot}10^{13}B_{19,20}(X) - 6.82353{\cdot}10^{13}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = 3.45595{\cdot}10^{14}X^2 - 1.30507{\cdot}10^{15}X + 8.5751{\cdot}10^{14}$$
$$= 8.5751{\cdot}10^{14}B_{0,2} + 2.04975{\cdot}10^{14}B_{1,2} - 1.01965{\cdot}10^{14}B_{2,2}$$

$$\widetilde{q}_2 = -5.06775{\cdot}10^{16}X^{20} + 5.10089{\cdot}10^{17}X^{19} - 2.37755{\cdot}10^{18}X^{18} + 6.80899{\cdot}10^{18}X^{17} - 1.34059{\cdot}10^{19}X^{16} + 1.92383{\cdot}10^{19}X^{15} - 2.08182{\cdot}10^{19}X^{14} + 1.73365{\cdot}10^{19}X^{13} - 1.12379{\cdot}10^{19}X^{12} + 5.697{\cdot}10^{18}X^{11} - 2.25601{\cdot}10^{18}X^{10} + 6.93262{\cdot}10^{17}X^{9} - 1.63448{\cdot}10^{17}X^{8} + 2.91454{\cdot}10^{16}X^{7} - 3.87183{\cdot}10^{15}X^{6} + 3.76607{\cdot}10^{14}X^{5} - 2.60907{\cdot}10^{13}X^{4} + 1.19479{\cdot}10^{12}X^{3} + 3.45563{\cdot}10^{14}X^{2} - 1.30507{\cdot}10^{15}X + 8.5751{\cdot}10^{14}$$
$$= 8.5751{\cdot}10^{14}B_{0,20} + 7.92257{\cdot}10^{14}B_{1,20} + 7.28822{\cdot}10^{14}B_{2,20} + 6.67207{\cdot}10^{14}B_{3,20} + 6.07407{\cdot}10^{14}B_{4,20} + 5.49438{\cdot}10^{14}B_{5,20} + 4.93257{\cdot}10^{14}B_{6,20} + 4.38965{\cdot}10^{14}B_{7,20} + 3.86363{\cdot}10^{14}B_{8,20} + 3.35768{\cdot}10^{14}B_{9,20} + 2.86751{\cdot}10^{14}B_{10,20} + 2.39817{\cdot}10^{14}B_{11,20} + 1.94471{\cdot}10^{14}B_{12,20} + 1.51115{\cdot}10^{14}B_{13,20} + 1.09468{\cdot}10^{14}B_{14,20} + 6.97002{\cdot}10^{13}B_{15,20} + 3.17227{\cdot}10^{13}B_{16,20} - 4.42593{\cdot}10^{12}B_{17,20} - 3.87583{\cdot}10^{13}B_{18,20} - 7.12711{\cdot}10^{13}B_{19,20} - 1.01965{\cdot}10^{14}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 6.50156{\cdot}10^{13}$.

Bounding polynomials M and m:

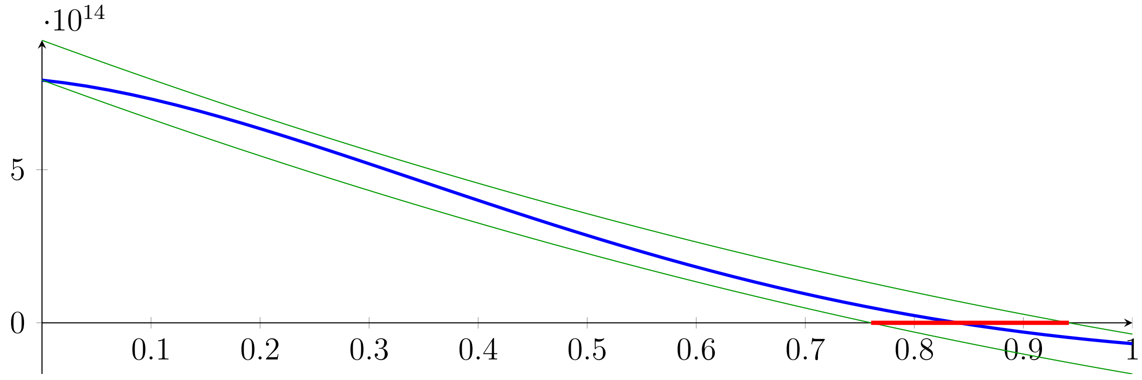
$$M = 3.45595{\cdot}10^{14}X^2 - 1.30507{\cdot}10^{15}X + 9.22526{\cdot}10^{14}$$

$$m = 3.45595{\cdot}10^{14}X^2 - 1.30507{\cdot}10^{15}X + 7.92494{\cdot}10^{14}$$

Root of M and m:

$$N(M) = \{0.941722, 2.83458\} \qquad N(m) = \{0.760329, 3.01597\}$$

Intersection intervals:



$[0.760329, 0.941722]$

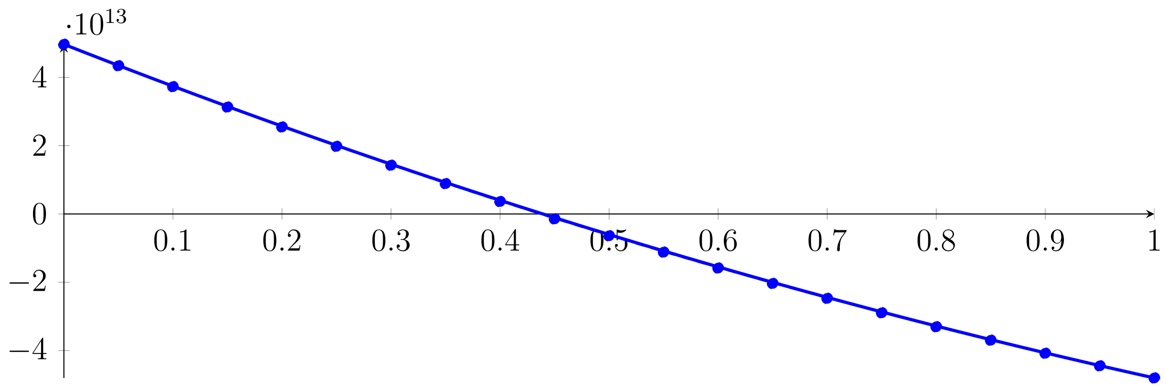
Longest intersection interval: 0.181393

$\Longrightarrow$ Selective recursion: interval 1: $[2.93776, 3.07947]$,

2.18 Recursion Branch 1 1 1 1 2 2 1 in Interval 1: $[2.93776, 3.07947]$

Normalized monomial und Bézier representations and the Bézier polygon:

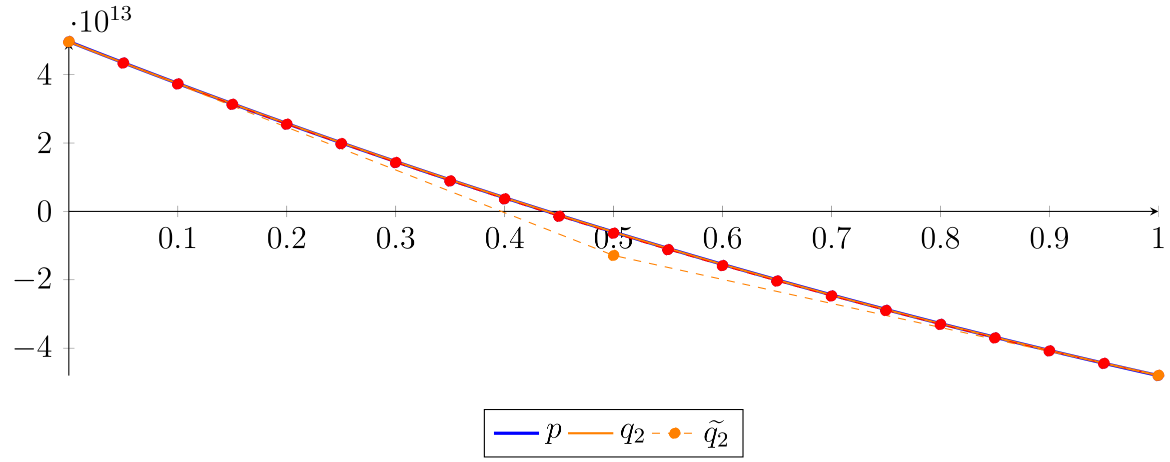
$$\begin{aligned} p &= 1145.25X^{20} - 98517.5X^{19} - 204511X^{18} - 744847X^{17} - 1.26333\cdot10^{6}X^{16} + 2.15506\cdot10^{6}X^{15} \\ &\quad - 2.44188\cdot10^{6}X^{14} - 2.29653\cdot10^{6}X^{13} - 6.86536\cdot10^{6}X^{12} - 2.5194\cdot10^{6}X^{11} - 2.80598\cdot10^{6}X^{10} \\ &\quad - 1.95253\cdot10^{6}X^{9} + 1.87302\cdot10^{7}X^{8} - 8.39493\cdot10^{7}X^{7} - 3.1278\cdot10^{9}X^{6} + 7.19255\cdot10^{10}X^{5} \\ &\quad - 5.82167\cdot10^{11}X^{4} - 4.5966\cdot10^{10}X^{3} + 2.83841\cdot10^{13}X^{2} - 1.25534\cdot10^{14}X + 4.96853\cdot10^{13} \\ &= 4.96853\cdot10^{13}B_{0,20}(X) + 4.34086\cdot10^{13}B_{1,20}(X) + 3.72813\cdot10^{13}B_{2,20}(X) + 3.13034 \\ &\quad \cdot10^{13}B_{3,20}(X) + 2.54746\cdot10^{13}B_{4,20}(X) + 1.97948\cdot10^{13}B_{5,20}(X) + 1.42635\cdot10^{13}B_{6,20}(X) \\ &\quad + 8.88017\cdot10^{12}B_{7,20}(X) + 3.64432\cdot10^{12}B_{8,20}(X) - 1.44479\cdot10^{12}B_{9,20}(X) - 6.38793 \\ &\quad \cdot10^{12}B_{10,20}(X) - 1.1186\cdot10^{13}B_{11,20}(X) - 1.58399\cdot10^{13}B_{12,20}(X) - 2.03508\cdot10^{13}B_{13,20}(X) \\ &\quad - 2.47197\cdot10^{13}B_{14,20}(X) - 2.89479\cdot10^{13}B_{15,20}(X) - 3.30365\cdot10^{13}B_{16,20}(X) - 3.6987 \\ &\quad \cdot10^{13}B_{17,20}(X) - 4.08007\cdot10^{13}B_{18,20}(X) - 4.44791\cdot10^{13}B_{19,20}(X) - 4.80237\cdot10^{13}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 2.74399\cdot10^{13}X^{2} - 1.25047\cdot10^{14}X + 4.96404\cdot10^{13} \\ &= 4.96404\cdot10^{13}B_{0,2} - 1.28832\cdot10^{13}B_{1,2} - 4.7967\cdot10^{13}B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 4.66367\cdot 10^{14}X^{20} - 4.6392\cdot 10^{15}X^{19} + 2.14635\cdot 10^{16}X^{18} - 6.11823\cdot 10^{16}X^{17} + 1.19873\cdot 10^{17}X^{16} \\ &\quad - 1.70374\cdot 10^{17}X^{15} + 1.80599\cdot 10^{17}X^{14} - 1.44565\cdot 10^{17}X^{13} + 8.75639\cdot 10^{16}X^{12} - 3.98941\cdot 10^{16}X^{11} \\ &\quad + 1.3495\cdot 10^{16}X^{10} - 3.3199\cdot 10^{15}X^{9} + 5.74232\cdot 10^{14}X^{8} - 6.53088\cdot 10^{13}X^{7} + 4.15942\cdot 10^{12}X^{6} - 1.15147 \\ &\quad \cdot 10^{11}X^{5} + 1.36831\cdot 10^{10}X^{4} - 2.03004\cdot 10^{9}X^{3} + 2.744\cdot 10^{13}X^{2} - 1.25047\cdot 10^{14}X + 4.96404\cdot 10^{13} \\ &= 4.96404\cdot 10^{13}B_{0,20} + 4.33881\cdot 10^{13}B_{1,20} + 3.72801\cdot 10^{13}B_{2,20} + 3.13166\cdot 10^{13}B_{3,20} + 2.54975 \\ &\quad \cdot 10^{13}B_{4,20} + 1.98228\cdot 10^{13}B_{5,20} + 1.42926\cdot 10^{13}B_{6,20} + 8.90651\cdot 10^{12}B_{7,20} + 3.6658\cdot 10^{12}B_{8,20} \\ &\quad - 1.43248\cdot 10^{12}B_{9,20} - 6.38382\cdot 10^{12}B_{10,20} - 1.11927\cdot 10^{13}B_{11,20} - 1.58555\cdot 10^{13}B_{12,20} \\ &\quad - 2.03759\cdot 10^{13}B_{13,20} - 2.47502\cdot 10^{13}B_{14,20} - 2.8981\cdot 10^{13}B_{15,20} - 3.30669\cdot 10^{13}B_{16,20} \\ &\quad - 3.70086\cdot 10^{13}B_{17,20} - 4.08058\cdot 10^{13}B_{18,20} - 4.44586\cdot 10^{13}B_{19,20} - 4.7967\cdot 10^{13}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 5.67512\cdot 10^{10}$.

Bounding polynomials M and m:

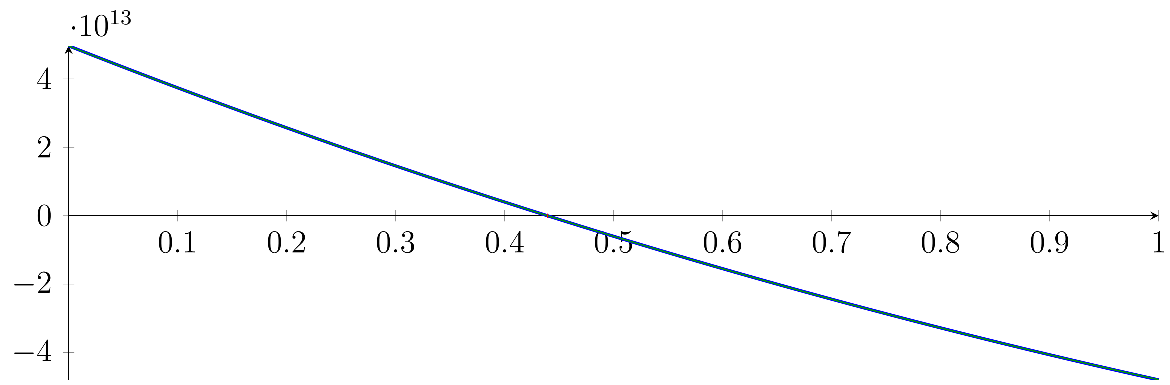
$$M = 2.74399\cdot 10^{13}X^2 - 1.25047\cdot 10^{14}X + 4.96972\cdot 10^{13}$$

$$m = 2.74399\cdot 10^{13}X^2 - 1.25047\cdot 10^{14}X + 4.95837\cdot 10^{13}$$

Root of M and m:

$N(M) = \{0.439888, 4.11724\}$ $\qquad$ $N(m) = \{0.438764, 4.11837\}$

Intersection intervals:



$[0.438764, 0.439888]$

Longest intersection interval: 0.00112448

$\Longrightarrow$ Selective recursion: interval 1: $[2.99994, 3.0001]$,

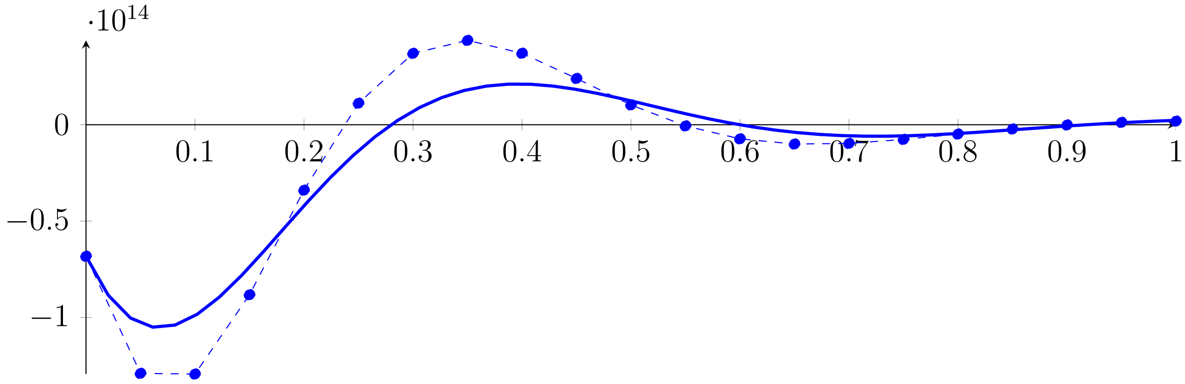
2.19 Recursion Branch 1 1 1 1 2 2 1 1 in Interval 1: $[2.99994, 3.0001]$

Found root in interval $[2.99994, 3.0001]$ at recursion depth 8!

2.20 Recursion Branch 1 1 1 2 on the Second Half $[3.125, 6.25]$

Normalized monomial und Bézier representations and the Bézier polygon:

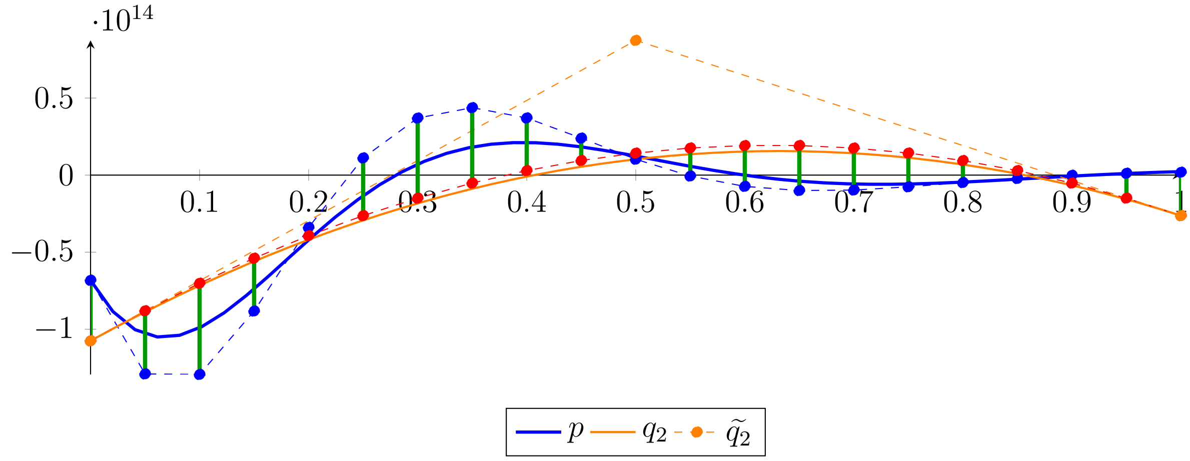
$$\begin{aligned}p &= 7.88859\cdot 10^{9}X^{20} - 3.72342\cdot 10^{11}X^{19} + 8.07932\cdot 10^{12}X^{18} - 1.06797\cdot 10^{14}X^{17} + 9.60483\cdot 10^{14}X^{16} \\ &\quad - 6.21458\cdot 10^{15}X^{15} + 2.98115\cdot 10^{16}X^{14} - 1.07566\cdot 10^{17}X^{13} + 2.92576\cdot 10^{17}X^{12} - 5.93362\cdot 10^{17}X^{11} \\ &\quad + 8.69791\cdot 10^{17}X^{10} - 8.52613\cdot 10^{17}X^{9} + 4.24784\cdot 10^{17}X^{8} + 1.26126\cdot 10^{17}X^{7} - 3.67434\cdot 10^{17}X^{6} + 2.40127 \\ &\quad \cdot 10^{17}X^{5} - 4.54599\cdot 10^{16}X^{4} - 2.16249\cdot 10^{16}X^{3} + 1.14835\cdot 10^{16}X^{2} - 1.2155\cdot 10^{15}X - 6.82353\cdot 10^{13} \\ &= -6.82353\cdot 10^{13}B_{0,20}(X) - 1.2901\cdot 10^{14}B_{1,20}(X) - 1.29346\cdot 10^{14}B_{2,20}(X) - 8.82108 \\ &\quad \cdot 10^{13}B_{3,20}(X) - 3.39572\cdot 10^{13}B_{4,20}(X) + 1.11681\cdot 10^{13}B_{5,20}(X) + 3.70318\cdot 10^{13}B_{6,20}(X) \\ &\quad + 4.37698\cdot 10^{13}B_{7,20}(X) + 3.70894\cdot 10^{13}B_{8,20}(X) + 2.40125\cdot 10^{13}B_{9,20}(X) + 1.02825 \\ &\quad \cdot 10^{13}B_{10,20}(X) - 6.08666\cdot 10^{11}B_{11,20}(X) - 7.31328\cdot 10^{12}B_{12,20}(X) - 1.00112\cdot 10^{13}B_{13,20}(X) \\ &\quad - 9.69955\cdot 10^{12}B_{14,20}(X) - 7.61291\cdot 10^{12}B_{15,20}(X) - 4.85196\cdot 10^{12}B_{16,20}(X) - 2.20819 \\ &\quad \cdot 10^{12}B_{17,20}(X) - 1.32423\cdot 10^{11}B_{18,20}(X) + 1.21228\cdot 10^{12}B_{19,20}(X) + 1.86285\cdot 10^{12}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= -3.08681\cdot 10^{14}X^{2} + 3.89921\cdot 10^{14}X - 1.07532\cdot 10^{14} \\ &= -1.07532\cdot 10^{14}B_{0,2} + 8.74285\cdot 10^{13}B_{1,2} - 2.62919\cdot 10^{13}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= -3.37498\cdot 10^{16}X^{20} + 3.37325\cdot 10^{17}X^{19} - 1.56043\cdot 10^{18}X^{18} + 4.43264\cdot 10^{18}X^{17} - 8.6508\cdot 10^{18}X^{16} \\ &\quad + 1.22952\cdot 10^{19}X^{15} - 1.31623\cdot 10^{19}X^{14} + 1.08275\cdot 10^{19}X^{13} - 6.92125\cdot 10^{18}X^{12} + 3.45445\cdot 10^{18}X^{11} \\ &\quad - 1.34556\cdot 10^{18}X^{10} + 4.06837\cdot 10^{17}X^{9} - 9.44772\cdot 10^{16}X^{8} + 1.65712\cdot 10^{16}X^{7} - 2.14281\cdot 10^{15}X^{6} + 1.97588 \\ &\quad \cdot 10^{14}X^{5} - 1.23903\cdot 10^{13}X^{4} + 4.86965\cdot 10^{11}X^{3} - 3.08691\cdot 10^{14}X^{2} + 3.89921\cdot 10^{14}X - 1.07532\cdot 10^{14} \\ &= -1.07532\cdot 10^{14}B_{0,20} - 8.8036\cdot 10^{13}B_{1,20} - 7.01646\cdot 10^{13}B_{2,20} - 5.39175\cdot 10^{13}B_{3,20} - 3.92968 \\ &\quad \cdot 10^{13}B_{4,20} - 2.62945\cdot 10^{13}B_{5,20} - 1.49347\cdot 10^{13}B_{6,20} - 5.15834\cdot 10^{12}B_{7,20} + 2.91579\cdot 10^{12}B_{8,20} \\ &\quad + 9.48696\cdot 10^{12}B_{9,20} + 1.42749\cdot 10^{13}B_{10,20} + 1.76105\cdot 10^{13}B_{11,20} + 1.91634\cdot 10^{13}B_{12,20} \\ &\quad + 1.92141\cdot 10^{13}B_{13,20} + 1.75612\cdot 10^{13}B_{14,20} + 1.43258\cdot 10^{13}B_{15,20} + 9.44719\cdot 10^{12}B_{16,20} \\ &\quad + 2.95056\cdot 10^{12}B_{17,20} - 5.17254\cdot 10^{12}B_{18,20} - 1.49199\cdot 10^{13}B_{19,20} - 2.62919\cdot 10^{13}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 5.91813 \cdot 10^{13}$.

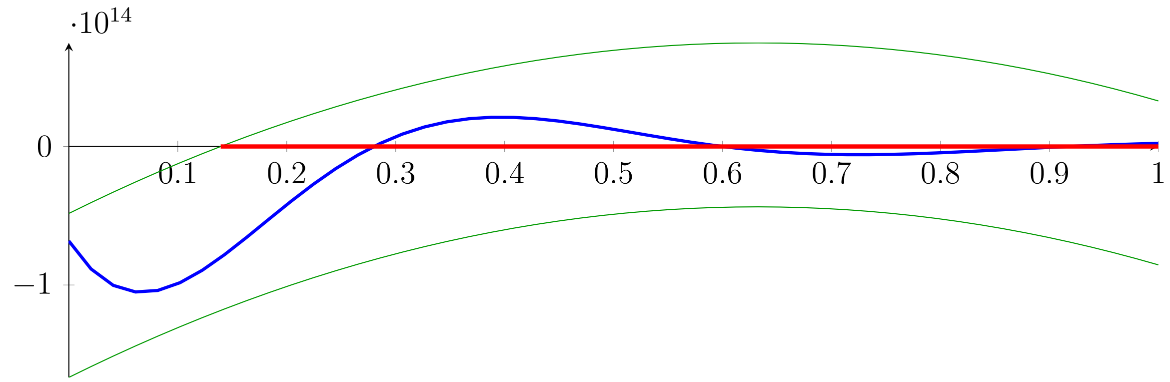
Bounding polynomials M and m:

$$M = -3.08681 \cdot 10^{14} X^2 + 3.89921 \cdot 10^{14} X - 4.83507 \cdot 10^{13}$$
$$m = -3.08681 \cdot 10^{14} X^2 + 3.89921 \cdot 10^{14} X - 1.66713 \cdot 10^{14}$$

Root of M and m:

$$N(M) = \{0.139381, 1.1238\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.139381, 1]$$

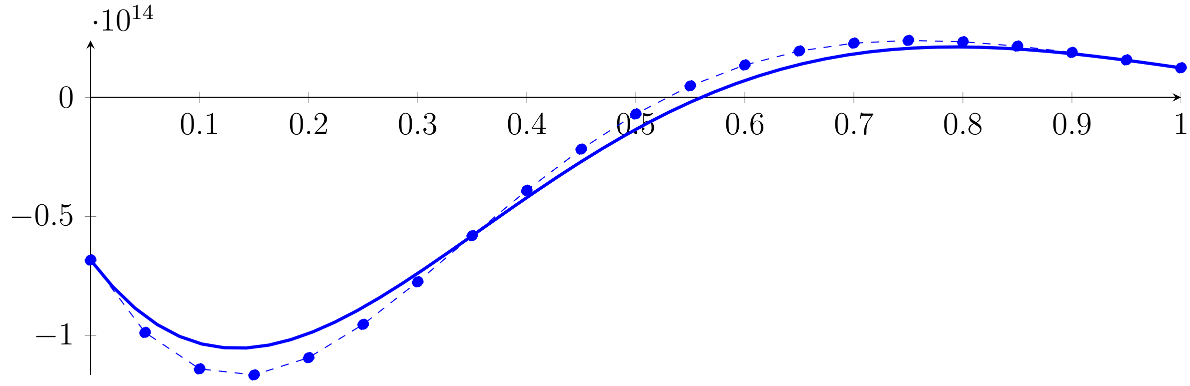
Longest intersection interval: 0.860619

$\Longrightarrow$ Bisection: first half $[3.125, 4.6875]$ und second half $[4.6875, 6.25]$

2.21 Recursion Branch 1 1 1 2 1 on the First Half $[3.125, 4.6875]$

Normalized monomial und Bézier representations and the Bézier polygon:

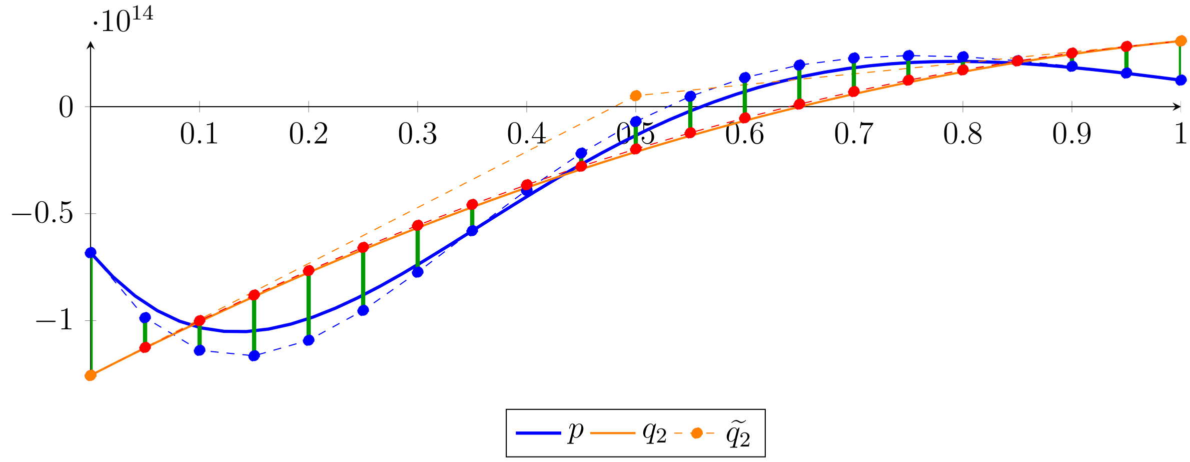
$$\begin{aligned} p = {} & 34696.8X^{20} - 584847X^{19} + 3.25592 \cdot 10^{7} X^{18} - 8.1589 \cdot 10^{8} X^{17} + 1.46784 \cdot 10^{10} X^{16} - 1.89675 \cdot 10^{11} X^{15} \\ & + 1.81956 \cdot 10^{12} X^{14} - 1.31307 \cdot 10^{13} X^{13} + 7.14297 \cdot 10^{13} X^{12} - 2.89728 \cdot 10^{14} X^{11} + 8.49406 \cdot 10^{14} X^{10} \\ & - 1.66526 \cdot 10^{15} X^{9} + 1.65931 \cdot 10^{15} X^{8} + 9.85362 \cdot 10^{14} X^{7} - 5.74116 \cdot 10^{15} X^{6} + 7.50397 \cdot 10^{15} X^{5} \\ & - 2.84125 \cdot 10^{15} X^{4} - 2.70311 \cdot 10^{15} X^{3} + 2.87089 \cdot 10^{15} X^{2} - 6.07752 \cdot 10^{14} X - 6.82353 \cdot 10^{13} \\ = {} & -6.82353 \cdot 10^{13} B_{0,20}(X) - 9.86229 \cdot 10^{13} B_{1,20}(X) - 1.13901 \cdot 10^{14} B_{2,20}(X) - 1.16439 \\ & \cdot 10^{14} B_{3,20}(X) - 1.09197 \cdot 10^{14} B_{4,20}(X) - 9.52335 \cdot 10^{13} B_{5,20}(X) - 7.73753 \cdot 10^{13} B_{6,20}(X) \\ & - 5.80151 \cdot 10^{13} B_{7,20}(X) - 3.90206 \cdot 10^{13} B_{8,20}(X) - 2.17241 \cdot 10^{13} B_{9,20}(X) - 6.96521 \\ & \cdot 10^{12} B_{10,20}(X) + 4.83558 \cdot 10^{12} B_{11,20}(X) + 1.35903 \cdot 10^{13} B_{12,20}(X) + 1.94553 \cdot 10^{13} B_{13,20}(X) \\ & + 2.27507 \cdot 10^{13} B_{14,20}(X) + 2.38903 \cdot 10^{13} B_{15,20}(X) + 2.33265 \cdot 10^{13} B_{16,20}(X) + 2.15075 \\ & \cdot 10^{13} B_{17,20}(X) + 1.88477 \cdot 10^{13} B_{18,20}(X) + 1.57094 \cdot 10^{13} B_{19,20}(X) + 1.23927 \cdot 10^{13} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = -1.05156 \cdot 10^{14} X^2 + 2.61476 \cdot 10^{14} X - 1.25611 \cdot 10^{14}$$
$$= -1.25611 \cdot 10^{14} B_{0,2} + 5.12694 \cdot 10^{12} B_{1,2} + 3.07095 \cdot 10^{13} B_{2,2}$$

$$\widetilde{q_2} = -2.07424 \cdot 10^{15} X^{20} + 2.03953 \cdot 10^{16} X^{19} - 9.27209 \cdot 10^{16} X^{18} + 2.58499 \cdot 10^{17} X^{17} - 4.94098 \cdot 10^{17} X^{16} + 6.85494 \cdot 10^{17} X^{15} - 7.12593 \cdot 10^{17} X^{14} + 5.6483 \cdot 10^{17} X^{13} - 3.44246 \cdot 10^{17} X^{12} + 1.61724 \cdot 10^{17} X^{11} - 5.85146 \cdot 10^{16} X^{10} + 1.62616 \cdot 10^{16} X^{9} - 3.44499 \cdot 10^{15} X^{8} + 5.41456 \cdot 10^{14} X^{7} - 5.83023 \cdot 10^{13} X^{6} + 3.45124 \cdot 10^{12} X^{5} + 4.51328 \cdot 10^{9} X^{4} - 1.47199 \cdot 10^{10} X^{3} - 1.05155 \cdot 10^{14} X^{2} + 2.61476 \cdot 10^{14} X - 1.25611 \cdot 10^{14}$$
$$= -1.25611 \cdot 10^{14} B_{0,20} - 1.12537 \cdot 10^{14} B_{1,20} - 1.00017 \cdot 10^{14} B_{2,20} - 8.80501 \cdot 10^{13} B_{3,20} - 7.66366 \cdot 10^{13} B_{4,20} - 6.57765 \cdot 10^{13} B_{5,20} - 5.54704 \cdot 10^{13} B_{6,20} - 4.57161 \cdot 10^{13} B_{7,20} - 3.65189 \cdot 10^{13} B_{8,20} - 2.78681 \cdot 10^{13} B_{9,20} - 1.97803 \cdot 10^{13} B_{10,20} - 1.22361 \cdot 10^{13} B_{11,20} - 5.25533 \cdot 10^{12} B_{12,20} + 1.18083 \cdot 10^{12} B_{13,20} + 7.05761 \cdot 10^{12} B_{14,20} + 1.2384 \cdot 10^{13} B_{15,20} + 1.71557 \cdot 10^{13} B_{16,20} + 2.13744 \cdot 10^{13} B_{17,20} + 2.50395 \cdot 10^{13} B_{18,20} + 2.81512 \cdot 10^{13} B_{19,20} + 3.07095 \cdot 10^{13} B_{20,20}$$



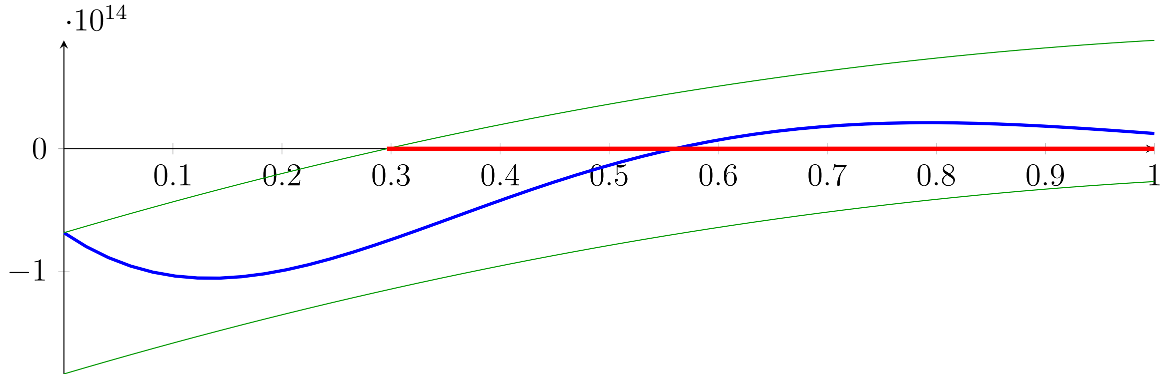
The maximum difference of the Bézier coefficients is $\delta = 5.73758 \cdot 10^{13}$.
Bounding polynomials M and m:

$$M = -1.05156 \cdot 10^{14} X^2 + 2.61476 \cdot 10^{14} X - 6.82353 \cdot 10^{13}$$
$$m = -1.05156 \cdot 10^{14} X^2 + 2.61476 \cdot 10^{14} X - 1.82987 \cdot 10^{14}$$

Root of M and m:

$$N(M) = \{0.296259, 2.19031\} \qquad N(m) = \{\}$$

Intersection intervals:



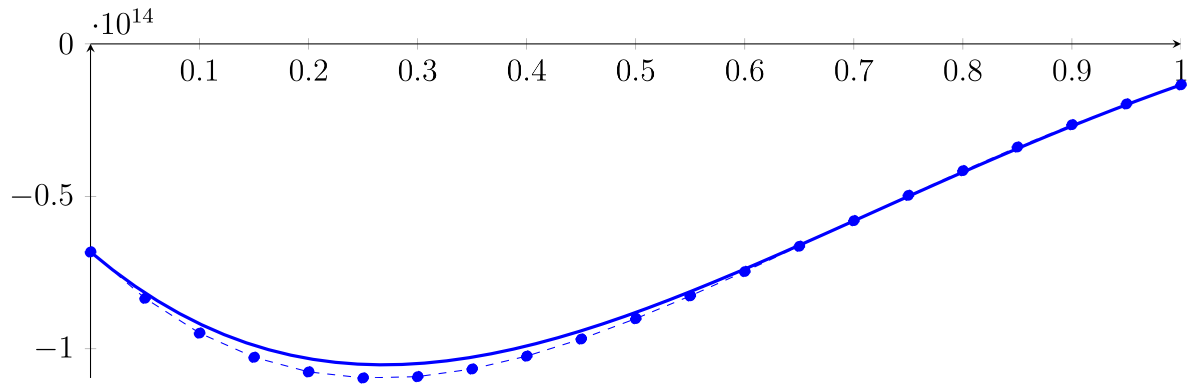
[0.296259, 1]

Longest intersection interval: 0.703741
$\Longrightarrow$ Bisection: first half [3.125, 3.90625] und second half [3.90625, 4.6875]
Bisection point is very near to a root?!?

2.22 Recursion Branch 1 1 1 2 1 1 on the First Half [3.125, 3.90625]

Normalized monomial und Bézier representations and the Bézier polygon:

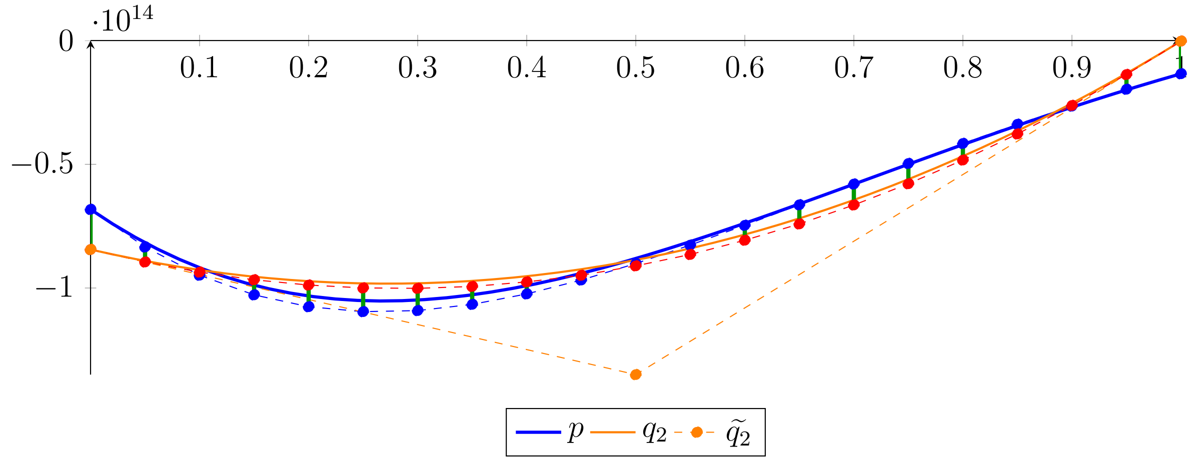
$$p = 92779.6X^{20} - 367537X^{19} + 2.95727\cdot 10^{6}X^{18} - 1.0394\cdot 10^{7}X^{17} + 6.23362\cdot 10^{7}X^{16} - 5.54359\cdot 10^{7}X^{15} + 1.31952\cdot 10^{8}X^{14} - 1.59062\cdot 10^{9}X^{13} + 1.74901\cdot 10^{10}X^{12} - 1.41461\cdot 10^{11}X^{11} + 8.29513\cdot 10^{11}X^{10} - 3.25246\cdot 10^{12}X^{9} + 6.48169\cdot 10^{12}X^{8} + 7.69814\cdot 10^{12}X^{7} - 8.97056\cdot 10^{13}X^{6} + 2.34499\cdot 10^{14}X^{5} - 1.77578\cdot 10^{14}X^{4} - 3.37889\cdot 10^{14}X^{3} + 7.17722\cdot 10^{14}X^{2} - 3.03876\cdot 10^{14}X - 6.82353\cdot 10^{13}$$
$$= -6.82353\cdot 10^{13}B_{0,20}(X) - 8.34291\cdot 10^{13}B_{1,20}(X) - 9.48454\cdot 10^{13}B_{2,20}(X) - 1.02781\cdot 10^{14}B_{3,20}(X) - 1.07568\cdot 10^{14}B_{4,20}(X) - 1.09562\cdot 10^{14}B_{5,20}(X) - 1.09125\cdot 10^{14}B_{6,20}(X) - 1.0662\cdot 10^{14}B_{7,20}(X) - 1.02397\cdot 10^{14}B_{8,20}(X) - 9.67898\cdot 10^{13}B_{9,20}(X) - 9.0111\cdot 10^{13}B_{10,20}(X) - 8.26467\cdot 10^{13}B_{11,20}(X) - 7.46554\cdot 10^{13}B_{12,20}(X) - 6.6367\cdot 10^{13}B_{13,20}(X) - 5.79821\cdot 10^{13}B_{14,20}(X) - 4.96725\cdot 10^{13}B_{15,20}(X) - 4.15827\cdot 10^{13}B_{16,20}(X) - 3.38307\cdot 10^{13}B_{17,20}(X) - 2.65105\cdot 10^{13}B_{18,20}(X) - 1.96935\cdot 10^{13}B_{19,20}(X) - 1.3431\cdot 10^{13}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 1.85293\cdot 10^{14}X^2 - 1.01103\cdot 10^{14}X - 8.44429\cdot 10^{13}$$
$$= -8.44429\cdot 10^{13}B_{0,2} - 1.34994\cdot 10^{14}B_{1,2} - 2.53219\cdot 10^{11}B_{2,2}$$

$$\begin{aligned}\widetilde{q}_2 &= 4.03866 \cdot 10^{16} X^{20} - 4.04551 \cdot 10^{17} X^{19} + 1.876 \cdot 10^{18} X^{18} - 5.3434 \cdot 10^{18} X^{17} + 1.04588 \cdot 10^{19} X^{16} - 1.4912 \\ &\quad \cdot 10^{19} X^{15} + 1.6019 \cdot 10^{19} X^{14} - 1.32275 \cdot 10^{19} X^{13} + 8.49052 \cdot 10^{18} X^{12} - 4.25646 \cdot 10^{18} X^{11} + 1.66539 \\ &\quad \cdot 10^{18} X^{10} - 5.05639 \cdot 10^{17} X^{9} + 1.17855 \cdot 10^{17} X^{8} - 2.07587 \cdot 10^{16} X^{7} + 2.70642 \cdot 10^{15} X^{6} - 2.54304 \\ &\quad \cdot 10^{14} X^{5} + 1.65862 \cdot 10^{13} X^{4} - 6.9667 \cdot 10^{11} X^{3} + 1.85309 \cdot 10^{14} X^{2} - 1.01103 \cdot 10^{14} X - 8.44429 \cdot 10^{13} \\ &= -8.44429 \cdot 10^{13} B_{0,20} - 8.94981 \cdot 10^{13} B_{1,20} - 9.35779 \cdot 10^{13} B_{2,20} - 9.66831 \cdot 10^{13} B_{3,20} - 9.88107 \\ &\quad \cdot 10^{13} B_{4,20} - 9.9971 \cdot 10^{13} B_{5,20} - 1.00134 \cdot 10^{14} B_{6,20} - 9.93725 \cdot 10^{13} B_{7,20} - 9.75402 \cdot 10^{13} B_{8,20} \\ &\quad - 9.488 \cdot 10^{13} B_{9,20} - 9.10537 \cdot 10^{13} B_{10,20} - 8.64592 \cdot 10^{13} B_{11,20} - 8.07027 \cdot 10^{13} B_{12,20} \\ &\quad - 7.41145 \cdot 10^{13} B_{13,20} - 6.6458 \cdot 10^{13} B_{14,20} - 5.78764 \cdot 10^{13} B_{15,20} - 4.82969 \cdot 10^{13} B_{16,20} \\ &\quad - 3.77503 \cdot 10^{13} B_{17,20} - 2.62262 \cdot 10^{13} B_{18,20} - 1.37273 \cdot 10^{13} B_{19,20} - 2.53219 \cdot 10^{11} B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.62076 \cdot 10^{13}$.
Bounding polynomials M and m:

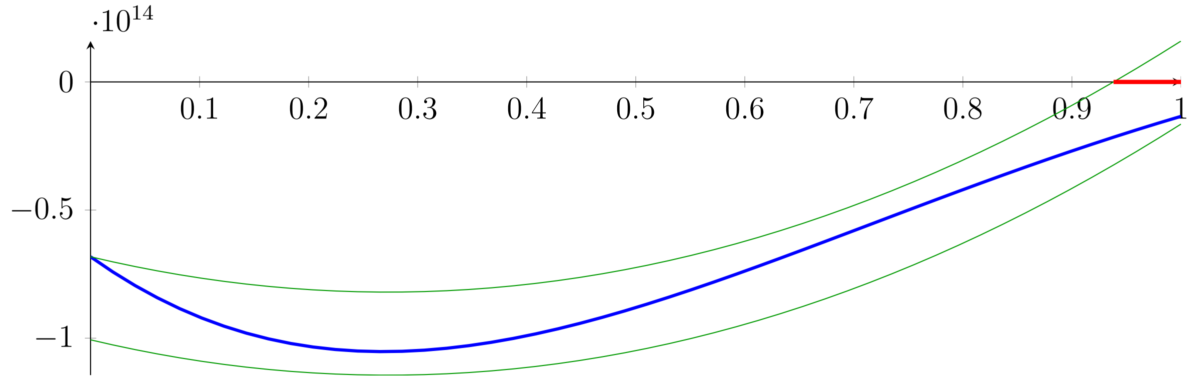
$$M = 1.85293 \cdot 10^{14} X^2 - 1.01103 \cdot 10^{14} X - 6.82353 \cdot 10^{13}$$

$$m = 1.85293 \cdot 10^{14} X^2 - 1.01103 \cdot 10^{14} X - 1.00651 \cdot 10^{14}$$

Root of M and m:

$$N(M) = \{-0.392528, 0.938167\} \qquad N(m) = \{-0.513073, 1.05871\}$$

Intersection intervals:



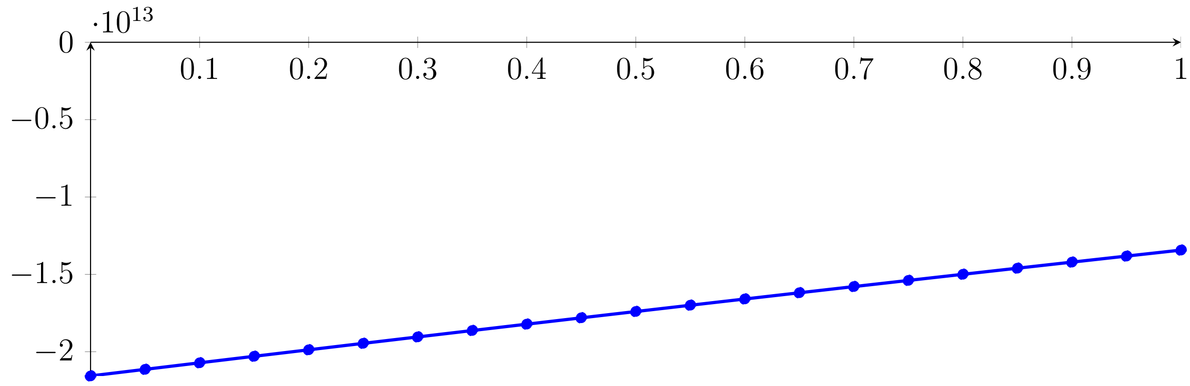
$$[0.938167, 1]$$

Longest intersection interval: 0.0618327
$\Longrightarrow$ Selective recursion: interval 1: $[3.85794, 3.90625]$,

2.23 Recursion Branch 1 1 1 2 1 1 1 in Interval 1: $[3.85794, 3.90625]$

Normalized monomial und Bézier representations and the Bézier polygon:

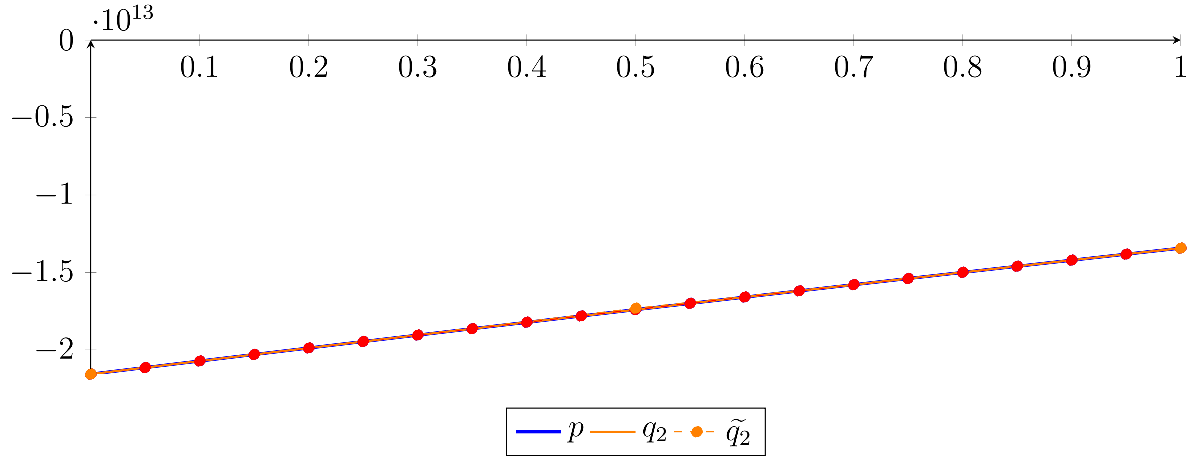
$$p = 17724.2X^{20} - 80042.4X^{19} + 556426X^{18} - 2.05648{\cdot}10^{6}X^{17} + 1.15323{\cdot}10^{7}X^{16} - 8.76164{\cdot}10^{6}X^{15} + 3.42769{\cdot}10^{6}X^{14} + 1.84867{\cdot}10^{6}X^{13} + 9.18744{\cdot}10^{6}X^{12} + 1.24461{\cdot}10^{6}X^{11} + 2.82835{\cdot}10^{6}X^{10} + 341825X^{9} + 54619.8X^{8} + 26950.3X^{7} - 440441X^{6} - 2.788{\cdot}10^{7}X^{5} + 1.57458{\cdot}10^{9}X^{4} - 1.9842{\cdot}10^{10}X^{3} - 3.52467{\cdot}10^{11}X^{2} + 8.5028{\cdot}10^{12}X - 2.15631{\cdot}10^{13}$$
$$= -2.15631{\cdot}10^{13}B_{0,20}(X) - 2.11379{\cdot}10^{13}B_{1,20}(X) - 2.07146{\cdot}10^{13}B_{2,20}(X) - 2.02932{\cdot}10^{13}B_{3,20}(X) - 1.98737{\cdot}10^{13}B_{4,20}(X) - 1.94561{\cdot}10^{13}B_{5,20}(X) - 1.90404{\cdot}10^{13}B_{6,20}(X) - 1.86266{\cdot}10^{13}B_{7,20}(X) - 1.82148{\cdot}10^{13}B_{8,20}(X) - 1.7805{\cdot}10^{13}B_{9,20}(X) - 1.73972{\cdot}10^{13}B_{10,20}(X) - 1.69913{\cdot}10^{13}B_{11,20}(X) - 1.65875{\cdot}10^{13}B_{12,20}(X) - 1.61857{\cdot}10^{13}B_{13,20}(X) - 1.57859{\cdot}10^{13}B_{14,20}(X) - 1.53882{\cdot}10^{13}B_{15,20}(X) - 1.49926{\cdot}10^{13}B_{16,20}(X) - 1.45991{\cdot}10^{13}B_{17,20}(X) - 1.42076{\cdot}10^{13}B_{18,20}(X) - 1.38183{\cdot}10^{13}B_{19,20}(X) - 1.3431{\cdot}10^{13}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = -3.79581{\cdot}10^{11}X^{2} + 8.5133{\cdot}10^{12}X - 2.15639{\cdot}10^{13}$$
$$= -2.15639{\cdot}10^{13}B_{0,2} - 1.73073{\cdot}10^{13}B_{1,2} - 1.34302{\cdot}10^{13}B_{2,2}$$

$$\widetilde{q}_2 = 3.43877{\cdot}10^{15}X^{20} - 3.45427{\cdot}10^{16}X^{19} + 1.60702{\cdot}10^{17}X^{18} - 4.59386{\cdot}10^{17}X^{17} + 9.02693{\cdot}10^{17}X^{16} - 1.29235{\cdot}10^{18}X^{15} + 1.39411{\cdot}10^{18}X^{14} - 1.15594{\cdot}10^{18}X^{13} + 7.44892{\cdot}10^{17}X^{12} - 3.74733{\cdot}10^{17}X^{11} + 1.47025{\cdot}10^{17}X^{10} - 4.47161{\cdot}10^{16}X^{9} + 1.04304{\cdot}10^{16}X^{8} - 1.83955{\cdot}10^{15}X^{7} + 2.41393{\cdot}10^{14}X^{6} - 2.31461{\cdot}10^{13}X^{5} + 1.57964{\cdot}10^{12}X^{4} - 7.12775{\cdot}10^{10}X^{3} - 3.77738{\cdot}10^{11}X^{2} + 8.51328{\cdot}10^{12}X - 2.15639{\cdot}10^{13}$$
$$= -2.15639{\cdot}10^{13}B_{0,20} - 2.11383{\cdot}10^{13}B_{1,20} - 2.07146{\cdot}10^{13}B_{2,20} - 2.0293{\cdot}10^{13}B_{3,20} - 1.98731{\cdot}10^{13}B_{4,20} - 1.9456{\cdot}10^{13}B_{5,20} - 1.90389{\cdot}10^{13}B_{6,20} - 1.86283{\cdot}10^{13}B_{7,20} - 1.82112{\cdot}10^{13}B_{8,20} - 1.7809{\cdot}10^{13}B_{9,20} - 1.73923{\cdot}10^{13}B_{10,20} - 1.69952{\cdot}10^{13}B_{11,20} - 1.65846{\cdot}10^{13}B_{12,20} - 1.61879{\cdot}10^{13}B_{13,20} - 1.57854{\cdot}10^{13}B_{14,20} - 1.53892{\cdot}10^{13}B_{15,20} - 1.49929{\cdot}10^{13}B_{16,20} - 1.45994{\cdot}10^{13}B_{17,20} - 1.42076{\cdot}10^{13}B_{18,20} - 1.38179{\cdot}10^{13}B_{19,20} - 1.34302{\cdot}10^{13}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 4.90912{\cdot}10^{9}$.

Bounding polynomials M and m:

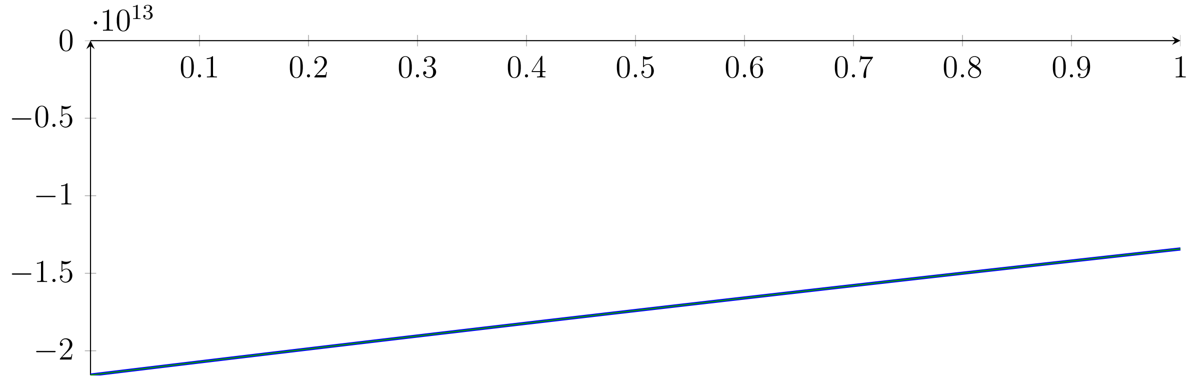
$$M = -3.79581{\cdot}10^{11}X^2 + 8.5133{\cdot}10^{12}X - 2.1559{\cdot}10^{13}$$

$$m = -3.79581{\cdot}10^{11}X^2 + 8.5133{\cdot}10^{12}X - 2.15688{\cdot}10^{13}$$

Root of M and m:

$$N(M) = \{2.90995, 19.5182\} \qquad N(m) = \{2.9115, 19.5166\}$$

Intersection intervals:

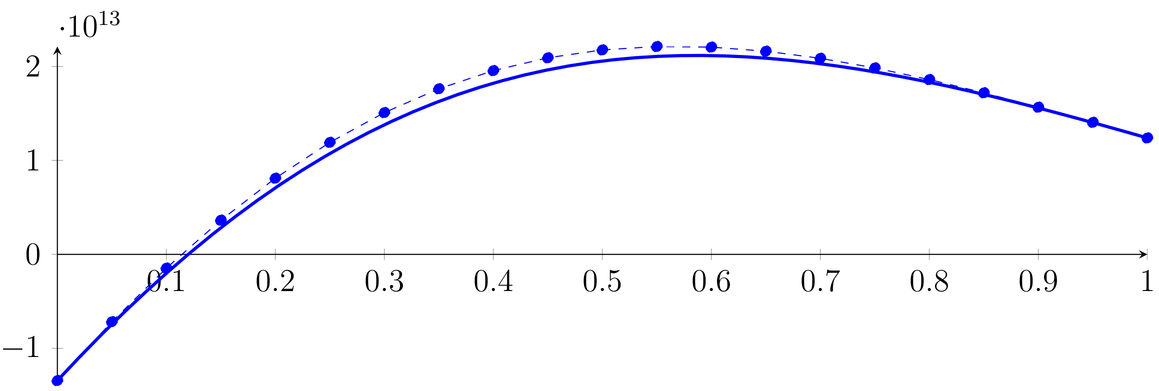


No intersection intervals with the x axis.

2.24 Recursion Branch 1 1 1 2 1 2 on the Second Half $[3.90625, 4.6875]$

Normalized monomial und Bézier representations and the Bézier polygon:

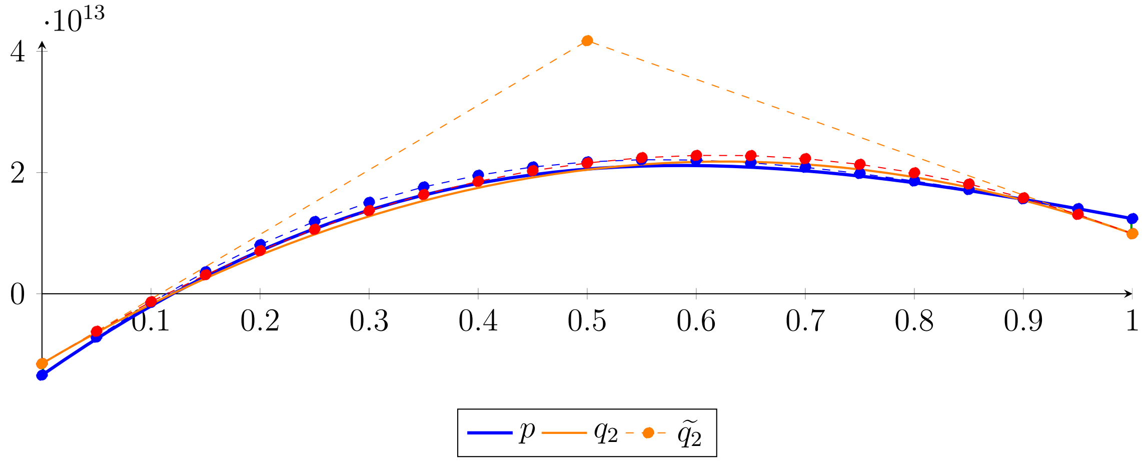
$$\begin{aligned} p = &-20467.2X^{20} + 127175X^{19} - 513767X^{18} + 2.51846{\cdot}10^{6}X^{17} - 1.22729{\cdot}10^{7}X^{16} + 6.47156{\cdot}10^{6}X^{15} \\ &+ 4.46537{\cdot}10^{7}X^{14} - 5.45367{\cdot}10^{8}X^{13} + 4.43795{\cdot}10^{9}X^{12} - 2.3798{\cdot}10^{10}X^{11} + 6.13497{\cdot}10^{10}X^{10} \\ &+ 1.48028{\cdot}10^{11}X^{9} - 1.93703{\cdot}10^{12}X^{8} + 6.72557{\cdot}10^{12}X^{7} - 5.03278{\cdot}10^{12}X^{6} - 3.32796{\cdot}10^{13}X^{5} \\ &+ 9.77763{\cdot}10^{13}X^{4} - 5.85049{\cdot}10^{13}X^{3} - 1.05363{\cdot}10^{14}X^{2} + 1.25249{\cdot}10^{14}X - 1.3431{\cdot}10^{13} \\ = &-1.3431{\cdot}10^{13}B_{0,20}(X) - 7.16856{\cdot}10^{12}B_{1,20}(X) - 1.46064{\cdot}10^{12}B_{2,20}(X) + 3.64143 \\ &{\cdot}10^{12}B_{3,20}(X) + 8.10649{\cdot}10^{12}B_{4,20}(X) + 1.19215{\cdot}10^{13}B_{5,20}(X) + 1.5089{\cdot}10^{13}B_{6,20}(X) \\ &+ 1.76251{\cdot}10^{13}B_{7,20}(X) + 1.95571{\cdot}10^{13}B_{8,20}(X) + 2.09212{\cdot}10^{13}B_{9,20}(X) + 2.17604 \\ &{\cdot}10^{13}B_{10,20}(X) + 2.21227{\cdot}10^{13}B_{11,20}(X) + 2.20593{\cdot}10^{13}B_{12,20}(X) + 2.16231{\cdot}10^{13}B_{13,20}(X) \\ &+ 2.08673{\cdot}10^{13}B_{14,20}(X) + 1.98442{\cdot}10^{13}B_{15,20}(X) + 1.86046{\cdot}10^{13}B_{16,20}(X) + 1.71964 \\ &{\cdot}10^{13}B_{17,20}(X) + 1.56648{\cdot}10^{13}B_{18,20}(X) + 1.40511{\cdot}10^{13}B_{19,20}(X) + 1.23927{\cdot}10^{13}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -8.51272{\cdot}10^{13}X^2 + 1.06556{\cdot}10^{14}X - 1.1522{\cdot}10^{13} \\ &= -1.1522{\cdot}10^{13}B_{0,2} + 4.17561{\cdot}10^{13}B_{1,2} + 9.90705{\cdot}10^{12}B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 = &-1.25591{\cdot}10^{16}X^{20} + 1.25696{\cdot}10^{17}X^{19} - 5.82359{\cdot}10^{17}X^{18} + 1.65716{\cdot}10^{18}X^{17} - 3.24026{\cdot}10^{18}X^{16} \\ &+ 4.61459{\cdot}10^{18}X^{15} - 4.95041{\cdot}10^{18}X^{14} + 4.08101{\cdot}10^{18}X^{13} - 2.61425{\cdot}10^{18}X^{12} + 1.30742{\cdot}10^{18}X^{11} \\ &- 5.10154{\cdot}10^{17}X^{10} + 1.54449{\cdot}10^{17}X^9 - 3.58972{\cdot}10^{16}X^8 + 6.30357{\cdot}10^{15}X^7 - 8.18268{\cdot}10^{14}X^6 + 7.63106 \\ &{\cdot}10^{13}X^5 - 4.91276{\cdot}10^{12}X^4 + 2.02305{\cdot}10^{11}X^3 - 8.51319{\cdot}10^{13}X^2 + 1.06556{\cdot}10^{14}X - 1.1522{\cdot}10^{13} \\ = &-1.1522{\cdot}10^{13}B_{0,20} - 6.19418{\cdot}10^{12}B_{1,20} - 1.31443{\cdot}10^{12}B_{2,20} + 3.11744{\cdot}10^{12}B_{3,20} + 7.10058 \\ &{\cdot}10^{12}B_{4,20} + 1.06381{\cdot}10^{13}B_{5,20} + 1.37207{\cdot}10^{13}B_{6,20} + 1.6371{\cdot}10^{13}B_{7,20} + 1.85438{\cdot}10^{13}B_{8,20} \\ &+ 2.03143{\cdot}10^{13}B_{9,20} + 2.15774{\cdot}10^{13}B_{10,20} + 2.24566{\cdot}10^{13}B_{11,20} + 2.28296{\cdot}10^{13}B_{12,20} \\ &+ 2.27996{\cdot}10^{13}B_{13,20} + 2.22923{\cdot}10^{13}B_{14,20} + 2.13527{\cdot}10^{13}B_{15,20} + 1.9958{\cdot}10^{13}B_{16,20} \\ &+ 1.81178{\cdot}10^{13}B_{17,20} + 1.58288{\cdot}10^{13}B_{18,20} + 1.3092{\cdot}10^{13}B_{19,20} + 9.90705{\cdot}10^{12}B_{20,20} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.48569{\cdot}10^{12}$.

Bounding polynomials M and m:

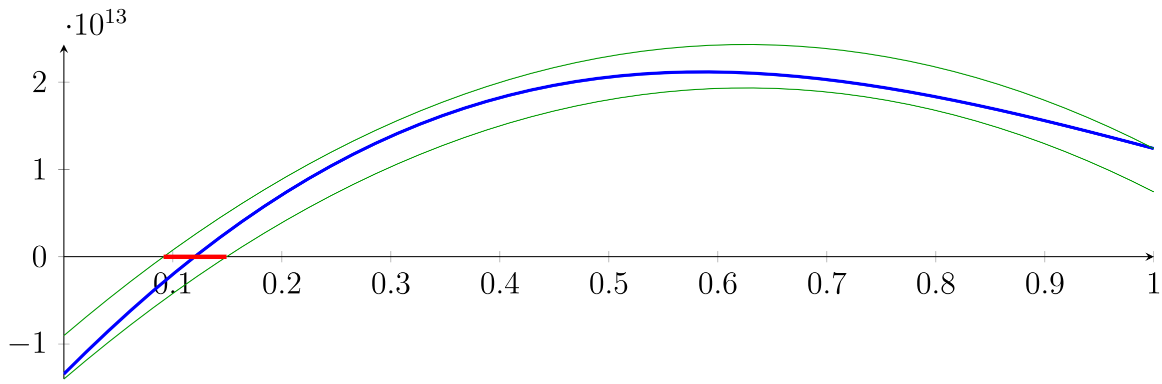
$$M = -8.51272{\cdot}10^{13}X^2 + 1.06556{\cdot}10^{14}X - 9.03631{\cdot}10^{12}$$

$$m = -8.51272{\cdot}10^{13}X^2 + 1.06556{\cdot}10^{14}X - 1.40077{\cdot}10^{13}$$

Root of M and m:

$$N(M) = \{0.0914902, 1.16024\} \qquad N(m) = \{0.149255, 1.10247\}$$

Intersection intervals:



$$[0.0914902, 0.149255]$$

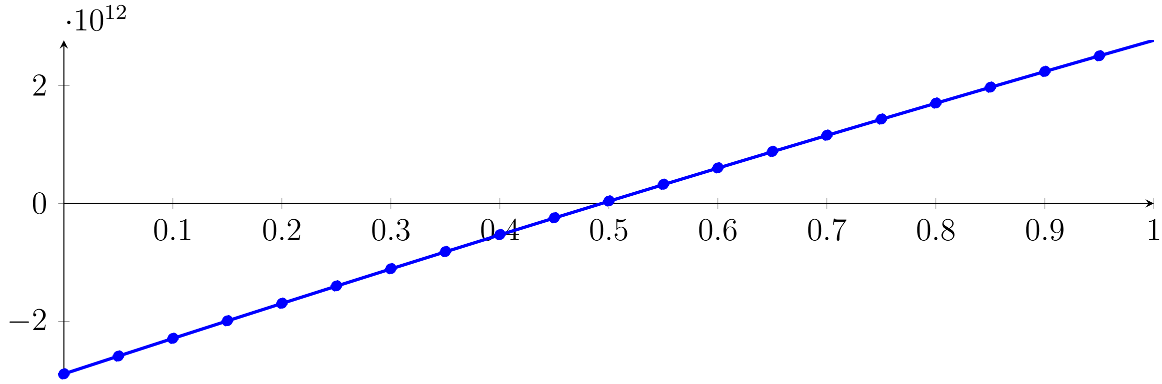
Longest intersection interval: 0.057765

$\Longrightarrow$ Selective recursion: interval 1: $[3.97773, 4.02286]$,

2.25 Recursion Branch 1 1 1 2 1 2 1 in Interval 1: $[3.97773, 4.02286]$

Normalized monomial und Bézier representations and the Bézier polygon:

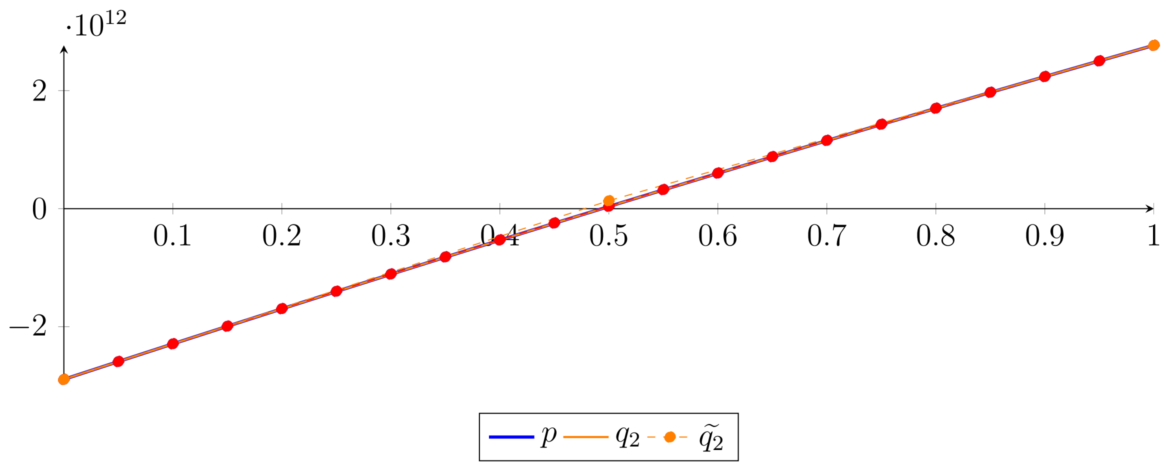
$$p = 250.402X^{20} + 4131.64X^{19} + 20698.9X^{18} + 8189.3X^{17} + 275200X^{16} - 295121X^{15} + 199705X^{14} + 127181X^{13} + 532420X^{12} + 167960X^{11} + 199190X^{10} + 38053.4X^{9} + 984.141X^{8} + 12112.5X^{7} - 42090.9X^{6} - 2.24734\cdot 10^{7}X^{5} + 9.14023\cdot 10^{8}X^{4} - 4.92852\cdot 10^{9}X^{3} - 3.89635\cdot 10^{11}X^{2} + 6.05311\cdot 10^{12}X - 2.89204\cdot 10^{12}$$
$$= -2.89204\cdot 10^{12}B_{0,20}(X) - 2.58939\cdot 10^{12}B_{1,20}(X) - 2.28878\cdot 10^{12}B_{2,20}(X) - 1.99023\cdot 10^{12}B_{3,20}(X) - 1.69374\cdot 10^{12}B_{4,20}(X) - 1.39931\cdot 10^{12}B_{5,20}(X) - 1.10695\cdot 10^{12}B_{6,20}(X) - 8.16663\cdot 10^{11}B_{7,20}(X) - 5.28446\cdot 10^{11}B_{8,20}(X) - 2.42307\cdot 10^{11}B_{9,20}(X) + 4.17522\cdot 10^{10}B_{10,20}(X) + 3.23728\cdot 10^{11}B_{11,20}(X) + 6.03619\cdot 10^{11}B_{12,20}(X) + 8.81422\cdot 10^{11}B_{13,20}(X) + 1.15713\cdot 10^{12}B_{14,20}(X) + 1.43075\cdot 10^{12}B_{15,20}(X) + 1.70228\cdot 10^{12}B_{16,20}(X) + 1.97171\cdot 10^{12}B_{17,20}(X) + 2.23904\cdot 10^{12}B_{18,20}(X) + 2.50427\cdot 10^{12}B_{19,20}(X) + 2.7674\cdot 10^{12}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = -3.95501\cdot 10^{11}X^{2} + 6.05526\cdot 10^{12}X - 2.89221\cdot 10^{12}$$
$$= -2.89221\cdot 10^{12}B_{0,2} + 1.35416\cdot 10^{11}B_{1,2} + 2.76754\cdot 10^{12}B_{2,2}$$

$$\widetilde{q}_2 = 1.65469\cdot 10^{14}X^{20} - 1.65797\cdot 10^{15}X^{19} + 7.68425\cdot 10^{15}X^{18} - 2.18626\cdot 10^{16}X^{17} + 4.27392\cdot 10^{16}X^{16} - 6.08968\cdot 10^{16}X^{15} + 6.54716\cdot 10^{16}X^{14} - 5.42464\cdot 10^{16}X^{13} + 3.50641\cdot 10^{16}X^{12} - 1.77775\cdot 10^{16}X^{11} + 7.06455\cdot 10^{15}X^{10} - 2.18591\cdot 10^{15}X^{9} + 5.2018\cdot 10^{14}X^{8} - 9.35281\cdot 10^{13}X^{7} + 1.24086\cdot 10^{13}X^{6} - 1.17474\cdot 10^{12}X^{5} + 7.53639\cdot 10^{10}X^{4} - 3.02201\cdot 10^{9}X^{3} - 3.95434\cdot 10^{11}X^{2} + 6.05526\cdot 10^{12}X - 2.89221\cdot 10^{12}$$
$$= -2.89221\cdot 10^{12}B_{0,20} - 2.58945\cdot 10^{12}B_{1,20} - 2.28877\cdot 10^{12}B_{2,20} - 1.99017\cdot 10^{12}B_{3,20} - 1.69364\cdot 10^{12}B_{4,20} - 1.39924\cdot 10^{12}B_{5,20} - 1.10681\cdot 10^{12}B_{6,20} - 8.16684\cdot 10^{11}B_{7,20} - 5.28246\cdot 10^{11}B_{8,20} - 2.42474\cdot 10^{11}B_{9,20} + 4.19787\cdot 10^{10}B_{10,20} + 3.23479\cdot 10^{11}B_{11,20} + 6.03697\cdot 10^{11}B_{12,20} + 8.81257\cdot 10^{11}B_{13,20} + 1.15709\cdot 10^{12}B_{14,20} + 1.43065\cdot 10^{12}B_{15,20} + 1.70221\cdot 10^{12}B_{16,20} + 1.97166\cdot 10^{12}B_{17,20} + 2.23904\cdot 10^{12}B_{18,20} + 2.50433\cdot 10^{12}B_{19,20} + 2.76754\cdot 10^{12}B_{20,20}$$



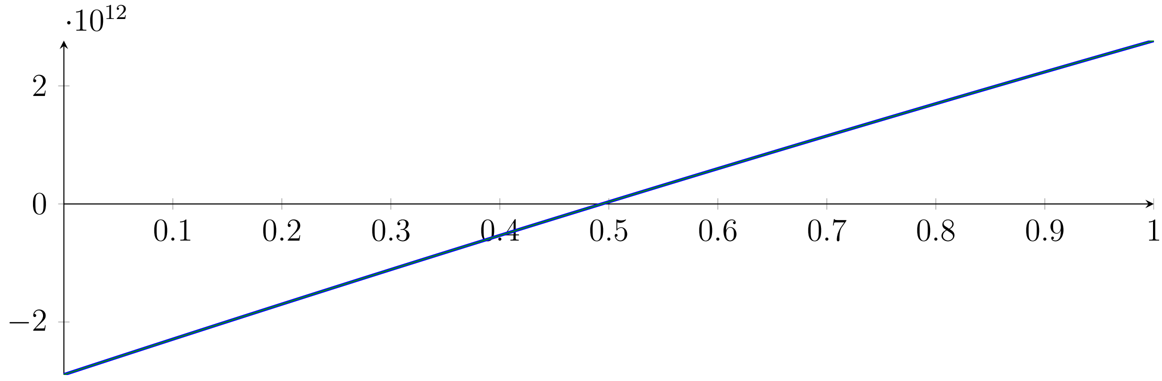
The maximum difference of the Bézier coefficients is $\delta = 2.49591 \cdot 10^8$.
Bounding polynomials M and m:

$$M = -3.95501 \cdot 10^{11} X^2 + 6.05526 \cdot 10^{12} X - 2.89196 \cdot 10^{12}$$
$$m = -3.95501 \cdot 10^{11} X^2 + 6.05526 \cdot 10^{12} X - 2.89246 \cdot 10^{12}$$

Root of M and m:

$$N(M) = \{0.493503, 14.8168\} \qquad N(m) = \{0.493591, 14.8168\}$$

Intersection intervals:



$$[0.493503, 0.493591]$$

Longest intersection interval: $8.81191 \cdot 10^{-05}$
$\Longrightarrow$ Selective recursion: interval 1: $[4, 4]$,

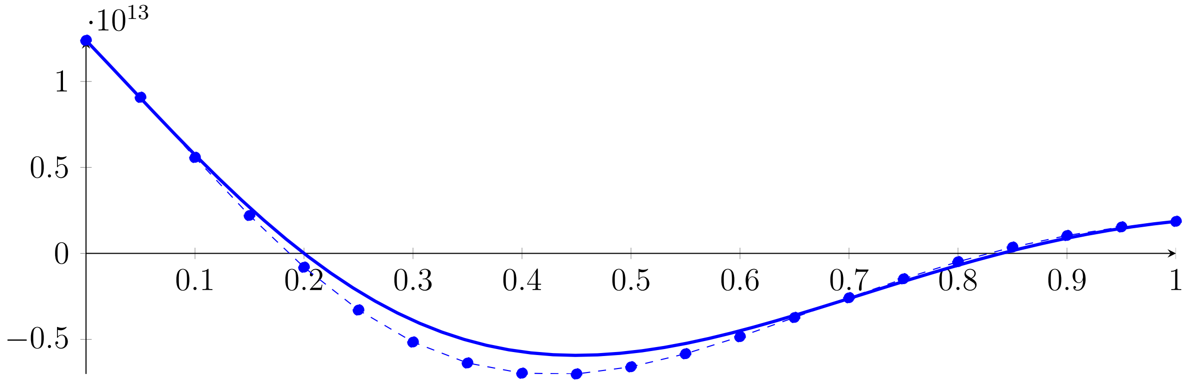
2.26 Recursion Branch 1 1 1 2 1 2 1 1 in Interval 1: $[4, 4]$

Found root in interval $[4, 4]$ at recursion depth 8!

2.27 Recursion Branch 1 1 1 2 2 on the Second Half $[4.6875, 6.25]$

Normalized monomial und Bézier representations and the Bézier polygon:

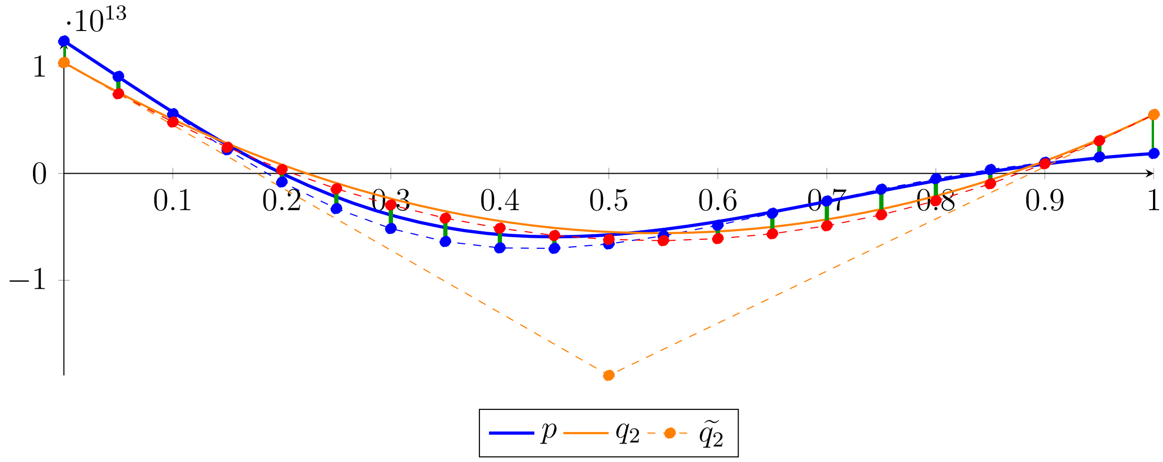
$$\begin{aligned} p = {} & 13943.5X^{20} - 587258X^{19} + 1.89008 \cdot 10^7 X^{18} - 3.73664 \cdot 10^8 X^{17} + 4.87208 \cdot 10^9 X^{16} - 4.34631 \cdot 10^{10} X^{15} \\ & + 2.65721 \cdot 10^{11} X^{14} - 1.05722 \cdot 10^{12} X^{13} + 2.18629 \cdot 10^{12} X^{12} + 1.53487 \cdot 10^{12} X^{11} - 2.39754 \cdot 10^{13} X^{10} \\ & + 6.26713 \cdot 10^{13} X^9 - 3.75532 \cdot 10^{13} X^8 - 1.53878 \cdot 10^{14} X^7 + 3.47765 \cdot 10^{14} X^6 - 1.50066 \cdot 10^{14} X^5 \\ & - 3.00387 \cdot 10^{14} X^4 + 3.42221 \cdot 10^{14} X^3 - 3.38862 \cdot 10^{13} X^2 - 6.63332 \cdot 10^{13} X + 1.23927 \cdot 10^{13} \\ = {} & 1.23927 \cdot 10^{13} B_{0,20}(X) + 9.07608 \cdot 10^{12} B_{1,20}(X) + 5.58107 \cdot 10^{12} B_{2,20}(X) + 2.20791 \\ & \cdot 10^{12} B_{3,20}(X) - 8.05212 \cdot 10^{11} B_{4,20}(X) - 3.29178 \cdot 10^{12} B_{5,20}(X) - 5.15766 \cdot 10^{12} B_{6,20}(X) \\ & - 6.37482 \cdot 10^{12} B_{7,20}(X) - 6.97037 \cdot 10^{12} B_{8,20}(X) - 7.01303 \cdot 10^{12} B_{9,20}(X) - 6.5991 \\ & \cdot 10^{12} B_{10,20}(X) - 5.83916 \cdot 10^{12} B_{11,20}(X) - 4.8467 \cdot 10^{12} B_{12,20}(X) - 3.72904 \cdot 10^{12} B_{13,20}(X) \\ & - 2.58078 \cdot 10^{12} B_{14,20}(X) - 1.47983 \cdot 10^{12} B_{15,20}(X) - 4.85456 \cdot 10^{11} B_{16,20}(X) + 3.6178 \\ & \cdot 10^{11} B_{17,20}(X) + 1.03875 \cdot 10^{12} B_{18,20}(X) + 1.53757 \cdot 10^{12} B_{19,20}(X) + 1.86285 \cdot 10^{12} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = 5.36715{\cdot}10^{13}X^2 - 5.85485{\cdot}10^{13}X + 1.03783{\cdot}10^{13}$$
$$= 1.03783{\cdot}10^{13}B_{0,2} - 1.8896{\cdot}10^{13}B_{1,2} + 5.50134{\cdot}10^{12}B_{2,2}$$

$$\widetilde{q}_2 = 6.89573{\cdot}10^{15}X^{20} - 6.89644{\cdot}10^{16}X^{19} + 3.19237{\cdot}10^{17}X^{18} - 9.07506{\cdot}10^{17}X^{17} + 1.77252{\cdot}10^{18}X^{16} - 2.52145{\cdot}10^{18}X^{15} + 2.70192{\cdot}10^{18}X^{14} - 2.22512{\cdot}10^{18}X^{13} + 1.42417{\cdot}10^{18}X^{12} - 7.1182{\cdot}10^{17}X^{11} + 2.77681{\cdot}10^{17}X^{10} - 8.40821{\cdot}10^{16}X^{9} + 1.95528{\cdot}10^{16}X^{8} - 3.43478{\cdot}10^{15}X^{7} + 4.4533{\cdot}10^{14}X^{6} - 4.12951{\cdot}10^{13}X^{5} + 2.61925{\cdot}10^{12}X^{4} - 1.04991{\cdot}10^{11}X^{3} + 5.36739{\cdot}10^{13}X^{2} - 5.85485{\cdot}10^{13}X + 1.03783{\cdot}10^{13}$$
$$= 1.03783{\cdot}10^{13}B_{0,20} + 7.45087{\cdot}10^{12}B_{1,20} + 4.80594{\cdot}10^{12}B_{2,20} + 2.44341{\cdot}10^{12}B_{3,20} + 3.6373{\cdot}10^{11}B_{4,20} - 1.43477{\cdot}10^{12}B_{5,20} - 2.94707{\cdot}10^{12}B_{6,20} - 4.18543{\cdot}10^{12}B_{7,20} - 5.12529{\cdot}10^{12}B_{8,20} - 5.80761{\cdot}10^{12}B_{9,20} - 6.175{\cdot}10^{12}B_{10,20} - 6.29517{\cdot}10^{12}B_{11,20} - 6.10064{\cdot}10^{12}B_{12,20} - 5.64852{\cdot}10^{12}B_{13,20} - 4.89785{\cdot}10^{12}B_{14,20} - 3.87329{\cdot}10^{12}B_{15,20} - 2.56244{\cdot}10^{12}B_{16,20} - 9.70458{\cdot}10^{11}B_{17,20} + 9.04376{\cdot}10^{11}B_{18,20} + 3.06161{\cdot}10^{12}B_{19,20} + 5.50134{\cdot}10^{12}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 3.63849{\cdot}10^{12}$.

Bounding polynomials M and m:

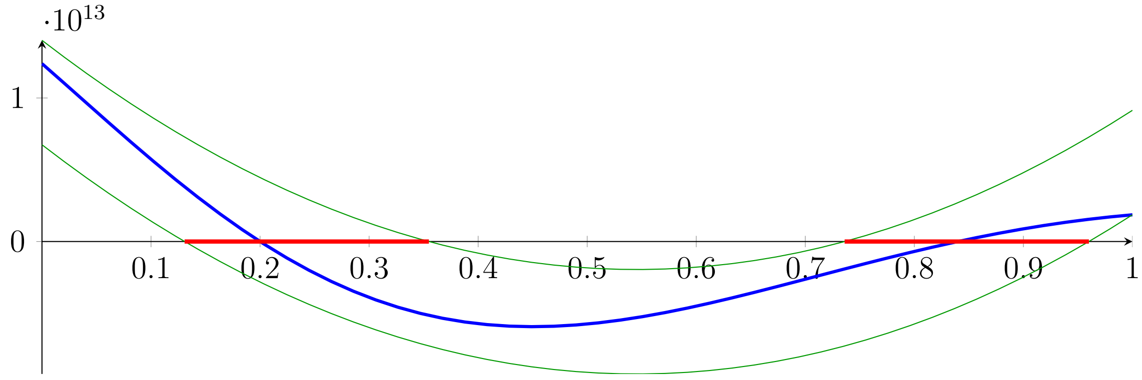
$$M = 5.36715{\cdot}10^{13}X^2 - 5.85485{\cdot}10^{13}X + 1.40168{\cdot}10^{13}$$

$$m = 5.36715{\cdot}10^{13}X^2 - 5.85485{\cdot}10^{13}X + 6.7398{\cdot}10^{12}$$

Root of M and m:

$$N(M) = \{0.354806, 0.736061\} \qquad N(m) = \{0.130798, 0.960069\}$$

Intersection intervals:



[0.130798, 0.354806], [0.736061, 0.960069]

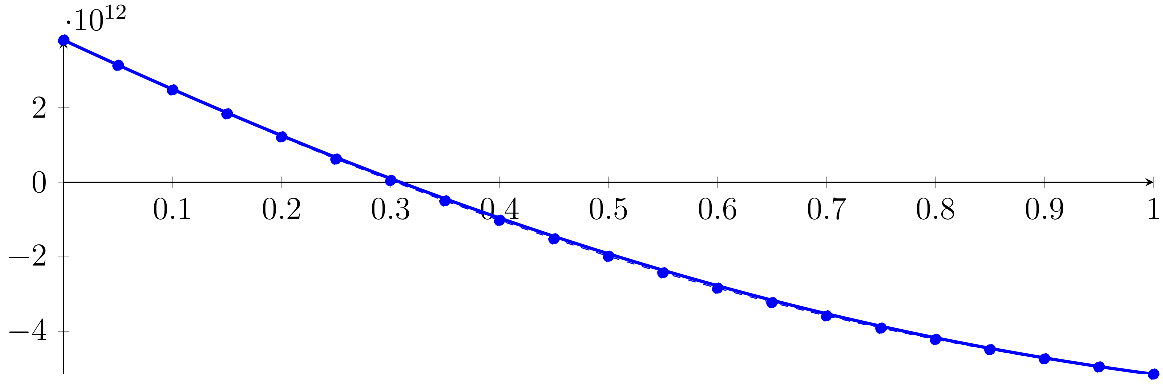
Longest intersection interval: 0.224008

$\Longrightarrow$ Selective recursion: interval 1: [4.89187, 5.24188], interval 2: [5.8376, 6.18761],

2.28 Recursion Branch 1 1 1 2 2 1 in Interval 1: [4.89187, 5.24188]

Normalized monomial und Bézier representations and the Bézier polygon:

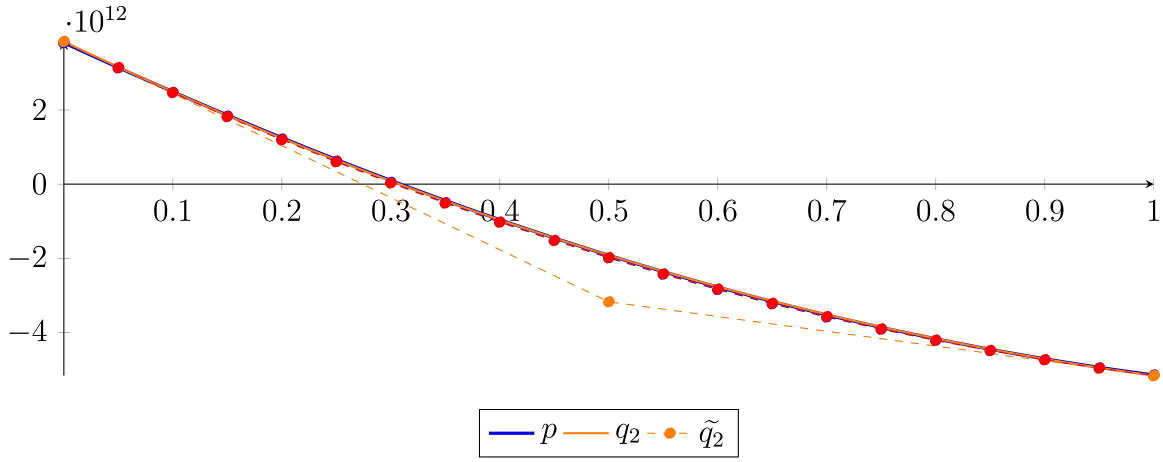
$$\begin{aligned} p = {} & 1413.76X^{20} - 15900.7X^{19} + 21303X^{18} - 227684X^{17} + 741266X^{16} - 454158X^{15} \\ & + 23770.8X^{14} - 86907.2X^{13} - 4428.63X^{12} + 93821.4X^{11} - 6.4311\cdot 10^{6}X^{10} + 4.7717 \\ & \cdot 10^{7}X^{9} + 1.17927\cdot 10^{8}X^{8} - 4.55142\cdot 10^{9}X^{7} + 2.5173\cdot 10^{10}X^{6} + 3.66356\cdot 10^{10}X^{5} \\ & - 8.10357\cdot 10^{11}X^{4} + 1.94822\cdot 10^{12}X^{3} + 3.39245\cdot 10^{12}X^{2} - 1.35452\cdot 10^{13}X + 3.81053\cdot 10^{12} \\ = {} & 3.81053\cdot 10^{12}B_{0,20}(X) + 3.13327\cdot 10^{12}B_{1,20}(X) + 2.47387\cdot 10^{12}B_{2,20}(X) + 1.83403 \\ & \cdot 10^{12}B_{3,20}(X) + 1.21529\cdot 10^{12}B_{4,20}(X) + 6.19042\cdot 10^{11}B_{5,20}(X) + 4.6488\cdot 10^{10}B_{6,20}(X) \\ & - 5.01312\cdot 10^{11}B_{7,20}(X) - 1.02346\cdot 10^{12}B_{8,20}(X) - 1.51919\cdot 10^{12}B_{9,20}(X) - 1.98791 \\ & \cdot 10^{12}B_{10,20}(X) - 2.42914\cdot 10^{12}B_{11,20}(X) - 2.84254\cdot 10^{12}B_{12,20}(X) - 3.22791\cdot 10^{12}B_{13,20}(X) \\ & - 3.58517\cdot 10^{12}B_{14,20}(X) - 3.91433\cdot 10^{12}B_{15,20}(X) - 4.21553\cdot 10^{12}B_{16,20}(X) - 4.48902 \\ & \cdot 10^{12}B_{17,20}(X) - 4.73512\cdot 10^{12}B_{18,20}(X) - 4.95425\cdot 10^{12}B_{19,20}(X) - 5.14692\cdot 10^{12}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 5.02827\cdot 10^{12}X^2 - 1.40361\cdot 10^{13}X + 3.84486\cdot 10^{12} \\ &= 3.84486\cdot 10^{12}B_{0,2} - 3.1732\cdot 10^{12}B_{1,2} - 5.16298\cdot 10^{12}B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 = {} & 5.73793{\cdot}10^{14}X^{20} - 5.74566{\cdot}10^{15}X^{19} + 2.66462{\cdot}10^{16}X^{18} - 7.59244{\cdot}10^{16}X^{17} + 1.4867{\cdot}10^{17}X^{16} \\ & - 2.11992{\cdot}10^{17}X^{15} + 2.27559{\cdot}10^{17}X^{14} - 1.87498{\cdot}10^{17}X^{13} + 1.19851{\cdot}10^{17}X^{12} - 5.96868{\cdot}10^{16}X^{11} \\ & + 2.31397{\cdot}10^{16}X^{10} - 6.9461{\cdot}10^{15}X^{9} + 1.59844{\cdot}10^{15}X^{8} - 2.77863{\cdot}10^{14}X^{7} + 3.5797{\cdot}10^{13}X^{6} - 3.34153 \\ & {\cdot}10^{12}X^{5} + 2.19562{\cdot}10^{11}X^{4} - 9.44223{\cdot}10^{9}X^{3} + 5.0285{\cdot}10^{12}X^{2} - 1.40361{\cdot}10^{13}X + 3.84486{\cdot}10^{12} \\ = {} & 3.84486{\cdot}10^{12}B_{0,20} + 3.14305{\cdot}10^{12}B_{1,20} + 2.46771{\cdot}10^{12}B_{2,20} + 1.81883{\cdot}10^{12}B_{3,20} + 1.19644 \\ & {\cdot}10^{12}B_{4,20} + 6.00415{\cdot}10^{11}B_{5,20} + 3.11548{\cdot}10^{10}B_{6,20} - 5.1235{\cdot}10^{11}B_{7,20} - 1.02803{\cdot}10^{12}B_{8,20} \\ & - 1.51938{\cdot}10^{12}B_{9,20} - 1.98152{\cdot}10^{12}B_{10,20} - 2.42008{\cdot}10^{12}B_{11,20} - 2.82959{\cdot}10^{12}B_{12,20} \\ & - 3.21471{\cdot}10^{12}B_{13,20} - 3.57197{\cdot}10^{12}B_{14,20} - 3.90352{\cdot}10^{12}B_{15,20} - 4.20826{\cdot}10^{12}B_{16,20} \\ & - 4.48666{\cdot}10^{12}B_{17,20} - 4.73856{\cdot}10^{12}B_{18,20} - 4.96401{\cdot}10^{12}B_{19,20} - 5.16298{\cdot}10^{12}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.43251{\cdot}10^{10}$.

Bounding polynomials M and m:

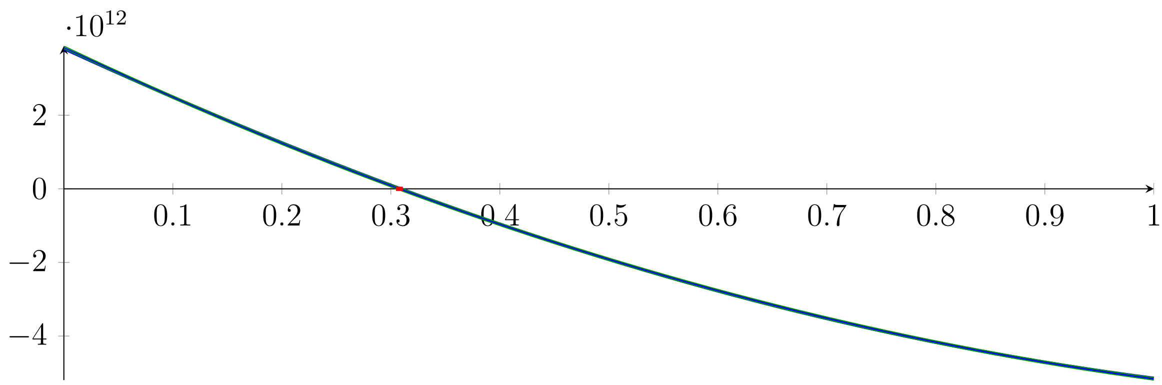
$$M = 5.02827{\cdot}10^{12}X^2 - 1.40361{\cdot}10^{13}X + 3.87918{\cdot}10^{12}$$

$$m = 5.02827{\cdot}10^{12}X^2 - 1.40361{\cdot}10^{13}X + 3.81053{\cdot}10^{12}$$

Root of M and m:

$$N(M) = \{0.311027, 2.48041\} \qquad N(m) = \{0.304751, 2.48669\}$$

Intersection intervals:



$$[0.304751, 0.311027]$$

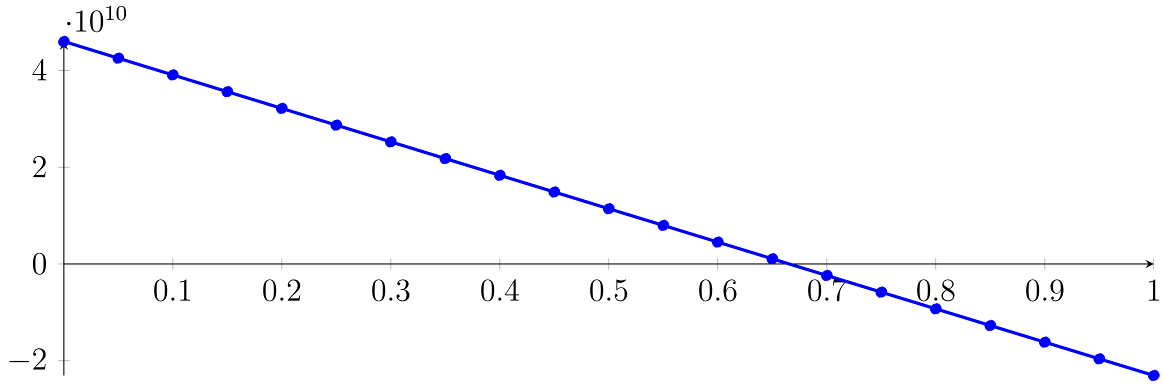
Longest intersection interval: 0.00627527

$\Longrightarrow$ Selective recursion: interval 1: $[4.99854, 5.00073]$,

2.29 Recursion Branch 1 1 1 2 2 1 1 in Interval 1: $[4.99854, 5.00073]$

Normalized monomial und Bézier representations and the Bézier polygon:

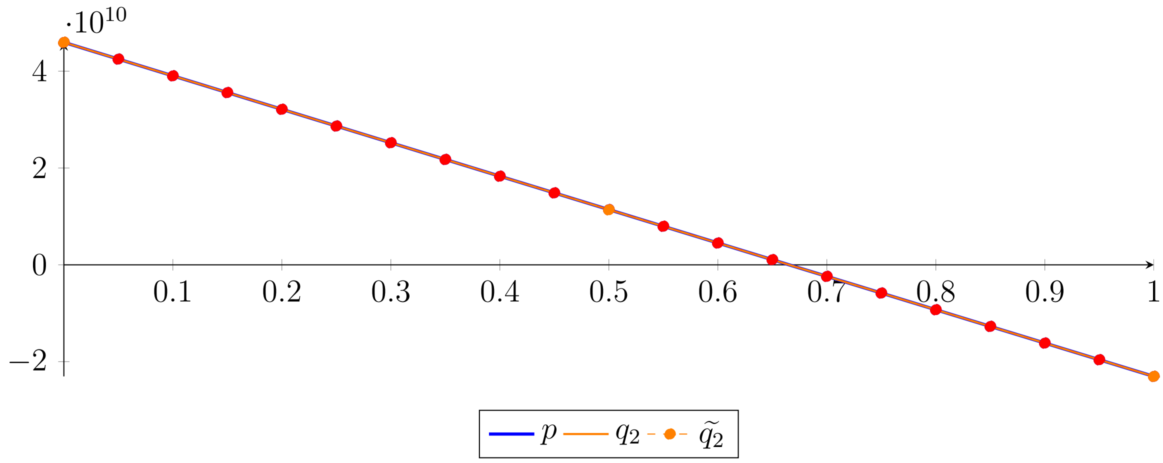
$$p = -14.838X^{20} + 7.59583X^{19} - 612.612X^{18} + 1473.18X^{17} - 10971.2X^{16} + 9529.6X^{15} - 5011.78X^{14} - 2599.93X^{13} - 12444X^{12} - 2373.21X^{11} - 4268.2X^{10} - 732.98X^{9} - 57.6645X^{8} - 7.09717X^{7} - 23.6572X^{6} + 0.473145X^{5} - 1122.68X^{4} + 248920X^{3} + 1.86477{\cdot}10^{8}X^{2} - 6.91814{\cdot}10^{10}X + 4.5954{\cdot}10^{10}$$
$$= 4.5954{\cdot}10^{10}B_{0,20}(X) + 4.24949{\cdot}10^{10}B_{1,20}(X) + 3.90368{\cdot}10^{10}B_{2,20}(X) + 3.55797{\cdot}10^{10}B_{3,20}(X) + 3.21236{\cdot}10^{10}B_{4,20}(X) + 2.86684{\cdot}10^{10}B_{5,20}(X) + 2.52143{\cdot}10^{10}B_{6,20}(X) + 2.17611{\cdot}10^{10}B_{7,20}(X) + 1.83089{\cdot}10^{10}B_{8,20}(X) + 1.48577{\cdot}10^{10}B_{9,20}(X) + 1.14075{\cdot}10^{10}B_{10,20}(X) + 7.95822{\cdot}10^{9}B_{11,20}(X) + 4.50995{\cdot}10^{9}B_{12,20}(X) + 1.06267{\cdot}10^{9}B_{13,20}(X) - 2.38362{\cdot}10^{9}B_{14,20}(X) - 5.82893{\cdot}10^{9}B_{15,20}(X) - 9.27326{\cdot}10^{9}B_{16,20}(X) - 1.27166{\cdot}10^{10}B_{17,20}(X) - 1.6159{\cdot}10^{10}B_{18,20}(X) - 1.96003{\cdot}10^{10}B_{19,20}(X) - 2.30407{\cdot}10^{10}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 1.86848{\cdot}10^{8}X^{2} - 6.91816{\cdot}10^{10}X + 4.5954{\cdot}10^{10}$$
$$= 4.5954{\cdot}10^{10}B_{0,2} + 1.13632{\cdot}10^{10}B_{1,2} - 2.30407{\cdot}10^{10}B_{2,2}$$

$$\widetilde{q}_2 = -4.70213{\cdot}10^{12}X^{20} + 4.71603{\cdot}10^{13}X^{19} - 2.18938{\cdot}10^{14}X^{18} + 6.24255{\cdot}10^{14}X^{17} - 1.22325{\cdot}10^{15}X^{16} + 1.74662{\cdot}10^{15}X^{15} - 1.88012{\cdot}10^{15}X^{14} + 1.55718{\cdot}10^{15}X^{13} - 1.00385{\cdot}10^{15}X^{12} + 5.06176{\cdot}10^{14}X^{11} - 1.99475{\cdot}10^{14}X^{10} + 6.10594{\cdot}10^{13}X^{9} - 1.43541{\cdot}10^{13}X^{8} + 2.55065{\cdot}10^{12}X^{7} - 3.35784{\cdot}10^{11}X^{6} + 3.19073{\cdot}10^{10}X^{5} - 2.10628{\cdot}10^{9}X^{4} + 8.96367{\cdot}10^{7}X^{3} + 1.84686{\cdot}10^{8}X^{2} - 6.91815{\cdot}10^{10}X + 4.5954{\cdot}10^{10}$$
$$= 4.5954{\cdot}10^{10}B_{0,20} + 4.24949{\cdot}10^{10}B_{1,20} + 3.90368{\cdot}10^{10}B_{2,20} + 3.55798{\cdot}10^{10}B_{3,20} + 3.21234{\cdot}10^{10}B_{4,20} + 2.8669{\cdot}10^{10}B_{5,20} + 2.52128{\cdot}10^{10}B_{6,20} + 2.17639{\cdot}10^{10}B_{7,20} + 1.83045{\cdot}10^{10}B_{8,20} + 1.48632{\cdot}10^{10}B_{9,20} + 1.14008{\cdot}10^{10}B_{10,20} + 7.96375{\cdot}10^{9}B_{11,20} + 4.50575{\cdot}10^{9}B_{12,20} + 1.06514{\cdot}10^{9}B_{13,20} - 2.38493{\cdot}10^{9}B_{14,20} - 5.82839{\cdot}10^{9}B_{15,20} - 9.27341{\cdot}10^{9}B_{16,20} - 1.27166{\cdot}10^{10}B_{17,20} - 1.6159{\cdot}10^{10}B_{18,20} - 1.96003{\cdot}10^{10}B_{19,20} - 2.30407{\cdot}10^{10}B_{20,20}$$



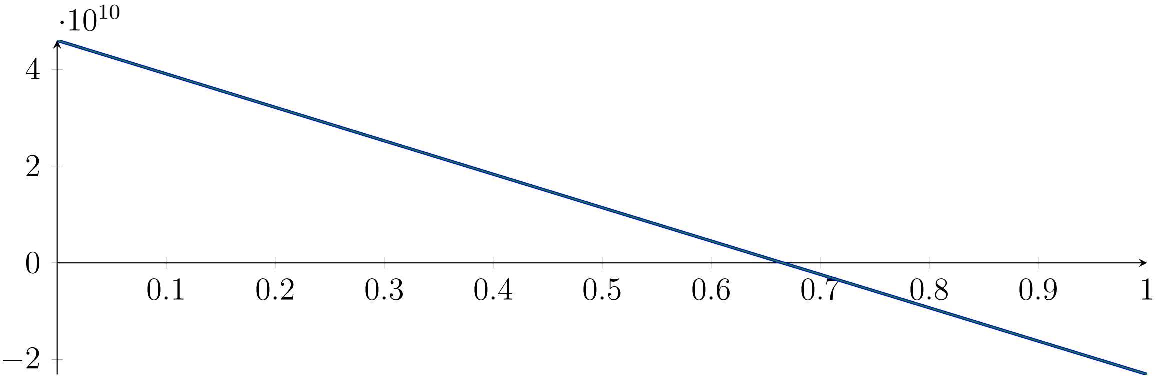
The maximum difference of the Bézier coefficients is $\delta = 6.64783 \cdot 10^6$.
Bounding polynomials M and m:

$$M = 1.86848 \cdot 10^8 X^2 - 6.91816 \cdot 10^{10} X + 4.59606 \cdot 10^{10}$$
$$m = 1.86848 \cdot 10^8 X^2 - 6.91816 \cdot 10^{10} X + 4.59473 \cdot 10^{10}$$

Root of M and m:

$$N(M) = \{0.665544, 369.59\} \qquad N(m) = \{0.665352, 369.59\}$$

Intersection intervals:



$$[0.665352, 0.665544]$$

Longest intersection interval: 0.000192878
$\Longrightarrow$ Selective recursion: interval 1: $[5, 5]$,

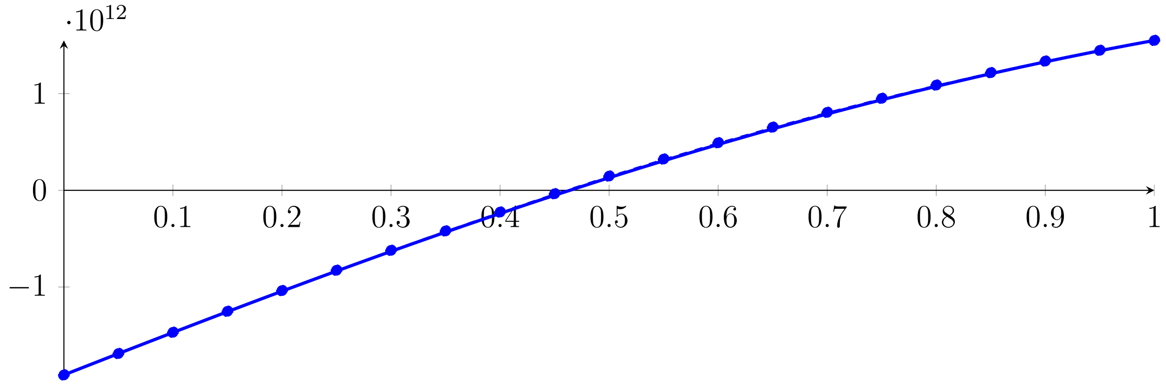
2.30 Recursion Branch 1 1 1 2 2 1 1 1 in Interval 1: $[5, 5]$

Found root in interval $[5, 5]$ at recursion depth 8!

2.31 Recursion Branch 1 1 1 2 2 2 in Interval 2: $[5.8376, 6.18761]$

Normalized monomial und Bézier representations and the Bézier polygon:

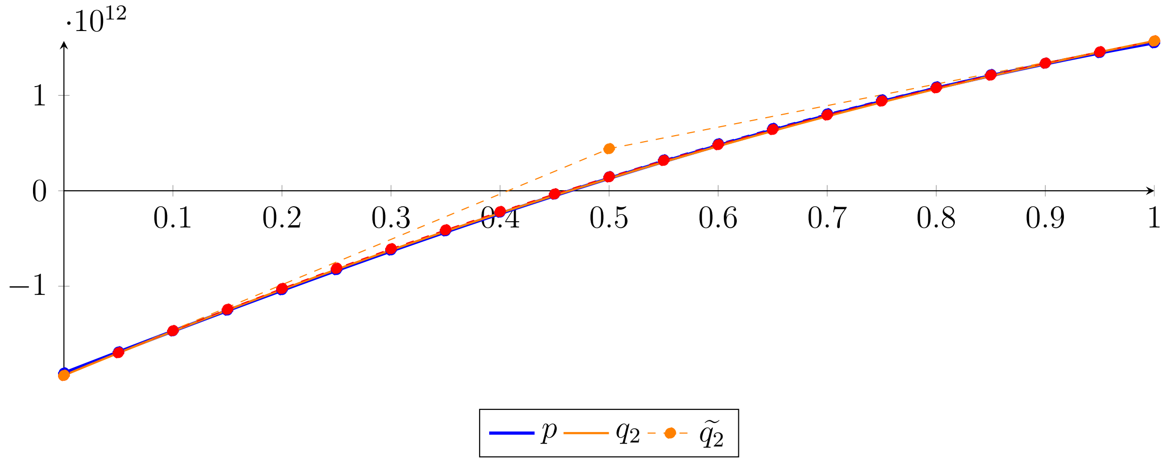
$$\begin{aligned} p = {} & 43.0291X^{20} + 3258.25X^{19} + 10385.8X^{18} + 18710.1X^{17} + 109115X^{16} - 125573X^{15} \\ & + 128005X^{14} + 65880.6X^{13} + 307759X^{12} + 180795X^{11} + 559636X^{10} - 1.70233 \cdot 10^7 X^9 \\ & + 1.06479 \cdot 10^8 X^8 + 5.70684 \cdot 10^8 X^7 - 9.41772 \cdot 10^9 X^6 + 2.00085 \cdot 10^{10} X^5 + 1.88253 \\ & \cdot 10^{11} X^4 - 8.46185 \cdot 10^{11} X^3 - 3.19904 \cdot 10^{11} X^2 + 4.42636 \cdot 10^{12} X - 1.90945 \cdot 10^{12} \\ = {} & -1.90945 \cdot 10^{12} B_{0,20}(X) - 1.68814 \cdot 10^{12} B_{1,20}(X) - 1.4685 \cdot 10^{12} B_{2,20}(X) - 1.25129 \\ & \cdot 10^{12} B_{3,20}(X) - 1.03721 \cdot 10^{12} B_{4,20}(X) - 8.26928 \cdot 10^{11} B_{5,20}(X) - 6.21057 \cdot 10^{11} B_{6,20}(X) \\ & - 4.2018 \cdot 10^{11} B_{7,20}(X) - 2.24836 \cdot 10^{11} B_{8,20}(X) - 3.55182 \cdot 10^{10} B_{9,20}(X) + 1.47321 \\ & \cdot 10^{11} B_{10,20}(X) + 3.23274 \cdot 10^{11} B_{11,20}(X) + 4.91975 \cdot 10^{11} B_{12,20}(X) + 6.53101 \cdot 10^{11} B_{13,20}(X) \\ & + 8.0637 \cdot 10^{11} B_{14,20}(X) + 9.51543 \cdot 10^{11} B_{15,20}(X) + 1.08842 \cdot 10^{12} B_{16,20}(X) + 1.21684 \\ & \cdot 10^{12} B_{17,20}(X) + 1.33668 \cdot 10^{12} B_{18,20}(X) + 1.44786 \cdot 10^{12} B_{19,20}(X) + 1.55032 \cdot 10^{12} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = -1.2464 \cdot 10^{12} X^2 + 4.75051 \cdot 10^{12} X - 1.93452 \cdot 10^{12}$$
$$= -1.93452 \cdot 10^{12} B_{0,2} + 4.40731 \cdot 10^{11} B_{1,2} + 1.56959 \cdot 10^{12} B_{2,2}$$

$$\widetilde{q}_2 = -3.05 \cdot 10^{13} X^{20} + 3.0319 \cdot 10^{14} X^{19} - 1.39794 \cdot 10^{15} X^{18} + 3.96358 \cdot 10^{15} X^{17} - 7.72002 \cdot 10^{15} X^{16} + 1.09241 \cdot 10^{16} X^{15} - 1.15778 \cdot 10^{16} X^{14} + 9.33845 \cdot 10^{15} X^{13} - 5.77059 \cdot 10^{15} X^{12} + 2.73284 \cdot 10^{15} X^{11} - 9.88128 \cdot 10^{14} X^{10} + 2.71044 \cdot 10^{14} X^9 - 5.58822 \cdot 10^{13} X^8 + 8.52169 \cdot 10^{12} X^7 - 9.35187 \cdot 10^{11} X^6 + 7.23623 \cdot 10^{10} X^5 - 4.10245 \cdot 10^9 X^4 + 1.65707 \cdot 10^8 X^3 - 1.2464 \cdot 10^{12} X^2 + 4.75051 \cdot 10^{12} X - 1.93452 \cdot 10^{12}$$
$$= -1.93452 \cdot 10^{12} B_{0,20} - 1.697 \cdot 10^{12} B_{1,20} - 1.46603 \cdot 10^{12} B_{2,20} - 1.24163 \cdot 10^{12} B_{3,20} - 1.02378 \cdot 10^{12} B_{4,20} - 8.12493 \cdot 10^{11} B_{5,20} - 6.07776 \cdot 10^{11} B_{6,20} - 4.0959 \cdot 10^{11} B_{7,20} - 2.18029 \cdot 10^{11} B_{8,20} - 3.29083 \cdot 10^{10} B_{9,20} + 1.45498 \cdot 10^{11} B_{10,20} + 3.17486 \cdot 10^{11} B_{11,20} + 4.82788 \cdot 10^{11} B_{12,20} + 6.41652 \cdot 10^{11} B_{13,20} + 7.93863 \cdot 10^{11} B_{14,20} + 9.39565 \cdot 10^{11} B_{15,20} + 1.07868 \cdot 10^{12} B_{16,20} + 1.21125 \cdot 10^{12} B_{17,20} + 1.33726 \cdot 10^{12} B_{18,20} + 1.4567 \cdot 10^{12} B_{19,20} + 1.56959 \cdot 10^{12} B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 2.50678 \cdot 10^{10}$.

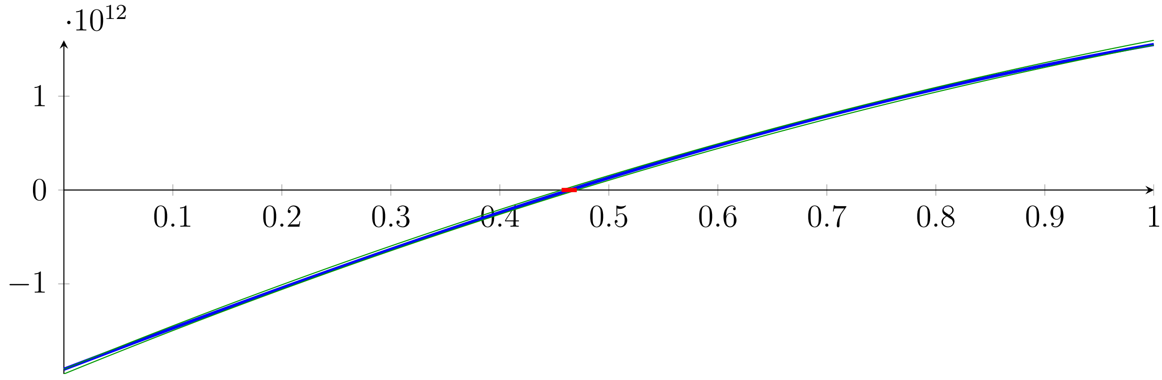
Bounding polynomials M and m:

$$M = -1.2464 \cdot 10^{12} X^2 + 4.75051 \cdot 10^{12} X - 1.90945 \cdot 10^{12}$$
$$m = -1.2464 \cdot 10^{12} X^2 + 4.75051 \cdot 10^{12} X - 1.95959 \cdot 10^{12}$$

Root of M and m:

$$N(M) = \{0.456662, 3.35473\} \qquad N(m) = \{0.470609, 3.34078\}$$

Intersection intervals:



$[0.456662, 0.470609]$

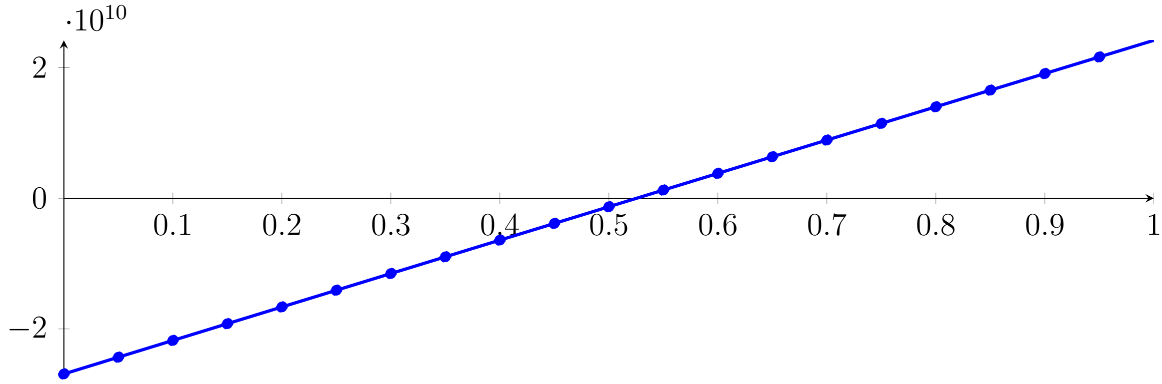
Longest intersection interval: 0.0139469

$\Longrightarrow$ Selective recursion: interval 1: $[5.99743, 6.00231]$,

2.32 Recursion Branch 1 1 1 2 2 2 1 in Interval 1: $[5.99743, 6.00231]$

Normalized monomial und Bézier representations and the Bézier polygon:

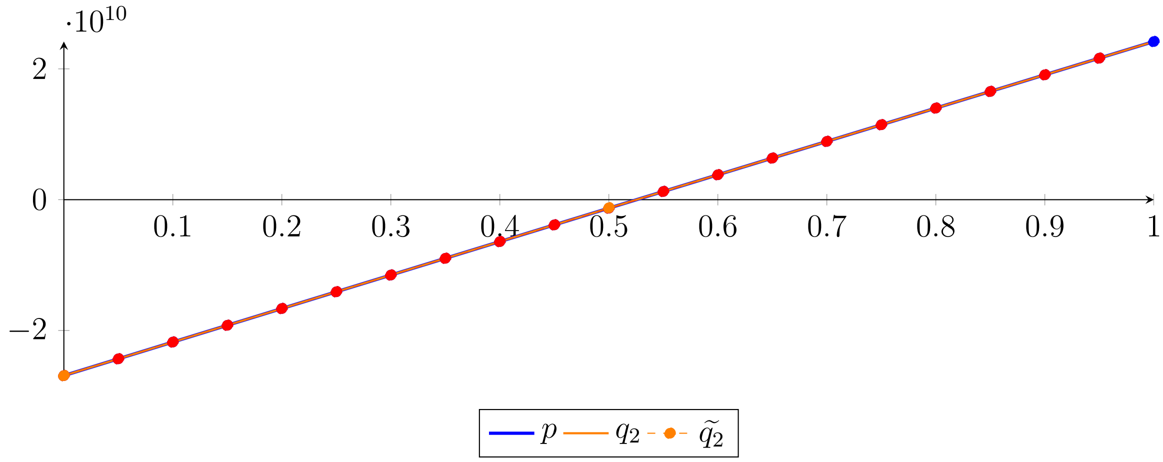
$$\begin{aligned} p &= 3.89121X^{20} + 30.5425X^{19} + 233.854X^{18} - 131.754X^{17} + 3366X^{16} - 3121.39X^{15} \\ &\quad + 1898.64X^{14} + 1596.57X^{13} + 5624.69X^{12} + 1721.61X^{11} + 1974.11X^{10} \\ &\quad + 424.795X^{9} - 22.5853X^{8} + 9.16718X^{7} + 13.1593X^{6} - 1.47858X^{5} \\ &\quad + 7819.55X^{4} - 1.29554\cdot 10^{6}X^{3} - 2.3934\cdot 10^{8}X^{2} + 5.13213\cdot 10^{10}X - 2.68979\cdot 10^{10} \\ &= -2.68979\cdot 10^{10}B_{0,20}(X) - 2.43318\cdot 10^{10}B_{1,20}(X) - 2.1767\cdot 10^{10}B_{2,20}(X) - 1.92035 \\ &\quad \cdot 10^{10}B_{3,20}(X) - 1.66412\cdot 10^{10}B_{4,20}(X) - 1.40802\cdot 10^{10}B_{5,20}(X) - 1.15204\cdot 10^{10}B_{6,20}(X) \\ &\quad - 8.96192\cdot 10^{9}B_{7,20}(X) - 6.4047\cdot 10^{9}B_{8,20}(X) - 3.84874\cdot 10^{9}B_{9,20}(X) - 1.29405 \\ &\quad \cdot 10^{9}B_{10,20}(X) + 1.25937\cdot 10^{9}B_{11,20}(X) + 3.81151\cdot 10^{9}B_{12,20}(X) + 6.36239\cdot 10^{9}B_{13,20}(X) \\ &\quad + 8.91199\cdot 10^{9}B_{14,20}(X) + 1.14603\cdot 10^{10}B_{15,20}(X) + 1.40074\cdot 10^{10}B_{16,20}(X) + 1.65531 \\ &\quad \cdot 10^{10}B_{17,20}(X) + 1.90976\cdot 10^{10}B_{18,20}(X) + 2.16409\cdot 10^{10}B_{19,20}(X) + 2.41828\cdot 10^{10}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -2.4127\cdot 10^{8}X^{2} + 5.13221\cdot 10^{10}X - 2.6898\cdot 10^{10} \\ &= -2.6898\cdot 10^{10}B_{0,2} - 1.23691\cdot 10^{9}B_{1,2} + 2.41829\cdot 10^{10}B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 = {} & 2.16711\cdot 10^{12}X^{20} - 2.17124\cdot 10^{13}X^{19} + 1.00651\cdot 10^{14}X^{18} - 2.86478\cdot 10^{14}X^{17} + 5.60281\cdot 10^{14}X^{16} \\ & - 7.98512\cdot 10^{14}X^{15} + 8.58288\cdot 10^{14}X^{14} - 7.10354\cdot 10^{14}X^{13} + 4.58116\cdot 10^{14}X^{12} - 2.31413\cdot 10^{14}X^{11} \\ & + 9.14987\cdot 10^{13}X^{10} - 2.814\cdot 10^{13}X^{9} + 6.65255\cdot 10^{12}X^{8} - 1.18846\cdot 10^{12}X^{7} + 1.56835\cdot 10^{11}X^{6} - 1.48157 \\ & \cdot 10^{10}X^{5} + 9.55607\cdot 10^{8}X^{4} - 3.89023\cdot 10^{7}X^{3} - 2.40385\cdot 10^{8}X^{2} + 5.13221\cdot 10^{10}X - 2.6898\cdot 10^{10} \\ = {} & -2.6898\cdot 10^{10}B_{0,20} - 2.43318\cdot 10^{10}B_{1,20} - 2.1767\cdot 10^{10}B_{2,20} - 1.92035\cdot 10^{10}B_{3,20} - 1.66411 \\ & \cdot 10^{10}B_{4,20} - 1.40804\cdot 10^{10}B_{5,20} - 1.15197\cdot 10^{10}B_{6,20} - 8.96316\cdot 10^{9}B_{7,20} - 6.40271 \\ & \cdot 10^{9}B_{8,20} - 3.85126\cdot 10^{9}B_{9,20} - 1.29101\cdot 10^{9}B_{10,20} + 1.25666\cdot 10^{9}B_{11,20} + 3.81338\cdot 10^{9}B_{12,20} \\ & + 6.36123\cdot 10^{9}B_{13,20} + 8.91253\cdot 10^{9}B_{14,20} + 1.14601\cdot 10^{10}B_{15,20} + 1.40074\cdot 10^{10}B_{16,20} \\ & + 1.65531\cdot 10^{10}B_{17,20} + 1.90976\cdot 10^{10}B_{18,20} + 2.16409\cdot 10^{10}B_{19,20} + 2.41829\cdot 10^{10}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.03925\cdot 10^{6}$.

Bounding polynomials M and m:

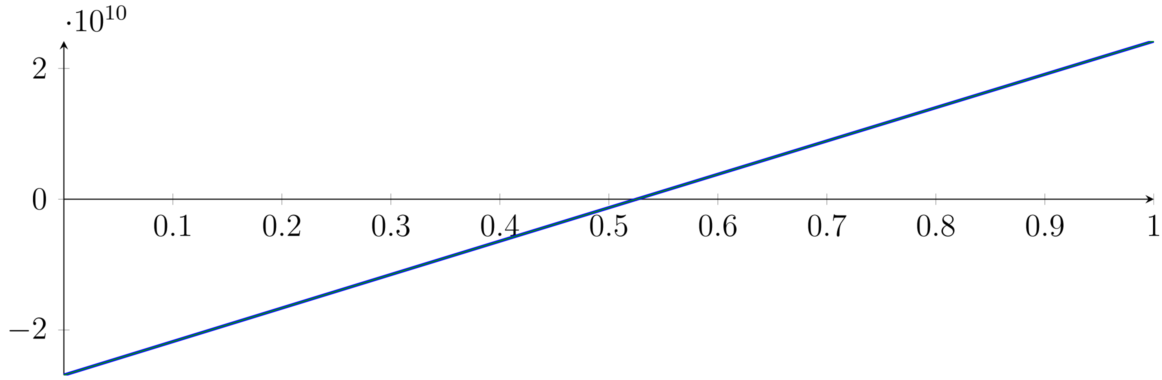
$$M = -2.4127\cdot 10^{8}X^2 + 5.13221\cdot 10^{10}X - 2.68949\cdot 10^{10}$$

$$m = -2.4127\cdot 10^{8}X^2 + 5.13221\cdot 10^{10}X - 2.6901\cdot 10^{10}$$

Root of M and m:

$$N(M) = \{0.525339, 212.191\} \qquad N(m) = \{0.525458, 212.191\}$$

Intersection intervals:



$$[0.525339, 0.525458]$$

Longest intersection interval: 0.000119026

$\Longrightarrow$ Selective recursion: interval 1: $[6, 6]$,

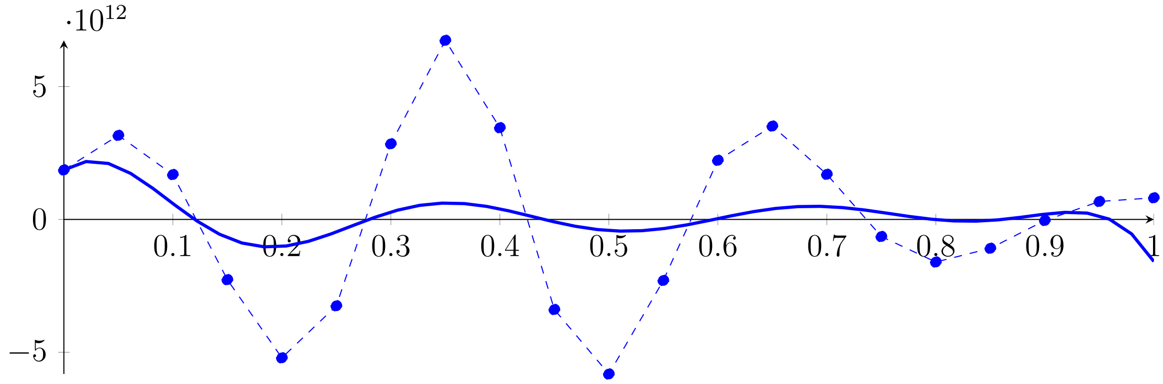
2.33 Recursion Branch 1 1 1 2 2 2 1 1 in Interval 1: $[6, 6]$

Found root in interval $[6, 6]$ at recursion depth 8!

2.34 Recursion Branch 1 1 2 on the Second Half $[6.25, 12.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

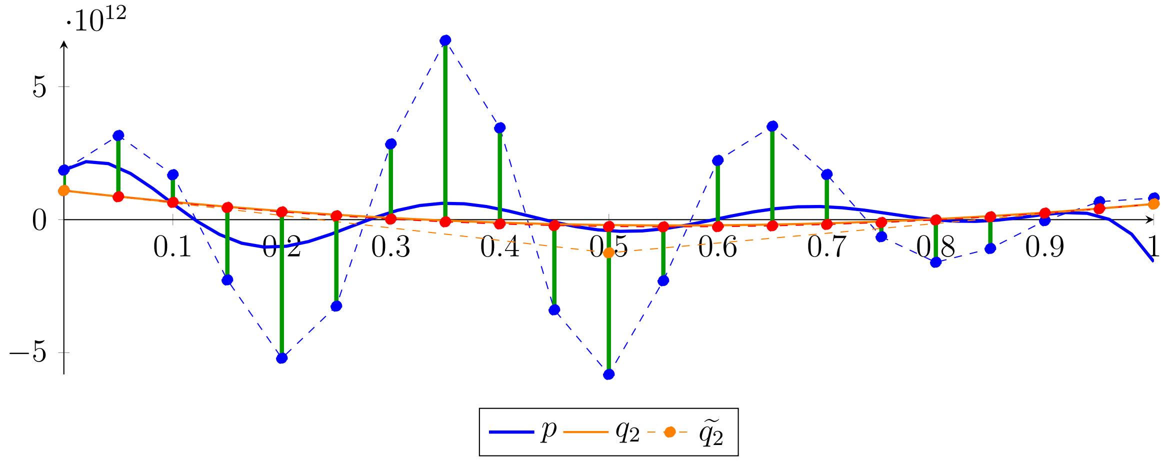
$$\begin{aligned}p &= 8.27181 \cdot 10^{15} X^{20} - 1.12497 \cdot 10^{17} X^{19} + 6.56318 \cdot 10^{17} X^{18} - 2.10324 \cdot 10^{18} X^{17} + 3.83361 \cdot 10^{18} X^{16} \\ &\quad - 3.25611 \cdot 10^{18} X^{15} - 1.18134 \cdot 10^{18} X^{14} + 5.65844 \cdot 10^{18} X^{13} - 4.66119 \cdot 10^{18} X^{12} - 3.70393 \cdot 10^{17} X^{11} \\ &\quad + 2.95436 \cdot 10^{18} X^{10} - 1.48062 \cdot 10^{18} X^{9} - 3.2208 \cdot 10^{17} X^{8} + 4.91145 \cdot 10^{17} X^{7} - 8.64752 \cdot 10^{16} X^{6} - 4.35417 \\ &\quad \cdot 10^{16} X^{5} + 1.55034 \cdot 10^{16} X^{4} + 3.36768 \cdot 10^{14} X^{3} - 5.27545 \cdot 10^{14} X^{2} + 2.60227 \cdot 10^{13} X + 1.86285 \cdot 10^{12} \\ &= 1.86285 \cdot 10^{12} B_{0,20}(X) + 3.16399 \cdot 10^{12} B_{1,20}(X) + 1.68857 \cdot 10^{12} B_{2,20}(X) - 2.268 \\ &\quad \cdot 10^{12} B_{3,20}(X) - 5.21041 \cdot 10^{12} B_{4,20}(X) - 3.25192 \cdot 10^{12} B_{5,20}(X) + 2.84625 \cdot 10^{12} B_{6,20}(X) \\ &\quad + 6.74009 \cdot 10^{12} B_{7,20}(X) + 3.45161 \cdot 10^{12} B_{8,20}(X) - 3.39194 \cdot 10^{12} B_{9,20}(X) - 5.81848 \\ &\quad \cdot 10^{12} B_{10,20}(X) - 2.29738 \cdot 10^{12} B_{11,20}(X) + 2.22447 \cdot 10^{12} B_{12,20}(X) + 3.51385 \cdot 10^{12} B_{13,20}(X) \\ &\quad + 1.69765 \cdot 10^{12} B_{14,20}(X) - 6.43381 \cdot 10^{11} B_{15,20}(X) - 1.60376 \cdot 10^{12} B_{16,20}(X) - 1.08654 \\ &\quad \cdot 10^{12} B_{17,20}(X) - 4.06339 \cdot 10^{10} B_{18,20}(X) + 6.75764 \cdot 10^{11} B_{19,20}(X) + 8.1419 \cdot 10^{11} B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 4.17863 \cdot 10^{12} X^{2} - 4.68238 \cdot 10^{12} X + 1.09435 \cdot 10^{12} \\ &= 1.09435 \cdot 10^{12} B_{0,2} - 1.24684 \cdot 10^{12} B_{1,2} + 5.90605 \cdot 10^{11} B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 4.91254 \cdot 10^{14} X^{20} - 4.91108 \cdot 10^{15} X^{19} + 2.27231 \cdot 10^{16} X^{18} - 6.45628 \cdot 10^{16} X^{17} + 1.26032 \cdot 10^{17} X^{16} \\ &\quad - 1.79178 \cdot 10^{17} X^{15} + 1.91882 \cdot 10^{17} X^{14} - 1.57919 \cdot 10^{17} X^{13} + 1.01009 \cdot 10^{17} X^{12} - 5.04536 \cdot 10^{16} X^{11} \\ &\quad + 1.96707 \cdot 10^{16} X^{10} - 5.95361 \cdot 10^{15} X^{9} + 1.38402 \cdot 10^{15} X^{8} - 2.43026 \cdot 10^{14} X^{7} + 3.14709 \cdot 10^{13} X^{6} - 2.90839 \\ &\quad \cdot 10^{12} X^{5} + 1.83035 \cdot 10^{11} X^{4} - 7.23447 \cdot 10^{9} X^{3} + 4.17879 \cdot 10^{12} X^{2} - 4.68238 \cdot 10^{12} X + 1.09435 \cdot 10^{12} \\ &= 1.09435 \cdot 10^{12} B_{0,20} + 8.60233 \cdot 10^{11} B_{1,20} + 6.48108 \cdot 10^{11} B_{2,20} + 4.5797 \cdot 10^{11} B_{3,20} + 2.89851 \\ &\quad \cdot 10^{11} B_{4,20} + 1.43632 \cdot 10^{11} B_{5,20} + 1.96691 \cdot 10^{10} B_{6,20} - 8.29049 \cdot 10^{10} B_{7,20} - 1.62353 \cdot 10^{11} B_{8,20} \\ &\quad - 2.21579 \cdot 10^{11} B_{9,20} - 2.56505 \cdot 10^{11} B_{10,20} - 2.71948 \cdot 10^{11} B_{11,20} - 2.63097 \cdot 10^{11} B_{12,20} \\ &\quad - 2.34033 \cdot 10^{11} B_{13,20} - 1.81829 \cdot 10^{11} B_{14,20} - 1.08245 \cdot 10^{11} B_{15,20} - 1.23971 \cdot 10^{10} B_{16,20} \\ &\quad + 1.05347 \cdot 10^{11} B_{17,20} + 2.4511 \cdot 10^{11} B_{18,20} + 4.06861 \cdot 10^{11} B_{19,20} + 5.90605 \cdot 10^{11} B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 6.823{\cdot}10^{12}$.

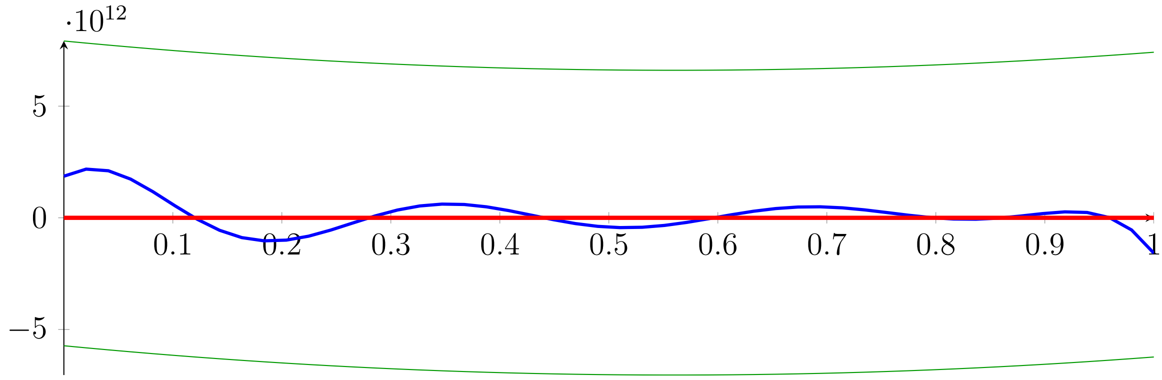
Bounding polynomials M and m:

$$M = 4.17863{\cdot}10^{12}X^2 - 4.68238{\cdot}10^{12}X + 7.91735{\cdot}10^{12}$$
$$m = 4.17863{\cdot}10^{12}X^2 - 4.68238{\cdot}10^{12}X - 5.72865{\cdot}10^{12}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{-0.737741, 1.85829\}$$

Intersection intervals:



$[0, 1]$

Longest intersection interval: 1

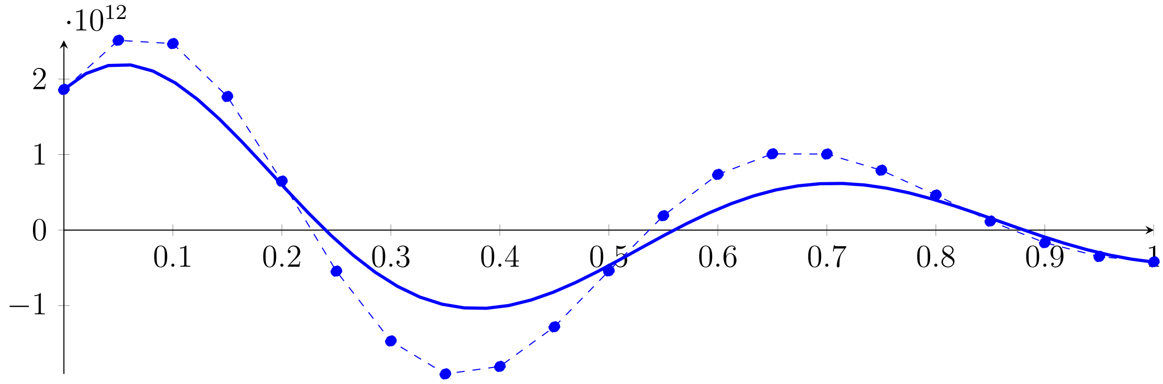
$\Longrightarrow$ Bisection: first half $[6.25, 9.375]$ und second half $[9.375, 12.5]$

Bisection point is very near to a root?!?

2.35 Recursion Branch 1 1 2 1 on the First Half $[6.25, 9.375]$

Normalized monomial und Bézier representations and the Bézier polygon:

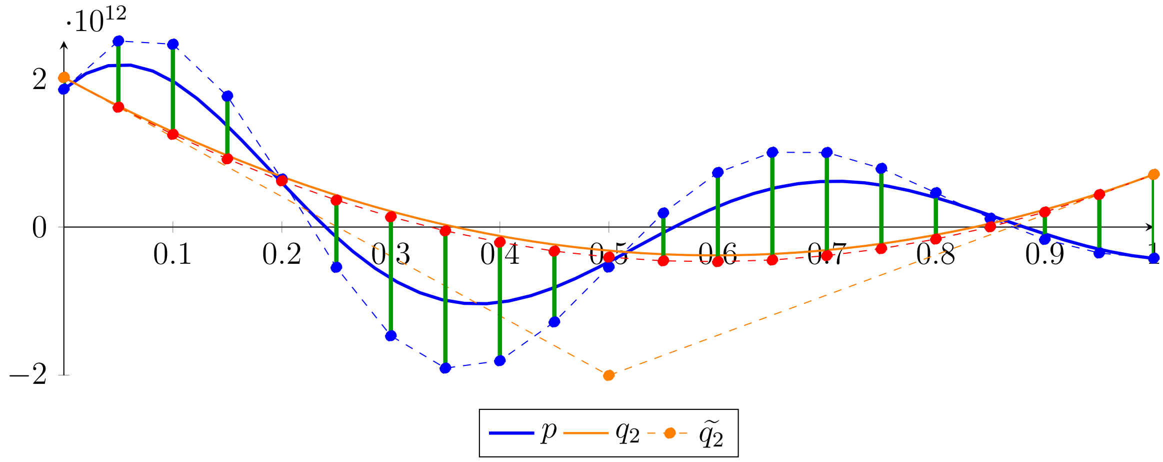
$$\begin{aligned} p = {} & 7.88861{\cdot}10^{9}X^{20} - 2.1457{\cdot}10^{11}X^{19} + 2.50366{\cdot}10^{12}X^{18} - 1.60464{\cdot}10^{13}X^{17} + 5.84963{\cdot}10^{13}X^{16} - 9.93687 \\ & {\cdot}10^{13}X^{15} - 7.21032{\cdot}10^{13}X^{14} + 6.90728{\cdot}10^{14}X^{13} - 1.13799{\cdot}10^{15}X^{12} - 1.80856{\cdot}10^{14}X^{11} + 2.88511 \\ & {\cdot}10^{15}X^{10} - 2.89183{\cdot}10^{15}X^{9} - 1.25813{\cdot}10^{15}X^{8} + 3.83707{\cdot}10^{15}X^{7} - 1.35117{\cdot}10^{15}X^{6} - 1.36068 \\ & {\cdot}10^{15}X^{5} + 9.68965{\cdot}10^{14}X^{4} + 4.2096{\cdot}10^{13}X^{3} - 1.31886{\cdot}10^{14}X^{2} + 1.30114{\cdot}10^{13}X + 1.86285{\cdot}10^{12} \\ = {} & 1.86285{\cdot}10^{12}B_{0,20}(X) + 2.51342{\cdot}10^{12}B_{1,20}(X) + 2.46985{\cdot}10^{12}B_{2,20}(X) + 1.76906 \\ & {\cdot}10^{12}B_{3,20}(X) + 6.47986{\cdot}10^{11}B_{4,20}(X) - 5.44235{\cdot}10^{11}B_{5,20}(X) - 1.46885{\cdot}10^{12}B_{6,20}(X) \\ & - 1.90547{\cdot}10^{12}B_{7,20}(X) - 1.80595{\cdot}10^{12}B_{8,20}(X) - 1.28171{\cdot}10^{12}B_{9,20}(X) - 5.41242 \\ & {\cdot}10^{11}B_{10,20}(X) + 1.90115{\cdot}10^{11}B_{11,20}(X) + 7.36986{\cdot}10^{11}B_{12,20}(X) + 1.00973{\cdot}10^{12}B_{13,20}(X) \\ & + 1.00677{\cdot}10^{12}B_{14,20}(X) + 7.92436{\cdot}10^{11}B_{15,20}(X) + 4.63782{\cdot}10^{11}B_{16,20}(X) + 1.1866 \\ & {\cdot}10^{11}B_{17,20}(X) - 1.67068{\cdot}10^{11}B_{18,20}(X) - 3.50344{\cdot}10^{11}B_{19,20}(X) - 4.20945{\cdot}10^{11}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = 6.73916\cdot10^{12}X^2 - 8.05023\cdot10^{12}X + 2.02139\cdot10^{12}$$
$$= 2.02139\cdot10^{12}B_{0,2} - 2.00373\cdot10^{12}B_{1,2} + 7.10318\cdot10^{11}B_{2,2}$$

$$\widetilde{q}_2 = 7.71864\cdot10^{14}X^{20} - 7.71594\cdot10^{15}X^{19} + 3.56993\cdot10^{16}X^{18} - 1.01428\cdot10^{17}X^{17} + 1.97988\cdot10^{17}X^{16} - 2.81459\cdot10^{17}X^{15} + 3.01388\cdot10^{17}X^{14} - 2.48007\cdot10^{17}X^{13} + 1.58596\cdot10^{17}X^{12} - 7.91936\cdot10^{16}X^{11} + 3.08635\cdot10^{16}X^{10} - 9.33696\cdot10^{15}X^{9} + 2.16946\cdot10^{15}X^{8} - 3.80749\cdot10^{14}X^{7} + 4.92776\cdot10^{13}X^{6} - 4.55112\cdot10^{12}X^{5} + 2.8623\cdot10^{11}X^{4} - 1.1305\cdot10^{10}X^{3} + 6.73941\cdot10^{12}X^{2} - 8.05023\cdot10^{12}X + 2.02139\cdot10^{12}$$
$$= 2.02139\cdot10^{12}B_{0,20} + 1.61887\cdot10^{12}B_{1,20} + 1.25183\cdot10^{12}B_{2,20} + 9.20253\cdot10^{11}B_{3,20} + 6.24183\cdot10^{11}B_{4,20} + 3.63437\cdot10^{11}B_{5,20} + 1.38573\cdot10^{11}B_{6,20} - 5.17686\cdot10^{10}B_{7,20} - 2.04861\cdot10^{11}B_{8,20} - 3.25269\cdot10^{11}B_{9,20} - 4.06581\cdot10^{11}B_{10,20} - 4.56365\cdot10^{11}B_{11,20} - 4.67066\cdot10^{11}B_{12,20} - 4.45096\cdot10^{11}B_{13,20} - 3.85852\cdot10^{11}B_{14,20} - 2.92101\cdot10^{11}B_{15,20} - 1.62457\cdot10^{11}B_{16,20} + 2.50589\cdot10^{9}B_{17,20} + 2.0298\cdot10^{11}B_{18,20} + 4.38914\cdot10^{11}B_{19,20} + 7.10318\cdot10^{11}B_{20,20}$$



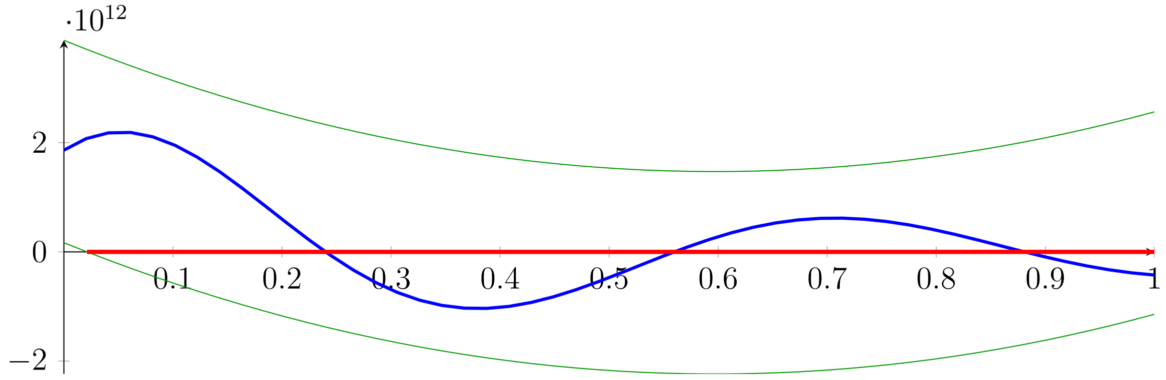
The maximum difference of the Bézier coefficients is $\delta = 1.8537\cdot10^{12}$.
Bounding polynomials M and m:

$$M = 6.73916\cdot10^{12}X^2 - 8.05023\cdot10^{12}X + 3.87509\cdot10^{12}$$
$$m = 6.73916\cdot10^{12}X^2 - 8.05023\cdot10^{12}X + 1.67684\cdot10^{11}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.0212062, 1.17334\}$$

Intersection intervals:



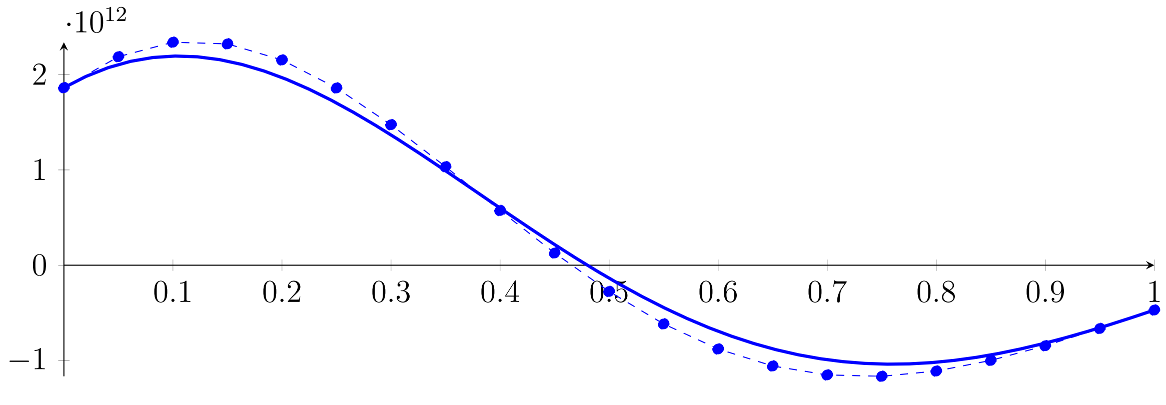
[0.0212062, 1]

Longest intersection interval: 0.978794
$\Longrightarrow$ Bisection: first half [6.25, 7.8125] und second half [7.8125, 9.375]
Bisection point is very near to a root?!?

2.36 Recursion Branch 1 1 2 1 1 on the First Half [6.25, 7.8125]

Normalized monomial und Bézier representations and the Bézier polygon:

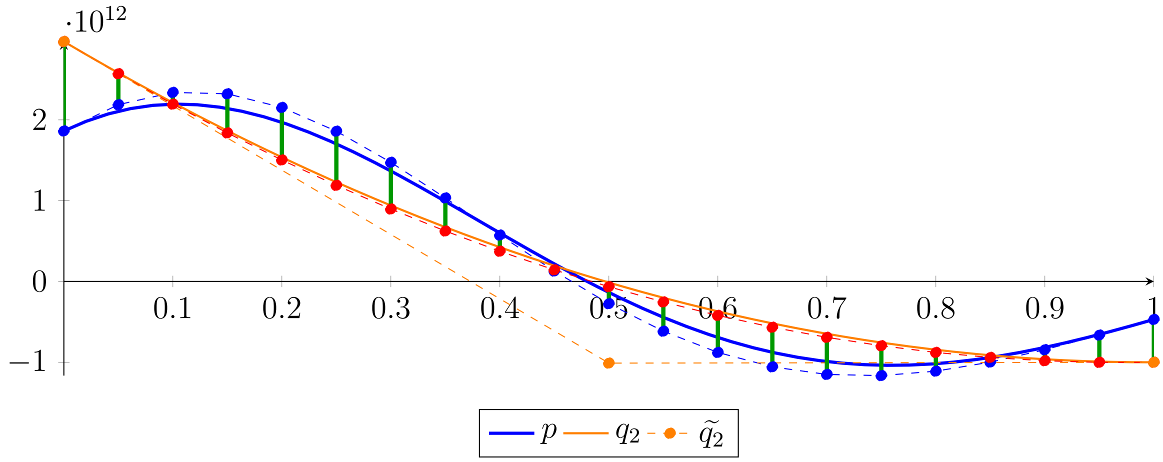
$$p = 7302.07X^{20} - 415766X^{19} + 9.52338\cdot10^{6}X^{18} - 1.22457\cdot10^{8}X^{17} + 8.92307\cdot10^{8}X^{16} - 3.03216\cdot10^{9}X^{15} - 4.40106\cdot10^{9}X^{14} + 8.43171\cdot10^{10}X^{13} - 2.77829\cdot10^{11}X^{12} - 8.83088\cdot10^{10}X^{11} + 2.81749\cdot10^{12}X^{10} - 5.64811\cdot10^{12}X^{9} - 4.91456\cdot10^{12}X^{8} + 2.99771\cdot10^{13}X^{7} - 2.11121\cdot10^{13}X^{6} - 4.25212\cdot10^{13}X^{5} + 6.05603\cdot10^{13}X^{4} + 5.262\cdot10^{12}X^{3} - 3.29716\cdot10^{13}X^{2} + 6.50568\cdot10^{12}X + 1.86285\cdot10^{12}$$
$$= 1.86285\cdot10^{12}B_{0,20}(X) + 2.18813\cdot10^{12}B_{1,20}(X) + 2.33988\cdot10^{12}B_{2,20}(X) + 2.32271\cdot10^{12}B_{3,20}(X) + 2.15374\cdot10^{12}B_{4,20}(X) + 1.85984\cdot10^{12}B_{5,20}(X) + 1.47434\cdot10^{12}B_{6,20}(X) + 1.03362\cdot10^{12}B_{7,20}(X) + 5.7382\cdot10^{11}B_{8,20}(X) + 1.28041\cdot10^{11}B_{9,20}(X) - 2.75764\cdot10^{11}B_{10,20}(X) - 6.16213\cdot10^{11}B_{11,20}(X) - 8.79156\cdot10^{11}B_{12,20}(X) - 1.05766\cdot10^{12}B_{13,20}(X) - 1.15145\cdot10^{12}B_{14,20}(X) - 1.1659\cdot10^{12}B_{15,20}(X) - 1.11081\cdot10^{12}B_{16,20}(X) - 9.99056\cdot10^{11}B_{17,20}(X) - 8.45188\cdot10^{11}B_{18,20}(X) - 6.64233\cdot10^{11}B_{19,20}(X) - 4.70618\cdot10^{11}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 3.99117\cdot10^{12}X^2 - 7.96214\cdot10^{12}X + 2.96977\cdot10^{12}$$
$$= 2.96977\cdot10^{12}B_{0,2} - 1.0113\cdot10^{12}B_{1,2} - 1.00119\cdot10^{12}B_{2,2}$$

$$\begin{aligned}\widetilde{q}_2 &= 3.02719{\cdot}10^{14}X^{20} - 3.0222{\cdot}10^{15}X^{19} + 1.3966{\cdot}10^{16}X^{18} - 3.96332{\cdot}10^{16}X^{17} + 7.72653{\cdot}10^{16}X^{16} - 1.09666 \\ &\quad \cdot 10^{17}X^{15} + 1.17176\cdot 10^{17}X^{14} - 9.61219\cdot 10^{16}X^{13} + 6.11992\cdot 10^{16}X^{12} - 3.03803\cdot 10^{16}X^{11} + 1.17536 \\ &\quad \cdot 10^{16}X^{10} - 3.52608\cdot 10^{15}X^{9} + 8.12025\cdot 10^{14}X^{8} - 1.41173\cdot 10^{14}X^{7} + 1.80666\cdot 10^{13}X^{6} - 1.64351 \\ &\quad \cdot 10^{12}X^{5} + 1.01221\cdot 10^{11}X^{4} - 3.87803\cdot 10^{9}X^{3} + 3.99126\cdot 10^{12}X^{2} - 7.96214\cdot 10^{12}X + 2.96977\cdot 10^{12} \\ &= 2.96977\cdot 10^{12}B_{0,20} + 2.57166\cdot 10^{12}B_{1,20} + 2.19456\cdot 10^{12}B_{2,20} + 1.83847\cdot 10^{12}B_{3,20} + 1.50339 \\ &\quad \cdot 10^{12}B_{4,20} + 1.18927\cdot 10^{12}B_{5,20} + 8.96303\cdot 10^{11}B_{6,20} + 6.23988\cdot 10^{11}B_{7,20} + 3.73364\cdot 10^{11}B_{8,20} \\ &\quad + 1.42651\cdot 10^{11}B_{9,20} - 6.56278\cdot 10^{10}B_{10,20} - 2.54431\cdot 10^{11}B_{11,20} - 4.20819\cdot 10^{11}B_{12,20} \\ &\quad - 5.67316\cdot 10^{11}B_{13,20} - 6.92077\cdot 10^{11}B_{14,20} - 7.96221\cdot 10^{11}B_{15,20} - 8.79188\cdot 10^{11}B_{16,20} \\ &\quad - 9.4121\cdot 10^{11}B_{17,20} - 9.82209\cdot 10^{11}B_{18,20} - 1.00221\cdot 10^{12}B_{19,20} - 1.00119\cdot 10^{12}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.10692\cdot 10^{12}$.

Bounding polynomials M and m:

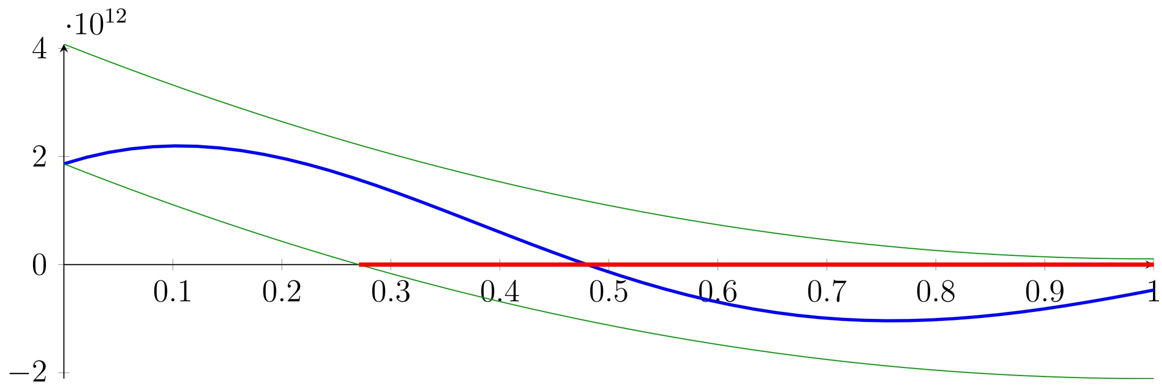
$$M = 3.99117\cdot 10^{12}X^2 - 7.96214\cdot 10^{12}X + 4.07669\cdot 10^{12}$$

$$m = 3.99117\cdot 10^{12}X^2 - 7.96214\cdot 10^{12}X + 1.86285\cdot 10^{12}$$

Root of M and m:

$$N(M) = \{\} \qquad\qquad N(m) = \{0.270694, 1.72424\}$$

Intersection intervals:



$$[0.270694, 1]$$

Longest intersection interval: 0.729306

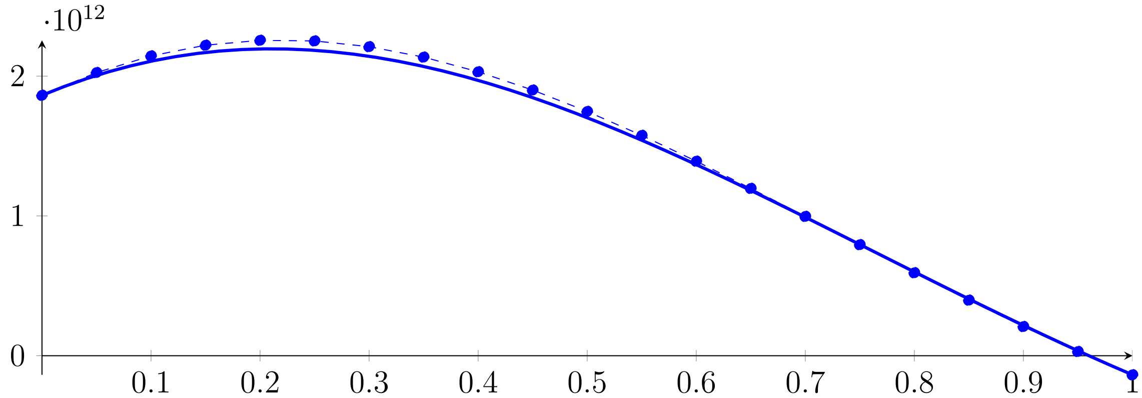
$\Longrightarrow$ Bisection: first half $[6.25, 7.03125]$ und second half $[7.03125, 7.8125]$

Bisection point is very near to a root?!?

2.37 Recursion Branch 1 1 2 1 1 1 on the First Half $[6.25, 7.03125]$

Normalized monomial und Bézier representations and the Bézier polygon:

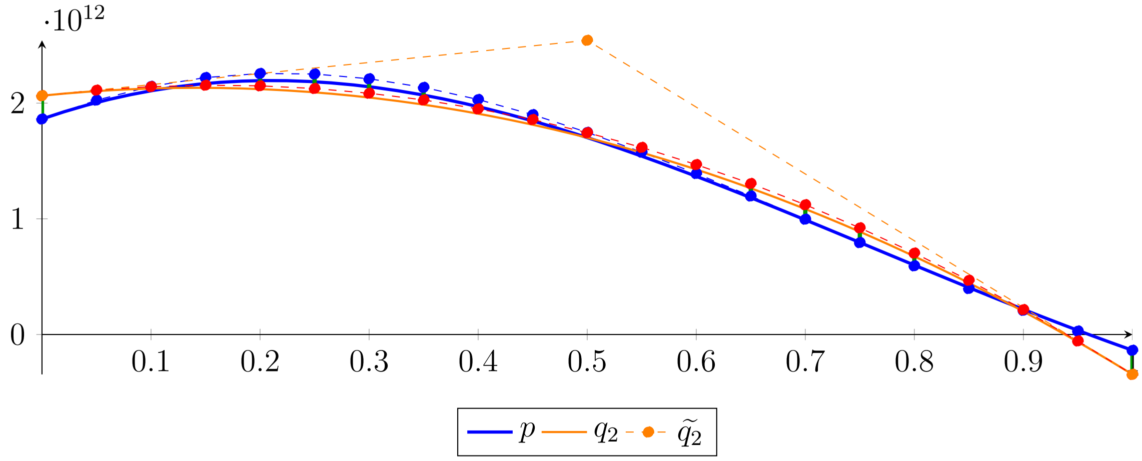
$$p = -1890.77X^{20} + 6963.11X^{19} - 62140.3X^{18} + 199499X^{17} - 1.21925\cdot10^{6}X^{16} + 910849X^{15} - 702042X^{14} + 1.01354\cdot10^{7}X^{13} - 6.89414\cdot10^{7}X^{12} - 4.32511\cdot10^{7}X^{11} + 2.75113\cdot10^{9}X^{10} - 1.10315\cdot10^{10}X^{9} - 1.91975\cdot10^{10}X^{8} + 2.34196\cdot10^{11}X^{7} - 3.29877\cdot10^{11}X^{6} - 1.32879\cdot10^{12}X^{5} + 3.78502\cdot10^{12}X^{4} + 6.5775\cdot10^{11}X^{3} - 8.2429\cdot10^{12}X^{2} + 3.25284\cdot10^{12}X + 1.86285\cdot10^{12}$$
$$= 1.86285\cdot10^{12}B_{0,20}(X) + 2.02549\cdot10^{12}B_{1,20}(X) + 2.14475\cdot10^{12}B_{2,20}(X) + 2.2212\cdot10^{12}B_{3,20}(X) + 2.25621\cdot10^{12}B_{4,20}(X) + 2.25181\cdot10^{12}B_{5,20}(X) + 2.21068\cdot10^{12}B_{6,20}(X) + 2.13597\cdot10^{12}B_{7,20}(X) + 2.03123\cdot10^{12}B_{8,20}(X) + 1.90031\cdot10^{12}B_{9,20}(X) + 1.74727\cdot10^{12}B_{10,20}(X) + 1.57625\cdot10^{12}B_{11,20}(X) + 1.39143\cdot10^{12}B_{12,20}(X) + 1.19689\cdot10^{12}B_{13,20}(X) + 9.96597\cdot10^{11}B_{14,20}(X) + 7.94288\cdot10^{11}B_{15,20}(X) + 5.93457\cdot10^{11}B_{16,20}(X) + 3.97291\cdot10^{11}B_{17,20}(X) + 2.08644\cdot10^{11}B_{18,20}(X) + 3.00113\cdot10^{10}B_{19,20}(X) - 1.36487\cdot10^{11}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = -3.36618\cdot10^{12}X^{2} + 9.57078\cdot10^{11}X + 2.06429\cdot10^{12}$$
$$= 2.06429\cdot10^{12}B_{0,2} + 2.54283\cdot10^{12}B_{1,2} - 3.44809\cdot10^{11}B_{2,2}$$

$$\widetilde{q}_2 = -7.83886\cdot10^{14}X^{20} + 7.85259\cdot10^{15}X^{19} - 3.64158\cdot10^{16}X^{18} + 1.03726\cdot10^{17}X^{17} - 2.03031\cdot10^{17}X^{16} + 2.89495\cdot10^{17}X^{15} - 3.11015\cdot10^{17}X^{14} + 2.56863\cdot10^{17}X^{13} - 1.64922\cdot10^{17}X^{12} + 8.27127\cdot10^{16}X^{11} - 3.23797\cdot10^{16}X^{10} + 9.83724\cdot10^{15}X^{9} - 2.29447\cdot10^{15}X^{8} + 4.04426\cdot10^{14}X^{7} - 5.27645\cdot10^{13}X^{6} + 4.96099\cdot10^{12}X^{5} - 3.23655\cdot10^{11}X^{4} + 1.35928\cdot10^{10}X^{3} - 3.3665\cdot10^{12}X^{2} + 9.57081\cdot10^{11}X + 2.06429\cdot10^{12}$$
$$= 2.06429\cdot10^{12}B_{0,20} + 2.11214\cdot10^{12}B_{1,20} + 2.14228\cdot10^{12}B_{2,20} + 2.15471\cdot10^{12}B_{3,20} + 2.14938\cdot10^{12}B_{4,20} + 2.12648\cdot10^{12}B_{5,20} + 2.08543\cdot10^{12}B_{6,20} + 2.02767\cdot10^{12}B_{7,20} + 1.95032\cdot10^{12}B_{8,20} + 1.85812\cdot10^{12}B_{9,20} + 1.74449\cdot10^{12}B_{10,20} + 1.61718\cdot10^{12}B_{11,20} + 1.46851\cdot10^{12}B_{12,20} + 1.30491\cdot10^{12}B_{13,20} + 1.1218\cdot10^{12}B_{14,20} + 9.21934\cdot10^{11}B_{15,20} + 7.03917\cdot10^{11}B_{16,20} + 4.68338\cdot10^{11}B_{17,20} + 2.15\cdot10^{11}B_{18,20} - 5.60455\cdot10^{10}B_{19,20} - 3.44809\cdot10^{11}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 2.08322 \cdot 10^{11}$.

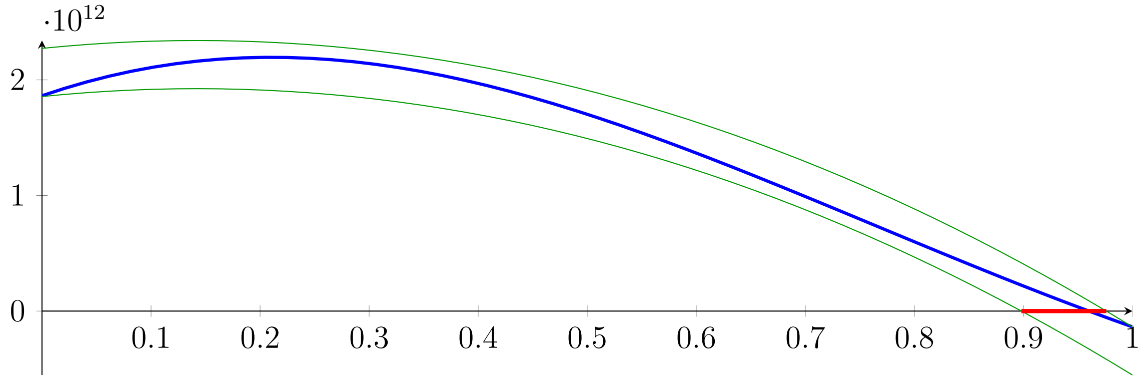
Bounding polynomials M and m:

$$M = -3.36618 \cdot 10^{12} X^2 + 9.57078 \cdot 10^{11} X + 2.27261 \cdot 10^{12}$$
$$m = -3.36618 \cdot 10^{12} X^2 + 9.57078 \cdot 10^{11} X + 1.85597 \cdot 10^{12}$$

Root of M and m:

$$N(M) = \{-0.69171, 0.976032\} \qquad N(m) = \{-0.61386, 0.898182\}$$

Intersection intervals:



$$[0.898182, 0.976032]$$

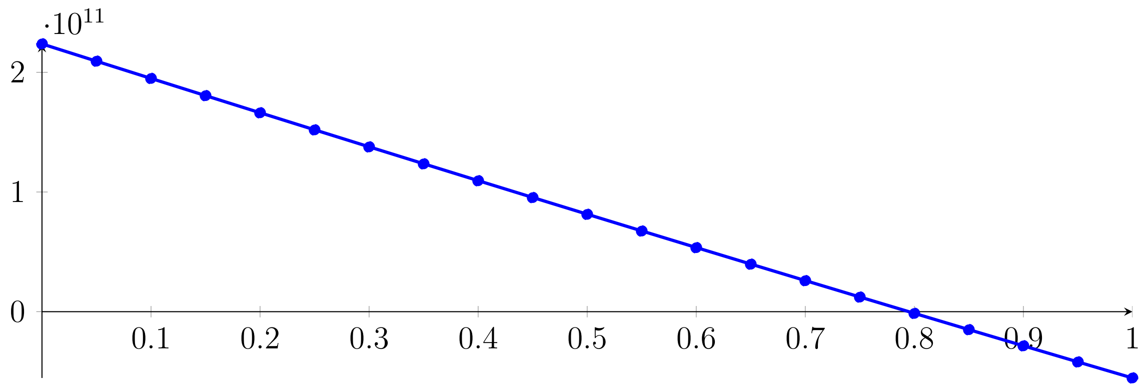
Longest intersection interval: 0.0778505

$\Longrightarrow$ Selective recursion: interval 1: $[6.9517, 7.01253]$,

2.38 Recursion Branch 1 1 2 1 1 1 1 in Interval 1: $[6.9517, 7.01253]$

Normalized monomial und Bézier representations and the Bézier polygon:

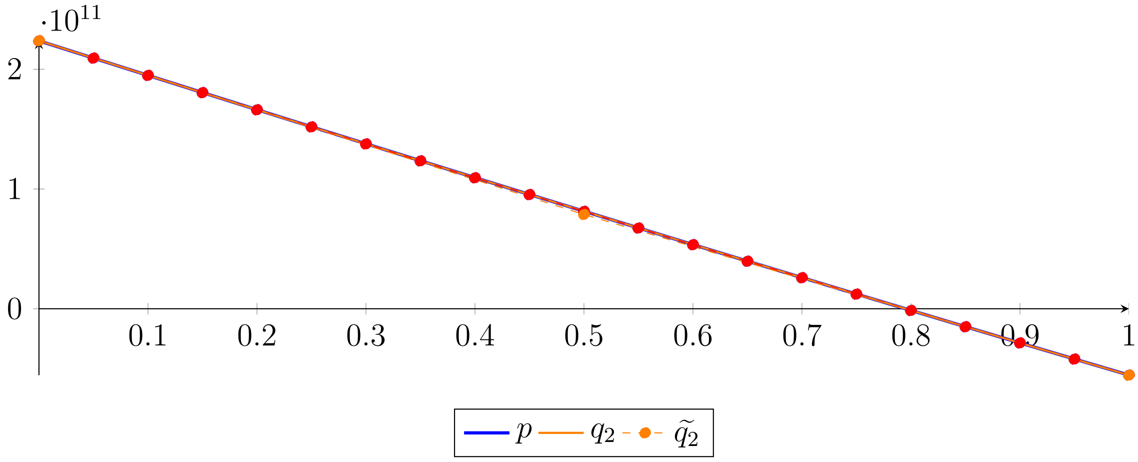
$$p = -98.2162X^{20} + 250.399X^{19} - 3715.84X^{18} + 9931.48X^{17} - 69192.2X^{16} + 57449.7X^{15} - 30902.3X^{14} - 12460.3X^{13} - 68447.7X^{12} - 11825.1X^{11} - 22626.5X^{10} - 4115.96X^9 - 234.502X^8 - 328.835X^7 + 83759.6X^6 - 1.28017 \cdot 10^6 X^5 - 6.01257 \cdot 10^7 X^4 + 1.47386 \cdot 10^9 X^3 + 8.83298 \cdot 10^9 X^2 - 2.89353 \cdot 10^{11} X + 2.23723 \cdot 10^{11}$$
$$= 2.23723 \cdot 10^{11} B_{0,20}(X) + 2.09255 \cdot 10^{11} B_{1,20}(X) + 1.94834 \cdot 10^{11} B_{2,20}(X) + 1.80461 \cdot 10^{11} B_{3,20}(X) + 1.66136 \cdot 10^{11} B_{4,20}(X) + 1.51862 \cdot 10^{11} B_{5,20}(X) + 1.3764 \cdot 10^{11} B_{6,20}(X) + 1.2347 \cdot 10^{11} B_{7,20}(X) + 1.09355 \cdot 10^{11} B_{8,20}(X) + 9.52947 \cdot 10^{10} B_{9,20}(X) + 8.1291 \cdot 10^{10} B_{10,20}(X) + 6.73449 \cdot 10^{10} B_{11,20}(X) + 5.34577 \cdot 10^{10} B_{12,20}(X) + 3.96305 \cdot 10^{10} B_{13,20}(X) + 2.58645 \cdot 10^{10} B_{14,20}(X) + 1.21607 \cdot 10^{10} B_{15,20}(X) - 1.47956 \cdot 10^9 B_{16,20}(X) - 1.50553 \cdot 10^{10} B_{17,20}(X) - 2.85654 \cdot 10^{10} B_{18,20}(X) - 4.20088 \cdot 10^{10} B_{19,20}(X) - 5.53844 \cdot 10^{10} B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 1.09386{\cdot}10^{10}X^2 - 2.90181{\cdot}10^{11}X + 2.23791{\cdot}10^{11}$$
$$= 2.23791{\cdot}10^{11}B_{0,2} + 7.87008{\cdot}10^{10}B_{1,2} - 5.5451{\cdot}10^{10}B_{2,2}$$

$$\widetilde{q_2} = -2.42988{\cdot}10^{13}X^{20} + 2.43862{\cdot}10^{14}X^{19} - 1.13304{\cdot}10^{15}X^{18} + 3.23377{\cdot}10^{15}X^{17} - 6.34339{\cdot}10^{15}X^{16} + 9.06684{\cdot}10^{15}X^{15} - 9.76877{\cdot}10^{15}X^{14} + 8.09597{\cdot}10^{15}X^{13} - 5.22025{\cdot}10^{15}X^{12} + 2.63137{\cdot}10^{15}X^{11} - 1.036{\cdot}10^{15}X^{10} + 3.16627{\cdot}10^{14}X^{9} - 7.42864{\cdot}10^{13}X^{8} + 1.31758{\cdot}10^{13}X^{7} - 1.73398{\cdot}10^{12}X^{6} + 1.65419{\cdot}10^{11}X^{5} - 1.10554{\cdot}10^{10}X^{4} + 4.80791{\cdot}10^{8}X^{3} + 1.09266{\cdot}10^{10}X^{2} - 2.90181{\cdot}10^{11}X + 2.23791{\cdot}10^{11}$$
$$= 2.23791{\cdot}10^{11}B_{0,20} + 2.09282{\cdot}10^{11}B_{1,20} + 1.94831{\cdot}10^{11}B_{2,20} + 1.80437{\cdot}10^{11}B_{3,20} + 1.661{\cdot}10^{11}B_{4,20} + 1.51825{\cdot}10^{11}B_{5,20} + 1.37593{\cdot}10^{11}B_{6,20} + 1.23451{\cdot}10^{11}B_{7,20} + 1.09308{\cdot}10^{11}B_{8,20} + 9.53111{\cdot}10^{10}B_{9,20} + 8.1257{\cdot}10^{10}B_{10,20} + 6.7386{\cdot}10^{10}B_{11,20} + 5.34605{\cdot}10^{10}B_{12,20} + 3.9677{\cdot}10^{10}B_{13,20} + 2.58967{\cdot}10^{10}B_{14,20} + 1.22035{\cdot}10^{10}B_{15,20} - 1.44568{\cdot}10^{9}B_{16,20} - 1.50325{\cdot}10^{10}B_{17,20} - 2.85631{\cdot}10^{10}B_{18,20} - 4.20358{\cdot}10^{10}B_{19,20} - 5.5451{\cdot}10^{10}B_{20,20}$$



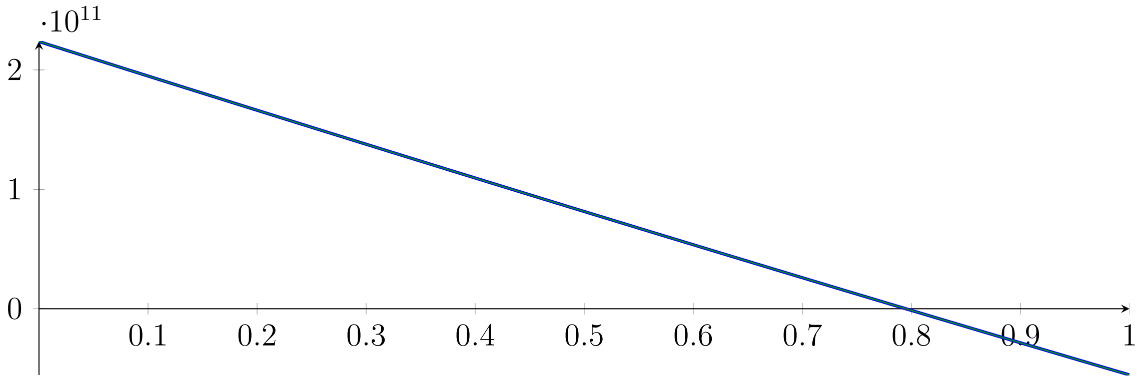
The maximum difference of the Bézier coefficients is $\delta = 6.84119{\cdot}10^{7}$.
Bounding polynomials M and m:

$$M = 1.09386{\cdot}10^{10}X^2 - 2.90181{\cdot}10^{11}X + 2.2386{\cdot}10^{11}$$
$$m = 1.09386{\cdot}10^{10}X^2 - 2.90181{\cdot}10^{11}X + 2.23723{\cdot}10^{11}$$

Root of M and m:

$$N(M) = \{0.795291, 25.733\} \qquad N(m) = \{0.794789, 25.7335\}$$

Intersection intervals:



$$[0.794789, 0.795291]$$

Longest intersection interval: 0.000501576
$\Longrightarrow$ Selective recursion: interval 1: $[7.00004, 7.00007]$,

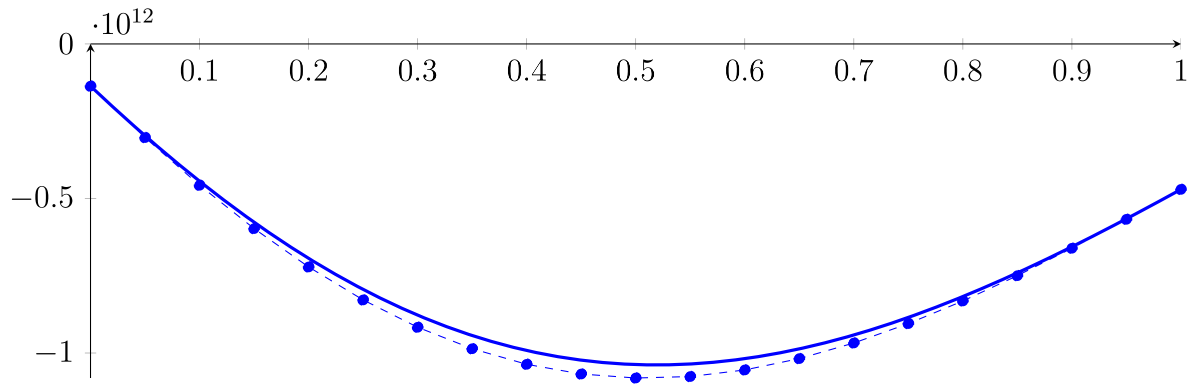
2.39 Recursion Branch 1 1 2 1 1 1 1 1 in Interval 1: $[7.00004, 7.00007]$

Found root in interval $[7.00004, 7.00007]$ at recursion depth 8!

2.40 Recursion Branch 1 1 2 1 1 2 on the Second Half $[7.03125, 7.8125]$

Normalized monomial und Bézier representations and the Bézier polygon:

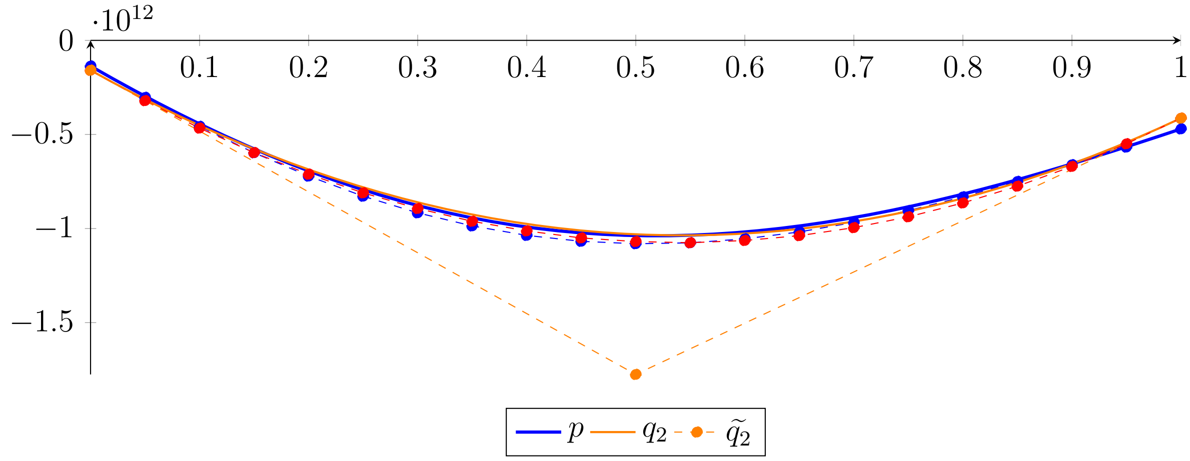
$$\begin{aligned} p &= 1035.03X^{20} - 5047.89X^{19} + 29665.4X^{18} - 125838X^{17} + 661490X^{16} - 487334X^{15} \\ &\quad - 385531X^{14} + 2.57003\cdot10^{6}X^{13} + 1.95142\cdot10^{7}X^{12} - 2.29284\cdot10^{8}X^{11} + 2.89862\cdot10^{8}X^{10} \\ &\quad + 5.68224\cdot10^{9}X^{9} - 2.33566\cdot10^{10}X^{8} - 3.80081\cdot10^{10}X^{7} + 3.57025\cdot10^{11}X^{6} - 2.22743\cdot10^{11}X^{5} \\ &\quad - 1.80701\cdot10^{12}X^{4} + 2.41853\cdot10^{12}X^{3} + 2.30563\cdot10^{12}X^{2} - 3.32997\cdot10^{12}X - 1.36487\cdot10^{11} \\ &= -1.36487\cdot10^{11}B_{0,20}(X) - 3.02985\cdot10^{11}B_{1,20}(X) - 4.57349\cdot10^{11}B_{2,20}(X) - 5.97455 \\ &\quad \cdot10^{11}B_{3,20}(X) - 7.21557\cdot10^{11}B_{4,20}(X) - 8.28293\cdot10^{11}B_{5,20}(X) - 9.16694\cdot10^{11}B_{6,20}(X) \\ &\quad - 9.8618\cdot10^{11}B_{7,20}(X) - 1.03655\cdot10^{12}B_{8,20}(X) - 1.06796\cdot10^{12}B_{9,20}(X) - 1.08089 \\ &\quad \cdot10^{12}B_{10,20}(X) - 1.07615\cdot10^{12}B_{11,20}(X) - 1.05479\cdot10^{12}B_{12,20}(X) - 1.0181\cdot10^{12}B_{13,20}(X) \\ &\quad - 9.67562\cdot10^{11}B_{14,20}(X) - 9.04818\cdot10^{11}B_{15,20}(X) - 8.31607\cdot10^{11}B_{16,20}(X) - 7.49742 \\ &\quad \cdot10^{11}B_{17,20}(X) - 6.61068\cdot10^{11}B_{18,20}(X) - 5.67425\cdot10^{11}B_{19,20}(X) - 4.70618\cdot10^{11}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 2.97878\cdot10^{12}X^{2} - 3.23342\cdot10^{12}X - 1.58799\cdot10^{11} \\ &= -1.58799\cdot10^{11}B_{0,2} - 1.77551\cdot10^{12}B_{1,2} - 4.13444\cdot10^{11}B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 5.1572\cdot10^{14}X^{20} - 5.16386\cdot10^{15}X^{19} + 2.39365\cdot10^{16}X^{18} - 6.81506\cdot10^{16}X^{17} + 1.33334\cdot10^{17}X^{16} - 1.90009 \\ &\quad \cdot10^{17}X^{15} + 2.03983\cdot10^{17}X^{14} - 1.68295\cdot10^{17}X^{13} + 1.07906\cdot10^{17}X^{12} - 5.40197\cdot10^{16}X^{11} + 2.11007 \\ &\quad \cdot10^{16}X^{10} - 6.39486\cdot10^{15}X^{9} + 1.48774\cdot10^{15}X^{8} - 2.61527\cdot10^{14}X^{7} + 3.40121\cdot10^{13}X^{6} - 3.18429 \\ &\quad \cdot10^{12}X^{5} + 2.06573\cdot10^{11}X^{4} - 8.61202\cdot10^{9}X^{3} + 2.97898\cdot10^{12}X^{2} - 3.23342\cdot10^{12}X - 1.58799\cdot10^{11} \\ &= -1.58799\cdot10^{11}B_{0,20} - 3.2047\cdot10^{11}B_{1,20} - 4.66462\cdot10^{11}B_{2,20} - 5.96783\cdot10^{11}B_{3,20} - 7.11398 \\ &\quad \cdot10^{11}B_{4,20} - 8.10434\cdot10^{11}B_{5,20} - 8.9351\cdot10^{11}B_{6,20} - 9.61558\cdot10^{11}B_{7,20} - 1.01271\cdot10^{12}B_{8,20} \\ &\quad - 1.05007\cdot10^{12}B_{9,20} - 1.0693\cdot10^{12}B_{10,20} - 1.0755\cdot10^{12}B_{11,20} - 1.06364\cdot10^{12}B_{12,20} \\ &\quad - 1.03794\cdot10^{12}B_{13,20} - 9.95368\cdot10^{11}B_{14,20} - 9.37761\cdot10^{11}B_{15,20} - 8.64185\cdot10^{11}B_{16,20} \\ &\quad - 7.75035\cdot10^{11}B_{17,20} - 6.70179\cdot10^{11}B_{18,20} - 5.49651\cdot10^{11}B_{19,20} - 4.13444\cdot10^{11}B_{20,20} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 5.71737{\cdot}10^{10}$.

Bounding polynomials M and m:

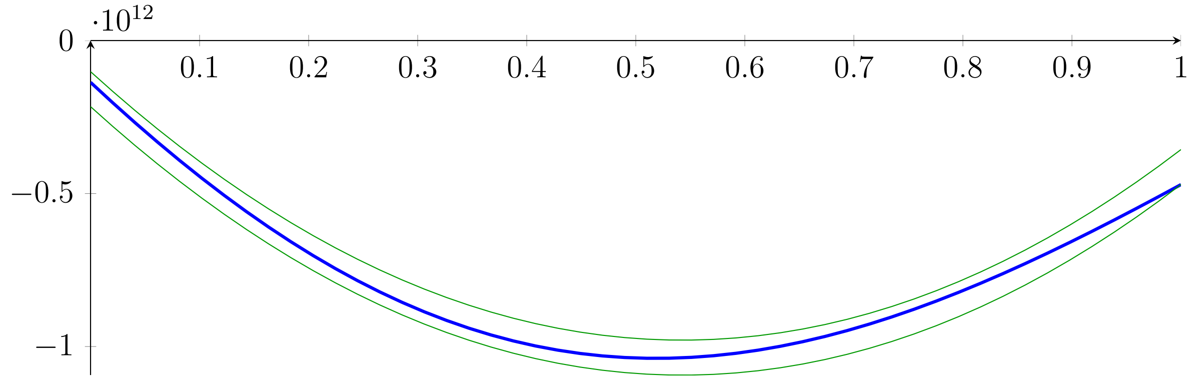
$$M = 2.97878{\cdot}10^{12}X^2 - 3.23342{\cdot}10^{12}X - 1.01625{\cdot}10^{11}$$

$$m = 2.97878{\cdot}10^{12}X^2 - 3.23342{\cdot}10^{12}X - 2.15973{\cdot}10^{11}$$

Root of M and m:

$$N(M) = \{-0.0305688, 1.11606\} \qquad N(m) = \{-0.0631231, 1.14861\}$$

Intersection intervals:

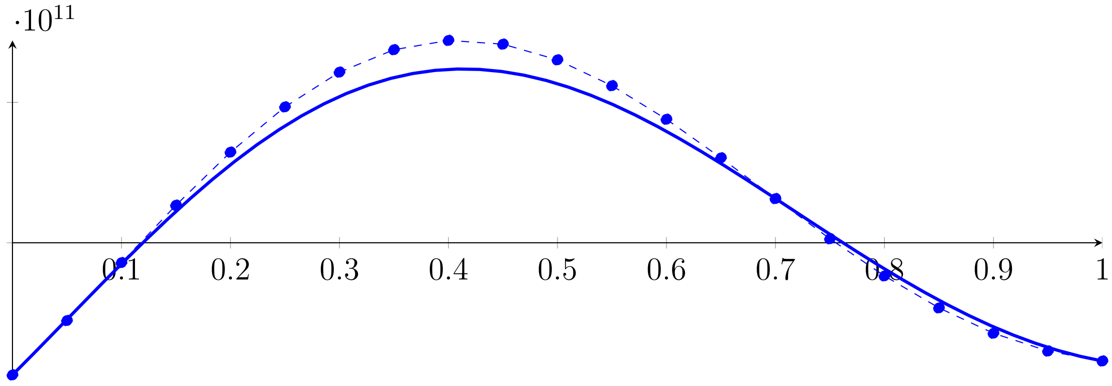


No intersection intervals with the x axis.

2.41 Recursion Branch 1 1 2 1 2 on the Second Half $[7.8125, 9.375]$

Normalized monomial und Bézier representations and the Bézier polygon:

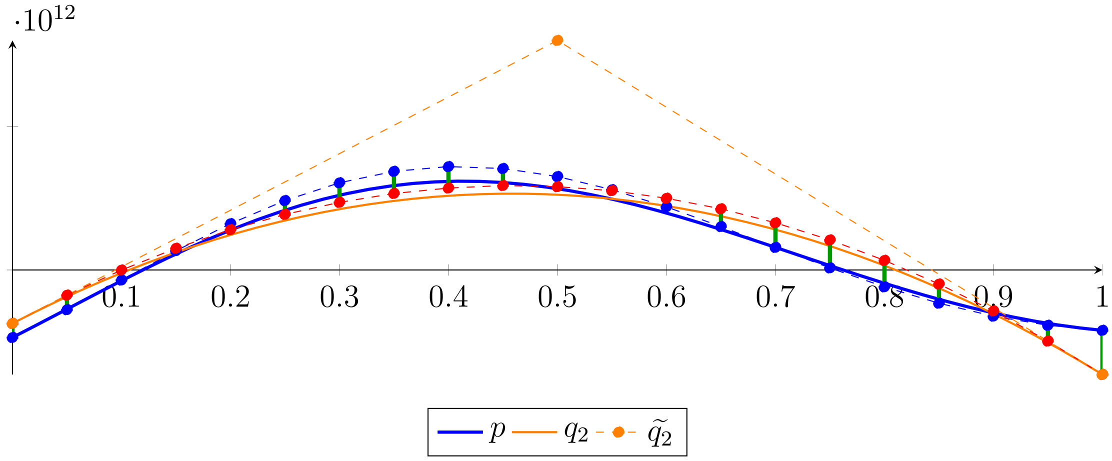
$$\begin{aligned} p = {} & 6877.98X^{20} - 256526X^{19} + 3.18676{\cdot}10^{6}X^{18} - 1.18426{\cdot}10^{7}X^{17} - 8.79174{\cdot}10^{7}X^{16} + 9.23149 \\ & {\cdot}10^{8}X^{15} - 1.26914{\cdot}10^{9}X^{14} - 1.59203{\cdot}10^{10}X^{13} + 6.26004{\cdot}10^{10}X^{12} + 7.11942{\cdot}10^{10}X^{11} - 7.3925 \\ & {\cdot}10^{11}X^{10} + 5.09162{\cdot}10^{11}X^{9} + 3.6295{\cdot}10^{12}X^{8} - 5.56929{\cdot}10^{12}X^{7} - 7.06545{\cdot}10^{12}X^{6} + 1.64355 \\ & {\cdot}10^{13}X^{5} + 2.9001{\cdot}10^{12}X^{4} - 1.64458{\cdot}10^{13}X^{3} + 2.40542{\cdot}10^{12}X^{2} + 3.8723{\cdot}10^{12}X - 4.70618{\cdot}10^{11} \\ = {} & -4.70618{\cdot}10^{11}B_{0,20}(X) - 2.77003{\cdot}10^{11}B_{1,20}(X) - 7.07277{\cdot}10^{10}B_{2,20}(X) + 1.33781 \\ & {\cdot}10^{11}B_{3,20}(X) + 3.22697{\cdot}10^{11}B_{4,20}(X) + 4.8385{\cdot}10^{11}B_{5,20}(X) + 6.07608{\cdot}10^{11}B_{6,20}(X) \\ & + 6.87499{\cdot}10^{11}B_{7,20}(X) + 7.20537{\cdot}10^{11}B_{8,20}(X) + 7.07242{\cdot}10^{11}B_{9,20}(X) + 6.51366 \\ & {\cdot}10^{11}B_{10,20}(X) + 5.59383{\cdot}10^{11}B_{11,20}(X) + 4.398{\cdot}10^{11}B_{12,20}(X) + 3.02359{\cdot}10^{11}B_{13,20}(X) \\ & + 1.57223{\cdot}10^{11}B_{14,20}(X) + 1.42063{\cdot}10^{10}B_{15,20}(X) - 1.17894{\cdot}10^{11}B_{16,20}(X) - 2.31815 \\ & {\cdot}10^{11}B_{17,20}(X) - 3.22175{\cdot}10^{11}B_{18,20}(X) - 3.85644{\cdot}10^{11}B_{19,20}(X) - 4.20945{\cdot}10^{11}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = -4.30088 \cdot 10^{12} X^2 + 3.94478 \cdot 10^{12} X - 3.72538 \cdot 10^{11}$$
$$= -3.72538 \cdot 10^{11} B_{0,2} + 1.59985 \cdot 10^{12} B_{1,2} - 7.28638 \cdot 10^{11} B_{2,2}$$

$$\widetilde{q}_2 = -5.96813 \cdot 10^{14} X^{20} + 5.96988 \cdot 10^{15} X^{19} - 2.76397 \cdot 10^{16} X^{18} + 7.85873 \cdot 10^{16} X^{17} - 1.53526 \cdot 10^{17} X^{16} + 2.18447 \cdot 10^{17} X^{15} - 2.34154 \cdot 10^{17} X^{14} + 1.92913 \cdot 10^{17} X^{13} - 1.23541 \cdot 10^{17} X^{12} + 6.17909 \cdot 10^{16} X^{11} - 2.41251 \cdot 10^{16} X^{10} + 7.31193 \cdot 10^{15} X^9 - 1.70199 \cdot 10^{15} X^8 + 2.9929 \cdot 10^{14} X^7 - 3.88535 \cdot 10^{13} X^6 + 3.60961 \cdot 10^{12} X^5 - 2.29601 \cdot 10^{11} X^4 + 9.2424 \cdot 10^9 X^3 - 4.30109 \cdot 10^{12} X^2 + 3.94478 \cdot 10^{12} X - 3.72538 \cdot 10^{11}$$
$$= -3.72538 \cdot 10^{11} B_{0,20} - 1.75299 \cdot 10^{11} B_{1,20} - 6.97072 \cdot 10^8 B_{2,20} + 1.51275 \cdot 10^{11} B_{3,20} + 2.8058 \cdot 10^{11} B_{4,20} + 3.87361 \cdot 10^{11} B_{5,20} + 4.71183 \cdot 10^{11} B_{6,20} + 5.3311 \cdot 10^{11} B_{7,20} + 5.71013 \cdot 10^{11} B_{8,20} + 5.88436 \cdot 10^{11} B_{9,20} + 5.80417 \cdot 10^{11} B_{10,20} + 5.52816 \cdot 10^{11} B_{11,20} + 4.99791 \cdot 10^{11} B_{12,20} + 4.26278 \cdot 10^{11} B_{13,20} + 3.28744 \cdot 10^{11} B_{14,20} + 2.09314 \cdot 10^{11} B_{15,20} + 6.69192 \cdot 10^{10} B_{16,20} - 9.79948 \cdot 10^{10} B_{17,20} - 2.85577 \cdot 10^{11} B_{18,20} - 4.95789 \cdot 10^{11} B_{19,20} - 7.28638 \cdot 10^{11} B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 3.07693 \cdot 10^{11}$.

Bounding polynomials M and m:

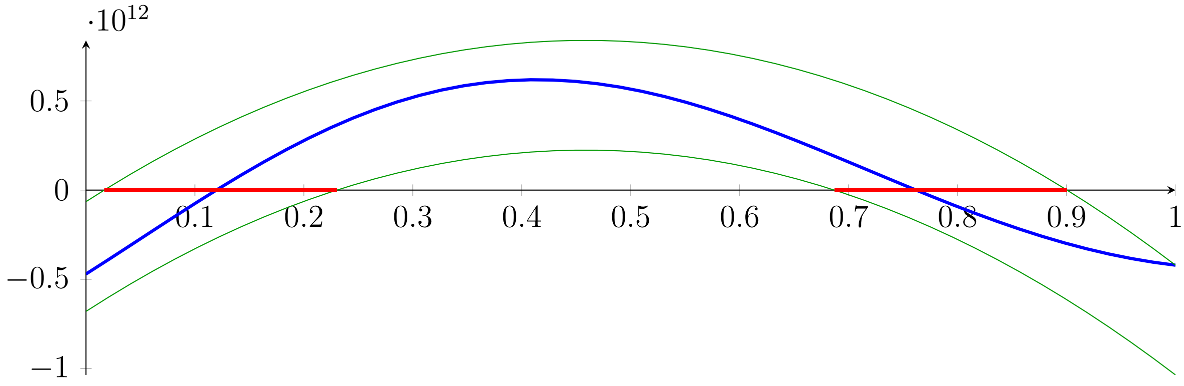
$$M = -4.30088 \cdot 10^{12} X^2 + 3.94478 \cdot 10^{12} X - 6.48444 \cdot 10^{10}$$

$$m = -4.30088 \cdot 10^{12} X^2 + 3.94478 \cdot 10^{12} X - 6.80231 \cdot 10^{11}$$

Root of M and m:

$$N(M) = \{0.0167437, 0.900459\} \qquad N(m) = \{0.230228, 0.686975\}$$

Intersection intervals:



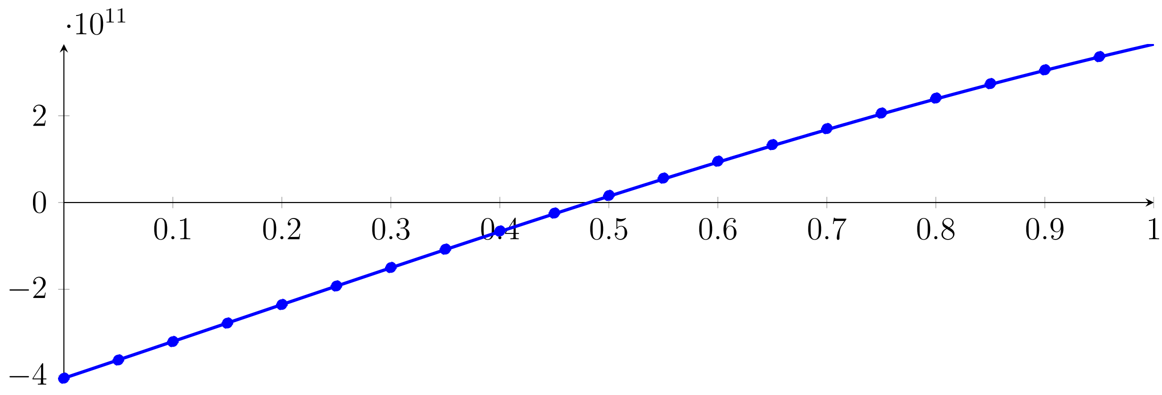
$[0.0167437, 0.230228]$, $[0.686975, 0.900459]$

Longest intersection interval: 0.213485
$\Longrightarrow$ Selective recursion: interval 1: $[7.83866, 8.17223]$, interval 2: $[8.8859, 9.21947]$,

2.42 Recursion Branch 1 1 2 1 2 1 in Interval 1: $[7.83866, 8.17223]$

Normalized monomial und Bézier representations and the Bézier polygon:

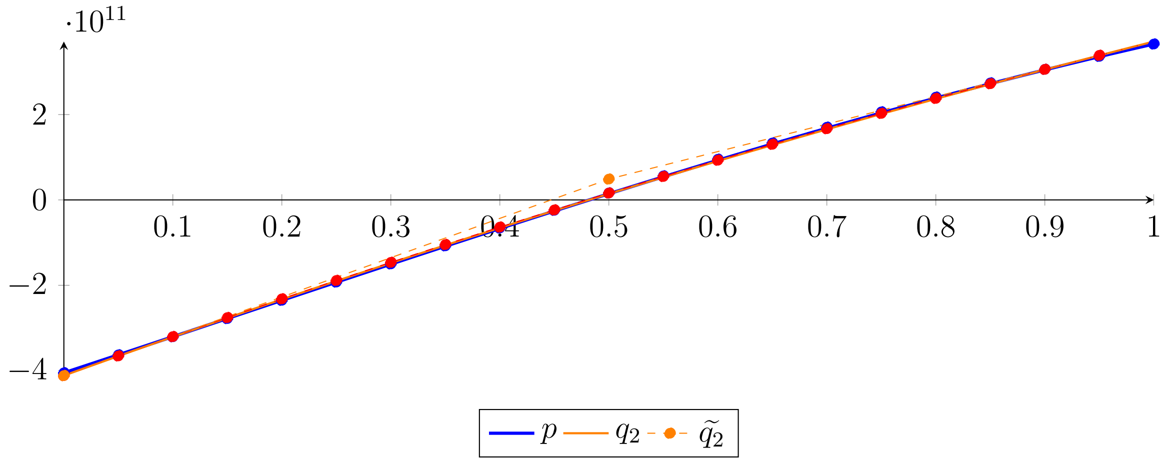
$$\begin{aligned} p = & \ 26.056X^{20} + 627.844X^{19} + 2761.01X^{18} + 2026.87X^{17} + 33798.5X^{16} - 37312.2X^{15} \\ & + 28819.2X^{14} + 15793.6X^{13} + 76194X^{12} + 21425.6X^{11} - 113961X^{10} + 361036X^{9} \\ & + 1.59509{\cdot}10^{7}X^{8} - 1.02634{\cdot}10^{8}X^{7} - 7.27945{\cdot}10^{8}X^{6} + 6.95926{\cdot}10^{9}X^{5} + 8.81839 \\ & \cdot 10^{9}X^{4} - 1.57681{\cdot}10^{11}X^{3} + 7.22363{\cdot}10^{10}X^{2} + 8.40933{\cdot}10^{11}X - 4.05184{\cdot}10^{11} \\ = & -4.05184{\cdot}10^{11}B_{0,20}(X) - 3.63137{\cdot}10^{11}B_{1,20}(X) - 3.2071{\cdot}10^{11}B_{2,20}(X) - 2.78042 \\ & \cdot 10^{11}B_{3,20}(X) - 2.35267{\cdot}10^{11}B_{4,20}(X) - 1.92522{\cdot}10^{11}B_{5,20}(X) - 1.49937{\cdot}10^{11}B_{6,20}(X) \\ & - 1.07641{\cdot}10^{11}B_{7,20}(X) - 6.5759{\cdot}10^{10}B_{8,20}(X) - 2.44115{\cdot}10^{10}B_{9,20}(X) + 1.62845 \\ & \cdot 10^{10}B_{10,20}(X) + 5.62164{\cdot}10^{10}B_{11,20}(X) + 9.5277{\cdot}10^{10}B_{12,20}(X) + 1.33364{\cdot}10^{11}B_{13,20}(X) \\ & + 1.70379{\cdot}10^{11}B_{14,20}(X) + 2.06232{\cdot}10^{11}B_{15,20}(X) + 2.40837{\cdot}10^{11}B_{16,20}(X) + 2.74114 \\ & \cdot 10^{11}B_{17,20}(X) + 3.05989{\cdot}10^{11}B_{18,20}(X) + 3.36394{\cdot}10^{11}B_{19,20}(X) + 3.65268{\cdot}10^{11}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -1.38193{\cdot}10^{11}X^{2} + 9.20955{\cdot}10^{11}X - 4.11664{\cdot}10^{11} \\ &= -4.11664{\cdot}10^{11}B_{0,2} + 4.88139{\cdot}10^{10}B_{1,2} + 3.71099{\cdot}10^{11}B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 1.1741{\cdot}10^{13}X^{20} - 1.17843{\cdot}10^{14}X^{19} + 5.46765{\cdot}10^{14}X^{18} - 1.55671{\cdot}10^{15}X^{17} + 3.0455{\cdot}10^{15}X^{16} - 4.34556\\ &\quad \cdot 10^{15}X^{15} + 4.68584{\cdot}10^{15}X^{14} - 3.90363{\cdot}10^{15}X^{13} + 2.54544{\cdot}10^{15}X^{12} - 1.30667{\cdot}10^{15}X^{11} + 5.27449\\ &\quad \cdot 10^{14}X^{10} - 1.66111{\cdot}10^{14}X^{9} + 4.02517{\cdot}10^{13}X^{8} - 7.36513{\cdot}10^{12}X^{7} + 9.9349{\cdot}10^{11}X^{6} - 9.53981\\ &\quad \cdot 10^{10}X^{5} + 6.16331{\cdot}10^{9}X^{4} - 2.4689{\cdot}10^{8}X^{3} - 1.38187{\cdot}10^{11}X^{2} + 9.20955{\cdot}10^{11}X - 4.11664{\cdot}10^{11}\\ &= -4.11664{\cdot}10^{11}B_{0,20} - 3.65616{\cdot}10^{11}B_{1,20} - 3.20295{\cdot}10^{11}B_{2,20} - 2.75703{\cdot}10^{11}B_{3,20} - 2.31836\\ &\quad \cdot 10^{11}B_{4,20} - 1.887{\cdot}10^{11}B_{5,20} - 1.46283{\cdot}10^{11}B_{6,20} - 1.04611{\cdot}10^{11}B_{7,20} - 6.36366{\cdot}10^{10}B_{8,20}\\ &\quad - 2.34304{\cdot}10^{10}B_{9,20} + 1.61014{\cdot}10^{10}B_{10,20} + 5.48425{\cdot}10^{10}B_{11,20} + 9.29148{\cdot}10^{10}B_{12,20}\\ &\quad + 1.3022{\cdot}10^{11}B_{13,20} + 1.66821{\cdot}10^{11}B_{14,20} + 2.02682{\cdot}10^{11}B_{15,20} + 2.37821{\cdot}10^{11}B_{16,20}\\ &\quad + 2.72231{\cdot}10^{11}B_{17,20} + 3.05915{\cdot}10^{11}B_{18,20} + 3.3887{\cdot}10^{11}B_{19,20} + 3.71099{\cdot}10^{11}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 6.47998{\cdot}10^{9}$.

Bounding polynomials M and m:

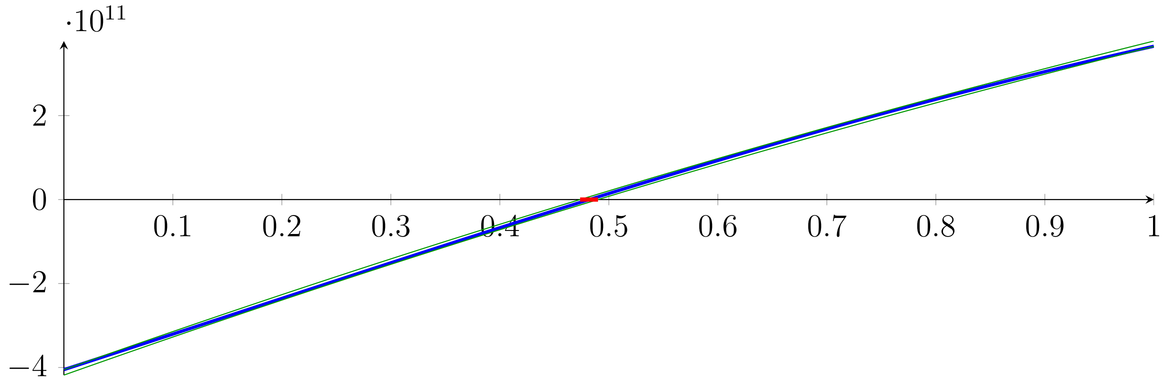
$$M = -1.38193{\cdot}10^{11}X^2 + 9.20955{\cdot}10^{11}X - 4.05184{\cdot}10^{11}$$

$$m = -1.38193{\cdot}10^{11}X^2 + 9.20955{\cdot}10^{11}X - 4.18144{\cdot}10^{11}$$

Root of M and m:

$$N(M) = \{0.47362, 6.19066\} \qquad N(m) = \{0.490071, 6.17421\}$$

Intersection intervals:



$$[0.47362, 0.490071]$$

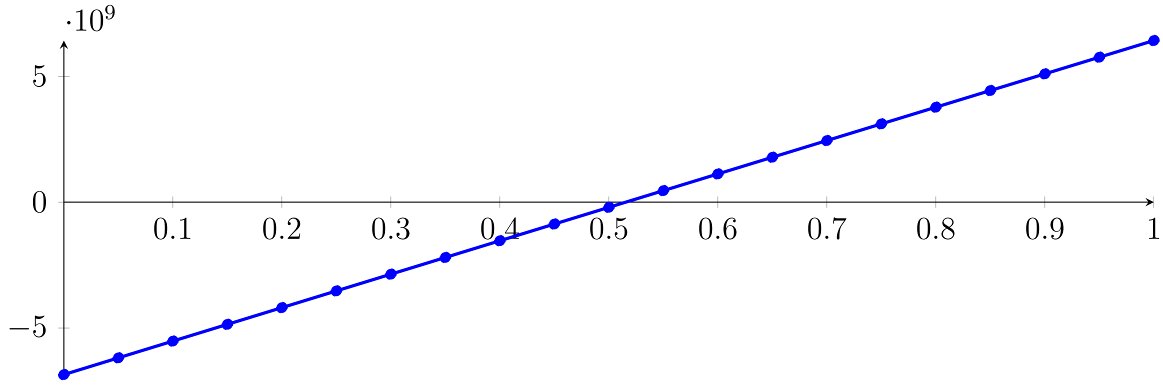
Longest intersection interval: 0.0164512

$\implies$ Selective recursion: interval 1: [7.99665, 8.00213],

2.43 Recursion Branch 1 1 2 1 2 1 1 in Interval 1: $[7.99665, 8.00213]$

Normalized monomial und Bézier representations and the Bézier polygon:

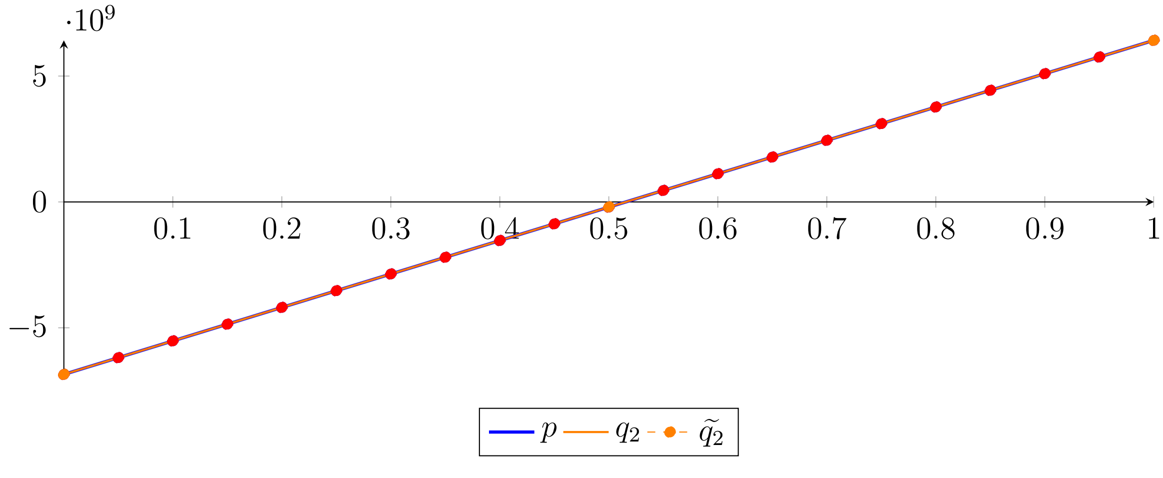
$$p = 0.881547X^{20} + 8.0728X^{19} + 55.9687X^{18} - 21.797X^{17} + 819.358X^{16} - 806.726X^{15} + 545.336X^{14} + 234.133X^{13} + 1420.59X^{12} + 400.928X^{11} + 521.719X^{10} + 90.8216X^{9} + 5.88658X^{8} - 0.0739288X^{7} + 3.88126X^{6} + 5.54466X^{5} + 1649.91X^{4} - 565769X^{3} - 3.60334\cdot 10^{7}X^{2} + 1.3303\cdot 10^{10}X - 6.84897\cdot 10^{9}$$
$$= -6.84897\cdot 10^{9}B_{0,20}(X) - 6.18382\cdot 10^{9}B_{1,20}(X) - 5.51886\cdot 10^{9}B_{2,20}(X) - 4.85409\cdot 10^{9}B_{3,20}(X) - 4.18951\cdot 10^{9}B_{4,20}(X) - 3.52512\cdot 10^{9}B_{5,20}(X) - 2.86092\cdot 10^{9}B_{6,20}(X) - 2.19691\cdot 10^{9}B_{7,20}(X) - 1.5331\cdot 10^{9}B_{8,20}(X) - 8.69478\cdot 10^{8}B_{9,20}(X) - 2.06051\cdot 10^{8}B_{10,20}(X) + 4.57182\cdot 10^{8}B_{11,20}(X) + 1.12022\cdot 10^{9}B_{12,20}(X) + 1.78306\cdot 10^{9}B_{13,20}(X) + 2.44571\cdot 10^{9}B_{14,20}(X) + 3.10816\cdot 10^{9}B_{15,20}(X) + 3.77042\cdot 10^{9}B_{16,20}(X) + 4.43247\cdot 10^{9}B_{17,20}(X) + 5.09433\cdot 10^{9}B_{18,20}(X) + 5.756\cdot 10^{9}B_{19,20}(X) + 6.41746\cdot 10^{9}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = -3.68792\cdot 10^{7}X^{2} + 1.33034\cdot 10^{10}X - 6.849\cdot 10^{9}$$
$$= -6.849\cdot 10^{9}B_{0,2} - 1.97317\cdot 10^{8}B_{1,2} + 6.41749\cdot 10^{9}B_{2,2}$$

$$\widetilde{q}_2 = 5.42005\cdot 10^{11}X^{20} - 5.42965\cdot 10^{12}X^{19} + 2.51654\cdot 10^{13}X^{18} - 7.16108\cdot 10^{13}X^{17} + 1.4002\cdot 10^{14}X^{16} - 1.99509\cdot 10^{14}X^{15} + 2.14402\cdot 10^{14}X^{14} - 1.77426\cdot 10^{14}X^{13} + 1.14424\cdot 10^{14}X^{12} - 5.78089\cdot 10^{13}X^{11} + 2.28644\cdot 10^{13}X^{10} - 7.03512\cdot 10^{12}X^{9} + 1.66411\cdot 10^{12}X^{8} - 2.97448\cdot 10^{11}X^{7} + 3.92597\cdot 10^{10}X^{6} - 3.70577\cdot 10^{9}X^{5} + 2.38332\cdot 10^{8}X^{4} - 9.64784\cdot 10^{6}X^{3} - 3.66616\cdot 10^{7}X^{2} + 1.33034\cdot 10^{10}X - 6.849\cdot 10^{9}$$
$$= -6.849\cdot 10^{9}B_{0,20} - 6.18383\cdot 10^{9}B_{1,20} - 5.51886\cdot 10^{9}B_{2,20} - 4.85408\cdot 10^{9}B_{3,20} - 4.18947\cdot 10^{9}B_{4,20} - 3.52517\cdot 10^{9}B_{5,20} - 2.86074\cdot 10^{9}B_{6,20} - 2.19722\cdot 10^{9}B_{7,20} - 1.5326\cdot 10^{9}B_{8,20} - 8.70105\cdot 10^{8}B_{9,20} - 2.05292\cdot 10^{8}B_{10,20} + 4.56497\cdot 10^{8}B_{11,20} + 1.12068\cdot 10^{9}B_{12,20} + 1.78277\cdot 10^{9}B_{13,20} + 2.44584\cdot 10^{9}B_{14,20} + 3.10809\cdot 10^{9}B_{15,20} + 3.77042\cdot 10^{9}B_{16,20} + 4.43246\cdot 10^{9}B_{17,20} + 5.09433\cdot 10^{9}B_{18,20} + 5.75601\cdot 10^{9}B_{19,20} + 6.41749\cdot 10^{9}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 759194$.
Bounding polynomials M and m:

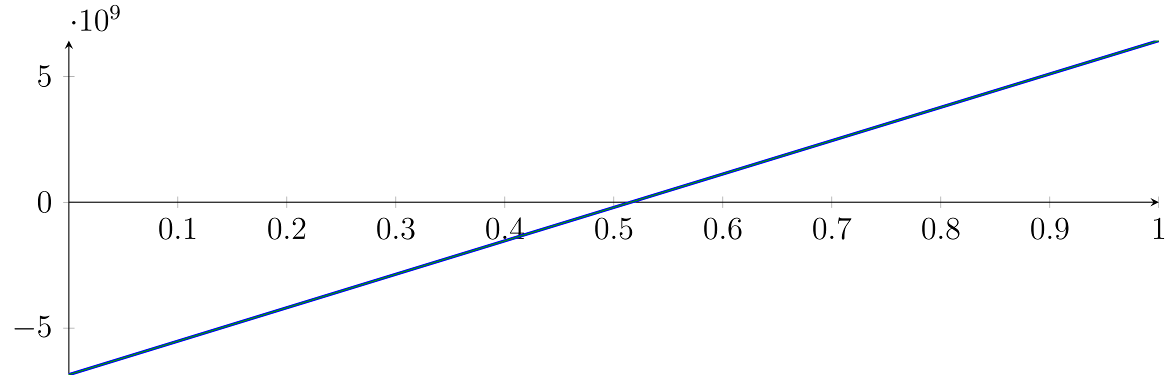
$$M = -3.68792{\cdot}10^{7}X^2 + 1.33034{\cdot}10^{10}X - 6.84824{\cdot}10^{9}$$

$$m = -3.68792{\cdot}10^{7}X^2 + 1.33034{\cdot}10^{10}X - 6.84976{\cdot}10^{9}$$

Root of M and m:

$$N(M) = \{0.515512, 360.213\} \qquad N(m) = \{0.515626, 360.213\}$$

Intersection intervals:



$$[0.515512, 0.515626]$$

Longest intersection interval: 0.000114463
$\Longrightarrow$ Selective recursion: interval 1: $[7.99948, 7.99948]$,

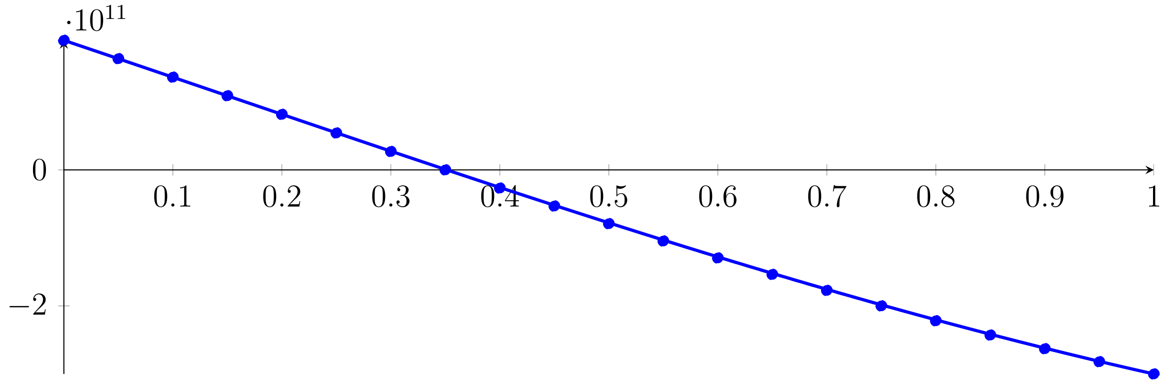
2.44 Recursion Branch 1 1 2 1 2 1 1 1 in Interval 1: $[7.99948, 7.99948]$

Found root in interval $[7.99948, 7.99948]$ at recursion depth 8!

2.45 Recursion Branch 1 1 2 1 2 2 in Interval 2: $[8.8859, 9.21947]$

Normalized monomial und Bézier representations and the Bézier polygon:

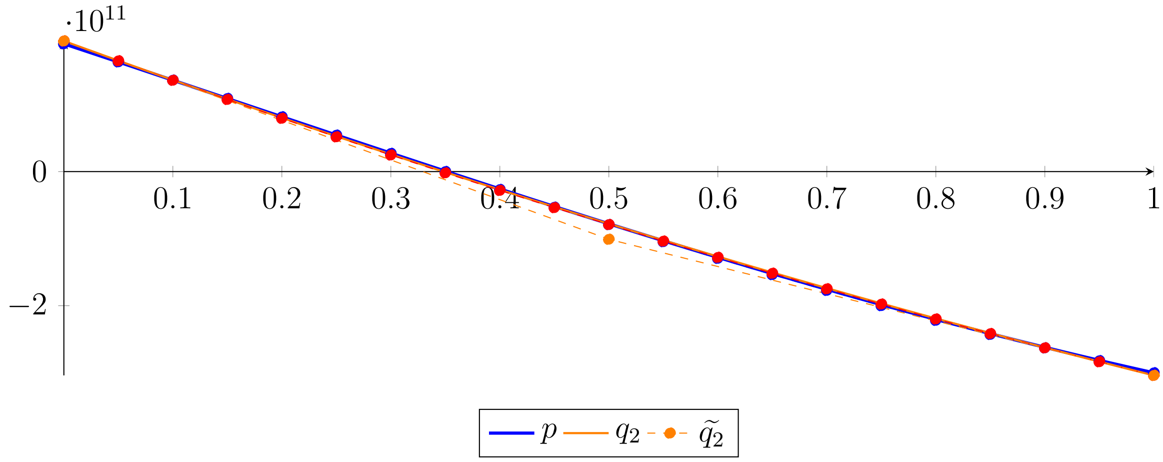
$$\begin{aligned}
p &= 52.8013X^{20} - 806.65X^{19} + 541.89X^{18} - 9437.29X^{17} + 22978.3X^{16} - 11484.2X^{15} \\
&\quad - 3501.27X^{14} - 6907.91X^{13} - 13224.4X^{12} - 7668.1X^{11} + 56743.9X^{10} - 718054X^{9} \\
&\quad - 5.96002{\cdot}10^{6}X^{8} + 8.40113{\cdot}10^{7}X^{7} + 2.36097{\cdot}10^{8}X^{6} - 4.52696{\cdot}10^{9}X^{5} - 1.95228 \\
&\quad {\cdot}10^{9}X^{4} + 9.72141{\cdot}10^{10}X^{3} - 4.21681{\cdot}10^{10}X^{2} - 5.39468{\cdot}10^{11}X + 1.90543{\cdot}10^{11} \\
&= 1.90543{\cdot}10^{11}B_{0,20}(X) + 1.6357{\cdot}10^{11}B_{1,20}(X) + 1.36375{\cdot}10^{11}B_{2,20}(X) + 1.09043 \\
&\quad {\cdot}10^{11}B_{3,20}(X) + 8.16588{\cdot}10^{10}B_{4,20}(X) + 5.43074{\cdot}10^{10}B_{5,20}(X) + 2.70715{\cdot}10^{10}B_{6,20}(X) \\
&\quad + 3.32557{\cdot}10^{7}B_{7,20}(X) - 2.67271{\cdot}10^{10}B_{8,20}(X) - 5.31309{\cdot}10^{10}B_{9,20}(X) - 7.91016 \\
&\quad {\cdot}10^{10}B_{10,20}(X) - 1.04565{\cdot}10^{11}B_{11,20}(X) - 1.29449{\cdot}10^{11}B_{12,20}(X) - 1.53685{\cdot}10^{11}B_{13,20}(X) \\
&\quad - 1.77206{\cdot}10^{11}B_{14,20}(X) - 1.9995{\cdot}10^{11}B_{15,20}(X) - 2.21858{\cdot}10^{11}B_{16,20}(X) - 2.42871 \\
&\quad {\cdot}10^{11}B_{17,20}(X) - 2.62939{\cdot}10^{11}B_{18,20}(X) - 2.82012{\cdot}10^{11}B_{19,20}(X) - 3.00044{\cdot}10^{11}B_{20,20}(X)
\end{aligned}$$



Degree reduction and raising:

$$q_2 = 9.27799 \cdot 10^{10} X^2 - 5.91521 \cdot 10^{11} X + 1.94789 \cdot 10^{11}$$
$$= 1.94789 \cdot 10^{11} B_{0,2} - 1.00971 \cdot 10^{11} B_{1,2} - 3.03952 \cdot 10^{11} B_{2,2}$$

$$\widetilde{q}_2 = 5.56739 \cdot 10^{12} X^{20} - 5.59868 \cdot 10^{13} X^{19} + 2.61448 \cdot 10^{14} X^{18} - 7.51579 \cdot 10^{14} X^{17} + 1.48584 \cdot 10^{15} X^{16} - 2.13691 \cdot 10^{15} X^{15} + 2.30624 \cdot 10^{15} X^{14} - 1.89965 \cdot 10^{15} X^{13} + 1.20374 \cdot 10^{15} X^{12} - 5.87807 \cdot 10^{14} X^{11} + 2.20584 \cdot 10^{14} X^{10} - 6.32266 \cdot 10^{13} X^9 + 1.37273 \cdot 10^{13} X^8 - 2.2405 \cdot 10^{12} X^7 + 2.75642 \cdot 10^{11} X^6 - 2.63214 \cdot 10^{10} X^5 + 2.02947 \cdot 10^9 X^4 - 1.12579 \cdot 10^8 X^3 + 9.27834 \cdot 10^{10} X^2 - 5.91521 \cdot 10^{11} X + 1.94789 \cdot 10^{11}$$
$$= 1.94789 \cdot 10^{11} B_{0,20} + 1.65213 \cdot 10^{11} B_{1,20} + 1.36126 \cdot 10^{11} B_{2,20} + 1.07526 \cdot 10^{11} B_{3,20} + 7.94151 \cdot 10^{10} B_{4,20} + 5.17918 \cdot 10^{10} B_{5,20} + 2.46592 \cdot 10^{10} B_{6,20} - 1.99166 \cdot 10^9 B_{7,20} - 2.81402 \cdot 10^{10} B_{8,20} - 5.38234 \cdot 10^{10} B_{9,20} - 7.89895 \cdot 10^{10} B_{10,20} - 1.03694 \cdot 10^{11} B_{11,20} - 1.27888 \cdot 10^{11} B_{12,20} - 1.51615 \cdot 10^{11} B_{13,20} - 1.74837 \cdot 10^{11} B_{14,20} - 1.9758 \cdot 10^{11} B_{15,20} - 2.1983 \cdot 10^{11} B_{16,20} - 2.41593 \cdot 10^{11} B_{17,20} - 2.62868 \cdot 10^{11} B_{18,20} - 2.83654 \cdot 10^{11} B_{19,20} - 3.03952 \cdot 10^{11} B_{20,20}$$



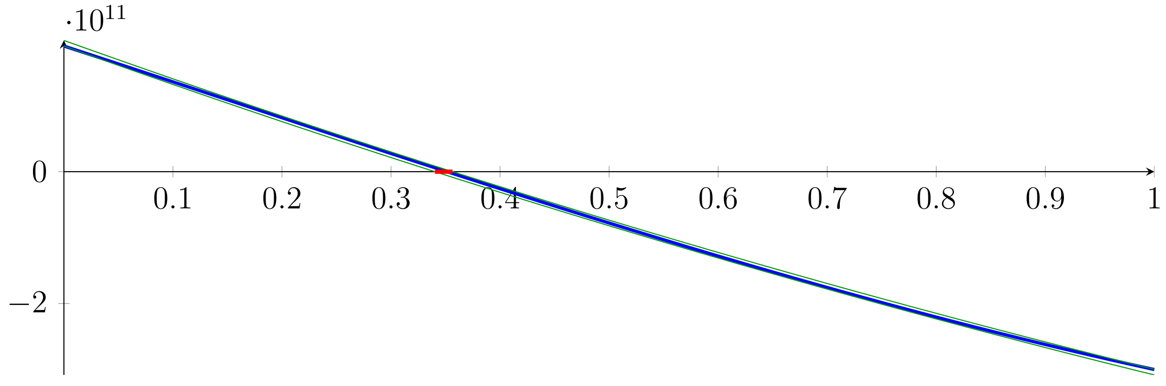
The maximum difference of the Bézier coefficients is $\delta = 4.24611 \cdot 10^9$.
Bounding polynomials M and m:

$$M = 9.27799 \cdot 10^{10} X^2 - 5.91521 \cdot 10^{11} X + 1.99035 \cdot 10^{11}$$
$$m = 9.27799 \cdot 10^{10} X^2 - 5.91521 \cdot 10^{11} X + 1.90543 \cdot 10^{11}$$

Root of M and m:

$$N(M) = \{0.356404, 6.01913\} \qquad N(m) = \{0.340286, 6.03525\}$$

Intersection intervals:



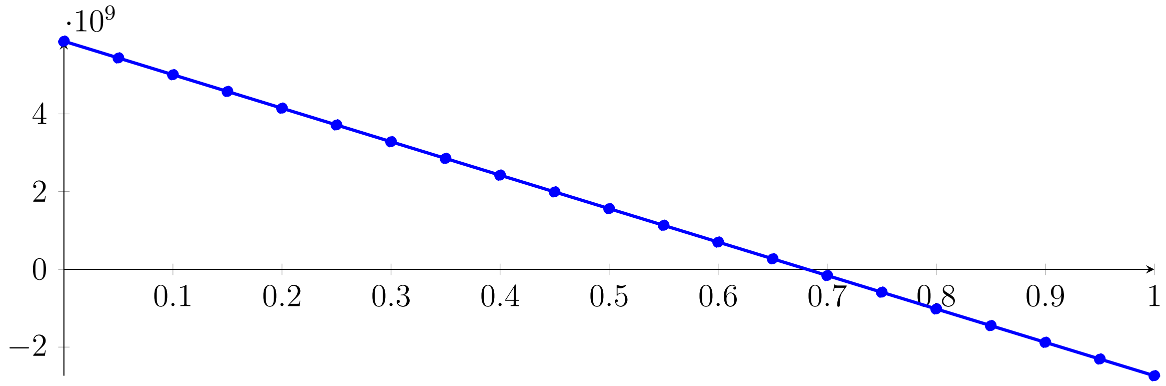
$[0.340286, 0.356404]$

Longest intersection interval: 0.0161178
$\Longrightarrow$ Selective recursion: interval 1: $[8.99941, 9.00478]$,

2.46 Recursion Branch 1 1 2 1 2 2 1 in Interval 1: $[8.99941, 9.00478]$

Normalized monomial und Bézier representations and the Bézier polygon:

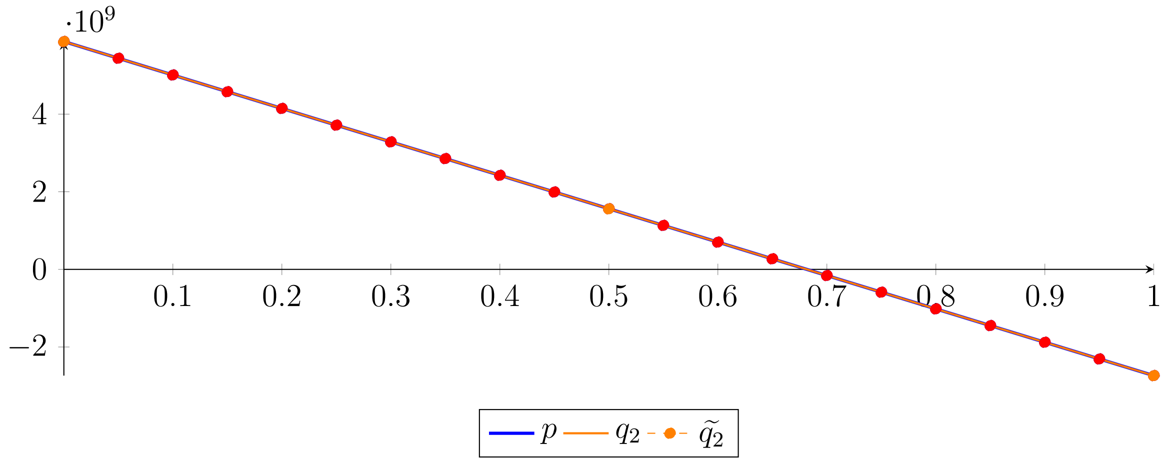
$$\begin{aligned}p &= -2.02995X^{20} + 2.53525X^{19} - 80.2302X^{18} + 196.225X^{17} - 1446.01X^{16} + 1181.32X^{15} - 602.372X^{14} \\ &\quad - 334.158X^{13} - 1645.36X^{12} - 331.891X^{11} - 527.182X^{10} - 93.5446X^{9} + 0.961075X^{8} + 0.591431X^{7} \\ &\quad - 3.10501X^{6} - 4.19916X^{5} - 616.53X^{4} + 374914X^{3} + 1.40255\cdot 10^{7}X^{2} - 8.62304\cdot 10^{9}X + 5.87092\cdot 10^{9} \\ &= 5.87092\cdot 10^{9}B_{0,20}(X) + 5.43977\cdot 10^{9}B_{1,20}(X) + 5.00869\cdot 10^{9}B_{2,20}(X) + 4.57769 \\ &\quad \cdot 10^{9}B_{3,20}(X) + 4.14676\cdot 10^{9}B_{4,20}(X) + 3.7159\cdot 10^{9}B_{5,20}(X) + 3.28512\cdot 10^{9}B_{6,20}(X) \\ &\quad + 2.85442\cdot 10^{9}B_{7,20}(X) + 2.42379\cdot 10^{9}B_{8,20}(X) + 1.99324\cdot 10^{9}B_{9,20}(X) + 1.56276 \\ &\quad \cdot 10^{9}B_{10,20}(X) + 1.13236\cdot 10^{9}B_{11,20}(X) + 7.02041\cdot 10^{8}B_{12,20}(X) + 2.71796\cdot 10^{8}B_{13,20}(X) \\ &\quad - 1.5837\cdot 10^{8}B_{14,20}(X) - 5.88459\cdot 10^{8}B_{15,20}(X) - 1.01847\cdot 10^{9}B_{16,20}(X) - 1.4484 \\ &\quad \cdot 10^{9}B_{17,20}(X) - 1.87825\cdot 10^{9}B_{18,20}(X) - 2.30803\cdot 10^{9}B_{19,20}(X) - 2.73772\cdot 10^{9}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 1.45868\cdot 10^{7}X^{2} - 8.62327\cdot 10^{9}X + 5.87094\cdot 10^{9} \\ &= 5.87094\cdot 10^{9}B_{0,2} + 1.55931\cdot 10^{9}B_{1,2} - 2.73774\cdot 10^{9}B_{2,2}\end{aligned}$$

$$
\begin{aligned}
\widetilde{q}_2 = &-6.13065\cdot 10^{11}X^{20} + 6.14918\cdot 10^{12}X^{19} - 2.85498\cdot 10^{13}X^{18} + 8.14132\cdot 10^{13}X^{17} - 1.59553\cdot 10^{14}X^{16} \\
&+ 2.27844\cdot 10^{14}X^{15} - 2.45283\cdot 10^{14}X^{14} + 2.0316\cdot 10^{14}X^{13} - 1.30964\cdot 10^{14}X^{12} + 6.60276\cdot 10^{13}X^{11} \\
&- 2.60142\cdot 10^{13}X^{10} + 7.96026\cdot 10^{12}X^{9} - 1.87059\cdot 10^{12}X^{8} + 3.32265\cdot 10^{11}X^{7} - 4.37333\cdot 10^{10}X^{6} \\
&+ 4.15725\cdot 10^{9}X^{5} - 2.74851\cdot 10^{8}X^{4} + 1.17302\cdot 10^{7}X^{3} + 1.43028\cdot 10^{7}X^{2} - 8.62326\cdot 10^{9}X + 5.87094\cdot 10^{9} \\
= &\ 5.87094\cdot 10^{9}B_{0,20} + 5.43978\cdot 10^{9}B_{1,20} + 5.00869\cdot 10^{9}B_{2,20} + 4.57769\cdot 10^{9}B_{3,20} + 4.14672 \\
&\cdot 10^{9}B_{4,20} + 3.71596\cdot 10^{9}B_{5,20} + 3.28493\cdot 10^{9}B_{6,20} + 2.85477\cdot 10^{9}B_{7,20} + 2.42321\cdot 10^{9}B_{8,20} \\
&+ 1.99396\cdot 10^{9}B_{9,20} + 1.56189\cdot 10^{9}B_{10,20} + 1.13308\cdot 10^{9}B_{11,20} + 7.01498\cdot 10^{8}B_{12,20} \\
&+ 2.72127\cdot 10^{8}B_{13,20} - 1.58532\cdot 10^{8}B_{14,20} - 5.88378\cdot 10^{8}B_{15,20} - 1.01848\cdot 10^{9}B_{16,20} \\
&- 1.44839\cdot 10^{9}B_{17,20} - 1.87825\cdot 10^{9}B_{18,20} - 2.30803\cdot 10^{9}B_{19,20} - 2.73774\cdot 10^{9}B_{20,20}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 867155$.

Bounding polynomials M and m:

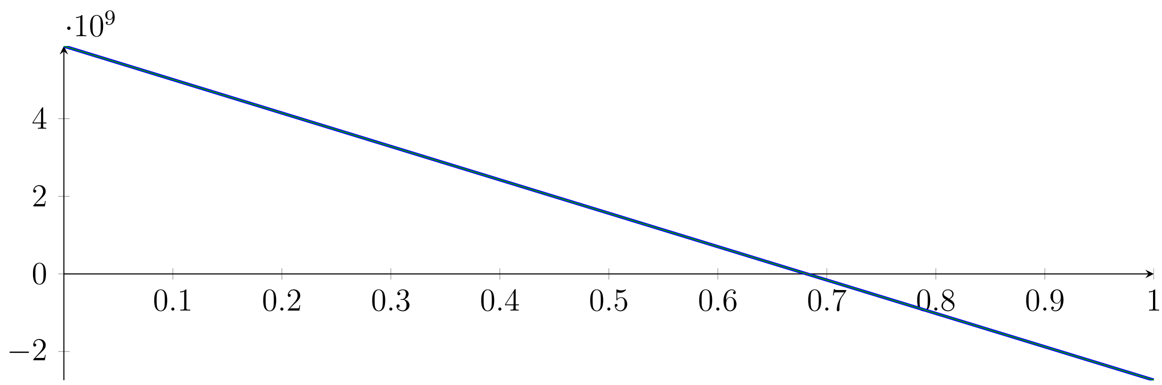
$$M = 1.45868\cdot 10^{7}X^2 - 8.62327\cdot 10^{9}X + 5.87181\cdot 10^{9}$$

$$m = 1.45868\cdot 10^{7}X^2 - 8.62327\cdot 10^{9}X + 5.87007\cdot 10^{9}$$

Root of M and m:

$$N(M) = \{0.681712, 590.487\} \qquad N(m) = \{0.681511, 590.487\}$$

Intersection intervals:



$$[0.681511, 0.681712]$$

Longest intersection interval: 0.000201585

$\Longrightarrow$ Selective recursion: interval 1: $[9.00307, 9.00307]$,

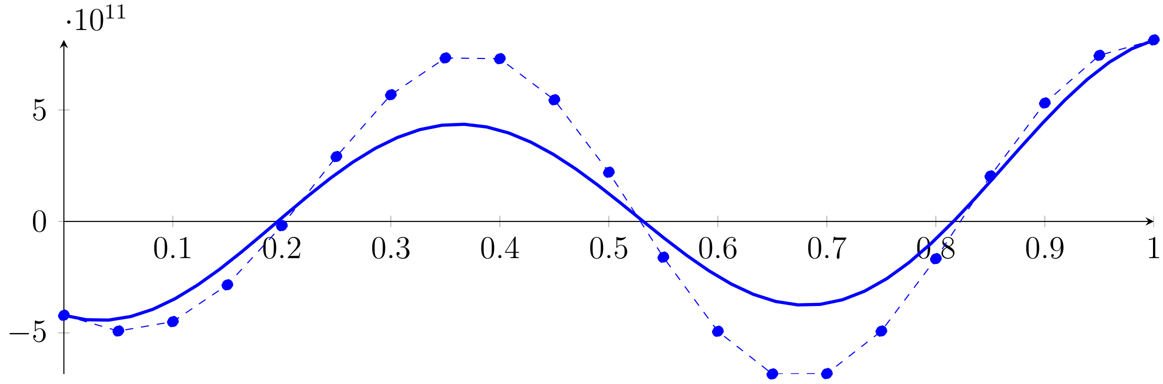
2.47 Recursion Branch 1 1 2 1 2 2 1 1 in Interval 1: $[9.00307, 9.00307]$

Found root in interval $[9.00307, 9.00307]$ at recursion depth 8!

2.48 Recursion Branch 1 1 2 2 on the Second Half $[9.375, 12.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

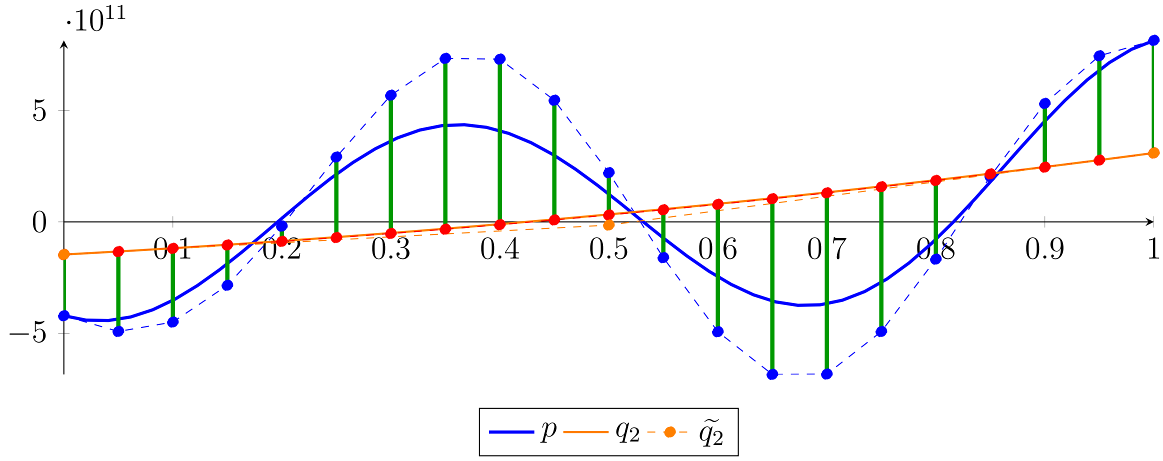
$$p = 7.88861{\cdot}10^{9}X^{20} - 5.6798{\cdot}10^{10}X^{19} - 7.43423{\cdot}10^{10}X^{18} + 1.32089{\cdot}10^{12}X^{17} - 9.3169{\cdot}10^{11}X^{16} - 1.21266{\cdot}10^{13}X^{15} + 1.72866{\cdot}10^{13}X^{14} + 5.61608{\cdot}10^{13}X^{13} - 1.04782{\cdot}10^{14}X^{12} - 1.38659{\cdot}10^{14}X^{11} + 3.15838{\cdot}10^{14}X^{10} + 1.75102{\cdot}10^{14}X^{9} - 5.05882{\cdot}10^{14}X^{8} - 9.20246{\cdot}10^{13}X^{7} + 4.17973{\cdot}10^{14}X^{6} - 4.84112{\cdot}10^{11}X^{5} - 1.59085{\cdot}10^{14}X^{4} + 1.16549{\cdot}10^{13}X^{3} + 2.14084{\cdot}10^{13}X^{2} - 1.41201{\cdot}10^{12}X - 4.20945{\cdot}10^{11}$$
$$= -4.20945{\cdot}10^{11}B_{0,20}(X) - 4.91545{\cdot}10^{11}B_{1,20}(X) - 4.4947{\cdot}10^{11}B_{2,20}(X) - 2.84495{\cdot}10^{11}B_{3,20}(X) - 1.92322{\cdot}10^{10}B_{4,20}(X) + 2.90841{\cdot}10^{11}B_{5,20}(X) + 5.68134{\cdot}10^{11}B_{6,20}(X) + 7.3329{\cdot}10^{11}B_{7,20}(X) + 7.29931{\cdot}10^{11}B_{8,20}(X) + 5.45616{\cdot}10^{11}B_{9,20}(X) + 2.20619{\cdot}10^{11}B_{10,20}(X) - 1.60453{\cdot}10^{11}B_{11,20}(X) - 4.92917{\cdot}10^{11}B_{12,20}(X) - 6.84241{\cdot}10^{11}B_{13,20}(X) - 6.82665{\cdot}10^{11}B_{14,20}(X) - 4.91903{\cdot}10^{11}B_{15,20}(X) - 1.67279{\cdot}10^{11}B_{16,20}(X) + 2.0413{\cdot}10^{11}B_{17,20}(X) + 5.31271{\cdot}10^{11}B_{18,20}(X) + 7.44977{\cdot}10^{11}B_{19,20}(X) + 8.1419{\cdot}10^{11}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 1.91358{\cdot}10^{11}X^{2} + 2.64143{\cdot}10^{11}X - 1.46436{\cdot}10^{11}$$
$$= -1.46436{\cdot}10^{11}B_{0,2} - 1.43643{\cdot}10^{10}B_{1,2} + 3.09065{\cdot}10^{11}B_{2,2}$$

$$\widetilde{q}_2 = 3.3058{\cdot}10^{13}X^{20} - 3.30426{\cdot}10^{14}X^{19} + 1.52792{\cdot}10^{15}X^{18} - 4.3373{\cdot}10^{15}X^{17} + 8.45821{\cdot}10^{15}X^{16} - 1.20156{\cdot}10^{16}X^{15} + 1.28664{\cdot}10^{16}X^{14} - 1.06009{\cdot}10^{16}X^{13} + 6.80008{\cdot}10^{15}X^{12} - 3.41366{\cdot}10^{15}X^{11} + 1.34056{\cdot}10^{15}X^{10} - 4.09458{\cdot}10^{14}X^{9} + 9.61689{\cdot}10^{13}X^{8} - 1.70617{\cdot}10^{13}X^{7} + 2.22825{\cdot}10^{12}X^{6} - 2.06427{\cdot}10^{11}X^{5} + 1.28274{\cdot}10^{10}X^{4} - 4.89934{\cdot}10^{8}X^{3} + 1.91368{\cdot}10^{11}X^{2} + 2.64143{\cdot}10^{11}X - 1.46436{\cdot}10^{11}$$
$$= -1.46436{\cdot}10^{11}B_{0,20} - 1.33229{\cdot}10^{11}B_{1,20} - 1.19014{\cdot}10^{11}B_{2,20} - 1.03793{\cdot}10^{11}B_{3,20} - 8.75632{\cdot}10^{10}B_{4,20} - 7.03325{\cdot}10^{10}B_{5,20} - 5.20763{\cdot}10^{10}B_{6,20} - 3.28543{\cdot}10^{10}B_{7,20} - 1.25491{\cdot}10^{10}B_{8,20} + 8.64623{\cdot}10^{9}B_{9,20} + 3.10019{\cdot}10^{10}B_{10,20} + 5.41937{\cdot}10^{10}B_{11,20} + 7.8551{\cdot}10^{10}B_{12,20} + 1.03797{\cdot}10^{11}B_{13,20} + 1.30124{\cdot}10^{11}B_{14,20} + 1.57418{\cdot}10^{11}B_{15,20} + 1.85737{\cdot}10^{11}B_{16,20} + 2.15058{\cdot}10^{11}B_{17,20} + 2.45387{\cdot}10^{11}B_{18,20} + 2.76722{\cdot}10^{11}B_{19,20} + 3.09065{\cdot}10^{11}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 8.12788 \cdot 10^{11}$.

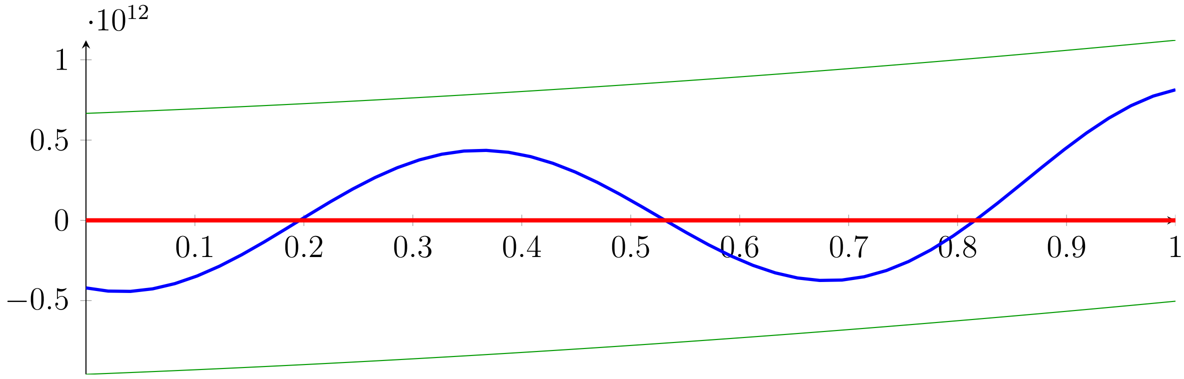
Bounding polynomials M and m:

$$M = 1.91358 \cdot 10^{11} X^2 + 2.64143 \cdot 10^{11} X + 6.66352 \cdot 10^{11}$$
$$m = 1.91358 \cdot 10^{11} X^2 + 2.64143 \cdot 10^{11} X - 9.59224 \cdot 10^{11}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{-3.03306, 1.6527\}$$

Intersection intervals:



$[0, 1]$

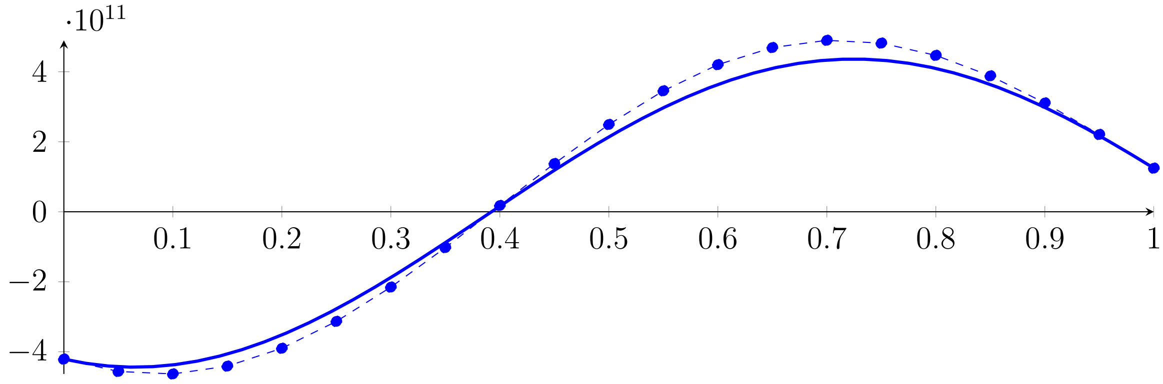
Longest intersection interval: 1

$\Longrightarrow$ Bisection: first half $[9.375, 10.9375]$ und second half $[10.9375, 12.5]$

2.49 Recursion Branch 1 1 2 2 1 on the First Half $[9.375, 10.9375]$

Normalized monomial und Bézier representations and the Bézier polygon:

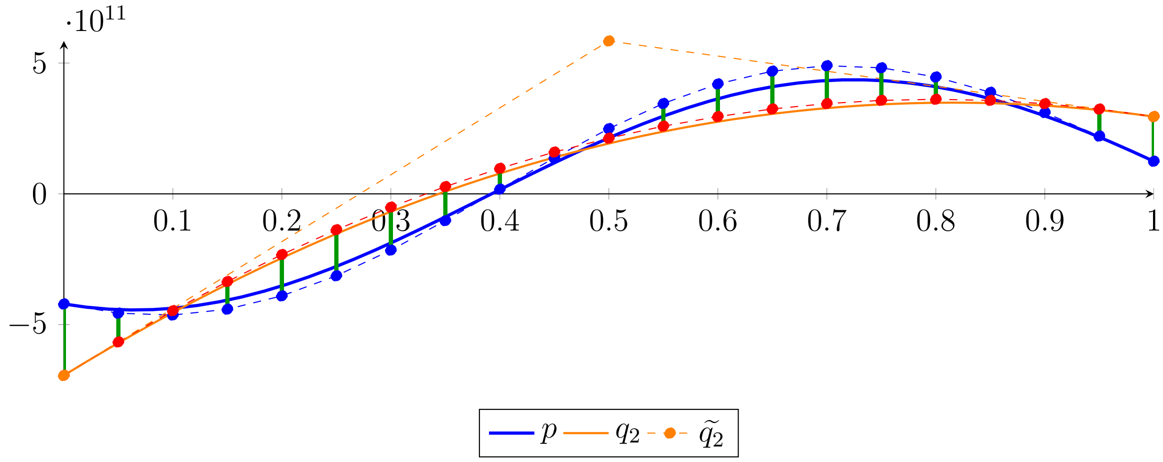
$$\begin{aligned} p = {} & 7412.69X^{20} - 105681X^{19} - 281108X^{18} + 1.01062 \cdot 10^{7} X^{17} - 1.42567 \cdot 10^{7} X^{16} - 3.70075 \cdot 10^{8} X^{15} \\ & + 1.05512 \cdot 10^{9} X^{14} + 6.8556 \cdot 10^{9} X^{13} - 2.55814 \cdot 10^{10} X^{12} - 6.77046 \cdot 10^{10} X^{11} + 3.08436 \cdot 10^{11} X^{10} \\ & + 3.41997 \cdot 10^{11} X^{9} - 1.9761 \cdot 10^{12} X^{8} - 7.18943 \cdot 10^{11} X^{7} + 6.53083 \cdot 10^{12} X^{6} - 1.51285 \cdot 10^{10} X^{5} \\ & - 9.94282 \cdot 10^{12} X^{4} + 1.45686 \cdot 10^{12} X^{3} + 5.3521 \cdot 10^{12} X^{2} - 7.06004 \cdot 10^{11} X - 4.20945 \cdot 10^{11} \\ = {} & -4.20945 \cdot 10^{11} B_{0,20}(X) - 4.56245 \cdot 10^{11} B_{1,20}(X) - 4.63376 \cdot 10^{11} B_{2,20}(X) - 4.4106 \\ & \cdot 10^{11} B_{3,20}(X) - 3.90072 \cdot 10^{11} B_{4,20}(X) - 3.13239 \cdot 10^{11} B_{5,20}(X) - 2.15273 \cdot 10^{11} B_{6,20}(X) \\ & - 1.02447 \cdot 10^{11} B_{7,20}(X) + 1.78698 \cdot 10^{10} B_{8,20}(X) + 1.37766 \cdot 10^{11} B_{9,20}(X) + 2.49392 \\ & \cdot 10^{11} B_{10,20}(X) + 3.45561 \cdot 10^{11} B_{11,20}(X) + 4.2028 \cdot 10^{11} B_{12,20}(X) + 4.69189 \cdot 10^{11} B_{13,20}(X) \\ & + 4.89846 \cdot 10^{11} B_{14,20}(X) + 4.81857 \cdot 10^{11} B_{15,20}(X) + 4.46838 \cdot 10^{11} B_{16,20}(X) + 3.88213 \\ & \cdot 10^{11} B_{17,20}(X) + 3.10886 \cdot 10^{11} B_{18,20}(X) + 2.20816 \cdot 10^{11} B_{19,20}(X) + 1.24532 \cdot 10^{11} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = -1.56779{\cdot}10^{12}X^2 + 2.5572{\cdot}10^{12}X - 6.9408{\cdot}10^{11}$$
$$= -6.9408{\cdot}10^{11}B_{0,2} + 5.84521{\cdot}10^{11}B_{1,2} + 2.95329{\cdot}10^{11}B_{2,2}$$

$$\widetilde{q}_2 = -1.74893{\cdot}10^{14}X^{20} + 1.74892{\cdot}10^{15}X^{19} - 8.09584{\cdot}10^{15}X^{18} + 2.30164{\cdot}10^{16}X^{17} - 4.49595{\cdot}10^{16}X^{16} + 6.39565{\cdot}10^{16}X^{15} - 6.85186{\cdot}10^{16}X^{14} + 5.63918{\cdot}10^{16}X^{13} - 3.60499{\cdot}10^{16}X^{12} + 1.79843{\cdot}10^{16}X^{11} - 6.9974{\cdot}10^{15}X^{10} + 2.11202{\cdot}10^{15}X^{9} - 4.8938{\cdot}10^{14}X^{8} + 8.56523{\cdot}10^{13}X^{7} - 1.10673{\cdot}10^{13}X^{6} + 1.02401{\cdot}10^{12}X^{5} - 6.50318{\cdot}10^{10}X^{4} + 2.62171{\cdot}10^{9}X^{3} - 1.56785{\cdot}10^{12}X^{2} + 2.5572{\cdot}10^{12}X - 6.9408{\cdot}10^{11}$$
$$= -6.9408{\cdot}10^{11}B_{0,20} - 5.6622{\cdot}10^{11}B_{1,20} - 4.46612{\cdot}10^{11}B_{2,20} - 3.35253{\cdot}10^{11}B_{3,20} - 2.32155{\cdot}10^{11}B_{4,20} - 1.37276{\cdot}10^{11}B_{5,20} - 5.07417{\cdot}10^{10}B_{6,20} + 2.77558{\cdot}10^{10}B_{7,20} + 9.75964{\cdot}10^{10}B_{8,20} + 1.59821{\cdot}10^{11}B_{9,20} + 2.12968{\cdot}10^{11}B_{10,20} + 2.58754{\cdot}10^{11}B_{11,20} + 2.95476{\cdot}10^{11}B_{12,20} + 3.24581{\cdot}10^{11}B_{13,20} + 3.45021{\cdot}10^{11}B_{14,20} + 3.57431{\cdot}10^{11}B_{15,20} + 3.6149{\cdot}10^{11}B_{16,20} + 3.57334{\cdot}10^{11}B_{17,20} + 3.44916{\cdot}10^{11}B_{18,20} + 3.24248{\cdot}10^{11}B_{19,20} + 2.95329{\cdot}10^{11}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 2.73136{\cdot}10^{11}$.

Bounding polynomials M and m:

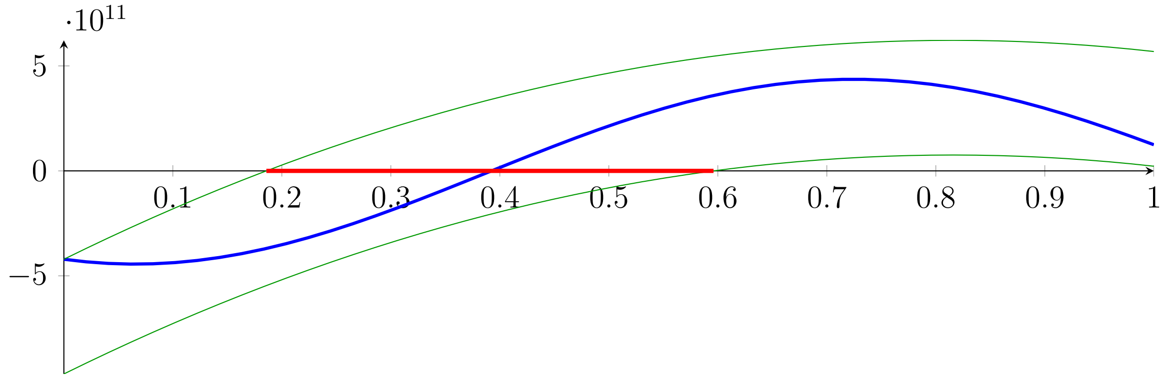
$$M = -1.56779{\cdot}10^{12}X^2 + 2.5572{\cdot}10^{12}X - 4.20945{\cdot}10^{11}$$

$$m = -1.56779{\cdot}10^{12}X^2 + 2.5572{\cdot}10^{12}X - 9.67216{\cdot}10^{11}$$

Root of M and m:

$$N(M) = \{0.185769, 1.44531\} \qquad N(m) = \{0.596041, 1.03504\}$$

Intersection intervals:



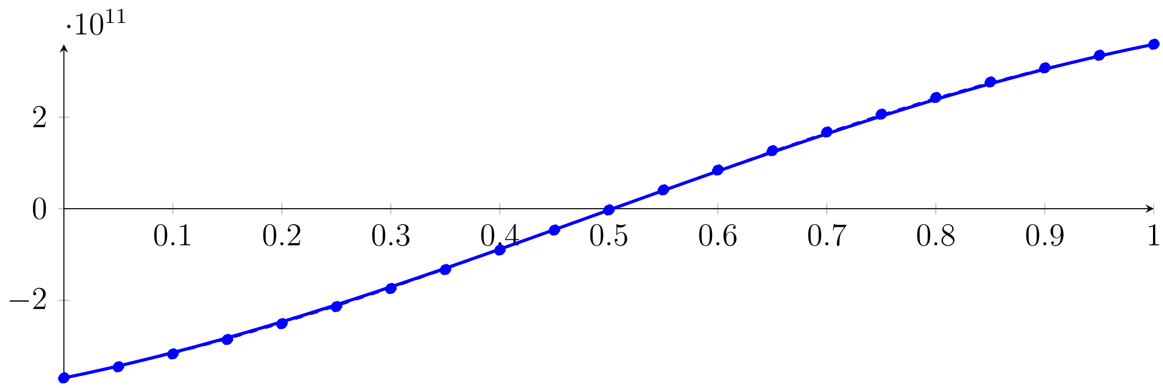
$[0.185769, 0.596041]$

Longest intersection interval: 0.410272
$\Longrightarrow$ Selective recursion: interval 1: $[9.66526, 10.3063]$,

2.50 Recursion Branch 1 1 2 2 1 1 in Interval 1: $[9.66526, 10.3063]$

Normalized monomial und Bézier representations and the Bézier polygon:

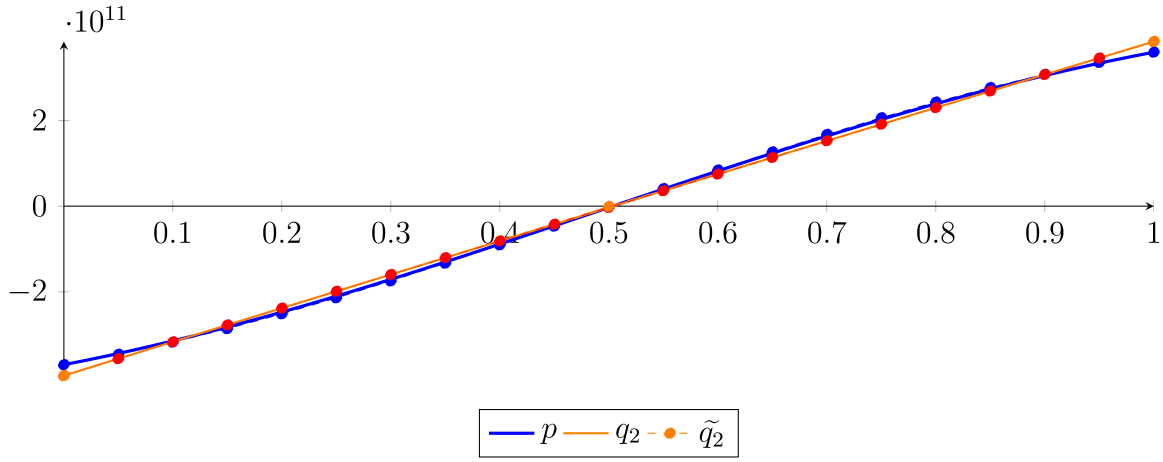
$$\begin{aligned} p &= 45.2096X^{20} + 593.445X^{19} + 3315.84X^{18} - 506.125X^{17} + 46243.4X^{16} - 46339.8X^{15} \\ &\quad + 28525.9X^{14} + 97382.6X^{13} - 72319X^{12} - 5.76495\cdot 10^{6}X^{11} + 1.69742\cdot 10^{7}X^{10} + 2.49191 \\ &\quad \cdot 10^{8}X^{9} - 8.10732\cdot 10^{8}X^{8} - 5.91706\cdot 10^{9}X^{7} + 1.87899\cdot 10^{10}X^{6} + 7.09169\cdot 10^{10}X^{5} \\ &\quad - 1.95218\cdot 10^{11}X^{4} - 3.55807\cdot 10^{11}X^{3} + 7.09557\cdot 10^{11}X^{2} + 4.86838\cdot 10^{11}X - 3.69641\cdot 10^{11} \\ &= -3.69641\cdot 10^{11}B_{0,20}(X) - 3.45299\cdot 10^{11}B_{1,20}(X) - 3.17223\cdot 10^{11}B_{2,20}(X) - 2.85724 \\ &\quad \cdot 10^{11}B_{3,20}(X) - 2.51155\cdot 10^{11}B_{4,20}(X) - 2.13905\cdot 10^{11}B_{5,20}(X) - 1.74391\cdot 10^{11}B_{6,20}(X) \\ &\quad - 1.33058\cdot 10^{11}B_{7,20}(X) - 9.03695\cdot 10^{10}B_{8,20}(X) - 4.68019\cdot 10^{10}B_{9,20}(X) - 2.8392 \\ &\quad \cdot 10^{9}B_{10,20}(X) + 4.10336\cdot 10^{10}B_{11,20}(X) + 8.4337\cdot 10^{10}B_{12,20}(X) + 1.26602\cdot 10^{11}B_{13,20}(X) \\ &\quad + 1.67379\cdot 10^{11}B_{14,20}(X) + 2.06238\cdot 10^{11}B_{15,20}(X) + 2.42778\cdot 10^{11}B_{16,20}(X) + 2.76632 \\ &\quad \cdot 10^{11}B_{17,20}(X) + 3.07469\cdot 10^{11}B_{18,20}(X) + 3.34996\cdot 10^{11}B_{19,20}(X) + 3.58967\cdot 10^{11}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -9.92798\cdot 10^{9}X^{2} + 7.88899\cdot 10^{11}X - 3.95139\cdot 10^{11} \\ &= -3.95139\cdot 10^{11}B_{0,2} - 6.89461\cdot 10^{8}B_{1,2} + 3.83832\cdot 10^{11}B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 2.93136\cdot10^{13}X^{20} - 2.93628\cdot10^{14}X^{19} + 1.36068\cdot10^{15}X^{18} - 3.87112\cdot10^{15}X^{17} + 7.56733\cdot10^{15}X^{16}\\ &\quad - 1.07801\cdot10^{16}X^{15} + 1.15835\cdot10^{16}X^{14} - 9.58637\cdot10^{15}X^{13} + 6.18424\cdot10^{15}X^{12} - 3.12626\cdot10^{15}X^{11}\\ &\quad + 1.23762\cdot10^{15}X^{10} - 3.81248\cdot10^{14}X^{9} + 9.03007\cdot10^{13}X^{8} - 1.61612\cdot10^{13}X^{7} + 2.13493\cdot10^{12}X^{6} - 2.01458\\ &\quad \cdot10^{11}X^{5} + 1.29191\cdot10^{10}X^{4} - 5.19674\cdot10^{8}X^{3} - 9.91636\cdot10^{9}X^{2} + 7.88899\cdot10^{11}X - 3.95139\cdot10^{11}\\ &= -3.95139\cdot10^{11}B_{0,20} - 3.55694\cdot10^{11}B_{1,20} - 3.16301\cdot10^{11}B_{2,20} - 2.76961\cdot10^{11}B_{3,20} - 2.37672\\ &\quad \cdot10^{11}B_{4,20} - 1.9844\cdot10^{11}B_{5,20} - 1.59244\cdot10^{11}B_{6,20} - 1.20139\cdot10^{11}B_{7,20} - 8.1016\cdot10^{10}B_{8,20}\\ &\quad - 4.20496\cdot10^{10}B_{9,20} - 2.99975\cdot10^{9}B_{10,20} + 3.58445\cdot10^{10}B_{11,20} + 7.47771\cdot10^{10}B_{12,20}\\ &\quad + 1.13555\cdot10^{11}B_{13,20} + 1.52343\cdot10^{11}B_{14,20} + 1.91046\cdot10^{11}B_{15,20} + 2.29711\cdot10^{11}B_{16,20}\\ &\quad + 2.68319\cdot10^{11}B_{17,20} + 3.06876\cdot10^{11}B_{18,20} + 3.4538\cdot10^{11}B_{19,20} + 3.83832\cdot10^{11}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.54979\cdot10^{10}$.

Bounding polynomials M and m:

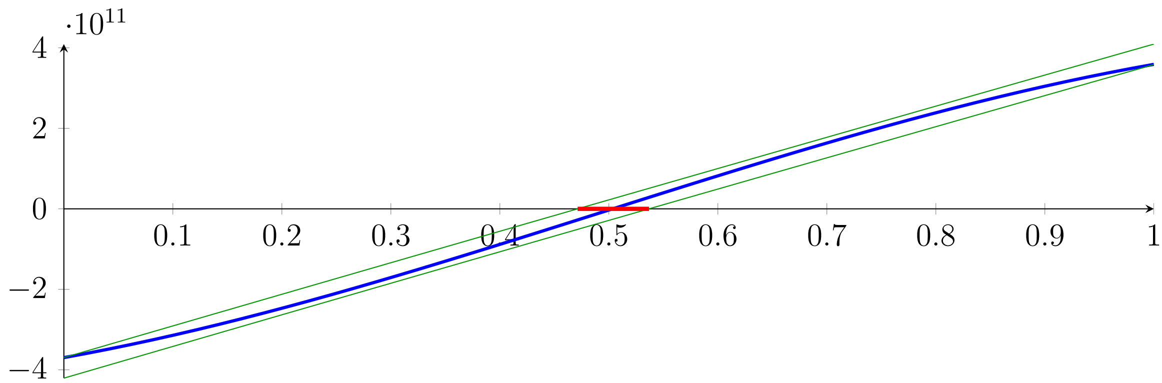
$$M = -9.92798\cdot10^{9}X^2 + 7.88899\cdot10^{11}X - 3.69641\cdot10^{11}$$

$$m = -9.92798\cdot10^{9}X^2 + 7.88899\cdot10^{11}X - 4.20637\cdot10^{11}$$

Root of M and m:

$$N(M) = \{0.471349, 78.9909\} \qquad N(m) = \{0.536821, 78.9254\}$$

Intersection intervals:



$$[0.471349, 0.536821]$$

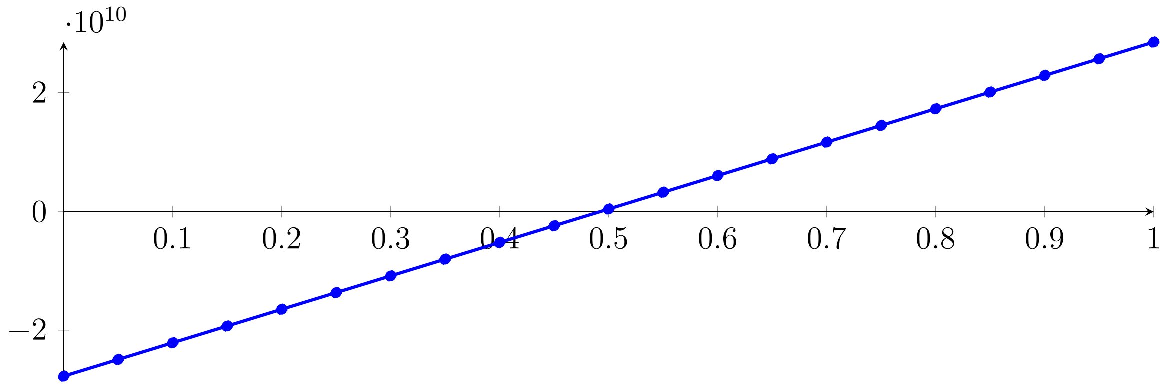
Longest intersection interval: 0.0654724

$\Longrightarrow$ Selective recursion: interval 1: $[9.96742, 10.0094]$,

2.51 Recursion Branch 1 1 2 2 1 1 1 in Interval 1: $[9.96742, 10.0094]$

Normalized monomial und Bézier representations and the Bézier polygon:

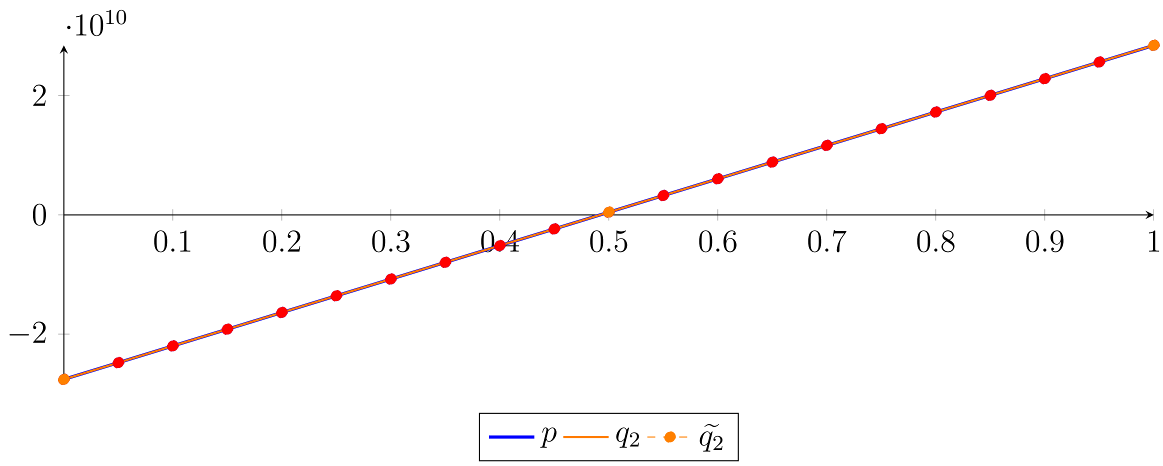
$$p = 2.51187X^{20} + 41.1164X^{19} + 199.561X^{18} + 52.1546X^{17} + 2674.87X^{16} - 2582.84X^{15} + 2061.58X^{14} + 1455.22X^{13} + 5370.01X^{12} + 1558.86X^{11} + 1995.26X^{10} + 390.196X^{9} + 11.0524X^{8} - 35.1901X^{7} - 261.56X^{6} + 112264X^{5} + 198526X^{4} - 1.50994\cdot 10^{8}X^{3} + 1.34099\cdot 10^{8}X^{2} + 5.60743\cdot 10^{10}X - 2.76007\cdot 10^{10}$$
$$= -2.76007\cdot 10^{10}B_{0,20}(X) - 2.4797\cdot 10^{10}B_{1,20}(X) - 2.19925\cdot 10^{10}B_{2,20}(X) - 1.91875\cdot 10^{10}B_{3,20}(X) - 1.63821\cdot 10^{10}B_{4,20}(X) - 1.35764\cdot 10^{10}B_{5,20}(X) - 1.07704\cdot 10^{10}B_{6,20}(X) - 7.96448\cdot 10^{9}B_{7,20}(X) - 5.1586\cdot 10^{9}B_{8,20}(X) - 2.35295\cdot 10^{9}B_{9,20}(X) + 4.52354\cdot 10^{8}B_{10,20}(X) + 3.25717\cdot 10^{9}B_{11,20}(X) + 6.06138\cdot 10^{9}B_{12,20}(X) + 8.86483\cdot 10^{9}B_{13,20}(X) + 1.16674\cdot 10^{10}B_{14,20}(X) + 1.4469\cdot 10^{10}B_{15,20}(X) + 1.72694\cdot 10^{10}B_{16,20}(X) + 2.00685\cdot 10^{10}B_{17,20}(X) + 2.28663\cdot 10^{10}B_{18,20}(X) + 2.56625\cdot 10^{10}B_{19,20}(X) + 2.8457\cdot 10^{10}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = -9.18514\cdot 10^{7}X^{2} + 5.61646\cdot 10^{10}X - 2.76082\cdot 10^{10}$$
$$= -2.76082\cdot 10^{10}B_{0,2} + 4.74104\cdot 10^{8}B_{1,2} + 2.84645\cdot 10^{10}B_{2,2}$$

$$\widetilde{q}_2 = 2.05692\cdot 10^{12}X^{20} - 2.05983\cdot 10^{13}X^{19} + 9.54226\cdot 10^{13}X^{18} - 2.71374\cdot 10^{14}X^{17} + 5.3027\cdot 10^{14}X^{16} - 7.55088\cdot 10^{14}X^{15} + 8.11041\cdot 10^{14}X^{14} - 6.7099\cdot 10^{14}X^{13} + 4.32767\cdot 10^{14}X^{12} - 2.18758\cdot 10^{14}X^{11} + 8.66111\cdot 10^{13}X^{10} - 2.66885\cdot 10^{13}X^{9} + 6.32406\cdot 10^{12}X^{8} - 1.13225\cdot 10^{12}X^{7} + 1.49546\cdot 10^{11}X^{6} - 1.40876\cdot 10^{10}X^{5} + 8.99004\cdot 10^{8}X^{4} - 3.58279\cdot 10^{7}X^{3} - 9.10613\cdot 10^{7}X^{2} + 5.61646\cdot 10^{10}X - 2.76082\cdot 10^{10}$$
$$= -2.76082\cdot 10^{10}B_{0,20} - 2.48\cdot 10^{10}B_{1,20} - 2.19922\cdot 10^{10}B_{2,20} - 1.9185\cdot 10^{10}B_{3,20} - 1.63781\cdot 10^{10}B_{4,20} - 1.35721\cdot 10^{10}B_{5,20} - 1.07654\cdot 10^{10}B_{6,20} - 7.96194\cdot 10^{9}B_{7,20} - 5.15404\cdot 10^{9}B_{8,20} - 2.35392\cdot 10^{9}B_{9,20} + 4.55223\cdot 10^{8}B_{10,20} + 3.25312\cdot 10^{9}B_{11,20} + 6.06043\cdot 10^{9}B_{12,20} + 8.86002\cdot 10^{9}B_{13,20} + 1.16636\cdot 10^{10}B_{14,20} + 1.44643\cdot 10^{10}B_{15,20} + 1.72655\cdot 10^{10}B_{16,20} + 2.00659\cdot 10^{10}B_{17,20} + 2.2866\cdot 10^{10}B_{18,20} + 2.56655\cdot 10^{10}B_{19,20} + 2.84645\cdot 10^{10}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 7.52066 \cdot 10^6$.

Bounding polynomials M and m:

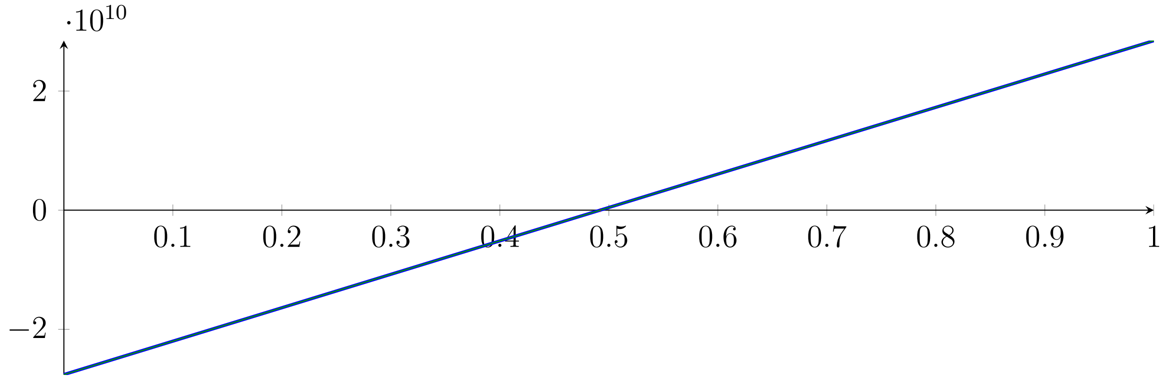
$$M = -9.18514 \cdot 10^7 X^2 + 5.61646 \cdot 10^{10} X - 2.76007 \cdot 10^{10}$$

$$m = -9.18514 \cdot 10^7 X^2 + 5.61646 \cdot 10^{10} X - 2.76157 \cdot 10^{10}$$

Root of M and m:

$$N(M) = \{0.49182, 610.981\} \qquad N(m) = \{0.492089, 610.98\}$$

Intersection intervals:



$$[0.49182, 0.492089]$$

Longest intersection interval: 0.000268239

$\Longrightarrow$ Selective recursion: interval 1: $[9.98806, 9.98808]$,

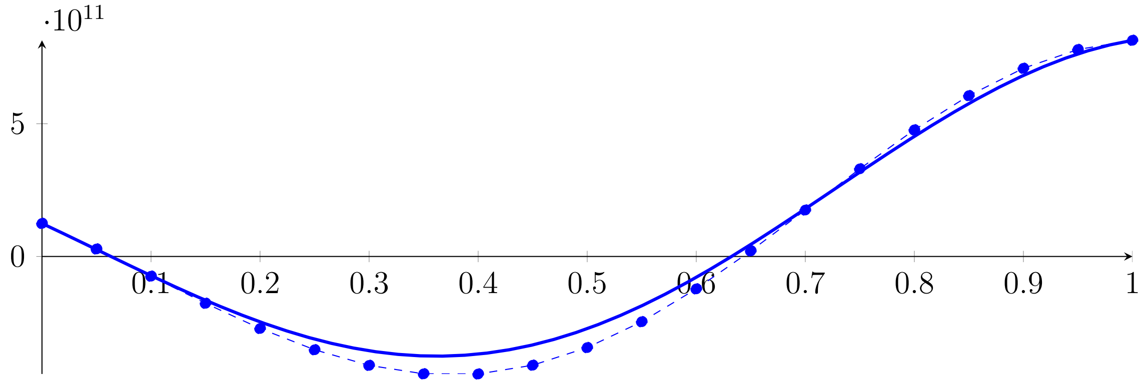
2.52 Recursion Branch 1 1 2 2 1 1 1 1 in Interval 1: $[9.98806, 9.98808]$

Found root in interval $[9.98806, 9.98808]$ at recursion depth 8!

2.53 Recursion Branch 1 1 2 2 2 on the Second Half $[10.9375, 12.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

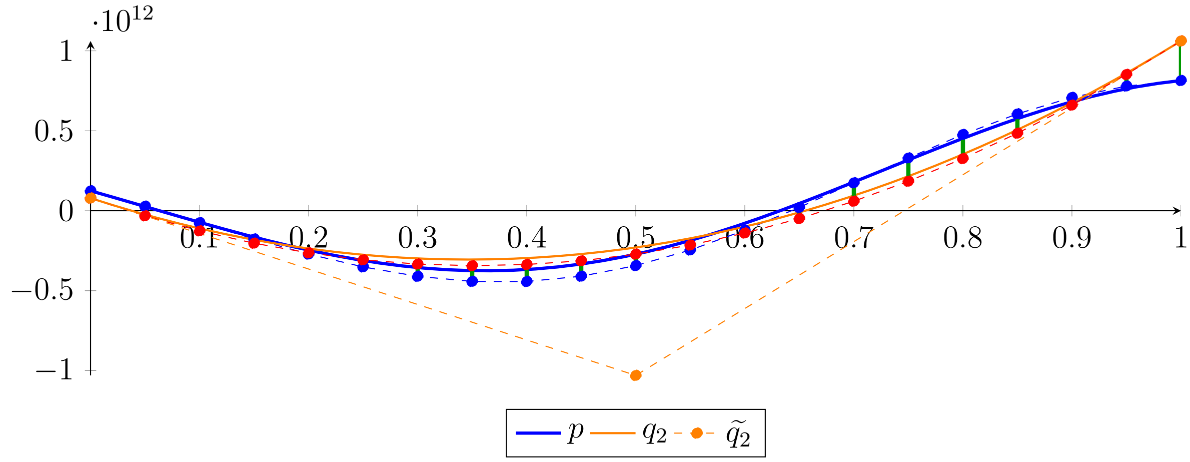
$$\begin{aligned} p = {} & 7874.96X^{20} + 41504.1X^{19} - 902121X^{18} - 5.01703 \cdot 10^6 X^{17} + 4.54393 \cdot 10^7 X^{16} + 2.38133 \cdot 10^8 X^{15} \\ & - 1.18503 \cdot 10^9 X^{14} - 5.9933 \cdot 10^9 X^{13} + 1.78815 \cdot 10^{10} X^{12} + 8.56274 \cdot 10^{10} X^{11} - 1.58071 \cdot 10^{11} X^{10} \\ & - 7.03711 \cdot 10^{11} X^9 + 7.99866 \cdot 10^{11} X^8 + 3.21659 \cdot 10^{12} X^7 - 2.16687 \cdot 10^{12} X^6 - 7.4915 \cdot 10^{12} X^5 \\ & + 2.76126 \cdot 10^{12} X^4 + 7.44201 \cdot 10^{12} X^3 - 1.18084 \cdot 10^{12} X^2 - 1.92569 \cdot 10^{12} X + 1.24532 \cdot 10^{11} \\ = {} & 1.24532 \cdot 10^{11} B_{0,20}(X) + 2.82469 \cdot 10^{10} B_{1,20}(X) - 7.42527 \cdot 10^{10} B_{2,20}(X) - 1.76439 \\ & \cdot 10^{11} B_{3,20}(X) - 2.71214 \cdot 10^{11} B_{4,20}(X) - 3.51394 \cdot 10^{11} B_{5,20}(X) - 4.10245 \cdot 10^{11} B_{6,20}(X) \\ & - 4.42042 \cdot 10^{11} B_{7,20}(X) - 4.42583 \cdot 10^{11} B_{8,20}(X) - 4.09631 \cdot 10^{11} B_{9,20}(X) - 3.43218 \\ & \cdot 10^{11} B_{10,20}(X) - 2.45778 \cdot 10^{11} B_{11,20}(X) - 1.22096 \cdot 10^{11} B_{12,20}(X) + 2.09582 \cdot 10^{10} B_{13,20}(X) \\ & + 1.74882 \cdot 10^{11} B_{14,20}(X) + 3.3015 \cdot 10^{11} B_{15,20}(X) + 4.76935 \cdot 10^{11} B_{16,20}(X) + 6.05883 \\ & \cdot 10^{11} B_{17,20}(X) + 7.08854 \cdot 10^{11} B_{18,20}(X) + 7.79584 \cdot 10^{11} B_{19,20}(X) + 8.1419 \cdot 10^{11} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = 3.20057\cdot10^{12}X^2 - 2.21796\cdot10^{12}X + 7.90412\cdot10^{10}$$
$$= 7.90412\cdot10^{10}B_{0,2} - 1.02994\cdot10^{12}B_{1,2} + 1.06166\cdot10^{12}B_{2,2}$$

$$\widetilde{q}_2 = 4.4238\cdot10^{14}X^{20} - 4.42413\cdot10^{15}X^{19} + 2.04771\cdot10^{16}X^{18} - 5.82009\cdot10^{16}X^{17} + 1.13655\cdot10^{17}X^{16} - 1.61654 \cdot10^{17}X^{15} + 1.73223\cdot10^{17}X^{14} - 1.42688\cdot10^{17}X^{13} + 9.13784\cdot10^{16}X^{12} - 4.57178\cdot10^{16}X^{11} + 1.78602 \cdot10^{16}X^{10} - 5.41794\cdot10^{15}X^{9} + 1.26251\cdot10^{15}X^{8} - 2.22245\cdot10^{14}X^{7} + 2.88648\cdot10^{13}X^{6} - 2.67799 \cdot10^{12}X^{5} + 1.69433\cdot10^{11}X^{4} - 6.74726\cdot10^{9}X^{3} + 3.20072\cdot10^{12}X^{2} - 2.21796\cdot10^{12}X + 7.90412\cdot10^{10}$$
$$= 7.90412\cdot10^{10}B_{0,20} - 3.18567\cdot10^{10}B_{1,20} - 1.25909\cdot10^{11}B_{2,20} - 2.03121\cdot10^{11}B_{3,20} - 2.63464 \cdot10^{11}B_{4,20} - 3.07046\cdot10^{11}B_{5,20} - 3.33543\cdot10^{11}B_{6,20} - 3.43745\cdot10^{11}B_{7,20} - 3.36075\cdot10^{11}B_{8,20} - 3.13153\cdot10^{11}B_{9,20} - 2.71311\cdot10^{11}B_{10,20} - 2.14891\cdot10^{11}B_{11,20} - 1.3955\cdot10^{11}B_{12,20} - 4.89587\cdot10^{10}B_{13,20} + 5.95009\cdot10^{10}B_{14,20} + 1.8426\cdot10^{11}B_{15,20} + 3.26105\cdot10^{11}B_{16,20} + 4.8471\cdot10^{11}B_{17,20} + 6.60183\cdot10^{11}B_{18,20} + 8.52497\cdot10^{11}B_{19,20} + 1.06166\cdot10^{12}B_{20,20}$$



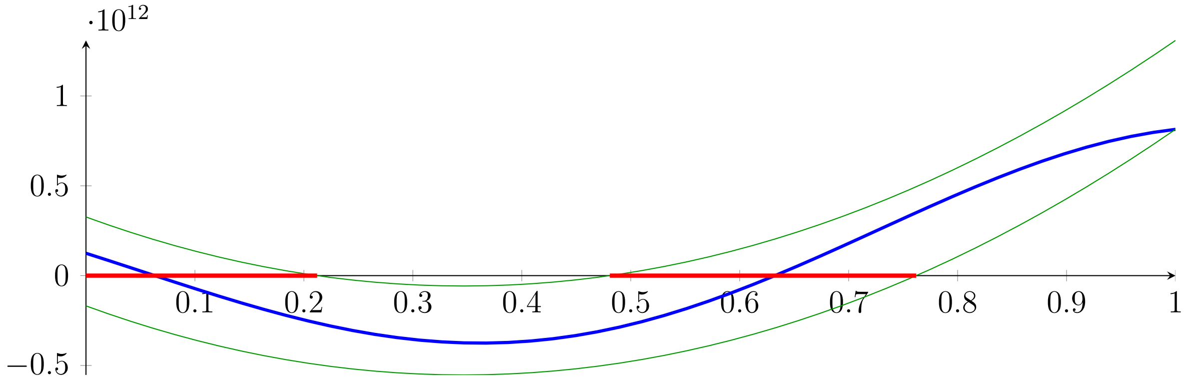
The maximum difference of the Bézier coefficients is $\delta = 2.47466\cdot10^{11}$.
Bounding polynomials M and m:

$$M = 3.20057\cdot10^{12}X^2 - 2.21796\cdot10^{12}X + 3.26507\cdot10^{11}$$
$$m = 3.20057\cdot10^{12}X^2 - 2.21796\cdot10^{12}X - 1.68425\cdot10^{11}$$

Root of M and m:

$$N(M) = \{0.21217, 0.480817\} \qquad N(m) = \{-0.0690555, 0.762043\}$$

Intersection intervals:



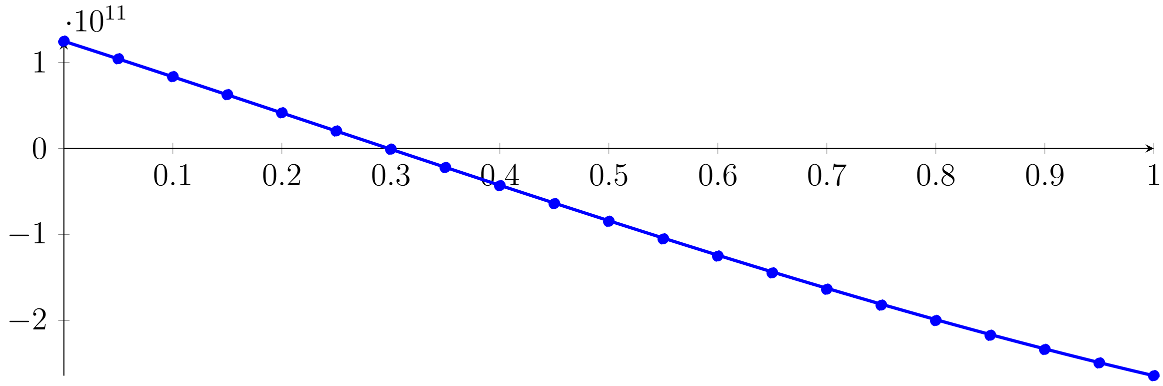
$$[0, 0.21217], [0.480817, 0.762043]$$

Longest intersection interval: 0.281226
$\Longrightarrow$ Selective recursion: interval 1: $[10.9375, 11.269]$, interval 2: $[11.6888, 12.1282]$,

2.54 Recursion Branch 1 1 2 2 2 1 in Interval 1: $[10.9375, 11.269]$

Normalized monomial und Bézier representations and the Bézier polygon:

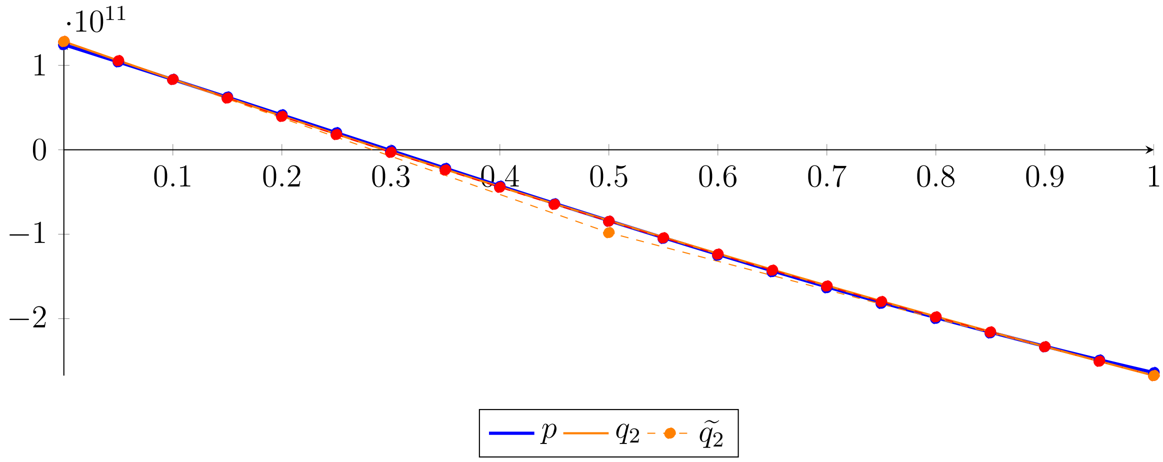
$$\begin{aligned} p = {} & 60.5342X^{20} - 718.228X^{19} + 945.927X^{18} - 9632.72X^{17} + 32272.2X^{16} - 20459.5X^{15} \\ & + 1856.5X^{14} - 2899.19X^{13} + 928.398X^{12} - 2878.1X^{11} - 32287.8X^{10} - 614686X^{9} \\ & + 3.28464{\cdot}10^{6}X^{8} + 6.22572{\cdot}10^{7}X^{7} - 1.9767{\cdot}10^{8}X^{6} - 3.22102{\cdot}10^{9}X^{5} + 5.59561 \\ & {\cdot}10^{9}X^{4} + 7.10796{\cdot}10^{10}X^{3} - 5.3157{\cdot}10^{10}X^{2} - 4.08575{\cdot}10^{11}X + 1.24532{\cdot}10^{11} \\ = {} & 1.24532{\cdot}10^{11}B_{0,20}(X) + 1.04103{\cdot}10^{11}B_{1,20}(X) + 8.33943{\cdot}10^{10}B_{2,20}(X) + 6.24683 \\ & {\cdot}10^{10}B_{3,20}(X) + 4.13885{\cdot}10^{10}B_{4,20}(X) + 2.02191{\cdot}10^{10}B_{5,20}(X) - 9.74473{\cdot}10^{8}B_{6,20}(X) \\ & - 2.21267{\cdot}10^{10}B_{7,20}(X) - 4.31714{\cdot}10^{10}B_{8,20}(X) - 6.40427{\cdot}10^{10}B_{9,20}(X) - 8.46745 \\ & {\cdot}10^{10}B_{10,20}(X) - 1.05001{\cdot}10^{11}B_{11,20}(X) - 1.24958{\cdot}10^{11}B_{12,20}(X) - 1.44481{\cdot}10^{11}B_{13,20}(X) \\ & - 1.63507{\cdot}10^{11}B_{14,20}(X) - 1.81974{\cdot}10^{11}B_{15,20}(X) - 1.99822{\cdot}10^{11}B_{16,20}(X) - 2.16992 \\ & {\cdot}10^{11}B_{17,20}(X) - 2.33427{\cdot}10^{11}B_{18,20}(X) - 2.49074{\cdot}10^{11}B_{19,20}(X) - 2.63879{\cdot}10^{11}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 5.70635{\cdot}10^{10}X^2 - 4.52737{\cdot}10^{11}X + 1.28205{\cdot}10^{11} \\ &= 1.28205{\cdot}10^{11}B_{0,2} - 9.81641{\cdot}10^{10}B_{1,2} - 2.67469{\cdot}10^{11}B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 6.5844{\cdot}10^{12}X^{20} - 6.6295{\cdot}10^{13}X^{19} + 3.09773{\cdot}10^{14}X^{18} - 8.90683{\cdot}10^{14}X^{17} + 1.76119{\cdot}10^{15}X^{16} - 2.53488\\ &\quad \cdot 10^{15}X^{15} + 2.74152\cdot 10^{15}X^{14} - 2.26805\cdot 10^{15}X^{13} + 1.44817\cdot 10^{15}X^{12} - 7.15592\cdot 10^{14}X^{11} + 2.73097\\ &\quad \cdot 10^{14}X^{10} - 8.00295\cdot 10^{13}X^{9} + 1.78535\cdot 10^{13}X^{8} - 3.00581\cdot 10^{12}X^{7} + 3.81262\cdot 10^{11}X^{6} - 3.69478\\ &\quad \cdot 10^{10}X^{5} + 2.77997\cdot 10^{9}X^{4} - 1.47109\cdot 10^{8}X^{3} + 5.7068\cdot 10^{10}X^{2} - 4.52738\cdot 10^{11}X + 1.28205\cdot 10^{11}\\ &= 1.28205\cdot 10^{11}B_{0,20} + 1.05568\cdot 10^{11}B_{1,20} + 8.32312\cdot 10^{10}B_{2,20} + 6.1195\cdot 10^{10}B_{3,20} + 3.94593\\ &\quad \cdot 10^{10}B_{4,20} + 1.8023\cdot 10^{10}B_{5,20} - 3.10969\cdot 10^{9}B_{6,20} - 2.39504\cdot 10^{10}B_{7,20} - 4.44739\cdot 10^{10}B_{8,20}\\ &\quad - 6.47237\cdot 10^{10}B_{9,20} - 8.46397\cdot 10^{10}B_{10,20} - 1.04288\cdot 10^{11}B_{11,20} - 1.23609\cdot 10^{11}B_{12,20}\\ &\quad - 1.42653\cdot 10^{11}B_{13,20} - 1.61379\cdot 10^{11}B_{14,20} - 1.79814\cdot 10^{11}B_{15,20} - 1.97945\cdot 10^{11}B_{16,20}\\ &\quad - 2.15777\cdot 10^{11}B_{17,20} - 2.33308\cdot 10^{11}B_{18,20} - 2.50539\cdot 10^{11}B_{19,20} - 2.67469\cdot 10^{11}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.67308\cdot 10^{9}$.

Bounding polynomials M and m:

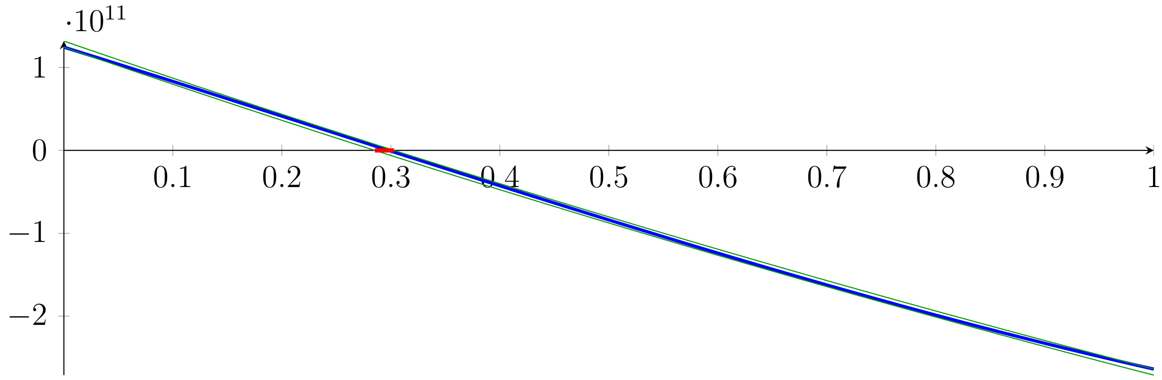
$$M = 5.70635\cdot 10^{10}X^2 - 4.52737\cdot 10^{11}X + 1.31878\cdot 10^{11}$$

$$m = 5.70635\cdot 10^{10}X^2 - 4.52737\cdot 10^{11}X + 1.24532\cdot 10^{11}$$

Root of M and m:

$$N(M) = \{0.30285, 7.63107\} \qquad N(m) = \{0.285325, 7.64859\}$$

Intersection intervals:



$$[0.285325, 0.30285]$$

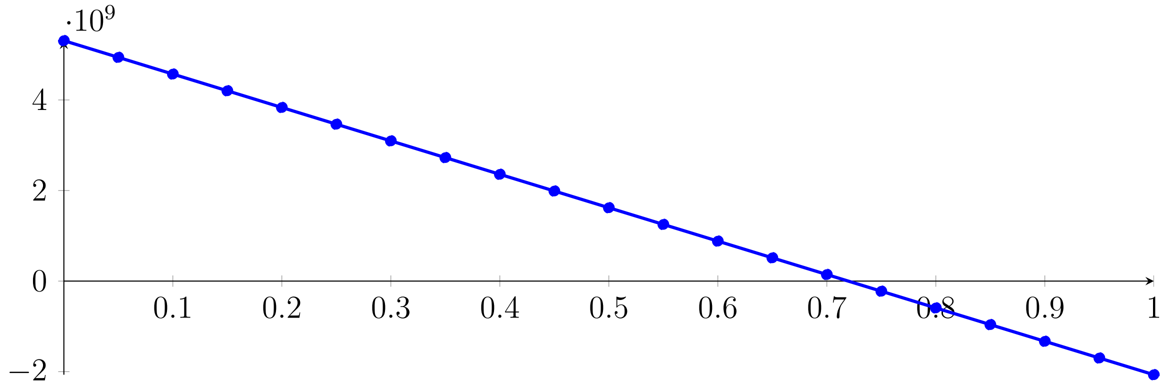
Longest intersection interval: 0.0175253

$\Longrightarrow$ Selective recursion: interval 1: $[11.0321, 11.0379]$,

2.55 Recursion Branch 1 1 2 2 2 1 1 in Interval 1: $[11.0321, 11.0379]$

Normalized monomial und Bézier representations and the Bézier polygon:

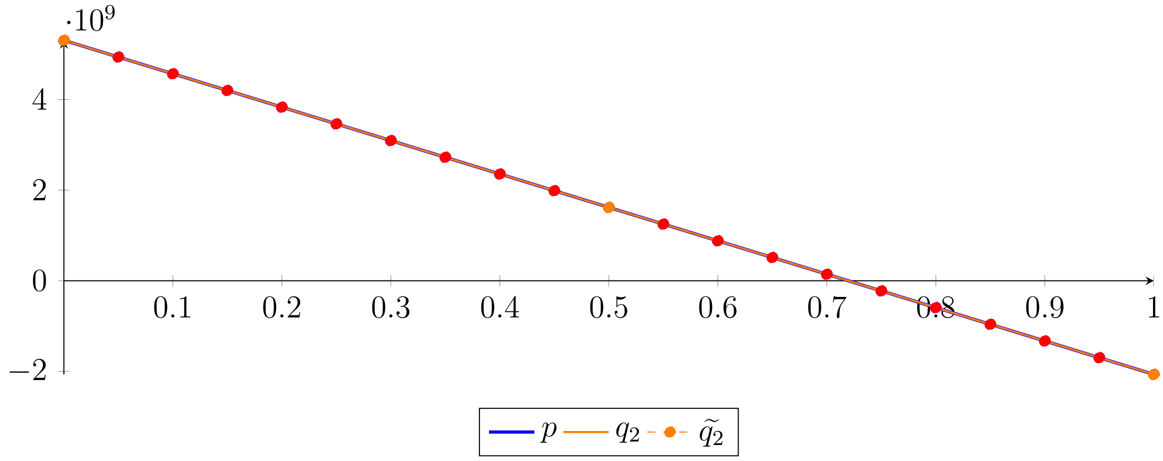
$$p = -2.04028X^{20} + 3.17499X^{19} - 75.8243X^{18} + 190.89X^{17} - 1437.38X^{16} + 1193.23X^{15} - 600.191X^{14} - 222.008X^{13} - 1541.2X^{12} - 310.587X^{11} - 506.919X^{10} - 77.3665X^{9} - 5.40605X^{8} - 0.221786X^{7} - 3.28983X^{6} - 5.78123X^{5} + 76.4563X^{4} + 402444X^{3} + 2.96485 \cdot 10^{6}X^{2} - 7.38058 \cdot 10^{9}X + 5.30953 \cdot 10^{9}$$
$$= 5.30953 \cdot 10^{9}B_{0,20}(X) + 4.9405 \cdot 10^{9}B_{1,20}(X) + 4.57148 \cdot 10^{9}B_{2,20}(X) + 4.20249 \cdot 10^{9}B_{3,20}(X) + 3.8335 \cdot 10^{9}B_{4,20}(X) + 3.46454 \cdot 10^{9}B_{5,20}(X) + 3.09559 \cdot 10^{9}B_{6,20}(X) + 2.72666 \cdot 10^{9}B_{7,20}(X) + 2.35775 \cdot 10^{9}B_{8,20}(X) + 1.98886 \cdot 10^{9}B_{9,20}(X) + 1.61998 \cdot 10^{9}B_{10,20}(X) + 1.25112 \cdot 10^{9}B_{11,20}(X) + 8.82286 \cdot 10^{8}B_{12,20}(X) + 5.13467 \cdot 10^{8}B_{13,20}(X) + 1.44669 \cdot 10^{8}B_{14,20}(X) - 2.24109 \cdot 10^{8}B_{15,20}(X) - 5.92867 \cdot 10^{8}B_{16,20}(X) - 9.61604 \cdot 10^{8}B_{17,20}(X) - 1.33032 \cdot 10^{9}B_{18,20}(X) - 1.69901 \cdot 10^{9}B_{19,20}(X) - 2.06769 \cdot 10^{9}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 3.56863 \cdot 10^{6}X^{2} - 7.38082 \cdot 10^{9}X + 5.30955 \cdot 10^{9}$$
$$= 5.30955 \cdot 10^{9}B_{0,2} + 1.61914 \cdot 10^{9}B_{1,2} - 2.06771 \cdot 10^{9}B_{2,2}$$

$$\widetilde{q}_2 = -5.77504 \cdot 10^{11}X^{20} + 5.79334 \cdot 10^{12}X^{19} - 2.69031 \cdot 10^{13}X^{18} + 7.67363 \cdot 10^{13}X^{17} - 1.50427 \cdot 10^{14}X^{16} + 2.14868 \cdot 10^{14}X^{15} - 2.31359 \cdot 10^{14}X^{14} + 1.91646 \cdot 10^{14}X^{13} - 1.23534 \cdot 10^{14}X^{12} + 6.22655 \cdot 10^{13}X^{11} - 2.45204 \cdot 10^{13}X^{10} + 7.49813 \cdot 10^{12}X^{9} - 1.76057 \cdot 10^{12}X^{8} + 3.12482 \cdot 10^{11}X^{7} - 4.11148 \cdot 10^{10}X^{6} + 3.91153 \cdot 10^{9}X^{5} - 2.59438 \cdot 10^{8}X^{4} + 1.1138 \cdot 10^{7}X^{3} + 3.297 \cdot 10^{6}X^{2} - 7.38082 \cdot 10^{9}X + 5.30955 \cdot 10^{9}$$
$$= 5.30955 \cdot 10^{9}B_{0,20} + 4.9405 \cdot 10^{9}B_{1,20} + 4.57148 \cdot 10^{9}B_{2,20} + 4.20248 \cdot 10^{9}B_{3,20} + 3.83347 \cdot 10^{9}B_{4,20} + 3.4646 \cdot 10^{9}B_{5,20} + 3.09541 \cdot 10^{9}B_{6,20} + 2.727 \cdot 10^{9}B_{7,20} + 2.3572 \cdot 10^{9}B_{8,20} + 1.98953 \cdot 10^{9}B_{9,20} + 1.61916 \cdot 10^{9}B_{10,20} + 1.2518 \cdot 10^{9}B_{11,20} + 8.81773 \cdot 10^{8}B_{12,20} + 5.13781 \cdot 10^{8}B_{13,20} + 1.44517 \cdot 10^{8}B_{14,20} - 2.24031 \cdot 10^{8}B_{15,20} - 5.92877 \cdot 10^{8}B_{16,20} - 9.61593 \cdot 10^{8}B_{17,20} - 1.33032 \cdot 10^{9}B_{18,20} - 1.69902 \cdot 10^{9}B_{19,20} - 2.06771 \cdot 10^{9}B_{20,20}$$



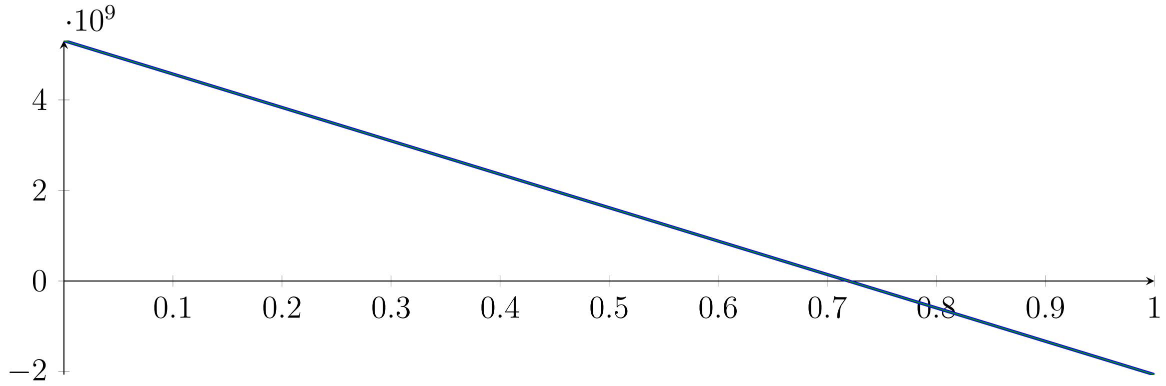
The maximum difference of the Bézier coefficients is $\delta = 817744$.
Bounding polynomials M and m:

$$M = 3.56863{\cdot}10^{6}X^2 - 7.38082{\cdot}10^{9}X + 5.31036{\cdot}10^{9}$$
$$m = 3.56863{\cdot}10^{6}X^2 - 7.38082{\cdot}10^{9}X + 5.30873{\cdot}10^{9}$$

Root of M and m:

$$N(M) = \{0.719732, 2067.53\} \qquad N(m) = \{0.71951, 2067.53\}$$

Intersection intervals:



$$[0.71951, 0.719732]$$

Longest intersection interval: 0.00022174
$\Longrightarrow$ Selective recursion: interval 1: $[11.0363, 11.0363]$,

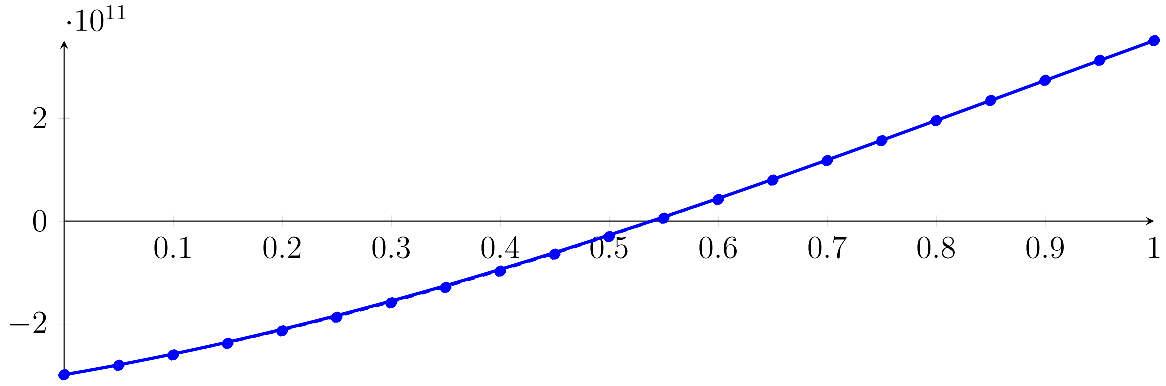
2.56 Recursion Branch 1 1 2 2 2 1 1 1 in Interval 1: $[11.0363, 11.0363]$

Found root in interval $[11.0363, 11.0363]$ at recursion depth 8!

2.57 Recursion Branch 1 1 2 2 2 2 in Interval 2: $[11.6888, 12.1282]$

Normalized monomial und Bézier representations and the Bézier polygon:

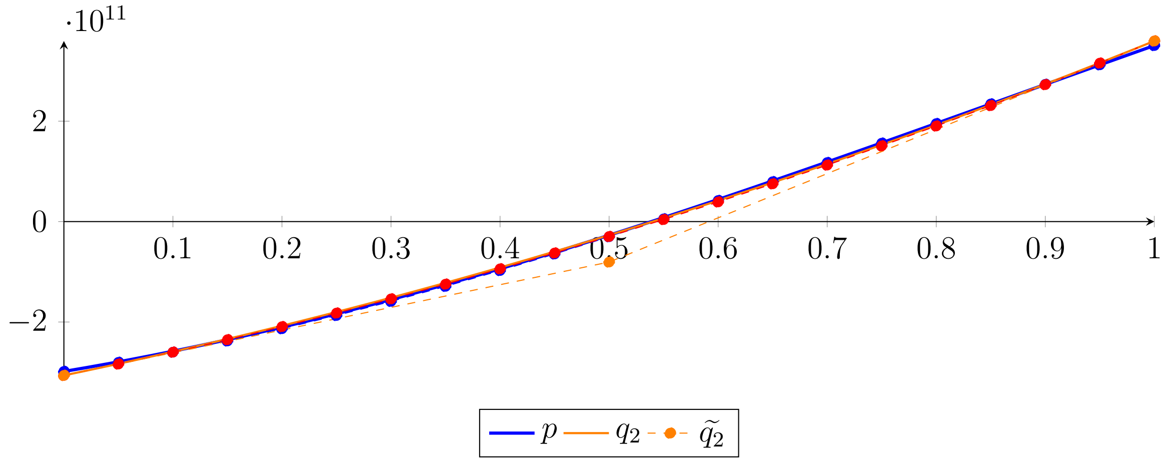
$$\begin{aligned}p &= 63.1291X^{20} + 343.805X^{19} + 3189.21X^{18} - 3423.55X^{17} + 51796X^{16} - 49126.6X^{15} \\ &\quad + 29266.4X^{14} + 18570.9X^{13} + 70327.6X^{12} + 64676.5X^{11} + 1.06581{\cdot}10^{6}X^{10} - 2.30899 \\ &\quad {\cdot}10^{6}X^{9} - 8.12936{\cdot}10^{7}X^{8} + 7.79133{\cdot}10^{6}X^{7} + 3.28643{\cdot}10^{9}X^{6} + 2.88144{\cdot}10^{9}X^{5} - 6.1039 \\ &\quad {\cdot}10^{10}X^{4} - 6.70429{\cdot}10^{10}X^{3} + 4.08756{\cdot}10^{11}X^{2} + 3.62367{\cdot}10^{11}X - 2.98487{\cdot}10^{11} \\ &= -2.98487{\cdot}10^{11}B_{0,20}(X) - 2.80368{\cdot}10^{11}B_{1,20}(X) - 2.60099{\cdot}10^{11}B_{2,20}(X) - 2.37736 \\ &\quad {\cdot}10^{11}B_{3,20}(X) - 2.13353{\cdot}10^{11}B_{4,20}(X) - 1.87032{\cdot}10^{11}B_{5,20}(X) - 1.5887{\cdot}10^{11}B_{6,20}(X) \\ &\quad - 1.28975{\cdot}10^{11}B_{7,20}(X) - 9.74646{\cdot}10^{10}B_{8,20}(X) - 6.44699{\cdot}10^{10}B_{9,20}(X) - 3.01307 \\ &\quad {\cdot}10^{10}B_{10,20}(X) + 5.40322{\cdot}10^{9}B_{11,20}(X) + 4.19735{\cdot}10^{10}B_{12,20}(X) + 7.94137{\cdot}10^{10}B_{13,20}(X) \\ &\quad + 1.1755{\cdot}10^{11}B_{14,20}(X) + 1.56204{\cdot}10^{11}B_{15,20}(X) + 1.9519{\cdot}10^{11}B_{16,20}(X) + 2.34319 \\ &\quad {\cdot}10^{11}B_{17,20}(X) + 2.73401{\cdot}10^{11}B_{18,20}(X) + 3.12242{\cdot}10^{11}B_{19,20}(X) + 3.50648{\cdot}10^{11}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 &= 2.14441\cdot 10^{11}X^2 + 4.51644\cdot 10^{11}X - 3.0638\cdot 10^{11}\\
&= -3.0638\cdot 10^{11}B_{0,2} - 8.05585\cdot 10^{10}B_{1,2} + 3.59704\cdot 10^{11}B_{2,2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_2 &= 5.47543\cdot 10^{13}X^{20} - 5.48017\cdot 10^{14}X^{19} + 2.53814\cdot 10^{15}X^{18} - 7.21816\cdot 10^{15}X^{17} + 1.41043\cdot 10^{16}X^{16} - 2.0078\\
&\quad \cdot 10^{16}X^{15} + 2.15438\cdot 10^{16}X^{14} - 1.7784\cdot 10^{16}X^{13} + 1.14256\cdot 10^{16}X^{12} - 5.74196\cdot 10^{15}X^{11} + 2.25591\\
&\quad \cdot 10^{15}X^{10} - 6.88824\cdot 10^{14}X^{9} + 1.61631\cdot 10^{14}X^{8} - 2.86565\cdot 10^{13}X^{7} + 3.75048\cdot 10^{12}X^{6} - 3.50909\\
&\quad \cdot 10^{11}X^{5} + 2.23904\cdot 10^{10}X^{4} - 8.99733\cdot 10^{8}X^{3} + 2.14461\cdot 10^{11}X^{2} + 4.51643\cdot 10^{11}X - 3.0638\cdot 10^{11}\\
&= -3.0638\cdot 10^{11}B_{0,20} - 2.83798\cdot 10^{11}B_{1,20} - 2.60087\cdot 10^{11}B_{2,20} - 2.35248\cdot 10^{11}B_{3,20} - 2.09278\\
&\quad \cdot 10^{11}B_{4,20} - 1.82189\cdot 10^{11}B_{5,20} - 1.53942\cdot 10^{11}B_{6,20} - 1.24635\cdot 10^{11}B_{7,20} - 9.40712\cdot 10^{10}B_{8,20}\\
&\quad - 6.25751\cdot 10^{10}B_{9,20} - 2.96947\cdot 10^{10}B_{10,20} + 4.03133\cdot 10^{9}B_{11,20} + 3.91449\cdot 10^{10}B_{12,20}\\
&\quad + 7.51922\cdot 10^{10}B_{13,20} + 1.12491\cdot 10^{11}B_{14,20} + 1.50853\cdot 10^{11}B_{15,20} + 1.90373\cdot 10^{11}B_{16,20}\\
&\quad + 2.31011\cdot 10^{11}B_{17,20} + 2.7278\cdot 10^{11}B_{18,20} + 3.15678\cdot 10^{11}B_{19,20} + 3.59704\cdot 10^{11}B_{20,20}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 9.05675\cdot 10^{9}$.

Bounding polynomials M and m:

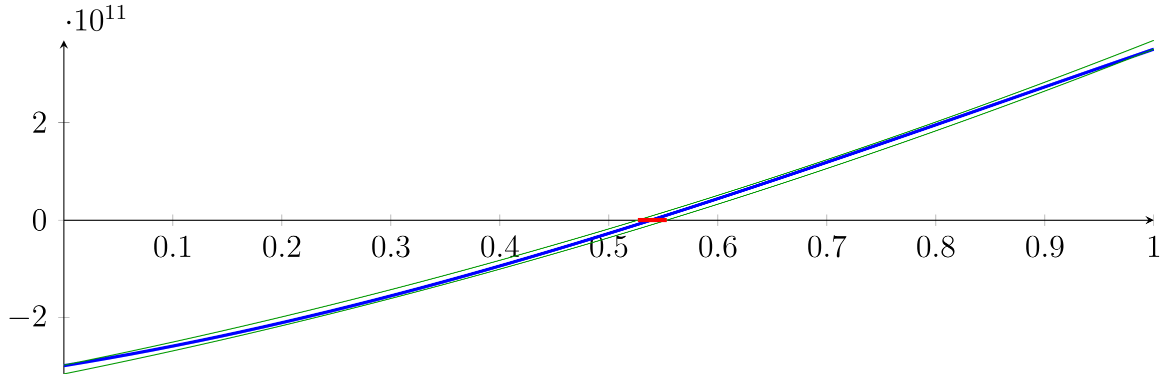
$$M = 2.14441\cdot 10^{11}X^2 + 4.51644\cdot 10^{11}X - 2.97324\cdot 10^{11}$$

$$m = 2.14441\cdot 10^{11}X^2 + 4.51644\cdot 10^{11}X - 3.15437\cdot 10^{11}$$

Root of M and m:

$$N(M) = \{-2.63278, 0.526632\} \qquad N(m) = \{-2.65929, 0.553145\}$$

Intersection intervals:



$[0.526632, 0.553145]$

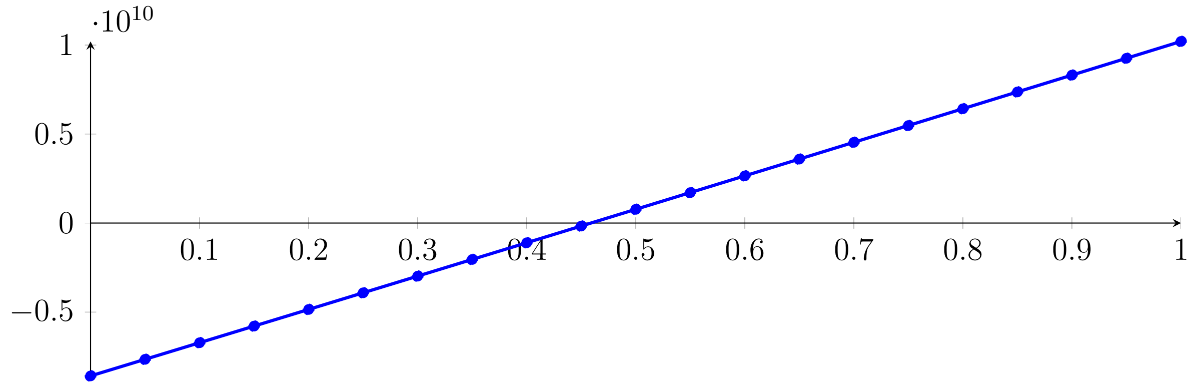
Longest intersection interval: 0.026513

$\Longrightarrow$ Selective recursion: interval 1: $[11.9202, 11.9318]$,

2.58 Recursion Branch 1 1 2 2 2 2 1 in Interval 1: $[11.9202, 11.9318]$

Normalized monomial und Bézier representations and the Bézier polygon:

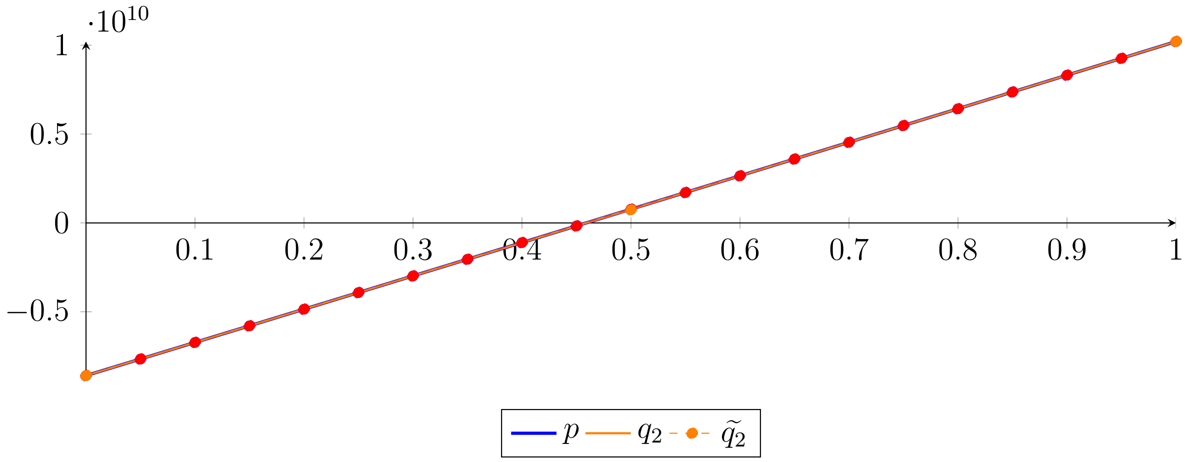
$$\begin{aligned} p = {} & 0.201217X^{20} + 17.3404X^{19} + 49.0567X^{18} + 90.3334X^{17} + 491.031X^{16} \\ & - 585.265X^{15} + 499.131X^{14} + 438.915X^{13} + 1474.17X^{12} + 494.793X^{11} \\ & + 566.474X^{10} + 105.238X^{9} + 12.3738X^{8} + 1.25679X^{7} + 5.28591X^{6} + 165.734X^{5} \\ & - 19856.3X^{4} - 3.3211 \cdot 10^{6} X^{3} + 1.4707 \cdot 10^{8} X^{2} + 1.86481 \cdot 10^{10} X - 8.58812 \cdot 10^{9} \\ = {} & -8.58812 \cdot 10^{9} B_{0,20}(X) - 7.65572 \cdot 10^{9} B_{1,20}(X) - 6.72254 \cdot 10^{9} B_{2,20}(X) - 5.78859 \\ & \cdot 10^{9} B_{3,20}(X) - 4.85388 \cdot 10^{9} B_{4,20}(X) - 3.9184 \cdot 10^{9} B_{5,20}(X) - 2.98215 \cdot 10^{9} B_{6,20}(X) \\ & - 2.04515 \cdot 10^{9} B_{7,20}(X) - 1.10739 \cdot 10^{9} B_{8,20}(X) - 1.68876 \cdot 10^{8} B_{9,20}(X) + 7.70388 \\ & \cdot 10^{8} B_{10,20}(X) + 1.7104 \cdot 10^{9} B_{11,20}(X) + 2.65116 \cdot 10^{9} B_{12,20}(X) + 3.59265 \cdot 10^{9} B_{13,20}(X) \\ & + 4.53489 \cdot 10^{9} B_{14,20}(X) + 5.47786 \cdot 10^{9} B_{15,20}(X) + 6.42157 \cdot 10^{9} B_{16,20}(X) + 7.36601 \\ & \cdot 10^{9} B_{17,20}(X) + 8.31117 \cdot 10^{9} B_{18,20}(X) + 9.25706 \cdot 10^{9} B_{19,20}(X) + 1.02037 \cdot 10^{10} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 1.42055 \cdot 10^{8} X^{2} + 1.86501 \cdot 10^{10} X - 8.58829 \cdot 10^{9} \\ &= -8.58829 \cdot 10^{9} B_{0,2} + 7.36744 \cdot 10^{8} B_{1,2} + 1.02038 \cdot 10^{10} B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 5.95815\cdot 10^{11}X^{20} - 5.96243\cdot 10^{12}X^{19} + 2.75956\cdot 10^{13}X^{18} - 7.83924\cdot 10^{13}X^{17} + 1.52994\cdot 10^{14}X^{16} \\ &\quad - 2.17597\cdot 10^{14}X^{15} + 2.33484\cdot 10^{14}X^{14} - 1.93042\cdot 10^{14}X^{13} + 1.24497\cdot 10^{14}X^{12} - 6.29733\cdot 10^{13}X^{11} \\ &\quad + 2.49691\cdot 10^{13}X^{10} - 7.71107\cdot 10^{12}X^{9} + 1.83214\cdot 10^{12}X^{8} - 3.28843\cdot 10^{11}X^{7} + 4.34659\cdot 10^{10}X^{6} \\ &\quad - 4.07781\cdot 10^{9}X^{5} + 2.56438\cdot 10^{8}X^{4} - 9.91793\cdot 10^{6}X^{3} + 1.42264\cdot 10^{8}X^{2} + 1.86501\cdot 10^{10}X - 8.58829\cdot 10^{9} \\ &= -8.58829\cdot 10^{9}B_{0,20} - 7.65579\cdot 10^{9}B_{1,20} - 6.72253\cdot 10^{9}B_{2,20} - 5.78854\cdot 10^{9}B_{3,20} - 4.85377 \\ &\quad \cdot 10^{9}B_{4,20} - 3.91837\cdot 10^{9}B_{5,20} - 2.98188\cdot 10^{9}B_{6,20} - 2.04541\cdot 10^{9}B_{7,20} - 1.1068\cdot 10^{9}B_{8,20} \\ &\quad - 1.69534\cdot 10^{8}B_{9,20} + 7.71215\cdot 10^{8}B_{10,20} + 1.70959\cdot 10^{9}B_{11,20} + 2.6516\cdot 10^{9}B_{12,20} \\ &\quad + 3.59226\cdot 10^{9}B_{13,20} + 4.53494\cdot 10^{9}B_{14,20} + 5.47771\cdot 10^{9}B_{15,20} + 6.4215\cdot 10^{9}B_{16,20} \\ &\quad + 7.36594\cdot 10^{9}B_{17,20} + 8.31116\cdot 10^{9}B_{18,20} + 9.25712\cdot 10^{9}B_{19,20} + 1.02038\cdot 10^{10}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 826891$.

Bounding polynomials M and m:

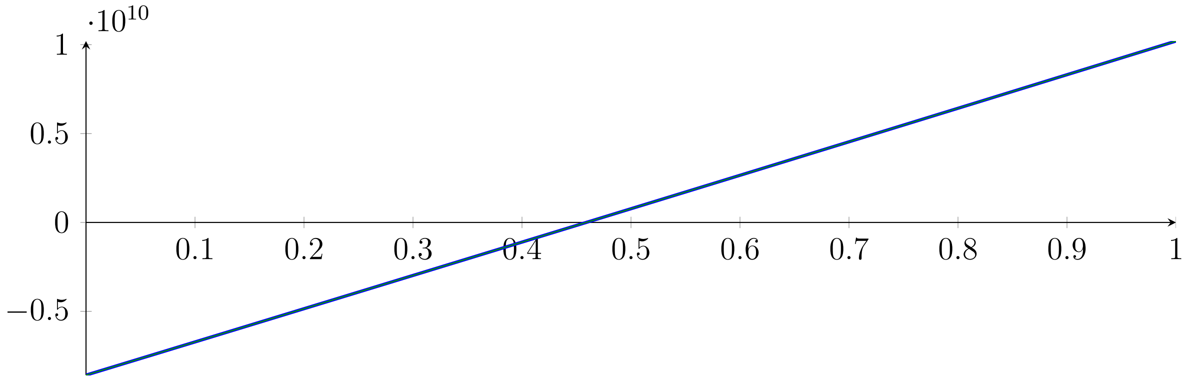
$$M = 1.42055\cdot 10^{8}X^2 + 1.86501\cdot 10^{10}X - 8.58746\cdot 10^{9}$$

$$m = 1.42055\cdot 10^{8}X^2 + 1.86501\cdot 10^{10}X - 8.58912\cdot 10^{9}$$

Root of M and m:

$$N(M) = \{-131.747, 0.458848\} \qquad N(m) = \{-131.747, 0.458936\}$$

Intersection intervals:



$$[0.458848, 0.458936]$$

Longest intersection interval: $8.80588\cdot 10^{-05}$
$\implies$ Selective recursion: interval 1: $[11.9255, 11.9255]$,

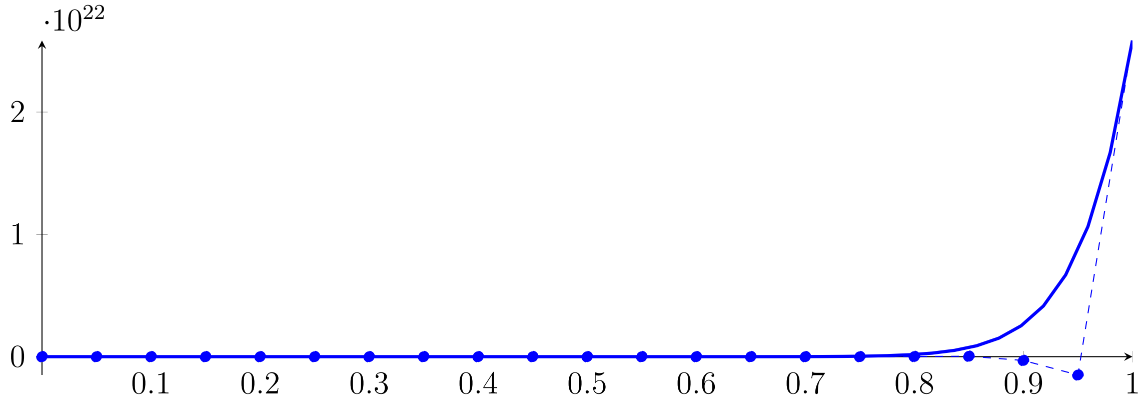
2.59 Recursion Branch 1 1 2 2 2 2 1 1 in Interval 1: $[11.9255, 11.9255]$

Found root in interval $[11.9255, 11.9255]$ at recursion depth 8!

2.60 Recursion Branch 1 2 on the Second Half $[12.5, 25]$

Normalized monomial und Bézier representations and the Bézier polygon:

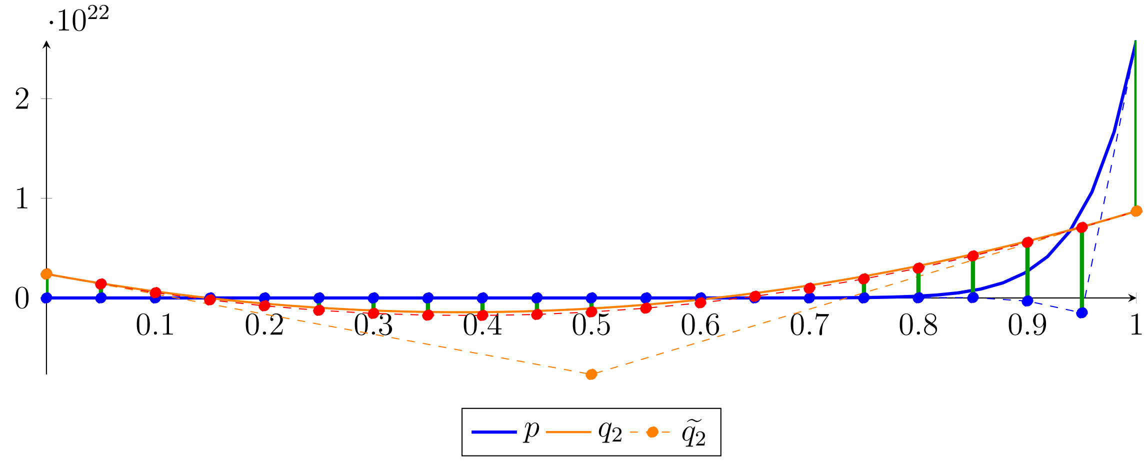
$$p = 8.67362\cdot10^{21}X^{20} + 2.77556\cdot10^{22}X^{19} + 2.3731\cdot10^{22}X^{18} - 1.26565\cdot10^{22}X^{17} - 2.8638\cdot10^{22}X^{16} - 6.33435\cdot10^{21}X^{15} + 1.06357\cdot10^{22}X^{14} + 5.39429\cdot10^{21}X^{13} - 1.50133\cdot10^{21}X^{12} - 1.39249\cdot10^{21}X^{11} + 1.05296\cdot10^{19}X^{10} + 1.67885\cdot10^{20}X^{9} + 1.71006\cdot10^{19}X^{8} - 9.83957\cdot10^{18}X^{7} - 1.53217\cdot10^{18}X^{6} + 2.57478\cdot10^{17}X^{5} + 4.72654\cdot10^{16}X^{4} - 2.266\cdot10^{15}X^{3} - 4.39258\cdot10^{14}X^{2} + 5.53708\cdot10^{12}X + 8.1419\cdot10^{11}$$
$$= 8.1419\cdot10^{11}B_{0,20}(X) + 1.09104\cdot10^{12}B_{1,20}(X) - 9.43984\cdot10^{11}B_{2,20}(X) - 7.27862\cdot10^{12}B_{3,20}(X) - 1.01451\cdot10^{13}B_{4,20}(X) + 2.45871\cdot10^{13}B_{5,20}(X) + 1.34488\cdot10^{14}B_{6,20}(X) + 1.71188\cdot10^{14}B_{7,20}(X) - 5.46645\cdot10^{14}B_{8,20}(X) - 2.59384\cdot10^{15}B_{9,20}(X) - 1.47677\cdot10^{15}B_{10,20}(X) + 2.00018\cdot10^{16}B_{11,20}(X) + 5.97972\cdot10^{16}B_{12,20}(X) - 8.43638\cdot10^{16}B_{13,20}(X) - 9.00155\cdot10^{17}B_{14,20}(X) - 6.30584\cdot10^{17}B_{15,20}(X) + 1.35026\cdot10^{19}B_{16,20}(X) + 3.45757\cdot10^{19}B_{17,20}(X) - 3.09468\cdot10^{20}B_{18,20}(X) - 1.49659\cdot10^{21}B_{19,20}(X) + 2.5852\cdot10^{22}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 2.64754\cdot10^{22}X^2 - 2.01665\cdot10^{22}X + 2.40539\cdot10^{21}$$
$$= 2.40539\cdot10^{21}B_{0,2} - 7.67787\cdot10^{21}B_{1,2} + 8.71426\cdot10^{21}B_{2,2}$$

$$\widetilde{q}_2 = 3.4363\cdot10^{24}X^{20} - 3.43574\cdot10^{25}X^{19} + 1.58981\cdot10^{26}X^{18} - 4.5173\cdot10^{26}X^{17} + 8.81856\cdot10^{26}X^{16} - 1.25385\cdot10^{27}X^{15} + 1.34308\cdot10^{27}X^{14} - 1.10588\cdot10^{27}X^{13} + 7.07914\cdot10^{26}X^{12} - 3.54021\cdot10^{26}X^{11} + 1.38243\cdot10^{26}X^{10} - 4.19202\cdot10^{25}X^{9} + 9.76526\cdot10^{24}X^{8} - 1.71837\cdot10^{24}X^{7} + 2.22995\cdot10^{23}X^{6} - 2.06467\cdot10^{22}X^{5} + 1.30046\cdot10^{21}X^{4} - 5.13771\cdot10^{19}X^{3} + 2.64765\cdot10^{22}X^{2} - 2.01665\cdot10^{22}X + 2.40539\cdot10^{21}$$
$$= 2.40539\cdot10^{21}B_{0,20} + 1.39707\cdot10^{21}B_{1,20} + 5.2809\cdot10^{20}B_{2,20} - 2.01581\cdot10^{20}B_{3,20} - 7.91724\cdot10^{20}B_{4,20} - 1.24318\cdot10^{21}B_{5,20} - 1.55343\cdot10^{21}B_{6,20} - 1.72858\cdot10^{21}B_{7,20} - 1.75645\cdot10^{21}B_{8,20} - 1.65734\cdot10^{21}B_{9,20} - 1.40277\cdot10^{21}B_{10,20} - 1.02646\cdot10^{21}B_{11,20} - 4.94658\cdot10^{20}B_{12,20} + 1.64073\cdot10^{20}B_{13,20} + 9.70107\cdot10^{20}B_{14,20} + 1.91125\cdot10^{21}B_{15,20} + 2.9936\cdot10^{21}B_{16,20} + 4.21463\cdot10^{21}B_{17,20} + 5.57518\cdot10^{21}B_{18,20} + 7.07505\cdot10^{21}B_{19,20} + 8.71426\cdot10^{21}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.71378 \cdot 10^{22}$.

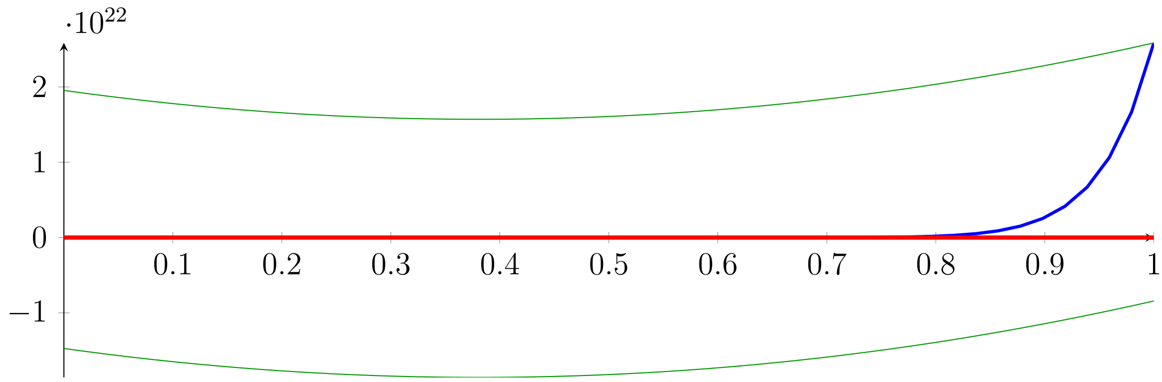
Bounding polynomials M and m:

$$M = 2.64754 \cdot 10^{22} X^2 - 2.01665 \cdot 10^{22} X + 1.95432 \cdot 10^{22}$$
$$m = 2.64754 \cdot 10^{22} X^2 - 2.01665 \cdot 10^{22} X - 1.47324 \cdot 10^{22}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{-0.456705, 1.21841\}$$

Intersection intervals:



$[0, 1]$

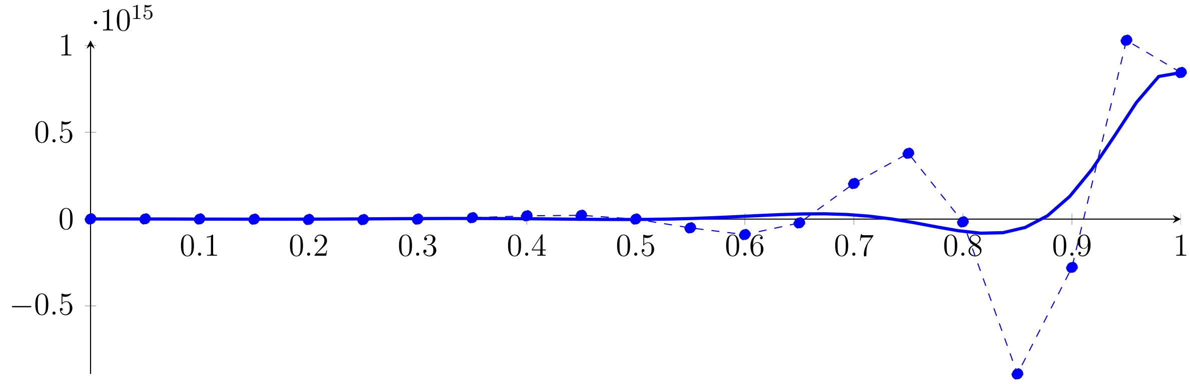
Longest intersection interval: 1

$\Longrightarrow$ Bisection: first half $[12.5, 18.75]$ und second half $[18.75, 25]$

2.61 Recursion Branch 1 2 1 on the First Half $[12.5, 18.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

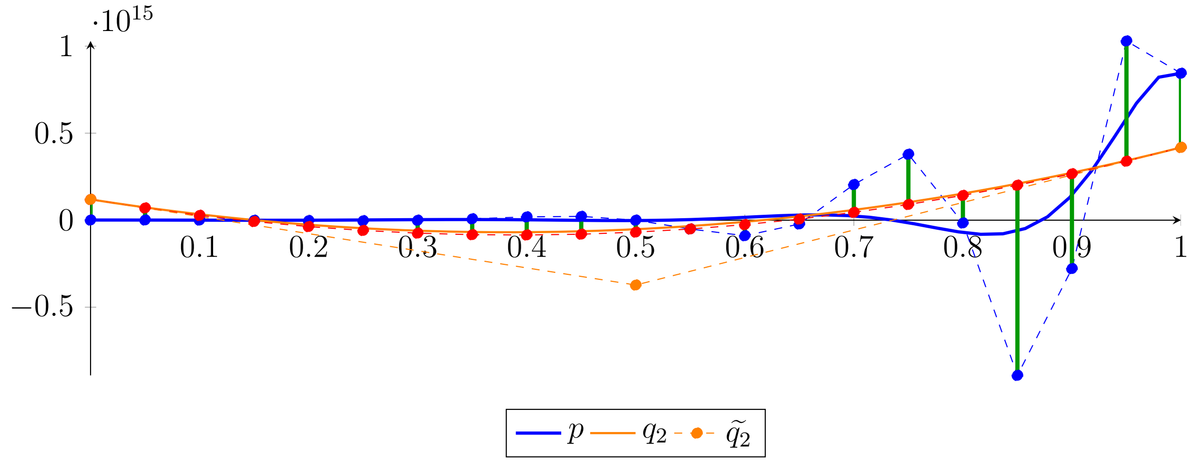
$$p = 8.27181 \cdot 10^{15} X^{20} + 5.29396 \cdot 10^{16} X^{19} + 9.05266 \cdot 10^{16} X^{18} - 9.65618 \cdot 10^{16} X^{17} - 4.36981 \cdot 10^{17} X^{16} - 1.93309 \cdot 10^{17} X^{15} + 6.49154 \cdot 10^{17} X^{14} + 6.58483 \cdot 10^{17} X^{13} - 3.66535 \cdot 10^{17} X^{12} - 6.79925 \cdot 10^{17} X^{11} + 1.02828 \cdot 10^{16} X^{10} + 3.279 \cdot 10^{17} X^9 + 6.67991 \cdot 10^{16} X^8 - 7.68717 \cdot 10^{16} X^7 - 2.39402 \cdot 10^{16} X^6 + 8.04618 \cdot 10^{15} X^5 + 2.95408 \cdot 10^{15} X^4 - 2.8325 \cdot 10^{14} X^3 - 1.09814 \cdot 10^{14} X^2 + 2.76854 \cdot 10^{12} X + 8.1419 \cdot 10^{11}$$
$$= 8.1419 \cdot 10^{11} B_{0,20}(X) + 9.52617 \cdot 10^{11} B_{1,20}(X) + 5.13074 \cdot 10^{11} B_{2,20}(X) - 7.52905 \cdot 10^{11} B_{3,20}(X) - 2.48407 \cdot 10^{12} B_{4,20}(X) - 3.19047 \cdot 10^{12} B_{5,20}(X) - 3.5214 \cdot 10^{11} B_{6,20}(X) + 7.87292 \cdot 10^{12} B_{7,20}(X) + 1.88702 \cdot 10^{13} B_{8,20}(X) + 2.17404 \cdot 10^{13} B_{9,20}(X) - 6.61543 \cdot 10^{10} B_{10,20}(X) - 5.06363 \cdot 10^{13} B_{11,20}(X) - 8.94122 \cdot 10^{13} B_{12,20}(X) - 2.20403 \cdot 10^{13} B_{13,20}(X) + 2.04834 \cdot 10^{14} B_{14,20}(X) + 3.789 \cdot 10^{14} B_{15,20}(X) - 1.62511 \cdot 10^{13} B_{16,20}(X) - 8.91971 \cdot 10^{14} B_{17,20}(X) - 2.7844 \cdot 10^{14} B_{18,20}(X) + 1.02974 \cdot 10^{15} B_{19,20}(X) + 8.43944 \cdot 10^{14} B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 1.28268\cdot 10^{15} X^2 - 9.84237\cdot 10^{14} X + 1.19443\cdot 10^{14}$$
$$= 1.19443\cdot 10^{14} B_{0,2} - 3.72676\cdot 10^{14} B_{1,2} + 4.17887\cdot 10^{14} B_{2,2}$$

$$\widetilde{q}_2 = 1.66333\cdot 10^{17} X^{20} - 1.66306\cdot 10^{18} X^{19} + 7.69544\cdot 10^{18} X^{18} - 2.1866\cdot 10^{19} X^{17} + 4.26864\cdot 10^{19} X^{16} - 6.0693 \cdot 10^{19} X^{15} + 6.50123\cdot 10^{19} X^{14} - 5.35305\cdot 10^{19} X^{13} + 3.42665\cdot 10^{19} X^{12} - 1.71361\cdot 10^{19} X^{11} + 6.6914 \cdot 10^{18} X^{10} - 2.02902\cdot 10^{18} X^9 + 4.72644\cdot 10^{17} X^8 - 8.31678\cdot 10^{16} X^7 + 1.07925\cdot 10^{16} X^6 - 9.99257 \cdot 10^{14} X^5 + 6.29424\cdot 10^{13} X^4 - 2.48694\cdot 10^{12} X^3 + 1.28274\cdot 10^{15} X^2 - 9.84238\cdot 10^{14} X + 1.19443\cdot 10^{14}$$
$$= 1.19443\cdot 10^{14} B_{0,20} + 7.02309\cdot 10^{13} B_{1,20} + 2.77703\cdot 10^{13} B_{2,20} - 7.9413\cdot 10^{12} B_{3,20} - 3.6893 \cdot 10^{13} B_{4,20} - 5.91255\cdot 10^{13} B_{5,20} - 7.45169\cdot 10^{13} B_{6,20} - 8.33631\cdot 10^{13} B_{7,20} - 8.50739\cdot 10^{13} B_{8,20} - 8.06321\cdot 10^{13} B_{9,20} - 6.86596\cdot 10^{13} B_{10,20} - 5.07879\cdot 10^{13} B_{11,20} - 2.5384\cdot 10^{13} B_{12,20} + 6.17012\cdot 10^{12} B_{13,20} + 4.48604\cdot 10^{13} B_{14,20} + 9.00965\cdot 10^{13} B_{15,20} + 1.42174\cdot 10^{14} B_{16,20} + 2.0097\cdot 10^{14} B_{17,20} + 2.66526\cdot 10^{14} B_{18,20} + 3.38831\cdot 10^{14} B_{19,20} + 4.17887\cdot 10^{14} B_{20,20}$$



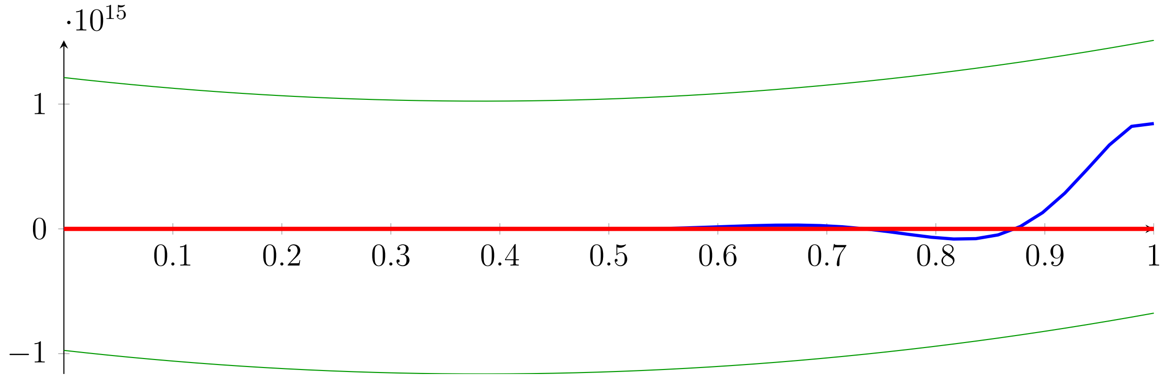
The maximum difference of the Bézier coefficients is $\delta = 1.09294\cdot 10^{15}$.
Bounding polynomials M and m:

$$M = 1.28268\cdot 10^{15} X^2 - 9.84237\cdot 10^{14} X + 1.21238\cdot 10^{15}$$
$$m = 1.28268\cdot 10^{15} X^2 - 9.84237\cdot 10^{14} X - 9.73498\cdot 10^{14}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{-0.568257, 1.33558\}$$

Intersection intervals:



$[0, 1]$

Longest intersection interval: 1

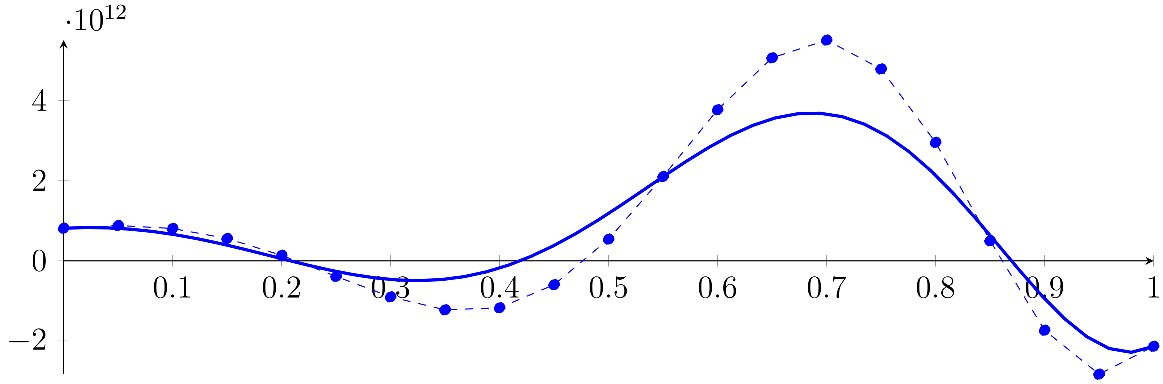
$\Longrightarrow$ Bisection: first half $[12.5, 15.625]$ und second half $[15.625, 18.75]$

Bisection point is very near to a root?!?

2.62 Recursion Branch 1 2 1 1 on the First Half $[12.5, 15.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

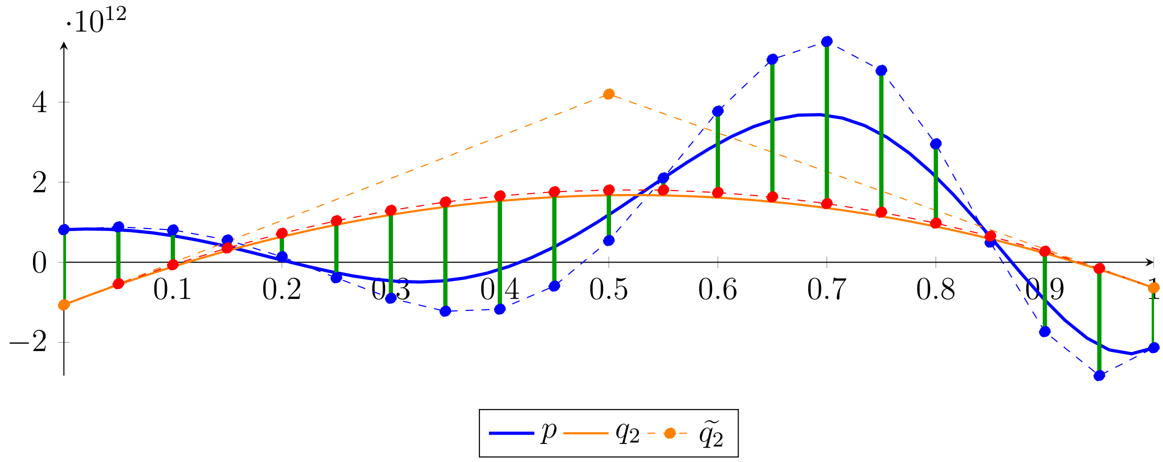
$$p = 7.88861 \cdot 10^{9} X^{20} + 1.00974 \cdot 10^{11} X^{19} + 3.45332 \cdot 10^{11} X^{18} - 7.36708 \cdot 10^{11} X^{17} - 6.66779 \cdot 10^{12} X^{16} - 5.89932 \cdot 10^{12} X^{15} + 3.96212 \cdot 10^{13} X^{14} + 8.03812 \cdot 10^{13} X^{13} - 8.94862 \cdot 10^{13} X^{12} - 3.31995 \cdot 10^{14} X^{11} + 1.00418 \cdot 10^{13} X^{10} + 6.4043 \cdot 10^{14} X^{9} + 2.60934 \cdot 10^{14} X^{8} - 6.0056 \cdot 10^{14} X^{7} - 3.74065 \cdot 10^{14} X^{6} + 2.51443 \cdot 10^{14} X^{5} + 1.8463 \cdot 10^{14} X^{4} - 3.54063 \cdot 10^{13} X^{3} - 2.74536 \cdot 10^{13} X^{2} + 1.38427 \cdot 10^{12} X + 8.1419 \cdot 10^{11}$$
$$= 8.1419 \cdot 10^{11} B_{0,20}(X) + 8.83404 \cdot 10^{11} B_{1,20}(X) + 8.08125 \cdot 10^{11} B_{2,20}(X) + 5.57295 \cdot 10^{11} B_{3,20}(X) + 1.37963 \cdot 10^{11} B_{4,20}(X) - 3.88495 \cdot 10^{11} B_{5,20}(X) - 8.99813 \cdot 10^{11} B_{6,20}(X) - 1.22366 \cdot 10^{12} B_{7,20}(X) - 1.17156 \cdot 10^{12} B_{8,20}(X) - 5.95624 \cdot 10^{11} B_{9,20}(X) + 5.41725 \cdot 10^{11} B_{10,20}(X) + 2.10687 \cdot 10^{12} B_{11,20}(X) + 3.77349 \cdot 10^{12} B_{12,20}(X) + 5.07064 \cdot 10^{12} B_{13,20}(X) + 5.51323 \cdot 10^{12} B_{14,20}(X) + 4.79225 \cdot 10^{12} B_{15,20}(X) + 2.95806 \cdot 10^{12} B_{16,20}(X) + 5.02527 \cdot 10^{11} B_{17,20}(X) - 1.7341 \cdot 10^{12} B_{18,20}(X) - 2.83115 \cdot 10^{12} B_{19,20}(X) - 2.1354 \cdot 10^{12} B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = -1.01001 \cdot 10^{13} X^2 + 1.05236 \cdot 10^{13} X - 1.06271 \cdot 10^{12}$$
$$= -1.06271 \cdot 10^{12} B_{0,2} + 4.19909 \cdot 10^{12} B_{1,2} - 6.39239 \cdot 10^{11} B_{2,2}$$

$$\begin{aligned}\widetilde{q}_2 &= -1.43378\cdot10^{15}X^{20} + 1.43449\cdot10^{16}X^{19} - 6.64327\cdot10^{16}X^{18} + 1.88946\cdot10^{17}X^{17} - 3.69245\cdot10^{17}X^{16} \\ &\quad + 5.25568\cdot10^{17}X^{15} - 5.63533\cdot10^{17}X^{14} + 4.64389\cdot10^{17}X^{13} - 2.97426\cdot10^{17}X^{12} + 1.48756\cdot10^{17}X^{11} \\ &\quad - 5.80653\cdot10^{16}X^{10} + 1.75911\cdot10^{16}X^{9} - 4.09229\cdot10^{15}X^{8} + 7.19225\cdot10^{14}X^{7} - 9.3366\cdot10^{13}X^{6} + 8.68669 \\ &\quad \cdot10^{12}X^{5} - 5.55119\cdot10^{11}X^{4} + 2.2545\cdot10^{10}X^{3} - 1.01006\cdot10^{13}X^{2} + 1.05236\cdot10^{13}X - 1.06271\cdot10^{12} \\ &= -1.06271\cdot10^{12}B_{0,20} - 5.36534\cdot10^{11}B_{1,20} - 6.35147\cdot10^{10}B_{2,20} + 3.56363\cdot10^{11}B_{3,20} + 7.23004 \\ &\quad \cdot10^{11}B_{4,20} + 1.03676\cdot10^{12}B_{5,20} + 1.29658\cdot10^{12}B_{6,20} + 1.50503\cdot10^{12}B_{7,20} + 1.65697\cdot10^{12}B_{8,20} \\ &\quad + 1.76095\cdot10^{12}B_{9,20} + 1.80501\cdot10^{12}B_{10,20} + 1.80325\cdot10^{12}B_{11,20} + 1.74166\cdot10^{12}B_{12,20} \\ &\quad + 1.63206\cdot10^{12}B_{13,20} + 1.46597\cdot10^{12}B_{14,20} + 1.24851\cdot10^{12}B_{15,20} + 9.77089\cdot10^{11}B_{16,20} \\ &\quad + 6.52796\cdot10^{11}B_{17,20} + 2.75266\cdot10^{11}B_{18,20} - 1.55406\cdot10^{11}B_{19,20} - 6.39239\cdot10^{11}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 4.04726\cdot10^{12}$.

Bounding polynomials M and m:

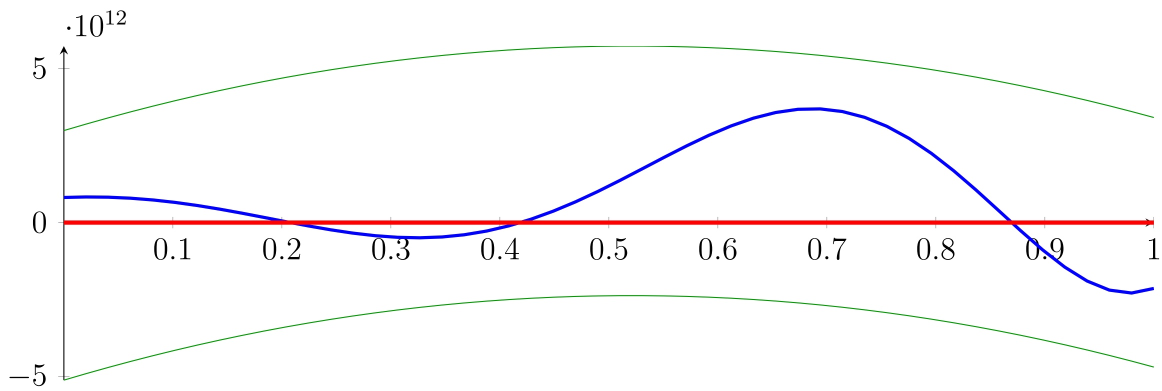
$$M = -1.01001\cdot10^{13}X^2 + 1.05236\cdot10^{13}X + 2.98454\cdot10^{12}$$

$$m = -1.01001\cdot10^{13}X^2 + 1.05236\cdot10^{13}X - 5.10997\cdot10^{12}$$

Root of M and m:

$$N(M) = \{-0.231963, 1.27389\} \qquad N(m) = \{\}$$

Intersection intervals:



$[0, 1]$

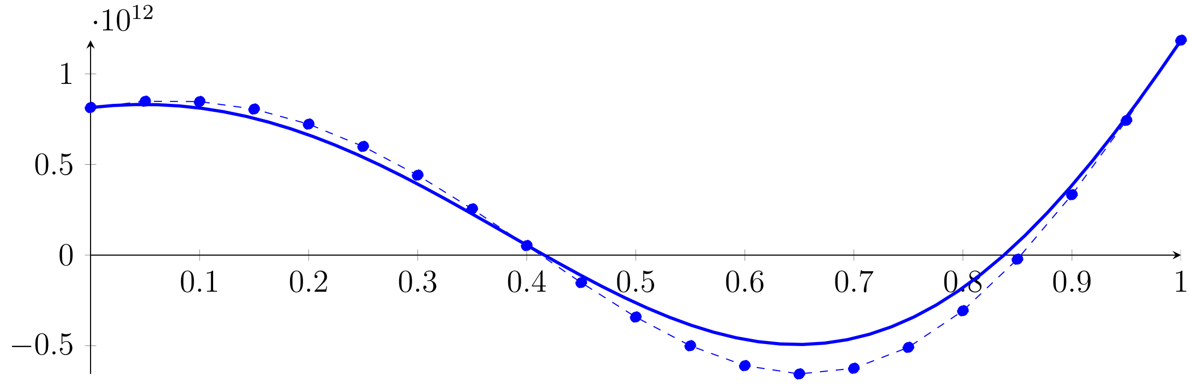
Longest intersection interval: 1

$\Longrightarrow$ Bisection: first half $[12.5, 14.0625]$ und second half $[14.0625, 15.625]$

2.63 Recursion Branch 1 2 1 1 1 on the First Half [12.5, 14.0625]

Normalized monomial und Bézier representations and the Bézier polygon:

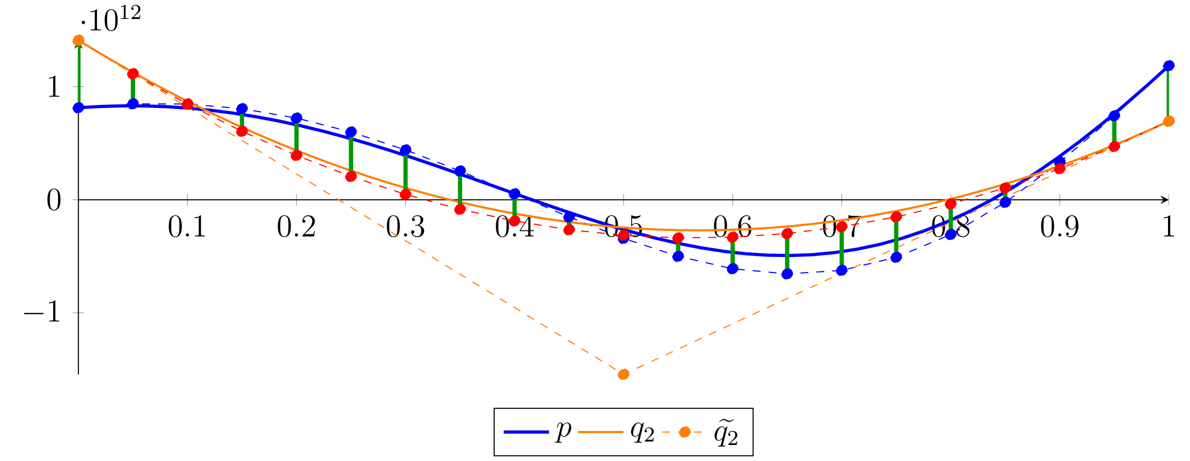
$$p = 7635.4X^{20} + 188595X^{19} + 1.31037{\cdot}10^{6}X^{18} - 5.65847{\cdot}10^{6}X^{17} - 1.0173{\cdot}10^{8}X^{16} - 1.7999 {\cdot}10^{8}X^{15} + 2.41822{\cdot}10^{9}X^{14} + 9.8121{\cdot}10^{9}X^{13} - 2.18474{\cdot}10^{10}X^{12} - 1.62107{\cdot}10^{11}X^{11} + 9.80637 {\cdot}10^{9}X^{10} + 1.25084{\cdot}10^{12}X^{9} + 1.01927{\cdot}10^{12}X^{8} - 4.69187{\cdot}10^{12}X^{7} - 5.84477{\cdot}10^{12}X^{6} + 7.8576 {\cdot}10^{12}X^{5} + 1.15394{\cdot}10^{13}X^{4} - 4.42579{\cdot}10^{12}X^{3} - 6.8634{\cdot}10^{12}X^{2} + 6.92135{\cdot}10^{11}X + 8.1419{\cdot}10^{11}$$
$$= 8.1419{\cdot}10^{11}B_{0,20}(X) + 8.48797{\cdot}10^{11}B_{1,20}(X) + 8.47281{\cdot}10^{11}B_{2,20}(X) + 8.05759 {\cdot}10^{11}B_{3,20}(X) + 7.22731{\cdot}10^{11}B_{4,20}(X) + 5.99585{\cdot}10^{11}B_{5,20}(X) + 4.40954{\cdot}10^{11}B_{6,20}(X) + 2.54859{\cdot}10^{11}B_{7,20}(X) + 5.25918{\cdot}10^{10}B_{8,20}(X) - 1.51705{\cdot}10^{11}B_{9,20}(X) - 3.41772 {\cdot}10^{11}B_{10,20}(X) - 5.00267{\cdot}10^{11}B_{11,20}(X) - 6.10048{\cdot}10^{11}B_{12,20}(X) - 6.55614{\cdot}10^{11}B_{13,20}(X) - 6.24598{\cdot}10^{11}B_{14,20}(X) - 5.09162{\cdot}10^{11}B_{15,20}(X) - 3.07139{\cdot}10^{11}B_{16,20}(X) - 2.27736 {\cdot}10^{10}B_{17,20}(X) + 3.33058{\cdot}10^{11}B_{18,20}(X) + 7.43235{\cdot}10^{11}B_{19,20}(X) + 1.1854{\cdot}10^{12}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 5.19514{\cdot}10^{12}X^{2} - 5.91052{\cdot}10^{12}X + 1.41047{\cdot}10^{12}$$
$$= 1.41047{\cdot}10^{12}B_{0,2} - 1.54479{\cdot}10^{12}B_{1,2} + 6.95092{\cdot}10^{11}B_{2,2}$$

$$\widetilde{q}_2 = 6.06368{\cdot}10^{14}X^{20} - 6.06177{\cdot}10^{15}X^{19} + 2.80467{\cdot}10^{16}X^{18} - 7.96875{\cdot}10^{16}X^{17} + 1.55554{\cdot}10^{17}X^{16} - 2.21143{\cdot}10^{17}X^{15} + 2.36816{\cdot}10^{17}X^{14} - 1.94891{\cdot}10^{17}X^{13} + 1.2465{\cdot}10^{17}X^{12} - 6.22573{\cdot}10^{16}X^{11} + 2.42703{\cdot}10^{16}X^{10} - 7.34493{\cdot}10^{15}X^{9} + 1.70726{\cdot}10^{15}X^{8} - 2.9975{\cdot}10^{14}X^{7} + 3.88107{\cdot}10^{13}X^{6} - 3.586 {\cdot}10^{12}X^{5} + 2.25617{\cdot}10^{11}X^{4} - 8.91402{\cdot}10^{9}X^{3} + 5.19534{\cdot}10^{12}X^{2} - 5.91052{\cdot}10^{12}X + 1.41047{\cdot}10^{12}$$
$$= 1.41047{\cdot}10^{12}B_{0,20} + 1.11494{\cdot}10^{12}B_{1,20} + 8.46758{\cdot}10^{11}B_{2,20} + 6.05912{\cdot}10^{11}B_{3,20} + 3.92441 {\cdot}10^{11}B_{4,20} + 2.06199{\cdot}10^{11}B_{5,20} + 4.76245{\cdot}10^{10}B_{6,20} - 8.43523{\cdot}10^{10}B_{7,20} - 1.87588{\cdot}10^{11}B_{8,20} - 2.65666{\cdot}10^{11}B_{9,20} - 3.13554{\cdot}10^{11}B_{10,20} - 3.37196{\cdot}10^{11}B_{11,20} - 3.30656{\cdot}10^{11}B_{12,20} - 2.9897{\cdot}10^{11}B_{13,20} - 2.38525{\cdot}10^{11}B_{14,20} - 1.51492{\cdot}10^{11}B_{15,20} - 3.67827{\cdot}10^{10}B_{16,20} + 1.0515{\cdot}10^{11}B_{17,20} + 2.74459{\cdot}10^{11}B_{18,20} + 4.71104{\cdot}10^{11}B_{19,20} + 6.95092{\cdot}10^{11}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 5.96276 \cdot 10^{11}$.

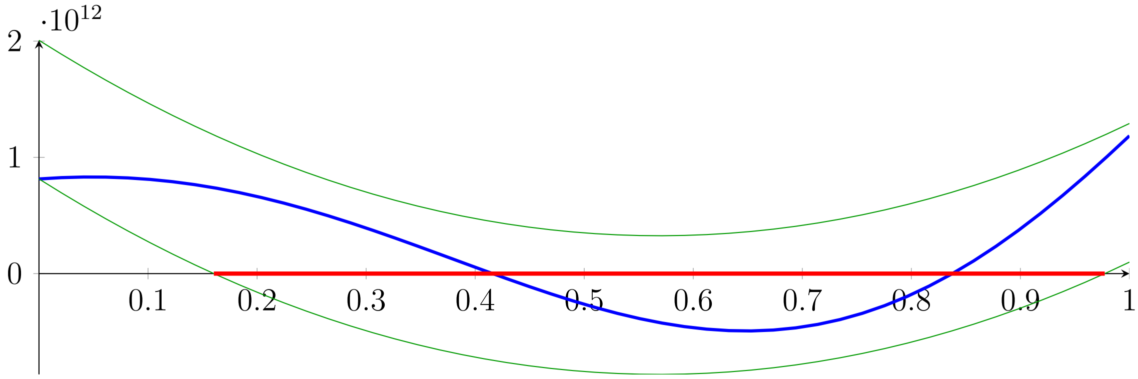
Bounding polynomials M and m:

$$M = 5.19514 \cdot 10^{12} X^2 - 5.91052 \cdot 10^{12} X + 2.00674 \cdot 10^{12}$$
$$m = 5.19514 \cdot 10^{12} X^2 - 5.91052 \cdot 10^{12} X + 8.1419 \cdot 10^{11}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.160354, 0.977347\}$$

Intersection intervals:



$$[0.160354, 0.977347]$$

Longest intersection interval: 0.816992

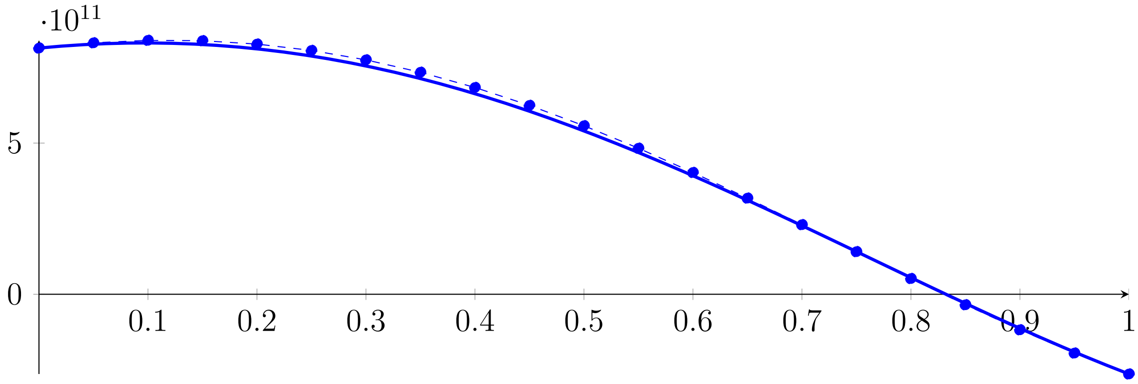
$\Longrightarrow$ Bisection: first half $[12.5, 13.2812]$ und second half $[13.2812, 14.0625]$

Bisection point is very near to a root?!?

2.64 Recursion Branch 1 2 1 1 1 1 on the First Half $[12.5, 13.2812]$

Normalized monomial und Bézier representations and the Bézier polygon:

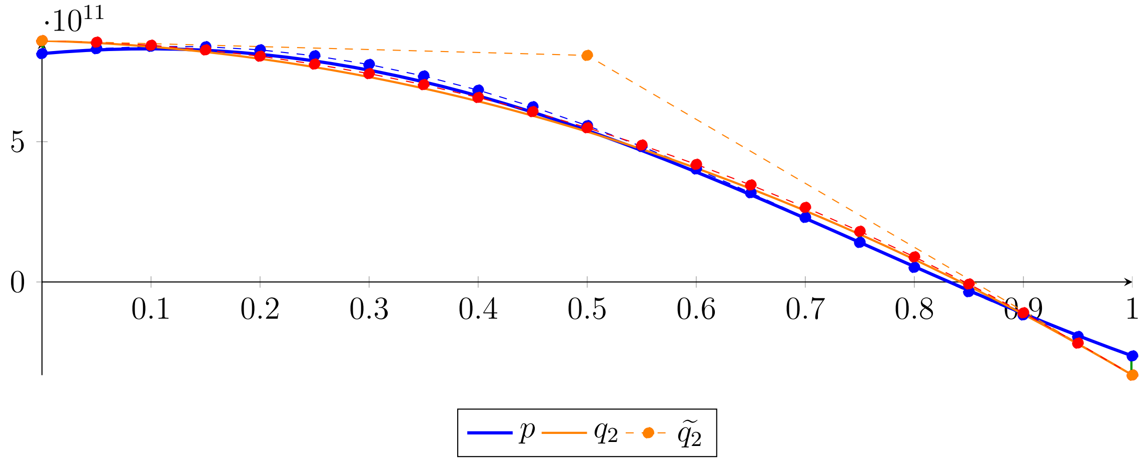
$$\begin{aligned} p = & -615.164 X^{20} + 1877.3 X^{19} - 21086.7 X^{18} + 65024.5 X^{17} - 421013 X^{16} + 344752 X^{15} \\ & - 5223.52 X^{14} + 1.14024 \cdot 10^6 X^{13} - 5.74098 \cdot 10^6 X^{12} - 7.91928 \cdot 10^7 X^{11} + 9.45612 \cdot 10^6 X^{10} \\ & + 2.44303 \cdot 10^9 X^9 + 3.98153 \cdot 10^9 X^8 - 3.66553 \cdot 10^{10} X^7 - 9.13245 \cdot 10^{10} X^6 + 2.4555 \cdot 10^{11} X^5 \\ & + 7.21212 \cdot 10^{11} X^4 - 5.53223 \cdot 10^{11} X^3 - 1.71585 \cdot 10^{12} X^2 + 3.46067 \cdot 10^{11} X + 8.1419 \cdot 10^{11} \\ = & \; 8.1419 \cdot 10^{11} B_{0,20}(X) + 8.31494 \cdot 10^{11} B_{1,20}(X) + 8.39766 \cdot 10^{11} B_{2,20}(X) + 8.38523 \\ & \cdot 10^{11} B_{3,20}(X) + 8.27427 \cdot 10^{11} B_{4,20}(X) + 8.06307 \cdot 10^{11} B_{5,20}(X) + 7.75169 \cdot 10^{11} B_{6,20}(X) \\ & + 7.34208 \cdot 10^{11} B_{7,20}(X) + 6.83817 \cdot 10^{11} B_{8,20}(X) + 6.24585 \cdot 10^{11} B_{9,20}(X) + 5.57306 \\ & \cdot 10^{11} B_{10,20}(X) + 4.82964 \cdot 10^{11} B_{11,20}(X) + 4.02731 \cdot 10^{11} B_{12,20}(X) + 3.17954 \cdot 10^{11} B_{13,20}(X) \\ & + 2.30131 \cdot 10^{11} B_{14,20}(X) + 1.40895 \cdot 10^{11} B_{15,20}(X) + 5.19842 \cdot 10^{10} B_{16,20}(X) - 3.47871 \\ & \cdot 10^{10} B_{17,20}(X) - 1.17561 \cdot 10^{11} B_{18,20}(X) - 1.94471 \cdot 10^{11} B_{19,20}(X) - 2.63682 \cdot 10^{11} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = -1.08742\cdot 10^{12}X^2 - 1.04698\cdot 10^{11}X + 8.60011\cdot 10^{11}$$
$$= 8.60011\cdot 10^{11}B_{0,2} + 8.07662\cdot 10^{11}B_{1,2} - 3.32109\cdot 10^{11}B_{2,2}$$

$$\widetilde{q}_2 = -2.66584\cdot 10^{14}X^{20} + 2.67041\cdot 10^{15}X^{19} - 1.23827\cdot 10^{16}X^{18} + 3.52665\cdot 10^{16}X^{17} - 6.90209\cdot 10^{16}X^{16} + 9.84039\cdot 10^{16}X^{15} - 1.05714\cdot 10^{17}X^{14} + 8.73137\cdot 10^{16}X^{13} - 5.60739\cdot 10^{16}X^{12} + 2.81346\cdot 10^{16}X^{11} - 1.10209\cdot 10^{16}X^{10} + 3.35104\cdot 10^{15}X^9 - 7.82345\cdot 10^{14}X^8 + 1.38028\cdot 10^{14}X^7 - 1.80209\cdot 10^{13}X^6 + 1.69428\cdot 10^{12}X^5 - 1.10342\cdot 10^{11}X^4 + 4.61671\cdot 10^9X^3 - 1.08753\cdot 10^{12}X^2 - 1.04697\cdot 10^{11}X + 8.60011\cdot 10^{11}$$
$$= 8.60011\cdot 10^{11}B_{0,20} + 8.54776\cdot 10^{11}B_{1,20} + 8.43818\cdot 10^{11}B_{2,20} + 8.27139\cdot 10^{11}B_{3,20} + 8.04722\cdot 10^{11}B_{4,20} + 7.76634\cdot 10^{11}B_{5,20} + 7.42674\cdot 10^{11}B_{6,20} + 7.03332\cdot 10^{11}B_{7,20} + 6.57632\cdot 10^{11}B_{8,20} + 6.0718\cdot 10^{11}B_{9,20} + 5.49747\cdot 10^{11}B_{10,20} + 4.8796\cdot 10^{11}B_{11,20} + 4.19212\cdot 10^{11}B_{12,20} + 3.45687\cdot 10^{11}B_{13,20} + 2.65829\cdot 10^{11}B_{14,20} + 1.80575\cdot 10^{11}B_{15,20} + 8.94504\cdot 10^{10}B_{16,20} - 7.34511\cdot 10^9B_{17,20} - 1.09878\cdot 10^{11}B_{18,20} - 2.18132\cdot 10^{11}B_{19,20} - 3.32109\cdot 10^{11}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 6.84266\cdot 10^{10}$.

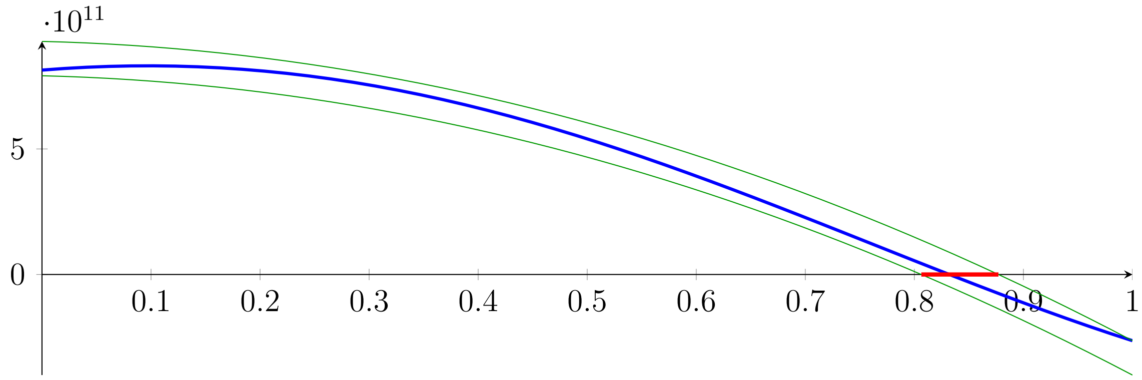
Bounding polynomials M and m:

$$M = -1.08742\cdot 10^{12}X^2 - 1.04698\cdot 10^{11}X + 9.28438\cdot 10^{11}$$
$$m = -1.08742\cdot 10^{12}X^2 - 1.04698\cdot 10^{11}X + 7.91585\cdot 10^{11}$$

Root of M and m:

$$N(M) = \{-0.973406, 0.877124\} \qquad N(m) = \{-0.902695, 0.806414\}$$

Intersection intervals:



$$[0.806414, 0.877124]$$

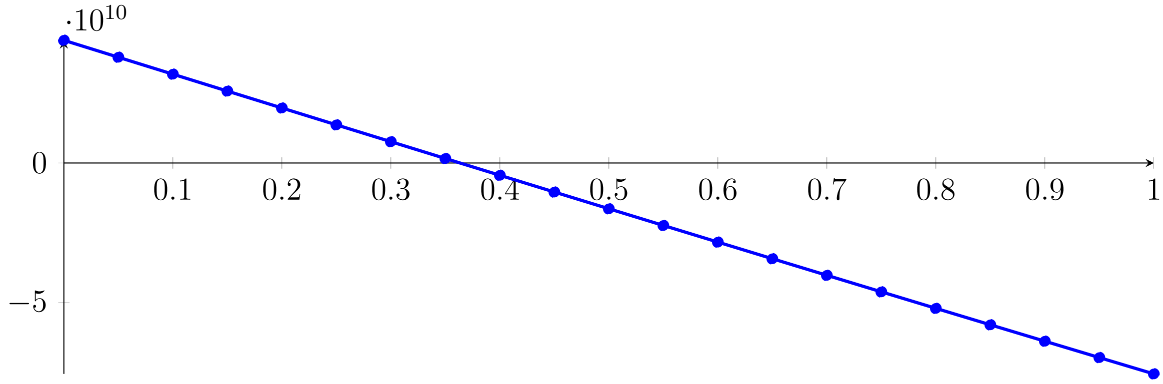
Longest intersection interval: 0.07071

$\Longrightarrow$ Selective recursion: interval 1: $[13.13, 13.1853]$,

2.65 Recursion Branch 1 2 1 1 1 1 1 in Interval 1: $[13.13, 13.1853]$

Normalized monomial und Bézier representations and the Bézier polygon:

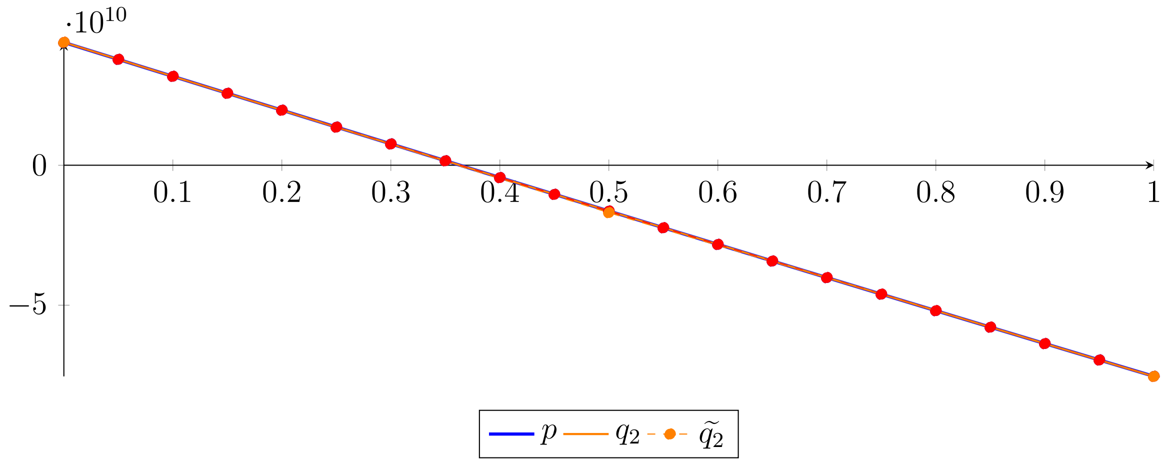
$$p = 9.0291X^{20} - 162.935X^{19} + 30.6935X^{18} - 1921.02X^{17} + 4140.24X^{16} - 1654.82X^{15} - 1155.06X^{14} - 1295.23X^{13} - 3715.52X^{12} - 2365.53X^{11} - 2021.33X^{10} - 540.765X^{9} - 9.61075X^{8} + 311.093X^{7} - 16339.5X^{6} - 812520X^{5} + 9.12308 \cdot 10^{6}X^{4} + 7.0704 \cdot 10^{8}X^{3} + 1.27633 \cdot 10^{9}X^{2} - 1.21272 \cdot 10^{11}X + 4.38722 \cdot 10^{10}$$
$$= 4.38722 \cdot 10^{10}B_{0,20}(X) + 3.78087 \cdot 10^{10}B_{1,20}(X) + 3.17518 \cdot 10^{10}B_{2,20}(X) + 2.57023 \cdot 10^{10}B_{3,20}(X) + 1.96607 \cdot 10^{10}B_{4,20}(X) + 1.36277 \cdot 10^{10}B_{5,20}(X) + 7.60396 \cdot 10^{9}B_{6,20}(X) + 1.59003 \cdot 10^{9}B_{7,20}(X) - 4.41344 \cdot 10^{9}B_{8,20}(X) - 1.04058 \cdot 10^{10}B_{9,20}(X) - 1.63865 \cdot 10^{10}B_{10,20}(X) - 2.23547 \cdot 10^{10}B_{11,20}(X) - 2.831 \cdot 10^{10}B_{12,20}(X) - 3.42517 \cdot 10^{10}B_{13,20}(X) - 4.0179 \cdot 10^{10}B_{14,20}(X) - 4.60915 \cdot 10^{10}B_{15,20}(X) - 5.19884 \cdot 10^{10}B_{16,20}(X) - 5.78691 \cdot 10^{10}B_{17,20}(X) - 6.3733 \cdot 10^{10}B_{18,20}(X) - 6.95794 \cdot 10^{10}B_{19,20}(X) - 7.54077 \cdot 10^{10}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 2.35105 \cdot 10^{9}X^{2} - 1.21703 \cdot 10^{11}X + 4.39083 \cdot 10^{10}$$
$$= 4.39083 \cdot 10^{10}B_{0,2} - 1.69433 \cdot 10^{10}B_{1,2} - 7.54439 \cdot 10^{10}B_{2,2}$$

$$\tilde{q}_2 = -1.32941 \cdot 10^{12}X^{20} + 1.32249 \cdot 10^{13}X^{19} - 6.06846 \cdot 10^{13}X^{18} + 1.70551 \cdot 10^{14}X^{17} - 3.28922 \cdot 10^{14}X^{16} + 4.6247 \cdot 10^{14}X^{15} - 4.91836 \cdot 10^{14}X^{14} + 4.05152 \cdot 10^{14}X^{13} - 2.62383 \cdot 10^{14}X^{12} + 1.34564 \cdot 10^{14}X^{11} - 5.46301 \cdot 10^{13}X^{10} + 1.74122 \cdot 10^{13}X^{9} - 4.28658 \cdot 10^{12}X^{8} + 7.94665 \cdot 10^{11}X^{7} - 1.06706 \cdot 10^{11}X^{6} + 9.7299 \cdot 10^{9}X^{5} - 5.32729 \cdot 10^{8}X^{4} + 1.39858 \cdot 10^{7}X^{3} + 2.35098 \cdot 10^{9}X^{2} - 1.21703 \cdot 10^{11}X + 4.39083 \cdot 10^{10}$$
$$= 4.39083 \cdot 10^{10}B_{0,20} + 3.78231 \cdot 10^{10}B_{1,20} + 3.17503 \cdot 10^{10}B_{2,20} + 2.56899 \cdot 10^{10}B_{3,20} + 1.96418 \cdot 10^{10}B_{4,20} + 1.36064 \cdot 10^{10}B_{5,20} + 7.58252 \cdot 10^{9}B_{6,20} + 1.57273 \cdot 10^{9}B_{7,20} - 4.42753 \cdot 10^{9}B_{8,20} - 1.04113 \cdot 10^{10}B_{9,20} - 1.63883 \cdot 10^{10}B_{10,20} - 2.23456 \cdot 10^{10}B_{11,20} - 2.82979 \cdot 10^{10}B_{12,20} - 3.4233 \cdot 10^{10}B_{13,20} - 4.01582 \cdot 10^{10}B_{14,20} - 4.60698 \cdot 10^{10}B_{15,20} - 5.19695 \cdot 10^{10}B_{16,20} - 5.78566 \cdot 10^{10}B_{17,20} - 6.37314 \cdot 10^{10}B_{18,20} - 6.95938 \cdot 10^{10}B_{19,20} - 7.54439 \cdot 10^{10}B_{20,20}$$



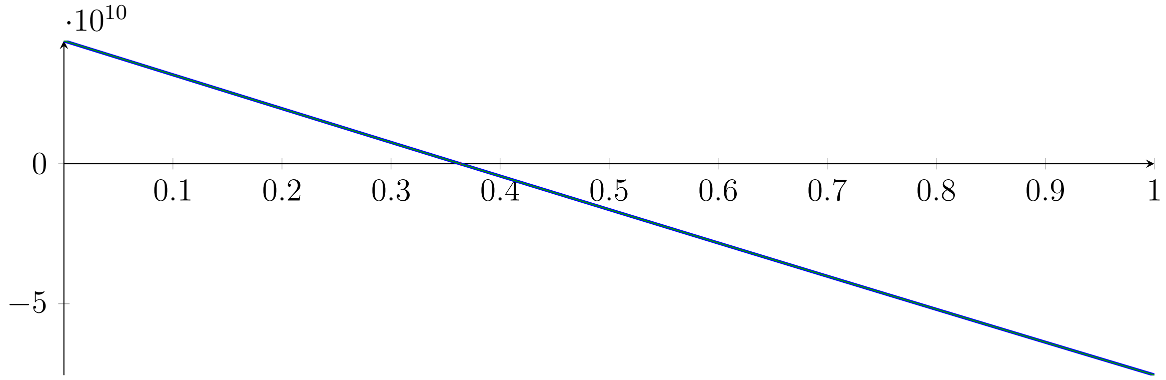
The maximum difference of the Bézier coefficients is $\delta = 3.62457{\cdot}10^7$.
Bounding polynomials M and m:

$$M = 2.35105{\cdot}10^9 X^2 - 1.21703{\cdot}10^{11} X + 4.39445{\cdot}10^{10}$$
$$m = 2.35105{\cdot}10^9 X^2 - 1.21703{\cdot}10^{11} X + 4.3872{\cdot}10^{10}$$

Root of M and m:

$$N(M) = \{0.363634, 51.4018\} \qquad N(m) = \{0.36303, 51.4024\}$$

Intersection intervals:



$$[0.36303, 0.363634]$$

Longest intersection interval: 0.000604122
$\Longrightarrow$ Selective recursion: interval 1: $[13.1501, 13.1501]$,

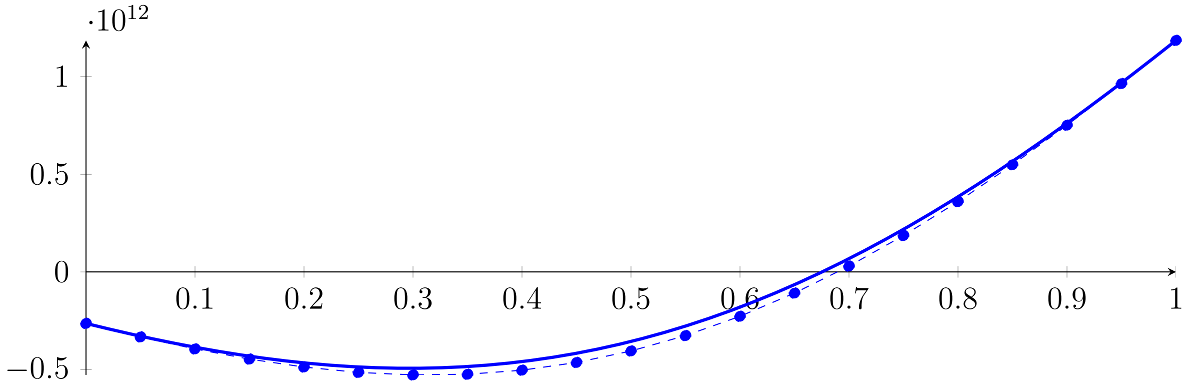
2.66 Recursion Branch 1 2 1 1 1 1 1 1 in Interval 1: $[13.1501, 13.1501]$

Found root in interval $[13.1501, 13.1501]$ at recursion depth 8!

2.67 Recursion Branch 1 2 1 1 1 2 on the Second Half $[13.2812, 14.0625]$

Normalized monomial und Bézier representations and the Bézier polygon:

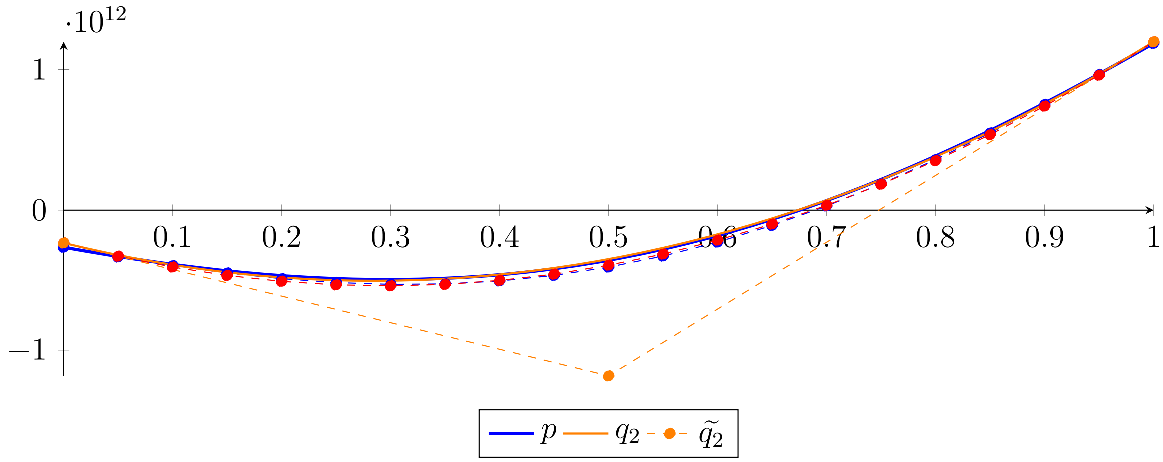
$$\begin{aligned} p = {} & 419.785X^{20} - 1104.09X^{19} + 13183.9X^{18} - 49424.6X^{17} + 297884X^{16} - 280941X^{15} \\ & - 27766.5X^{14} + 1.81264{\cdot}10^6X^{13} + 1.84521{\cdot}10^7X^{12} - 1.05999{\cdot}10^7X^{11} - 7.5227{\cdot}10^8X^{10} \\ & - 1.88218{\cdot}10^9X^9 + 1.2628{\cdot}10^{10}X^8 + 5.6459{\cdot}10^{10}X^7 - 6.82406{\cdot}10^{10}X^6 - 5.77934{\cdot}10^{11}X^5 \\ & - 1.43065{\cdot}10^{11}X^4 + 2.09319{\cdot}10^{12}X^3 + 1.46289{\cdot}10^{12}X^2 - 1.38422{\cdot}10^{12}X - 2.63682{\cdot}10^{11} \\ = {} & -2.63682{\cdot}10^{11}B_{0,20}(X) - 3.32893{\cdot}10^{11}B_{1,20}(X) - 3.94405{\cdot}10^{11}B_{2,20}(X) - 4.46381 \\ & {\cdot}10^{11}B_{3,20}(X) - 4.87015{\cdot}10^{11}B_{4,20}(X) - 5.14566{\cdot}10^{11}B_{5,20}(X) - 5.27402{\cdot}10^{11}B_{6,20}(X) \\ & - 5.24034{\cdot}10^{11}B_{7,20}(X) - 5.03161{\cdot}10^{11}B_{8,20}(X) - 4.63705{\cdot}10^{11}B_{9,20}(X) - 4.04855 \\ & {\cdot}10^{11}B_{10,20}(X) - 3.26096{\cdot}10^{11}B_{11,20}(X) - 2.27246{\cdot}10^{11}B_{12,20}(X) - 1.08476{\cdot}10^{11}B_{13,20}(X) \\ & + 2.96648{\cdot}10^{10}B_{14,20}(X) + 1.86236{\cdot}10^{11}B_{15,20}(X) + 3.59903{\cdot}10^{11}B_{16,20}(X) + 5.48938 \\ & {\cdot}10^{11}B_{17,20}(X) + 7.51232{\cdot}10^{11}B_{18,20}(X) + 9.64317{\cdot}10^{11}B_{19,20}(X) + 1.1854{\cdot}10^{12}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 &= 3.31952{\cdot}10^{12}X^2 - 1.8897{\cdot}10^{12}X - 2.32999{\cdot}10^{11} \\
&= -2.32999{\cdot}10^{11}B_{0,2} - 1.17785{\cdot}10^{12}B_{1,2} + 1.19682{\cdot}10^{12}B_{2,2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_2 &= 4.94535{\cdot}10^{14}X^{20} - 4.94688{\cdot}10^{15}X^{19} + 2.29024{\cdot}10^{16}X^{18} - 6.51124{\cdot}10^{16}X^{17} + 1.2719{\cdot}10^{17}X^{16} \\
&\quad - 1.80965{\cdot}10^{17}X^{15} + 1.93988{\cdot}10^{17}X^{14} - 1.5986{\cdot}10^{17}X^{13} + 1.02425{\cdot}10^{17}X^{12} - 5.12721{\cdot}10^{16}X^{11} \\
&\quad + 2.00414{\cdot}10^{16}X^{10} - 6.08302{\cdot}10^{15}X^{9} + 1.41823{\cdot}10^{15}X^{8} - 2.498{\cdot}10^{14}X^{7} + 3.24754{\cdot}10^{13}X^{6} - 3.01917 \\
&\quad \cdot 10^{12}X^{5} + 1.91812{\cdot}10^{11}X^{4} - 7.69263{\cdot}10^{9}X^{3} + 3.31969{\cdot}10^{12}X^{2} - 1.8897{\cdot}10^{12}X - 2.32999{\cdot}10^{11} \\
&= -2.32999{\cdot}10^{11}B_{0,20} - 3.27484{\cdot}10^{11}B_{1,20} - 4.04498{\cdot}10^{11}B_{2,20} - 4.64045{\cdot}10^{11}B_{3,20} - 5.06095 \\
&\quad \cdot 10^{11}B_{4,20} - 5.30769{\cdot}10^{11}B_{5,20} - 5.37701{\cdot}10^{11}B_{6,20} - 5.2778{\cdot}10^{11}B_{7,20} - 4.99236{\cdot}10^{11}B_{8,20} \\
&\quad - 4.55004{\cdot}10^{11}B_{9,20} - 3.9098{\cdot}10^{11}B_{10,20} - 3.12021{\cdot}10^{11}B_{11,20} - 2.13272{\cdot}10^{11}B_{12,20} \\
&\quad - 9.88307{\cdot}10^{10}B_{13,20} + 3.42229{\cdot}10^{10}B_{14,20} + 1.84138{\cdot}10^{11}B_{15,20} + 3.51795{\cdot}10^{11}B_{16,20} \\
&\quad + 5.36826{\cdot}10^{11}B_{17,20} + 7.39356{\cdot}10^{11}B_{18,20} + 9.5935{\cdot}10^{11}B_{19,20} + 1.19682{\cdot}10^{12}B_{20,20}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 3.06828{\cdot}10^{10}$.
Bounding polynomials M and m:

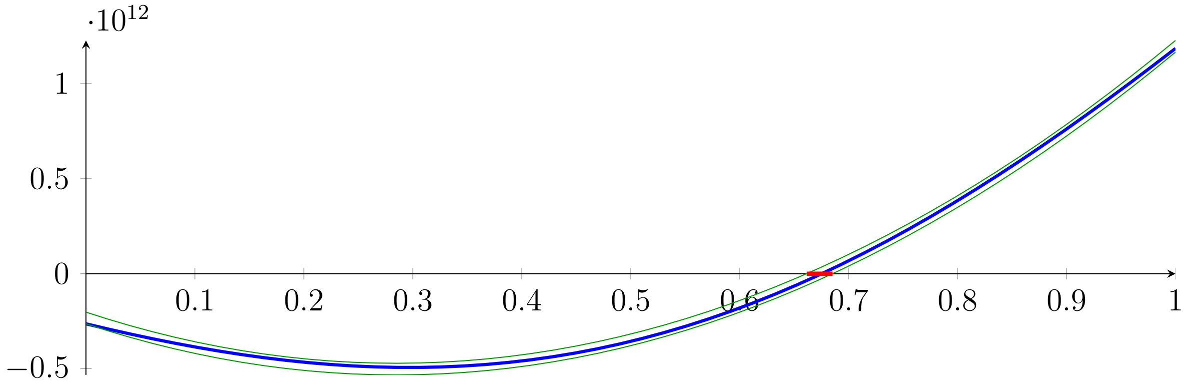
$$M = 3.31952{\cdot}10^{12}X^2 - 1.8897{\cdot}10^{12}X - 2.02316{\cdot}10^{11}$$

$$m = 3.31952{\cdot}10^{12}X^2 - 1.8897{\cdot}10^{12}X - 2.63682{\cdot}10^{11}$$

Root of M and m:

$$N(M) = \{-0.0921469, 0.661417\} \qquad N(m) = \{-0.115928, 0.685198\}$$

Intersection intervals:



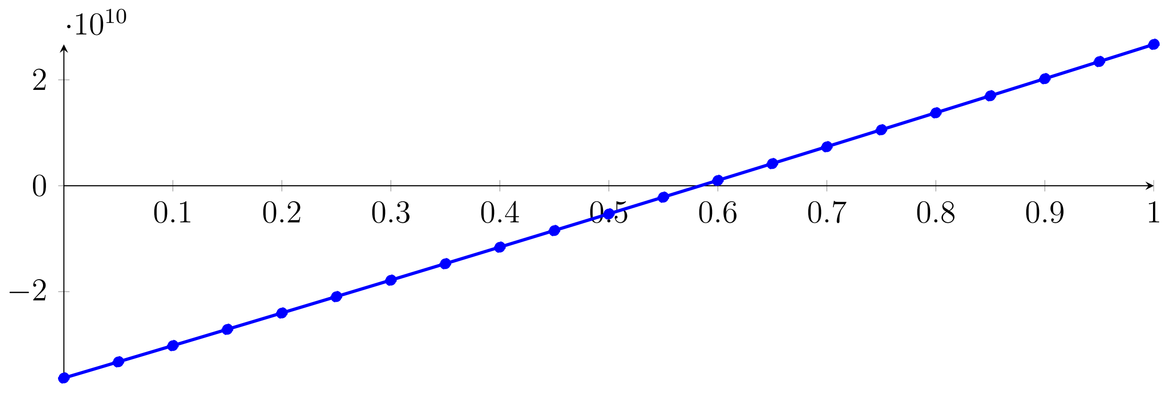
$[0.661417, 0.685198]$

Longest intersection interval: 0.0237813
$\Longrightarrow$ Selective recursion: interval 1: $[13.798, 13.8166]$,

2.68 Recursion Branch 1 2 1 1 1 2 1 in Interval 1: $[13.798, 13.8166]$

Normalized monomial und Bézier representations and the Bézier polygon:

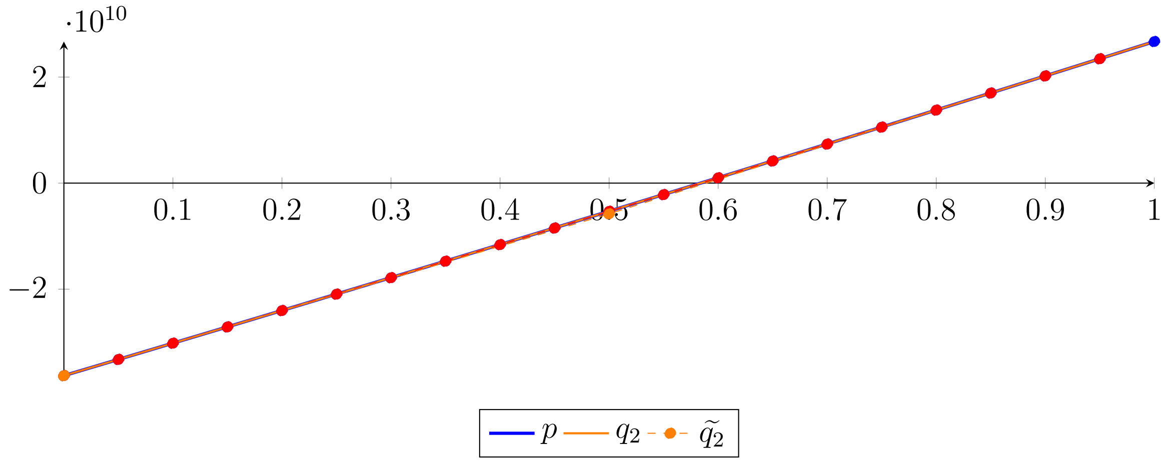
$$\begin{aligned}p &= 8.44749X^{20} + 12.1634X^{19} + 404.907X^{18} - 589.604X^{17} + 6737.21X^{16} \\ &\quad - 5732.38X^{15} + 3148.19X^{14} + 2012.05X^{13} + 8939.92X^{12} + 1940.09X^{11} \\ &\quad + 3030.59X^{10} + 535.639X^{9} + 1.92215X^{8} + 5.91431X^{7} + 69.1974X^{6} - 1478.34X^{5} \\ &\quad - 577095X^{4} - 1.02085\cdot 10^{7}X^{3} + 2.00619\cdot 10^{9}X^{2} + 6.10114\cdot 10^{10}X - 3.63059\cdot 10^{10} \\ &= -3.63059\cdot 10^{10}B_{0,20}(X) - 3.32553\cdot 10^{10}B_{1,20}(X) - 3.01942\cdot 10^{10}B_{2,20}(X) - 2.71225 \\ &\quad \cdot 10^{10}B_{3,20}(X) - 2.40403\cdot 10^{10}B_{4,20}(X) - 2.09476\cdot 10^{10}B_{5,20}(X) - 1.78443\cdot 10^{10}B_{6,20}(X) \\ &\quad - 1.47305\cdot 10^{10}B_{7,20}(X) - 1.16062\cdot 10^{10}B_{8,20}(X) - 8.47145\cdot 10^{9}B_{9,20}(X) - 5.32618 \\ &\quad \cdot 10^{9}B_{10,20}(X) - 2.17044\cdot 10^{9}B_{11,20}(X) + 9.95759\cdot 10^{8}B_{12,20}(X) + 4.17242\cdot 10^{9}B_{13,20}(X) \\ &\quad + 7.35952\cdot 10^{9}B_{14,20}(X) + 1.05571\cdot 10^{10}B_{15,20}(X) + 1.3765\cdot 10^{10}B_{16,20}(X) + 1.69834 \\ &\quad \cdot 10^{10}B_{17,20}(X) + 2.02121\cdot 10^{10}B_{18,20}(X) + 2.34513\cdot 10^{10}B_{19,20}(X) + 2.67008\cdot 10^{10}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 1.98989\cdot 10^{9}X^{2} + 6.1018\cdot 10^{10}X - 3.63065\cdot 10^{10} \\ &= -3.63065\cdot 10^{10}B_{0,2} - 5.79747\cdot 10^{9}B_{1,2} + 2.67014\cdot 10^{10}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 = {} & 3.60642\cdot 10^{12}X^{20} - 3.61455\cdot 10^{13}X^{19} + 1.67653\cdot 10^{14}X^{18} - 4.77532\cdot 10^{14}X^{17} + 9.34686\cdot 10^{14}X^{16} \\ & - 1.33308\cdot 10^{15}X^{15} + 1.43352\cdot 10^{15}X^{14} - 1.18637\cdot 10^{15}X^{13} + 7.64513\cdot 10^{14}X^{12} - 3.85543\cdot 10^{14}X^{11} \\ & + 1.52046\cdot 10^{14}X^{10} - 4.66026\cdot 10^{13}X^{9} + 1.09747\cdot 10^{13}X^{8} - 1.95326\cdot 10^{12}X^{7} + 2.57139\cdot 10^{11}X^{6} \\ & - 2.43259\cdot 10^{10}X^{5} + 1.58449\cdot 10^{9}X^{4} - 6.5832\cdot 10^{7}X^{3} + 1.99143\cdot 10^{9}X^{2} + 6.1018\cdot 10^{10}X - 3.63065\cdot 10^{10} \\ = {} & -3.63065\cdot 10^{10}B_{0,20} - 3.32556\cdot 10^{10}B_{1,20} - 3.01942\cdot 10^{10}B_{2,20} - 2.71224\cdot 10^{10}B_{3,20} - 2.40399 \\ & \cdot 10^{10}B_{4,20} - 2.09477\cdot 10^{10}B_{5,20} - 1.78429\cdot 10^{10}B_{6,20} - 1.47324\cdot 10^{10}B_{7,20} - 1.16027 \\ & \cdot 10^{10}B_{8,20} - 8.47558\cdot 10^{9}B_{9,20} - 5.32112\cdot 10^{9}B_{10,20} - 2.17491\cdot 10^{9}B_{11,20} + 9.98754\cdot 10^{8}B_{12,20} \\ & + 4.17024\cdot 10^{9}B_{13,20} + 7.36017\cdot 10^{9}B_{14,20} + 1.05563\cdot 10^{10}B_{15,20} + 1.37648\cdot 10^{10}B_{16,20} \\ & + 1.69831\cdot 10^{10}B_{17,20} + 2.02121\cdot 10^{10}B_{18,20} + 2.34515\cdot 10^{10}B_{19,20} + 2.67014\cdot 10^{10}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 5.06532\cdot 10^{6}$.

Bounding polynomials M and m:

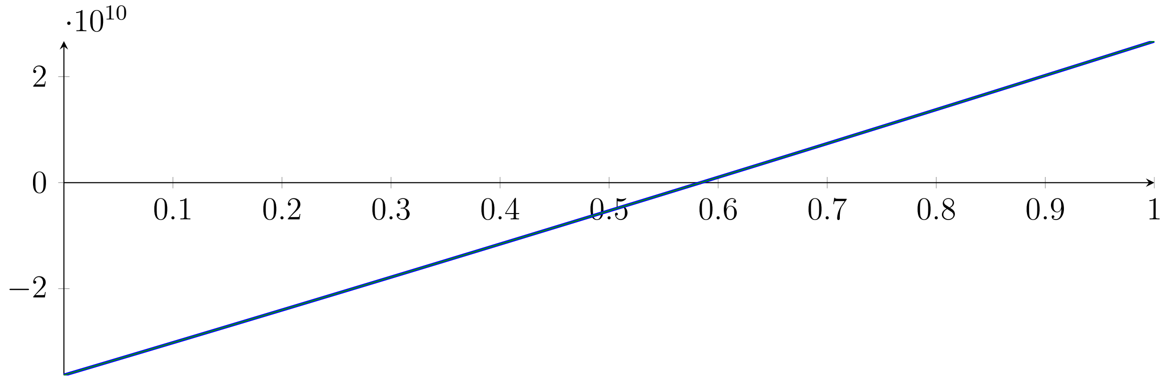
$$M = 1.98989\cdot 10^{9}X^2 + 6.1018\cdot 10^{10}X - 3.63014\cdot 10^{10}$$

$$m = 1.98989\cdot 10^{9}X^2 + 6.1018\cdot 10^{10}X - 3.63115\cdot 10^{10}$$

Root of M and m:

$$N(M) = \{-31.2479, 0.583814\} \qquad N(m) = \{-31.248, 0.583974\}$$

Intersection intervals:



$$[0.583814, 0.583974]$$

Longest intersection interval: 0.000159936

$\Longrightarrow$ Selective recursion: interval 1: $[13.8088, 13.8088]$,

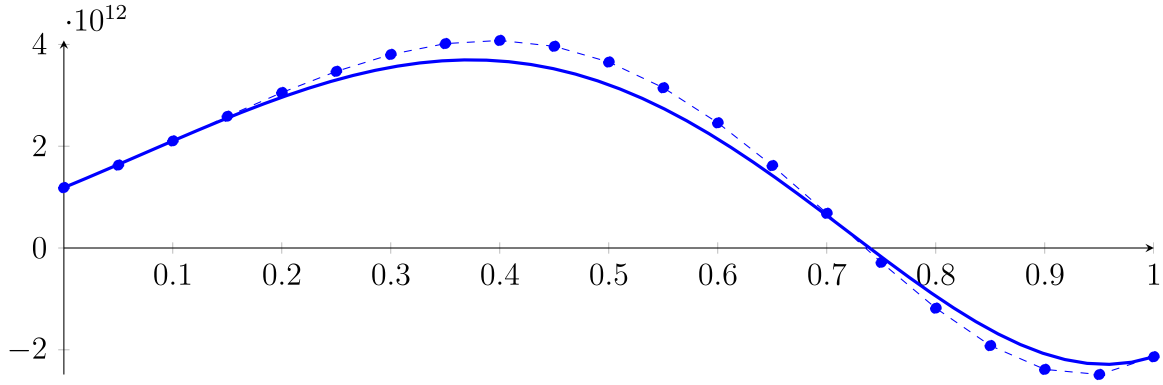
2.69 Recursion Branch 1 2 1 1 1 2 1 1 in Interval 1: $[13.8088, 13.8088]$

Found root in interval $[13.8088, 13.8088]$ at recursion depth 8!

2.70 Recursion Branch 1 2 1 1 2 on the Second Half $[14.0625, 15.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

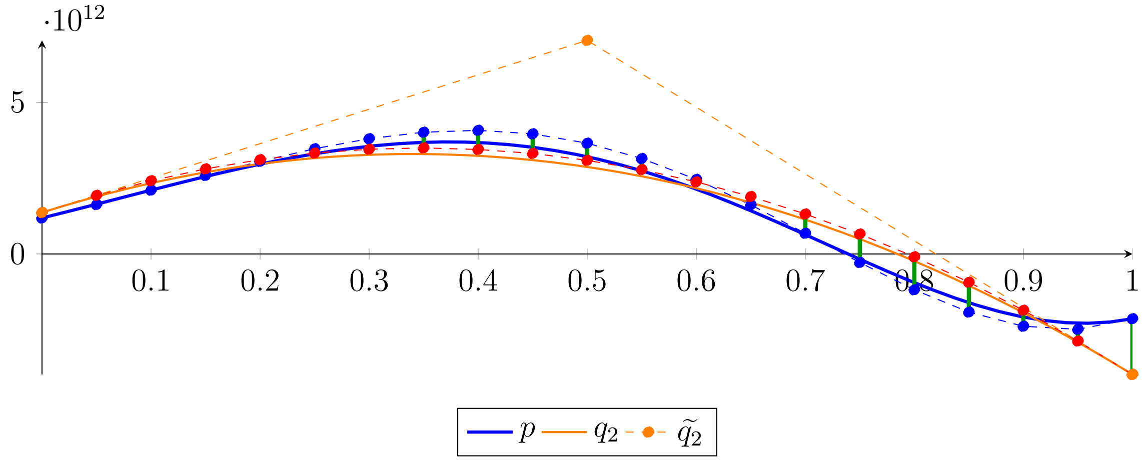
$$\begin{aligned} p = & 3986.15X^{20} + 355982X^{19} + 6.295\cdot10^{6}X^{18} + 6.00178\cdot10^{7}X^{17} + 2.24902\cdot10^{8}X^{16} - 6.32265 \\ & \cdot10^{8}X^{15} - 9.75189\cdot10^{9}X^{14} - 2.84929\cdot10^{10}X^{13} + 5.90133\cdot10^{10}X^{12} + 5.19357\cdot10^{11}X^{11} + 6.2382 \\ & \cdot10^{11}X^{10} - 2.63478\cdot10^{12}X^{9} - 7.48493\cdot10^{12}X^{8} + 1.62878\cdot10^{12}X^{7} + 2.42459\cdot10^{13}X^{6} + 1.56831 \\ & \cdot10^{13}X^{5} - 2.53581\cdot10^{13}X^{4} - 2.54855\cdot10^{13}X^{3} + 6.07786\cdot10^{12}X^{2} + 8.8433\cdot10^{12}X + 1.1854\cdot10^{12} \\ = & 1.1854\cdot10^{12}B_{0,20}(X) + 1.62756\cdot10^{12}B_{1,20}(X) + 2.10172\cdot10^{12}B_{2,20}(X) + 2.58551 \\ & \cdot10^{12}B_{3,20}(X) + 3.05134\cdot10^{12}B_{4,20}(X) + 3.4674\cdot10^{12}B_{5,20}(X) + 3.79929\cdot10^{12}B_{6,20}(X) \\ & + 4.01233\cdot10^{12}B_{7,20}(X) + 4.07439\cdot10^{12}B_{8,20}(X) + 3.95934\cdot10^{12}B_{9,20}(X) + 3.65071 \\ & \cdot10^{12}B_{10,20}(X) + 3.14537\cdot10^{12}B_{11,20}(X) + 2.4568\cdot10^{12}B_{12,20}(X) + 1.61759\cdot10^{12}B_{13,20}(X) \\ & + 6.80535\cdot10^{11}B_{14,20}(X) - 2.82012\cdot10^{11}B_{15,20}(X) - 1.18103\cdot10^{12}B_{16,20}(X) - 1.91608 \\ & \cdot10^{12}B_{17,20}(X) - 2.38295\cdot10^{12}B_{18,20}(X) - 2.48328\cdot10^{12}B_{19,20}(X) - 2.1354\cdot10^{12}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 = & -1.66838\cdot10^{13}X^{2} + 1.13476\cdot10^{13}X + 1.3653\cdot10^{12} \\ = & 1.3653\cdot10^{12}B_{0,2} + 7.0391\cdot10^{12}B_{1,2} - 3.9709\cdot10^{12}B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 = & -2.61849\cdot10^{15}X^{20} + 2.62011\cdot10^{16}X^{19} - 1.21349\cdot10^{17}X^{18} + 3.45155\cdot10^{17}X^{17} - 6.74551\cdot10^{17}X^{16} \\ & + 9.60226\cdot10^{17}X^{15} - 1.02981\cdot10^{18}X^{14} + 8.48975\cdot10^{17}X^{13} - 5.44101\cdot10^{17}X^{12} + 2.72394\cdot10^{17}X^{11} \\ & - 1.06462\cdot10^{17}X^{10} + 3.23025\cdot10^{16}X^{9} - 7.52718\cdot10^{15}X^{8} + 1.32518\cdot10^{15}X^{7} - 1.72323\cdot10^{14}X^{6} + 1.60571 \\ & \cdot10^{13}X^{5} - 1.02685\cdot10^{12}X^{4} + 4.16915\cdot10^{10}X^{3} - 1.66848\cdot10^{13}X^{2} + 1.13476\cdot10^{13}X + 1.3653\cdot10^{12} \\ = & 1.3653\cdot10^{12}B_{0,20} + 1.93268\cdot10^{12}B_{1,20} + 2.41225\cdot10^{12}B_{2,20} + 2.80404\cdot10^{12}B_{3,20} + 3.10787 \\ & \cdot10^{12}B_{4,20} + 3.3244\cdot10^{12}B_{5,20} + 3.45169\cdot10^{12}B_{6,20} + 3.49445\cdot10^{12}B_{7,20} + 3.44327\cdot10^{12}B_{8,20} \\ & + 3.31376\cdot10^{12}B_{9,20} + 3.08412\cdot10^{12}B_{10,20} + 2.7801\cdot10^{12}B_{11,20} + 2.37603\cdot10^{12}B_{12,20} \\ & + 1.89356\cdot10^{12}B_{13,20} + 1.31722\cdot10^{12}B_{14,20} + 6.56311\cdot10^{11}B_{15,20} - 9.38488\cdot10^{10}B_{16,20} \\ & - 9.31305\cdot10^{11}B_{17,20} - 1.85671\cdot10^{12}B_{18,20} - 2.8699\cdot10^{12}B_{19,20} - 3.9709\cdot10^{12}B_{20,20} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.8355{\cdot}10^{12}$.

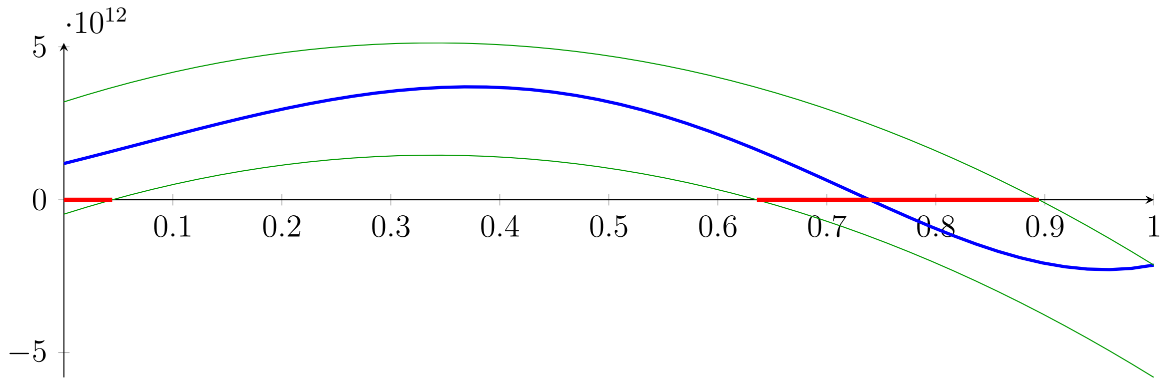
Bounding polynomials M and m:

$$M = -1.66838{\cdot}10^{13}X^2 + 1.13476{\cdot}10^{13}X + 3.2008{\cdot}10^{12}$$
$$m = -1.66838{\cdot}10^{13}X^2 + 1.13476{\cdot}10^{13}X - 4.70198{\cdot}10^{11}$$

Root of M and m:

$$N(M) = \{-0.214452, 0.894609\} \qquad N(m) = \{0.0443244, 0.635833\}$$

Intersection intervals:



$$[0, 0.0443244],\ [0.635833, 0.894609]$$

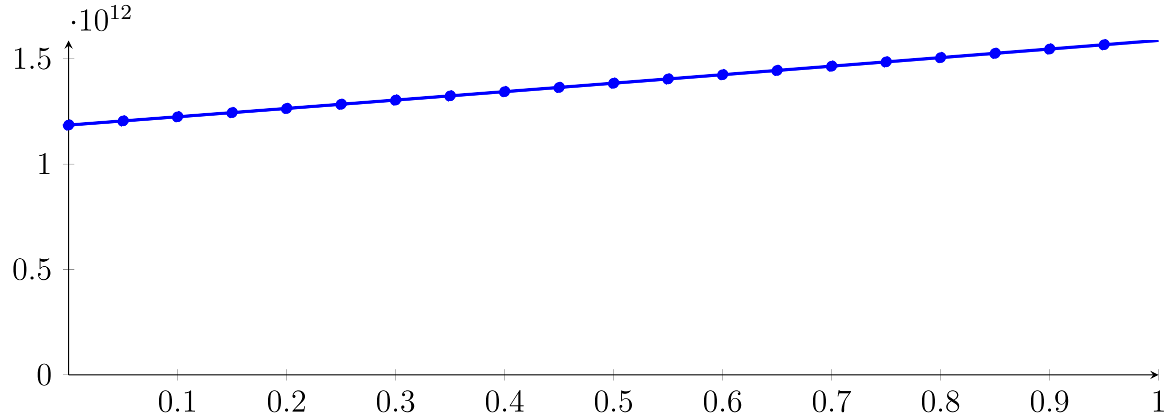
Longest intersection interval: 0.258776

$\Longrightarrow$ Selective recursion: interval 1: $[14.0625, 14.1318]$, interval 2: $[15.056, 15.4603]$,

2.71 Recursion Branch 1 2 1 1 2 1 in Interval 1: $[14.0625, 14.1318]$

Normalized monomial und Bézier representations and the Bézier polygon:

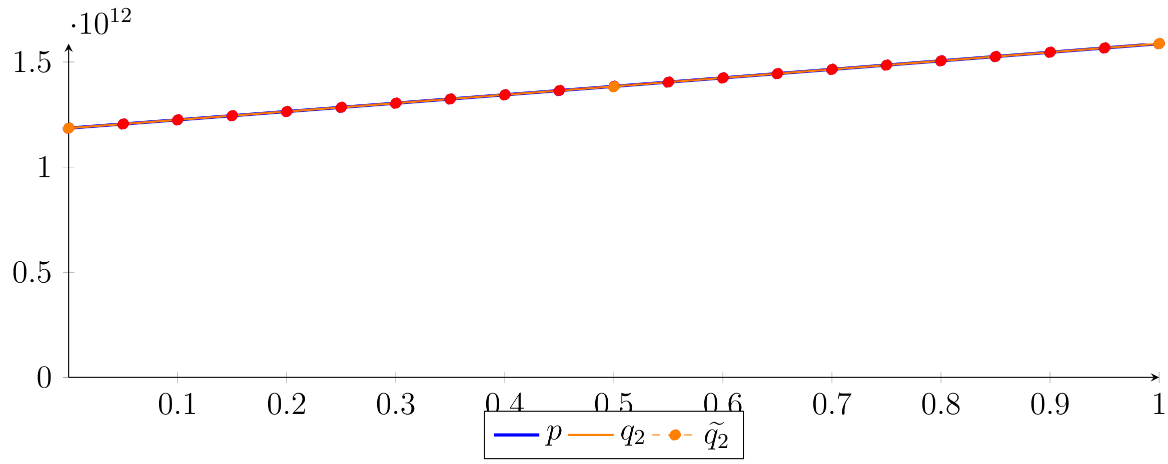
$$\begin{aligned}p = &-1375.19X^{20} + 7528.21X^{19} - 40129.4X^{18} + 162651X^{17} - 860633X^{16}\\ &+ 669405X^{15} - 300485X^{14} - 60373.2X^{13} - 642705X^{12} - 73072.4X^{11} - 199280X^{10}\\ &- 18944.7X^9 - 3998.07X^8 + 719.18X^7 + 182842X^6 + 2.6832{\cdot}10^6X^5 - 9.78792\\ &{\cdot}10^7X^4 - 2.21933{\cdot}10^9X^3 + 1.19409{\cdot}10^{10}X^2 + 3.91974{\cdot}10^{11}X + 1.1854{\cdot}10^{12}\\ = &\ 1.1854{\cdot}10^{12}B_{0,20}(X) + 1.205{\cdot}10^{12}B_{1,20}(X) + 1.22466{\cdot}10^{12}B_{2,20}(X) + 1.24438\\ &{\cdot}10^{12}B_{3,20}(X) + 1.26416{\cdot}10^{12}B_{4,20}(X) + 1.284{\cdot}10^{12}B_{5,20}(X) + 1.3039{\cdot}10^{12}B_{6,20}(X)\\ &+ 1.32384{\cdot}10^{12}B_{7,20}(X) + 1.34384{\cdot}10^{12}B_{8,20}(X) + 1.36388{\cdot}10^{12}B_{9,20}(X) + 1.38398\\ &{\cdot}10^{12}B_{10,20}(X) + 1.40411{\cdot}10^{12}B_{11,20}(X) + 1.42429{\cdot}10^{12}B_{12,20}(X) + 1.44451{\cdot}10^{12}B_{13,20}(X)\\ &+ 1.46477{\cdot}10^{12}B_{14,20}(X) + 1.48507{\cdot}10^{12}B_{15,20}(X) + 1.50539{\cdot}10^{12}B_{16,20}(X) + 1.52575\\ &{\cdot}10^{12}B_{17,20}(X) + 1.54614{\cdot}10^{12}B_{18,20}(X) + 1.56656{\cdot}10^{12}B_{19,20}(X) + 1.587{\cdot}10^{12}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$q_2 = 8.44923{\cdot}10^{9}X^2 + 3.93392{\cdot}10^{11}X + 1.18528{\cdot}10^{12}$$
$$= 1.18528{\cdot}10^{12}B_{0,2} + 1.38198{\cdot}10^{12}B_{1,2} + 1.58712{\cdot}10^{12}B_{2,2}$$

$$\widetilde{q}_2 = -2.35765{\cdot}10^{14}X^{20} + 2.36935{\cdot}10^{15}X^{19} - 1.10298{\cdot}10^{16}X^{18} + 3.15543{\cdot}10^{16}X^{17} - 6.20555{\cdot}10^{16}X^{16} + 8.89126{\cdot}10^{16}X^{15} - 9.5972{\cdot}10^{16}X^{14} + 7.95982{\cdot}10^{16}X^{13} - 5.12826{\cdot}10^{16}X^{12} + 2.57776{\cdot}10^{16}X^{11} - 1.00986{\cdot}10^{16}X^{10} + 3.0648{\cdot}10^{15}X^{9} - 7.1303{\cdot}10^{14}X^{8} + 1.25434{\cdot}10^{14}X^{7} - 1.644{\cdot}10^{13}X^{6} + 1.58052{\cdot}10^{12}X^{5} - 1.08952{\cdot}10^{11}X^{4} + 4.99936{\cdot}10^{9}X^{3} + 8.31758{\cdot}10^{9}X^{2} + 3.93394{\cdot}10^{11}X + 1.18528{\cdot}10^{12}$$
$$= 1.18528{\cdot}10^{12}B_{0,20} + 1.20495{\cdot}10^{12}B_{1,20} + 1.22466{\cdot}10^{12}B_{2,20} + 1.24443{\cdot}10^{12}B_{3,20} + 1.26422{\cdot}10^{12}B_{4,20} + 1.2841{\cdot}10^{12}B_{5,20} + 1.30389{\cdot}10^{12}B_{6,20} + 1.32404{\cdot}10^{12}B_{7,20} + 1.34365{\cdot}10^{12}B_{8,20} + 1.36419{\cdot}10^{12}B_{9,20} + 1.38364{\cdot}10^{12}B_{10,20} + 1.40434{\cdot}10^{12}B_{11,20} + 1.42403{\cdot}10^{12}B_{12,20} + 1.44458{\cdot}10^{12}B_{13,20} + 1.46463{\cdot}10^{12}B_{14,20} + 1.48502{\cdot}10^{12}B_{15,20} + 1.50532{\cdot}10^{12}B_{16,20} + 1.52571{\cdot}10^{12}B_{17,20} + 1.54614{\cdot}10^{12}B_{18,20} + 1.56661{\cdot}10^{12}B_{19,20} + 1.58712{\cdot}10^{12}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 3.3779{\cdot}10^{8}$.
Bounding polynomials M and m:

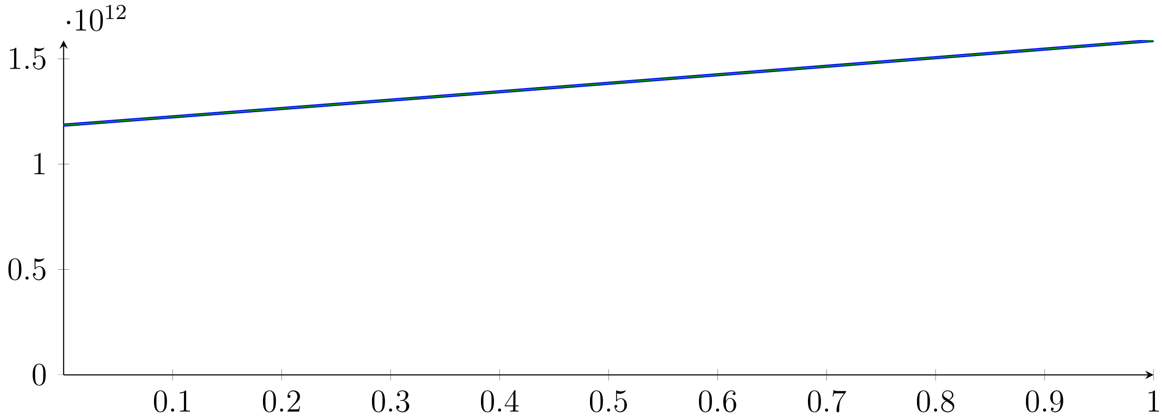
$$M = 8.44923{\cdot}10^{9}X^2 + 3.93392{\cdot}10^{11}X + 1.18562{\cdot}10^{12}$$

$$m = 8.44923{\cdot}10^{9}X^2 + 3.93392{\cdot}10^{11}X + 1.18494{\cdot}10^{12}$$

Root of M and m:

$$N(M) = \{-43.3204, -3.23918\} \qquad N(m) = \{-43.3224, -3.23719\}$$

Intersection intervals:

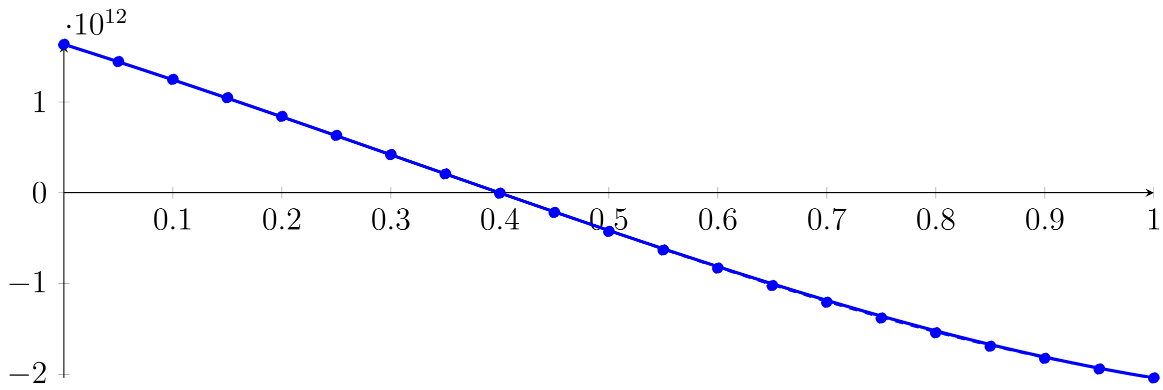


No intersection intervals with the x axis.

2.72 Recursion Branch 1 2 1 1 2 2 in Interval 2: $[15.056, 15.4603]$

Normalized monomial und Bézier representations and the Bézier polygon:

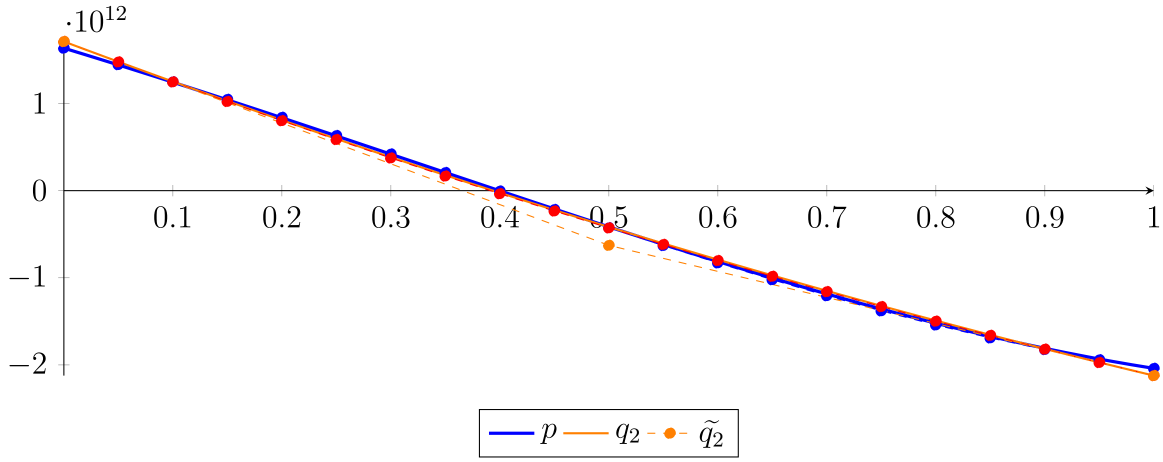
$$\begin{aligned} p &= 206.527X^{20} - 4998.83X^{19} - 3990.74X^{18} - 48067X^{17} + 47787.6X^{16} + 12657.6X^{15} \\ &\quad - 57534.4X^{14} - 69344.1X^{13} - 245789X^{12} - 380206X^{11} + 2.76521\cdot 10^{6}X^{10} + 5.61882 \\ &\quad \cdot 10^{7}X^{9} + 1.96629\cdot 10^{8}X^{8} - 2.12664\cdot 10^{9}X^{7} - 1.89306\cdot 10^{10}X^{6} - 8.31527\cdot 10^{9}X^{5} \\ &\quad + 3.68763\cdot 10^{11}X^{4} + 9.48769\cdot 10^{11}X^{3} - 1.15921\cdot 10^{12}X^{2} - 3.80456\cdot 10^{12}X + 1.63572\cdot 10^{12} \\ &= 1.63572\cdot 10^{12}B_{0,20}(X) + 1.4455\cdot 10^{12}B_{1,20}(X) + 1.24917\cdot 10^{12}B_{2,20}(X) + 1.04757 \\ &\quad \cdot 10^{12}B_{3,20}(X) + 8.41611\cdot 10^{11}B_{4,20}(X) + 6.32276\cdot 10^{11}B_{5,20}(X) + 4.20623\cdot 10^{11}B_{6,20}(X) \\ &\quad + 2.07784\cdot 10^{11}B_{7,20}(X) - 5.03947\cdot 10^{9}B_{8,20}(X) - 2.16576\cdot 10^{11}B_{9,20}(X) - 4.25491 \\ &\quad \cdot 10^{11}B_{10,20}(X) - 6.30386\cdot 10^{11}B_{11,20}(X) - 8.29808\cdot 10^{11}B_{12,20}(X) - 1.02225\cdot 10^{12}B_{13,20}(X) \\ &\quad - 1.20617\cdot 10^{12}B_{14,20}(X) - 1.37996\cdot 10^{12}B_{15,20}(X) - 1.54201\cdot 10^{12}B_{16,20}(X) - 1.69067 \\ &\quad \cdot 10^{12}B_{17,20}(X) - 1.82427\cdot 10^{12}B_{18,20}(X) - 1.94115\cdot 10^{12}B_{19,20}(X) - 2.03962\cdot 10^{12}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 8.44166\cdot 10^{11}X^2 - 4.67824\cdot 10^{12}X + 1.71139\cdot 10^{12} \\ &= 1.71139\cdot 10^{12}B_{0,2} - 6.27729\cdot 10^{11}B_{1,2} - 2.12268\cdot 10^{12}B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 = {} & 2.46311\cdot 10^{13}X^{20} - 2.47064\cdot 10^{14}X^{19} + 1.15243\cdot 10^{15}X^{18} - 3.31209\cdot 10^{15}X^{17} + 6.54652\cdot 10^{15}X^{16} \\ & - 9.40121\cdot 10^{15}X^{15} + 1.01006\cdot 10^{16}X^{14} - 8.23961\cdot 10^{15}X^{13} + 5.13058\cdot 10^{15}X^{12} - 2.43544\cdot 10^{15}X^{11} \\ & + 8.76008\cdot 10^{14}X^{10} - 2.36491\cdot 10^{14}X^{9} + 4.73539\cdot 10^{13}X^{8} - 6.96229\cdot 10^{12}X^{7} + 7.61444\cdot 10^{11}X^{6} - 6.8396 \\ & \cdot 10^{10}X^{5} + 5.87602\cdot 10^{9}X^{4} - 3.86809\cdot 10^{8}X^{3} + 8.4418\cdot 10^{11}X^{2} - 4.67824\cdot 10^{12}X + 1.71139\cdot 10^{12} \\ = {} & 1.71139\cdot 10^{12}B_{0,20} + 1.47748\cdot 10^{12}B_{1,20} + 1.24801\cdot 10^{12}B_{2,20} + 1.02298\cdot 10^{12}B_{3,20} + 8.02402 \\ & \cdot 10^{11}B_{4,20} + 5.8626\cdot 10^{11}B_{5,20} + 3.7457\cdot 10^{11}B_{6,20} + 1.67297\cdot 10^{11}B_{7,20} - 3.54733\cdot 10^{10}B_{8,20} \\ & - 2.33907\cdot 10^{11}B_{9,20} - 4.27764\cdot 10^{11}B_{10,20} - 6.17289\cdot 10^{11}B_{11,20} - 8.02285\cdot 10^{11}B_{12,20} \\ & - 9.82932\cdot 10^{11}B_{13,20} - 1.15906\cdot 10^{12}B_{14,20} - 1.33078\cdot 10^{12}B_{15,20} - 1.49804\cdot 10^{12}B_{16,20} \\ & - 1.66087\cdot 10^{12}B_{17,20} - 1.81925\cdot 10^{12}B_{18,20} - 1.97319\cdot 10^{12}B_{19,20} - 2.12268\cdot 10^{12}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 8.30596\cdot 10^{10}$.
Bounding polynomials M and m:

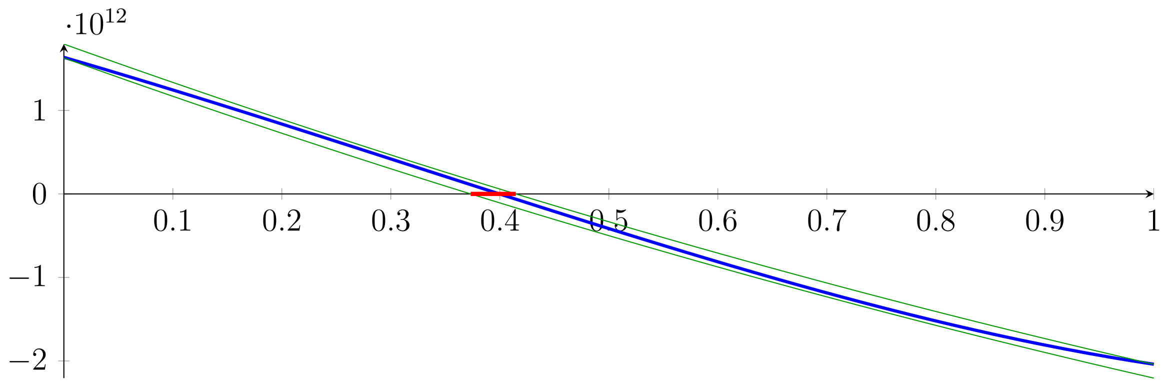
$$M = 8.44166\cdot 10^{11}X^2 - 4.67824\cdot 10^{12}X + 1.79445\cdot 10^{12}$$

$$m = 8.44166\cdot 10^{11}X^2 - 4.67824\cdot 10^{12}X + 1.62833\cdot 10^{12}$$

Root of M and m:

$$N(M) = \{0.41459, 5.12726\} \qquad N(m) = \{0.373197, 5.16865\}$$

Intersection intervals:



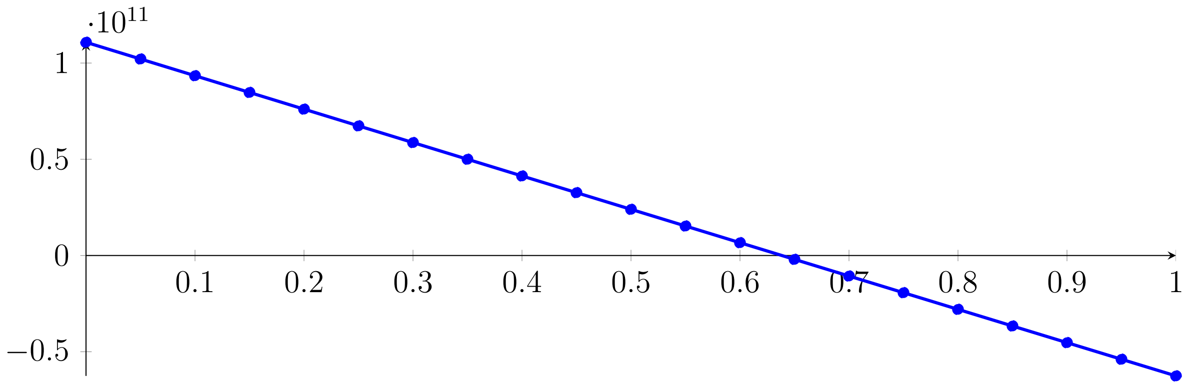
$$[0.373197, 0.41459]$$

Longest intersection interval: 0.041393
$\Longrightarrow$ Selective recursion: interval 1: $[15.2069, 15.2236]$,

2.73 Recursion Branch 1 2 1 1 2 2 1 in Interval 1: $[15.2069, 15.2236]$

Normalized monomial und Bézier representations and the Bézier polygon:

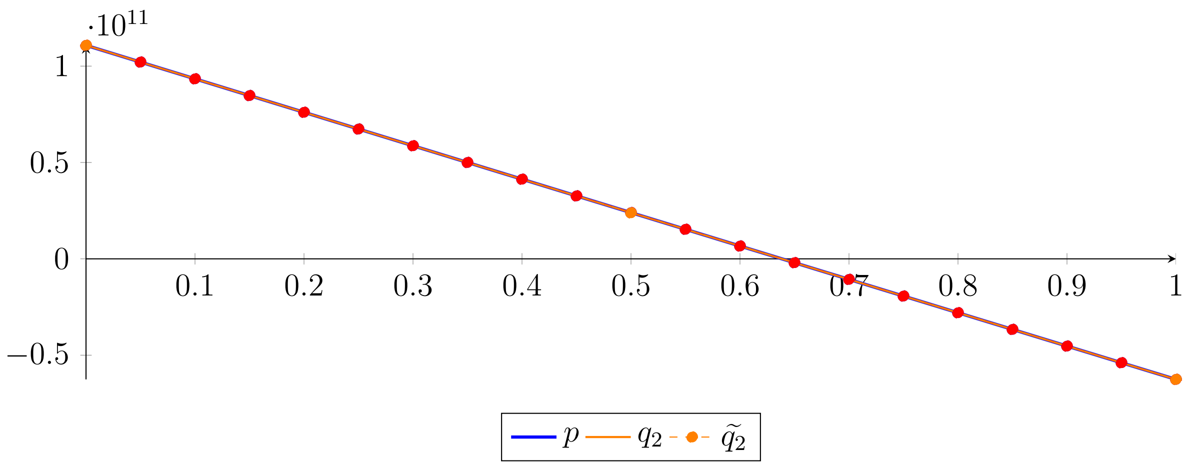
$$\begin{aligned}p &= -32.1604X^{20} + 7.05505X^{19} - 1327.4X^{18} + 2837.16X^{17} - 24488.6X^{16} \\ &\quad + 20935.2X^{15} - 10682.4X^{14} - 5282.66X^{13} - 29305.1X^{12} - 5827.96X^{11} \\ &\quad - 9737.35X^{10} - 1522.34X^{9} - 149.928X^{8} - 21.2915X^{7} - 188.075X^{6} - 6828.9X^{5} \\ &\quad + 910498X^{4} + 1.04017\cdot 10^{8}X^{3} + 3.4447\cdot 10^{8}X^{2} - 1.73783\cdot 10^{11}X + 1.10781\cdot 10^{11} \\ &= 1.10781\cdot 10^{11}B_{0,20}(X) + 1.02092\cdot 10^{11}B_{1,20}(X) + 9.34043\cdot 10^{10}B_{2,20}(X) + 8.47188 \\ &\quad \cdot 10^{10}B_{3,20}(X) + 7.60354\cdot 10^{10}B_{4,20}(X) + 6.73541\cdot 10^{10}B_{5,20}(X) + 5.86749\cdot 10^{10}B_{6,20}(X) \\ &\quad + 4.99981\cdot 10^{10}B_{7,20}(X) + 4.13235\cdot 10^{10}B_{8,20}(X) + 3.26515\cdot 10^{10}B_{9,20}(X) + 2.3982 \\ &\quad \cdot 10^{10}B_{10,20}(X) + 1.53151\cdot 10^{10}B_{11,20}(X) + 6.65096\cdot 10^{9}B_{12,20}(X) - 2.01035\cdot 10^{9}B_{13,20}(X) \\ &\quad - 1.06687\cdot 10^{10}B_{14,20}(X) - 1.93241\cdot 10^{10}B_{15,20}(X) - 2.79764\cdot 10^{10}B_{16,20}(X) - 3.66255 \\ &\quad \cdot 10^{10}B_{17,20}(X) - 4.52712\cdot 10^{10}B_{18,20}(X) - 5.39136\cdot 10^{10}B_{19,20}(X) - 6.25525\cdot 10^{10}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 5.02044\cdot 10^{8}X^{2} - 1.73846\cdot 10^{11}X + 1.10786\cdot 10^{11} \\ &= 1.10786\cdot 10^{11}B_{0,2} + 2.38631\cdot 10^{10}B_{1,2} - 6.25578\cdot 10^{10}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= -1.09579\cdot 10^{13}X^{20} + 1.09888\cdot 10^{14}X^{19} - 5.10049\cdot 10^{14}X^{18} + 1.45396\cdot 10^{15}X^{17} - 2.84838\cdot 10^{15}X^{16} \\ &\quad + 4.06608\cdot 10^{15}X^{15} - 4.37604\cdot 10^{15}X^{14} + 3.62405\cdot 10^{15}X^{13} - 2.33638\cdot 10^{15}X^{12} + 1.17835\cdot 10^{15}X^{11} \\ &\quad - 4.64565\cdot 10^{14}X^{10} + 1.42289\cdot 10^{14}X^{9} - 3.34743\cdot 10^{13}X^{8} + 5.95232\cdot 10^{12}X^{7} - 7.83845\cdot 10^{11}X^{6} + 7.44258 \\ &\quad \cdot 10^{10}X^{5} - 4.89828\cdot 10^{9}X^{4} + 2.07295\cdot 10^{8}X^{3} + 4.97079\cdot 10^{8}X^{2} - 1.73846\cdot 10^{11}X + 1.10786\cdot 10^{11} \\ &= 1.10786\cdot 10^{11}B_{0,20} + 1.02094\cdot 10^{11}B_{1,20} + 9.3404\cdot 10^{10}B_{2,20} + 8.47171\cdot 10^{10}B_{3,20} + 7.60322 \\ &\quad \cdot 10^{10}B_{4,20} + 6.73523\cdot 10^{10}B_{5,20} + 5.86685\cdot 10^{10}B_{6,20} + 5.00019\cdot 10^{10}B_{7,20} + 4.13115\cdot 10^{10}B_{8,20} \\ &\quad + 3.26634\cdot 10^{10}B_{9,20} + 2.39665\cdot 10^{10}B_{10,20} + 1.53291\cdot 10^{10}B_{11,20} + 6.64309\cdot 10^{9}B_{12,20} \\ &\quad - 2.00198\cdot 10^{9}B_{13,20} - 1.06687\cdot 10^{10}B_{14,20} - 1.93197\cdot 10^{10}B_{15,20} - 2.7974\cdot 10^{10}B_{16,20} \\ &\quad - 3.66235\cdot 10^{10}B_{17,20} - 4.5271\cdot 10^{10}B_{18,20} - 5.39157\cdot 10^{10}B_{19,20} - 6.25578\cdot 10^{10}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.54837 \cdot 10^7$.

Bounding polynomials M and m:

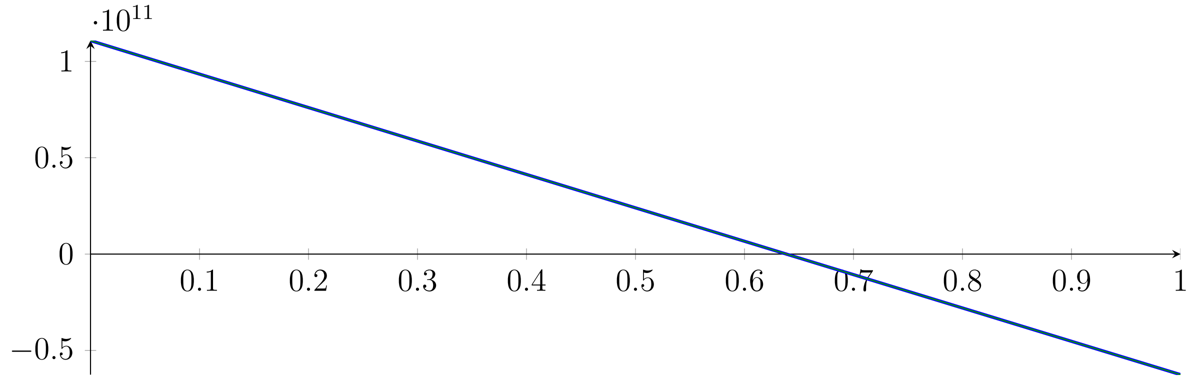
$$M = 5.02044 \cdot 10^8 X^2 - 1.73846 \cdot 10^{11} X + 1.10801 \cdot 10^{11}$$

$$m = 5.02044 \cdot 10^8 X^2 - 1.73846 \cdot 10^{11} X + 1.1077 \cdot 10^{11}$$

Root of M and m:

$$N(M) = \{0.638532, 345.637\} \qquad N(m) = \{0.638353, 345.638\}$$

Intersection intervals:



$$[0.638353, 0.638532]$$

Longest intersection interval: 0.00017879

$\Longrightarrow$ Selective recursion: interval 1: $[15.2176, 15.2176]$,

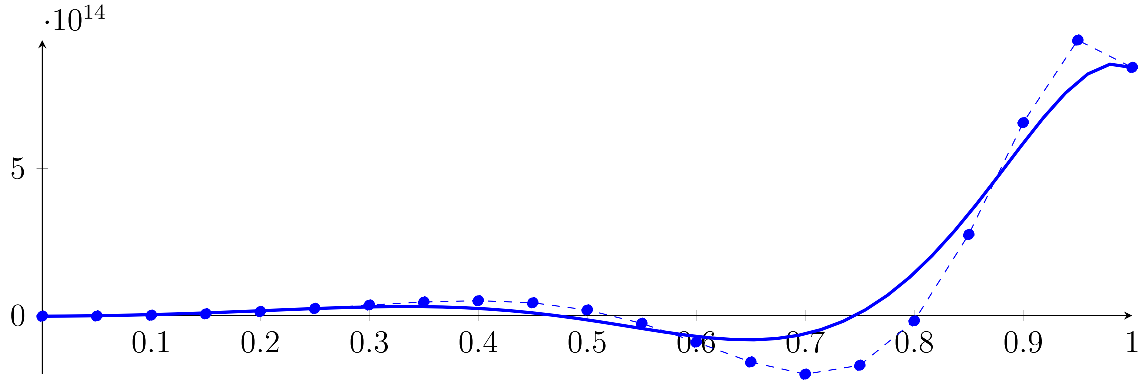
2.74 Recursion Branch 1 2 1 1 2 2 1 1 in Interval 1: $[15.2176, 15.2176]$

Found root in interval $[15.2176, 15.2176]$ at recursion depth 8!

2.75 Recursion Branch 1 2 1 2 on the Second Half $[15.625, 18.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

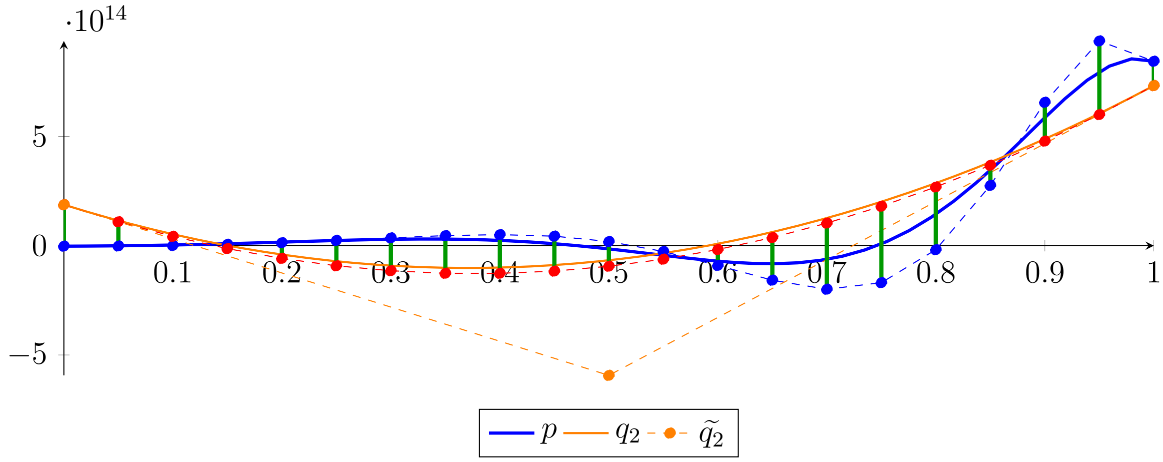
$$\begin{aligned}
p = {} & 7.88858 \cdot 10^9 X^{20} + 2.58746 \cdot 10^{11} X^{19} + 3.76268 \cdot 10^{12} X^{18} + 3.17389 \cdot 10^{13} X^{17} + 1.69708 \cdot 10^{14} X^{16} \\
& + 5.82695 \cdot 10^{14} X^{15} + 1.18664 \cdot 10^{15} X^{14} + 8.38279 \cdot 10^{14} X^{13} - 2.32497 \cdot 10^{15} X^{12} - 7.06233 \cdot 10^{15} X^{11} \\
& - 6.4407 \cdot 10^{15} X^{10} + 3.31615 \cdot 10^{15} X^9 + 1.18856 \cdot 10^{16} X^8 + 7.3503 \cdot 10^{15} X^7 - 3.10022 \cdot 10^{15} X^6 - 5.3941 \\
& \cdot 10^{15} X^5 - 1.29591 \cdot 10^{15} X^4 + 7.44661 \cdot 10^{14} X^3 + 3.40631 \cdot 10^{14} X^2 + 1.3915 \cdot 10^{13} X - 2.1354 \cdot 10^{12} \\
= {} & -2.1354 \cdot 10^{12} B_{0,20}(X) - 1.43965 \cdot 10^{12} B_{1,20}(X) + 1.04889 \cdot 10^{12} B_{2,20}(X) + 5.98344 \\
& \cdot 10^{12} B_{3,20}(X) + 1.37497 \cdot 10^{13} B_{4,20}(X) + 2.41181 \cdot 10^{13} B_{5,20}(X) + 3.58157 \cdot 10^{13} B_{6,20}(X) \\
& + 4.61131 \cdot 10^{13} B_{7,20}(X) + 5.06156 \cdot 10^{13} B_{8,20}(X) + 4.35612 \cdot 10^{13} B_{9,20}(X) + 1.90286 \\
& \cdot 10^{13} B_{10,20}(X) - 2.65368 \cdot 10^{13} B_{11,20}(X) - 9.02907 \cdot 10^{13} B_{12,20}(X) - 1.57924 \cdot 10^{14} B_{13,20}(X) \\
& - 1.99362 \cdot 10^{14} B_{14,20}(X) - 1.69182 \cdot 10^{14} B_{15,20}(X) - 1.82426 \cdot 10^{13} B_{16,20}(X) + 2.75733 \\
& \cdot 10^{14} B_{17,20}(X) + 6.56245 \cdot 10^{14} B_{18,20}(X) + 9.36841 \cdot 10^{14} B_{19,20}(X) + 8.43944 \cdot 10^{14} B_{20,20}(X)
\end{aligned}$$



Degree reduction and raising:

$$q_2 = 2.10702{\cdot}10^{15}X^2 - 1.56229{\cdot}10^{15}X + 1.87745{\cdot}10^{14}$$
$$= 1.87745{\cdot}10^{14}B_{0,2} - 5.93403{\cdot}10^{14}B_{1,2} + 7.3247{\cdot}10^{14}B_{2,2}$$

$$\widetilde{q_2} = 2.71897{\cdot}10^{17}X^{20} - 2.7184{\cdot}10^{18}X^{19} + 1.2578{\cdot}10^{19}X^{18} - 3.57368{\cdot}10^{19}X^{17} + 6.97591{\cdot}10^{19}X^{16} - 9.91779{\cdot}10^{19}X^{15} + 1.06228{\cdot}10^{20}X^{14} - 8.74625{\cdot}10^{19}X^{13} + 5.59859{\cdot}10^{19}X^{12} - 2.79979{\cdot}10^{19}X^{11} + 1.09334{\cdot}10^{19}X^{10} - 3.31565{\cdot}10^{18}X^{9} + 7.72459{\cdot}10^{17}X^{8} - 1.35942{\cdot}10^{17}X^{7} + 1.7641{\cdot}10^{16}X^{6} - 1.63278{\cdot}10^{15}X^{5} + 1.02733{\cdot}10^{14}X^{4} - 4.05016{\cdot}10^{12}X^{3} + 2.10711{\cdot}10^{15}X^{2} - 1.5623{\cdot}10^{15}X + 1.87745{\cdot}10^{14}$$
$$= 1.87745{\cdot}10^{14}B_{0,20} + 1.0963{\cdot}10^{14}B_{1,20} + 4.2605{\cdot}10^{13}B_{2,20} - 1.33332{\cdot}10^{13}B_{3,20} - 5.81673{\cdot}10^{13}B_{4,20} - 9.19637{\cdot}10^{13}B_{5,20} - 1.14523{\cdot}10^{14}B_{6,20} - 1.26329{\cdot}10^{14}B_{7,20} - 1.26418{\cdot}10^{14}B_{8,20} - 1.16394{\cdot}10^{14}B_{9,20} - 9.40071{\cdot}10^{13}B_{10,20} - 6.1923{\cdot}10^{13}B_{11,20} - 1.74711{\cdot}10^{13}B_{12,20} + 3.7088{\cdot}10^{13}B_{13,20} + 1.03366{\cdot}10^{14}B_{14,20} + 1.80399{\cdot}10^{14}B_{15,20} + 2.68668{\cdot}10^{14}B_{16,20} + 3.67975{\cdot}10^{14}B_{17,20} + 4.78386{\cdot}10^{14}B_{18,20} + 5.99883{\cdot}10^{14}B_{19,20} + 7.3247{\cdot}10^{14}B_{20,20}$$



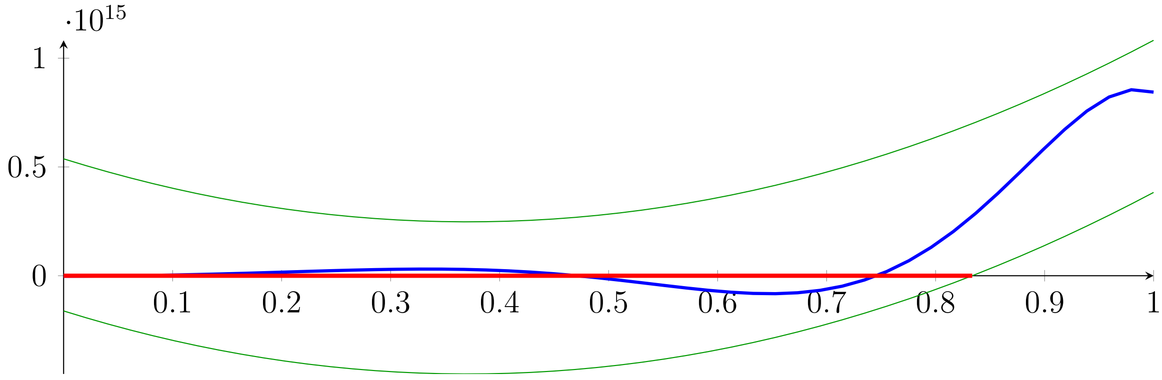
The maximum difference of the Bézier coefficients is $\delta = 3.49581{\cdot}10^{14}$.
Bounding polynomials M and m:

$$M = 2.10702{\cdot}10^{15}X^2 - 1.56229{\cdot}10^{15}X + 5.37325{\cdot}10^{14}$$
$$m = 2.10702{\cdot}10^{15}X^2 - 1.56229{\cdot}10^{15}X - 1.61836{\cdot}10^{14}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{-0.0921392, 0.83361\}$$

Intersection intervals:



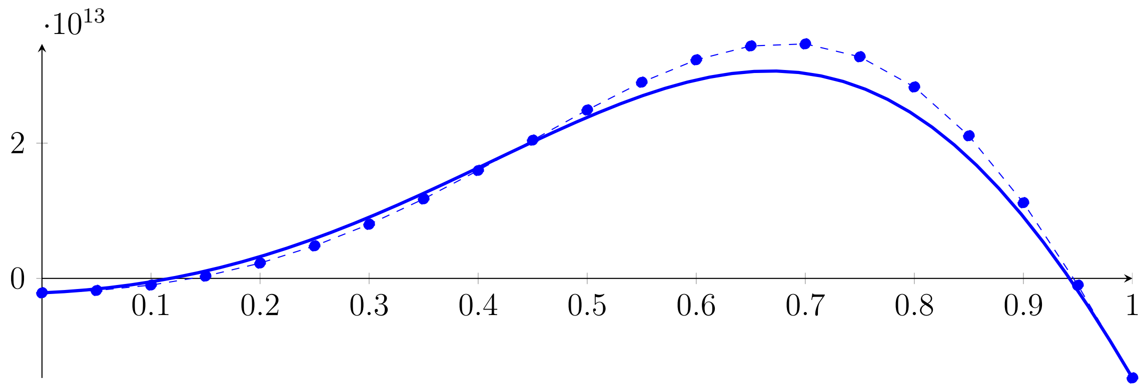
$[0, 0.83361]$

Longest intersection interval: 0.83361
$\Longrightarrow$ Bisection: first half $[15.625, 17.1875]$ und second half $[17.1875, 18.75]$
Bisection point is very near to a root?!?

2.76 Recursion Branch 1 2 1 2 1 on the First Half $[15.625, 17.1875]$

Normalized monomial und Bézier representations and the Bézier polygon:

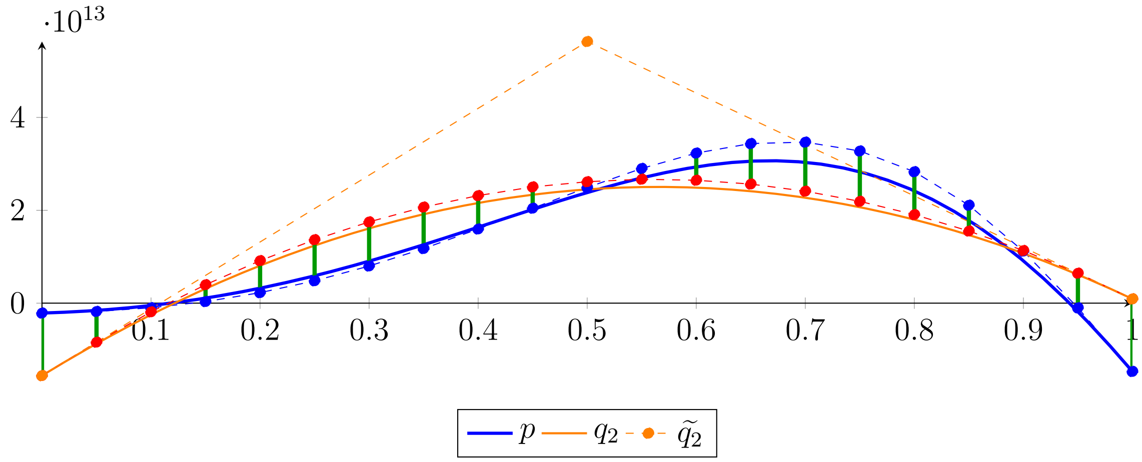
$$\begin{aligned}p &= -12255.1X^{20} + 678571X^{19} + 1.39217\cdot 10^{7}X^{18} + 2.45056\cdot 10^{8}X^{17} + 2.57809\cdot 10^{9}X^{16} + 1.77899\\&\quad\cdot 10^{10}X^{15} + 7.24246\cdot 10^{10}X^{14} + 1.02329\cdot 10^{11}X^{13} - 5.67624\cdot 10^{11}X^{12} - 3.4484\cdot 10^{12}X^{11} - 6.28975\\&\quad\cdot 10^{12}X^{10} + 6.47685\cdot 10^{12}X^{9} + 4.6428\cdot 10^{13}X^{8} + 5.74242\cdot 10^{13}X^{7} - 4.8441\cdot 10^{13}X^{6} - 1.68566\cdot 10^{14}X^{5}\\&\quad - 8.09942\cdot 10^{13}X^{4} + 9.30826\cdot 10^{13}X^{3} + 8.51578\cdot 10^{13}X^{2} + 6.95749\cdot 10^{12}X - 2.1354\cdot 10^{12}\\&= -2.1354\cdot 10^{12}B_{0,20}(X) - 1.78753\cdot 10^{12}B_{1,20}(X) - 9.91453\cdot 10^{11}B_{2,20}(X) + 3.34471\\&\quad\cdot 10^{11}B_{3,20}(X) + 2.25518\cdot 10^{12}B_{4,20}(X) + 4.80802\cdot 10^{12}B_{5,20}(X) + 7.99062\cdot 10^{12}B_{6,20}(X)\\&\quad + 1.17483\cdot 10^{13}B_{7,20}(X) + 1.59619\cdot 10^{13}B_{8,20}(X) + 2.04381\cdot 10^{13}B_{9,20}(X) + 2.49034\\&\quad\cdot 10^{13}B_{10,20}(X) + 2.90046\cdot 10^{13}B_{11,20}(X) + 3.2318\cdot 10^{13}B_{12,20}(X) + 3.43704\cdot 10^{13}B_{13,20}(X)\\&\quad + 3.46731\cdot 10^{13}B_{14,20}(X) + 3.27707\cdot 10^{13}B_{15,20}(X) + 2.83048\cdot 10^{13}B_{16,20}(X) + 2.10885\\&\quad\cdot 10^{13}B_{17,20}(X) + 1.11867\cdot 10^{13}B_{18,20}(X) - 1.00816\cdot 10^{12}B_{19,20}(X) - 1.47196\cdot 10^{13}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= -1.27364\cdot 10^{14}X^{2} + 1.4388\cdot 10^{14}X - 1.56036\cdot 10^{13}\\&= -1.56036\cdot 10^{13}B_{0,2} + 5.63363\cdot 10^{13}B_{1,2} + 9.12267\cdot 10^{11}B_{2,2}\end{aligned}$$

$$\widetilde{q}_2 = -1.82683{\cdot}10^{16}X^{20} + 1.82796{\cdot}10^{17}X^{19} - 8.46679{\cdot}10^{17}X^{18} + 2.40854{\cdot}10^{18}X^{17} - 4.7078{\cdot}10^{18}X^{16} + 6.70222{\cdot}10^{18}X^{15} - 7.18764{\cdot}10^{18}X^{14} + 5.92383{\cdot}10^{18}X^{13} - 3.79419{\cdot}10^{18}X^{12} + 1.89753{\cdot}10^{18}X^{11} - 7.40545{\cdot}10^{17}X^{10} + 2.24282{\cdot}10^{17}X^{9} - 5.21548{\cdot}10^{16}X^{8} + 9.16276{\cdot}10^{15}X^{7} - 1.18937{\cdot}10^{15}X^{6} + 1.10748{\cdot}10^{14}X^{5} - 7.09647{\cdot}10^{12}X^{4} + 2.89704{\cdot}10^{11}X^{3} - 1.27371{\cdot}10^{14}X^{2} + 1.4388{\cdot}10^{14}X - 1.56036{\cdot}10^{13}$$
$$= -1.56036{\cdot}10^{13}B_{0,20} - 8.40963{\cdot}10^{12}B_{1,20} - 1.88601{\cdot}10^{12}B_{2,20} + 3.9675{\cdot}10^{12}B_{3,20} + 9.14968{\cdot}10^{12}B_{4,20} + 1.3665{\cdot}10^{13}B_{5,20} + 1.75001{\cdot}10^{13}B_{6,20} + 2.06876{\cdot}10^{13}B_{7,20} + 2.3162{\cdot}10^{13}B_{8,20} + 2.50325{\cdot}10^{13}B_{9,20} + 2.61464{\cdot}10^{13}B_{10,20} + 2.66834{\cdot}10^{13}B_{11,20} + 2.64652{\cdot}10^{13}B_{12,20} + 2.56422{\cdot}10^{13}B_{13,20} + 2.41064{\cdot}10^{13}B_{14,20} + 2.19231{\cdot}10^{13}B_{15,20} + 1.90592{\cdot}10^{13}B_{16,20} + 1.55286{\cdot}10^{13}B_{17,20} + 1.13267{\cdot}10^{13}B_{18,20} + 6.45467{\cdot}10^{12}B_{19,20} + 9.12266{\cdot}10^{11}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.56319{\cdot}10^{13}$.

Bounding polynomials M and m:

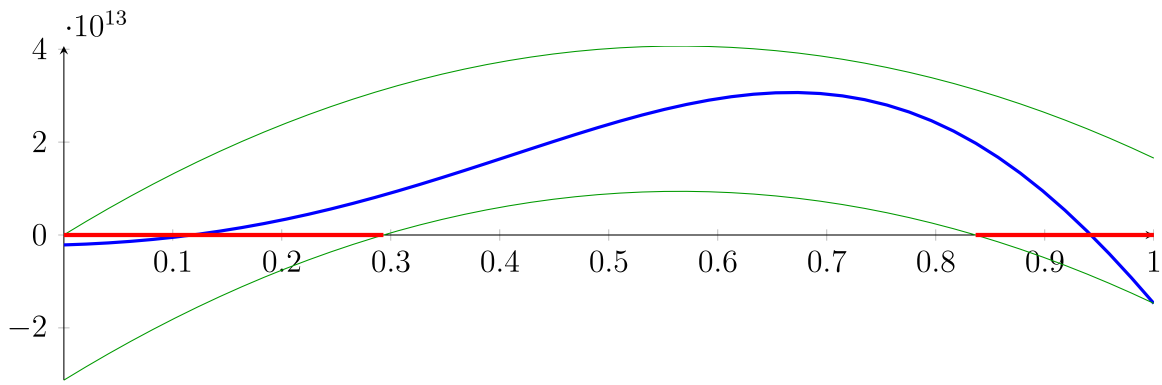
$$M = -1.27364{\cdot}10^{14}X^2 + 1.4388{\cdot}10^{14}X + 2.82641{\cdot}10^{10}$$

$$m = -1.27364{\cdot}10^{14}X^2 + 1.4388{\cdot}10^{14}X - 3.12355{\cdot}10^{13}$$

Root of M and m:

$$N(M) = \{-0.000196408, 1.12987\} \qquad N(m) = \{0.293185, 0.83649\}$$

Intersection intervals:



$$[0, 0.293185],\ [0.83649, 1]$$

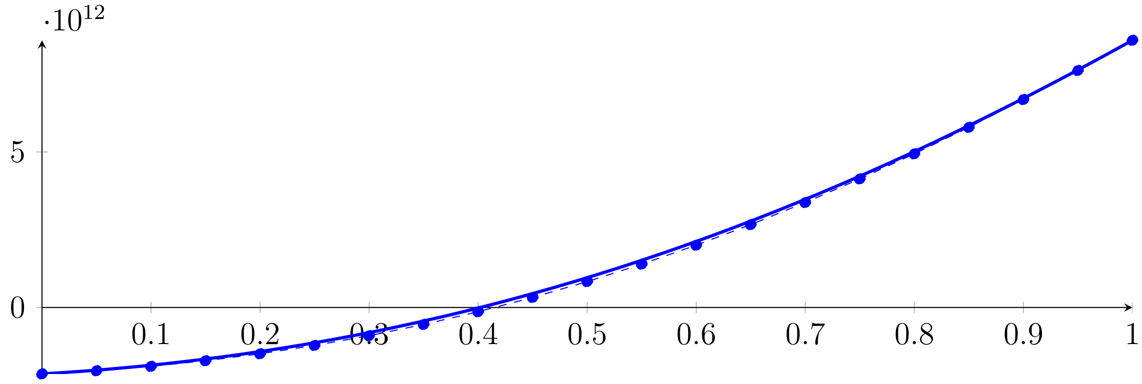
Longest intersection interval: 0.293185

$\Longrightarrow$ Selective recursion: interval 1: $[15.625, 16.0831]$, interval 2: $[16.932, 17.1875]$,

2.77 Recursion Branch 1 2 1 2 1 1 in Interval 1: $[15.625, 16.0831]$

Normalized monomial und Bézier representations and the Bézier polygon:

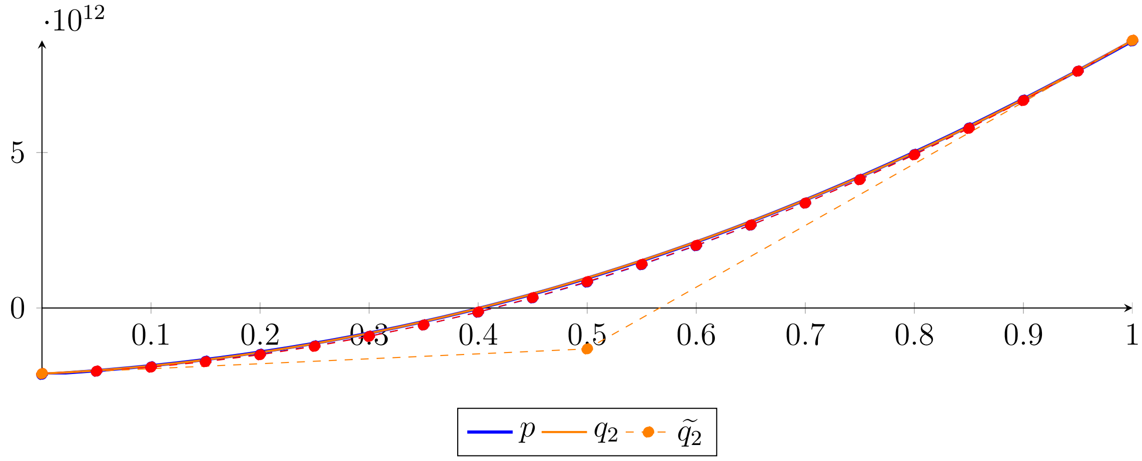
$$p = -394.961X^{20} + 11649.1X^{19} + 6663.27X^{18} + 97069.8X^{17} - 65239.5X^{16} - 94106.6X^{15} + 132916X^{14} + 128430X^{13} + 207100X^{12} - 4.59585\cdot10^{6}X^{11} - 2.93402\cdot10^{7}X^{10} + 1.03694\cdot10^{8}X^{9} + 2.53461\cdot10^{9}X^{8} + 1.06926\cdot10^{10}X^{7} - 3.07653\cdot10^{10}X^{6} - 3.65154\cdot10^{11}X^{5} - 5.98438\cdot10^{11}X^{4} + 2.34581\cdot10^{12}X^{3} + 7.31993\cdot10^{12}X^{2} + 2.03983\cdot10^{12}X - 2.1354\cdot10^{12}$$
$$= -2.1354\cdot10^{12}B_{0,20}(X) - 2.03341\cdot10^{12}B_{1,20}(X) - 1.89289\cdot10^{12}B_{2,20}(X) - 1.71179\cdot10^{12}B_{3,20}(X) - 1.48817\cdot10^{12}B_{4,20}(X) - 1.22025\cdot10^{12}B_{5,20}(X) - 9.06403\cdot10^{11}B_{6,20}(X) - 5.45217\cdot10^{11}B_{7,20}(X) - 1.35495\cdot10^{11}B_{8,20}(X) + 3.23715\cdot10^{11}B_{9,20}(X) + 8.33087\cdot10^{11}B_{10,20}(X) + 1.393\cdot10^{12}B_{11,20}(X) + 2.0035\cdot10^{12}B_{12,20}(X) + 2.6643\cdot10^{12}B_{13,20}(X) + 3.37472\cdot10^{12}B_{14,20}(X) + 4.13368\cdot10^{12}B_{15,20}(X) + 4.93972\cdot10^{12}B_{16,20}(X) + 5.79089\cdot10^{12}B_{17,20}(X) + 6.68483\cdot10^{12}B_{18,20}(X) + 7.61868\cdot10^{12}B_{19,20}(X) + 8.58911\cdot10^{12}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 9.12889\cdot10^{12}X^{2} + 1.59038\cdot10^{12}X - 2.11052\cdot10^{12}$$
$$= -2.11052\cdot10^{12}B_{0,2} - 1.31533\cdot10^{12}B_{1,2} + 8.60874\cdot10^{12}B_{2,2}$$

$$\widetilde{q}_2 = 1.26974\cdot10^{15}X^{20} - 1.2689\cdot10^{16}X^{19} + 5.86706\cdot10^{16}X^{18} - 1.66546\cdot10^{17}X^{17} + 3.24783\cdot10^{17}X^{16} - 4.6133\cdot10^{17}X^{15} + 4.93814\cdot10^{17}X^{14} - 4.0654\cdot10^{17}X^{13} + 2.60413\cdot10^{17}X^{12} - 1.3045\cdot10^{17}X^{11} + 5.1083\cdot10^{16}X^{10} - 1.55498\cdot10^{16}X^{9} + 3.63875\cdot10^{15}X^{8} - 6.43167\cdot10^{14}X^{7} + 8.36968\cdot10^{13}X^{6} - 7.73113\cdot10^{12}X^{5} + 4.80056\cdot10^{11}X^{4} - 1.83749\cdot10^{10}X^{3} + 9.12927\cdot10^{12}X^{2} + 1.59038\cdot10^{12}X - 2.11052\cdot10^{12}$$
$$= -2.11052\cdot10^{12}B_{0,20} - 2.031\cdot10^{12}B_{1,20} - 1.90344\cdot10^{12}B_{2,20} - 1.72784\cdot10^{12}B_{3,20} - 1.50412\cdot10^{12}B_{4,20} - 1.23261\cdot10^{12}B_{5,20} - 9.12347\cdot10^{11}B_{6,20} - 5.45616\cdot10^{11}B_{7,20} - 1.27924\cdot10^{11}B_{8,20} + 3.33297\cdot10^{11}B_{9,20} + 8.48471\cdot10^{11}B_{10,20} + 1.40515\cdot10^{12}B_{11,20} + 2.01593\cdot10^{12}B_{12,20} + 2.67017\cdot10^{12}B_{13,20} + 3.37534\cdot10^{12}B_{14,20} + 4.12703\cdot10^{12}B_{15,20} + 4.92744\cdot10^{12}B_{16,20} + 5.77565\cdot10^{12}B_{17,20} + 6.67198\cdot10^{12}B_{18,20} + 7.61633\cdot10^{12}B_{19,20} + 8.60874\cdot10^{12}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 2.48777 \cdot 10^{10}$.
Bounding polynomials M and m:

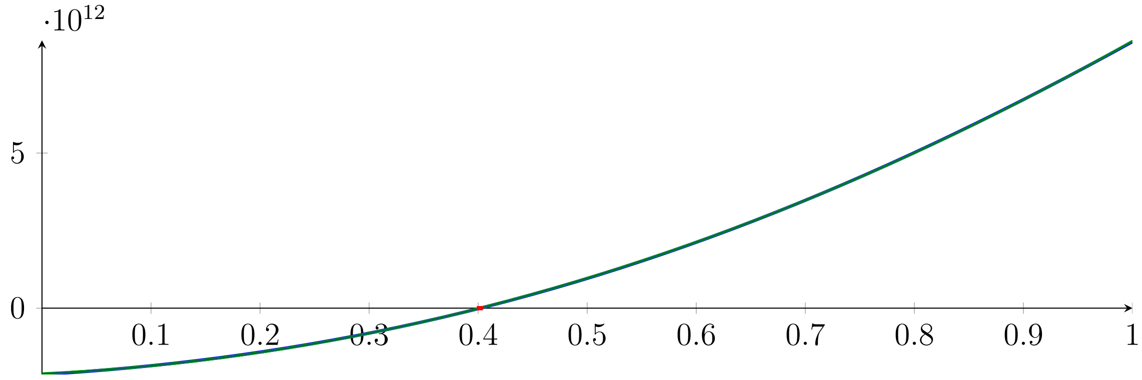
$$M = 9.12889 \cdot 10^{12} X^2 + 1.59038 \cdot 10^{12} X - 2.08565 \cdot 10^{12}$$

$$m = 9.12889 \cdot 10^{12} X^2 + 1.59038 \cdot 10^{12} X - 2.1354 \cdot 10^{12}$$

Root of M and m:

$$N(M) = \{-0.572961, 0.398747\} \qquad N(m) = \{-0.578538, 0.404324\}$$

Intersection intervals:



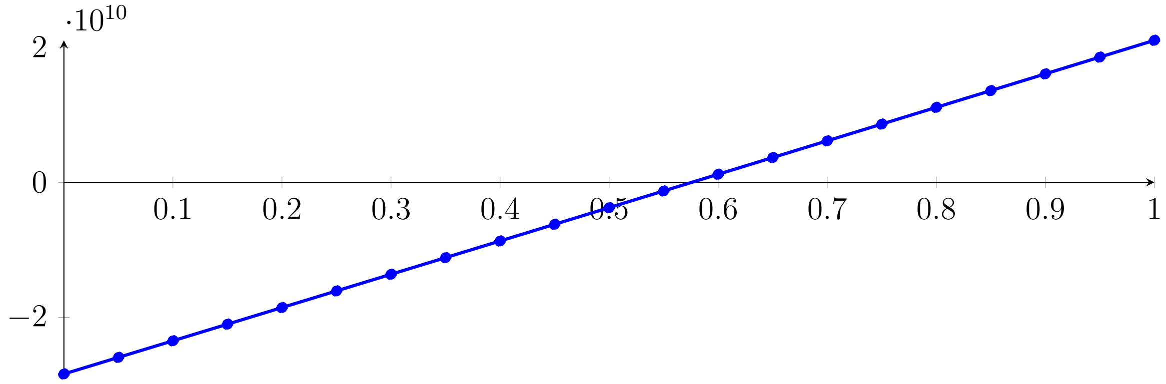
$$[0.398747, 0.404324]$$

Longest intersection interval: 0.005577
$\Longrightarrow$ Selective recursion: interval 1: $[15.8077, 15.8102]$,

2.78 Recursion Branch 1 2 1 2 1 1 1 in Interval 1: $[15.8077, 15.8102]$

Normalized monomial und Bézier representations and the Bézier polygon:

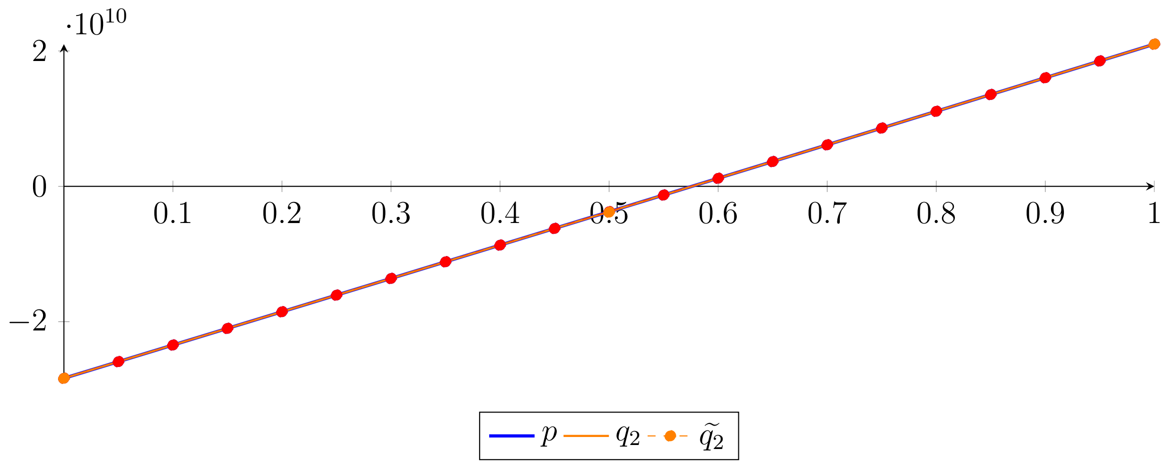
$$\begin{aligned} p = & 6.21301 X^{20} + 13.595 X^{19} + 307.289 X^{18} - 408.992 X^{17} + 5075.36 X^{16} - 4586.66 X^{15} + 2569.17 X^{14} \\ & + 1745.9 X^{13} + 6642.95 X^{12} + 1619.73 X^{11} + 2351.17 X^{10} + 404.933 X^{9} + 26.9101 X^{8} + 14.1943 X^{7} \\ & + 13.0115 X^{6} - 1.41943 X^{5} - 1326.58 X^{4} + 135755 X^{3} + 2.8971 \cdot 10^{8} X^{2} + 4.90605 \cdot 10^{10} X - 2.83496 \cdot 10^{10} \\ = & -2.83496 \cdot 10^{10} B_{0,20}(X) - 2.58966 \cdot 10^{10} B_{1,20}(X) - 2.3442 \cdot 10^{10} B_{2,20}(X) - 2.09859 \\ & \cdot 10^{10} B_{3,20}(X) - 1.85283 \cdot 10^{10} B_{4,20}(X) - 1.60692 \cdot 10^{10} B_{5,20}(X) - 1.36086 \cdot 10^{10} B_{6,20}(X) \\ & - 1.11464 \cdot 10^{10} B_{7,20}(X) - 8.68269 \cdot 10^{9} B_{8,20}(X) - 6.21747 \cdot 10^{9} B_{9,20}(X) - 3.75072 \\ & \cdot 10^{9} B_{10,20}(X) - 1.28244 \cdot 10^{9} B_{11,20}(X) + 1.18736 \cdot 10^{9} B_{12,20}(X) + 3.65869 \cdot 10^{9} B_{13,20}(X) \\ & + 6.13154 \cdot 10^{9} B_{14,20}(X) + 8.60592 \cdot 10^{9} B_{15,20}(X) + 1.10818 \cdot 10^{10} B_{16,20}(X) + 1.35593 \\ & \cdot 10^{10} B_{17,20}(X) + 1.60382 \cdot 10^{10} B_{18,20}(X) + 1.85187 \cdot 10^{10} B_{19,20}(X) + 2.10007 \cdot 10^{10} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = 2.89911{\cdot}10^{8}X^2 + 4.90604{\cdot}10^{10}X - 2.83496{\cdot}10^{10}$$
$$= -2.83496{\cdot}10^{10}B_{0,2} - 3.81938{\cdot}10^{9}B_{1,2} + 2.10007{\cdot}10^{10}B_{2,2}$$

$$\widetilde{q}_2 = 2.60722{\cdot}10^{12}X^{20} - 2.61333{\cdot}10^{13}X^{19} + 1.21222{\cdot}10^{14}X^{18} - 3.45295{\cdot}10^{14}X^{17} + 6.75889{\cdot}10^{14}X^{16} - 9.64053{\cdot}10^{14}X^{15} + 1.03684{\cdot}10^{15}X^{14} - 8.58315{\cdot}10^{14}X^{13} + 5.53339{\cdot}10^{14}X^{12} - 2.79215{\cdot}10^{14}X^{11} + 1.10197{\cdot}10^{14}X^{10} - 3.38058{\cdot}10^{13}X^{9} + 7.96859{\cdot}10^{12}X^{8} - 1.41958{\cdot}10^{12}X^{7} + 1.87056{\cdot}10^{11}X^{6} - 1.77109{\cdot}10^{10}X^{5} + 1.15415{\cdot}10^{9}X^{4} - 4.79542{\cdot}10^{7}X^{3} + 2.91032{\cdot}10^{8}X^{2} + 4.90604{\cdot}10^{10}X - 2.83496{\cdot}10^{10}$$
$$= -2.83496{\cdot}10^{10}B_{0,20} - 2.58966{\cdot}10^{10}B_{1,20} - 2.3442{\cdot}10^{10}B_{2,20} - 2.0986{\cdot}10^{10}B_{3,20} - 1.85282{\cdot}10^{10}B_{4,20} - 1.60695{\cdot}10^{10}B_{5,20} - 1.36078{\cdot}10^{10}B_{6,20} - 1.11479{\cdot}10^{10}B_{7,20} - 8.6803{\cdot}10^{9}B_{8,20} - 6.22053{\cdot}10^{9}B_{9,20} - 3.74705{\cdot}10^{9}B_{10,20} - 1.2856{\cdot}10^{9}B_{11,20} + 1.18967{\cdot}10^{9}B_{12,20} + 3.65733{\cdot}10^{9}B_{13,20} + 6.13226{\cdot}10^{9}B_{14,20} + 8.60564{\cdot}10^{9}B_{15,20} + 1.10819{\cdot}10^{10}B_{16,20} + 1.35592{\cdot}10^{10}B_{17,20} + 1.60382{\cdot}10^{10}B_{18,20} + 1.85187{\cdot}10^{10}B_{19,20} + 2.10007{\cdot}10^{10}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 3.66638{\cdot}10^{6}$.

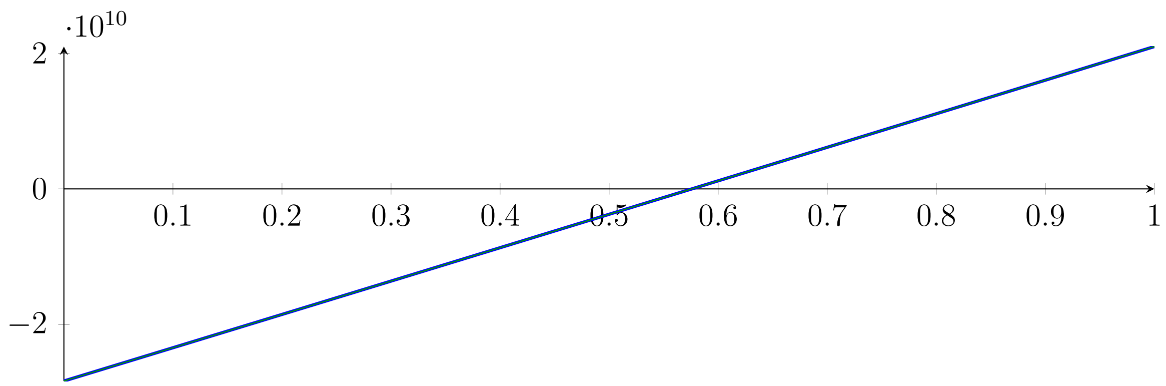
Bounding polynomials M and m:

$$M = 2.89911{\cdot}10^{8}X^2 + 4.90604{\cdot}10^{10}X - 2.83459{\cdot}10^{10}$$
$$m = 2.89911{\cdot}10^{8}X^2 + 4.90604{\cdot}10^{10}X - 2.83532{\cdot}10^{10}$$

Root of M and m:

$$N(M) = \{-169.801, 0.575817\} \qquad N(m) = \{-169.802, 0.575965\}$$

Intersection intervals:



$$[0.575817, 0.575965]$$

Longest intersection interval: 0.000148454

$\Longrightarrow$ Selective recursion: interval 1: $[15.8091, 15.8091]$,

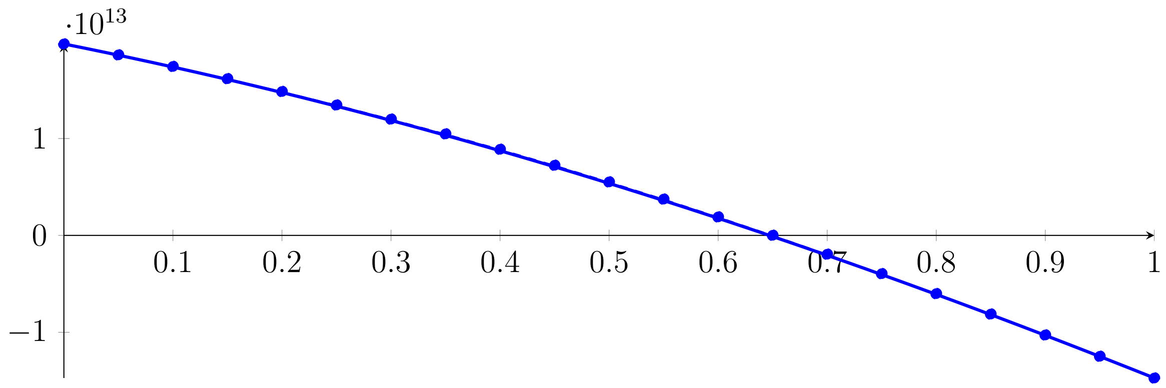
2.79 Recursion Branch 1 2 1 2 1 1 1 1 in Interval 1: $[15.8091, 15.8091]$

Found root in interval $[15.8091, 15.8091]$ at recursion depth 8!

2.80 Recursion Branch 1 2 1 2 1 2 in Interval 2: $[16.932, 17.1875]$

Normalized monomial und Bézier representations and the Bézier polygon:

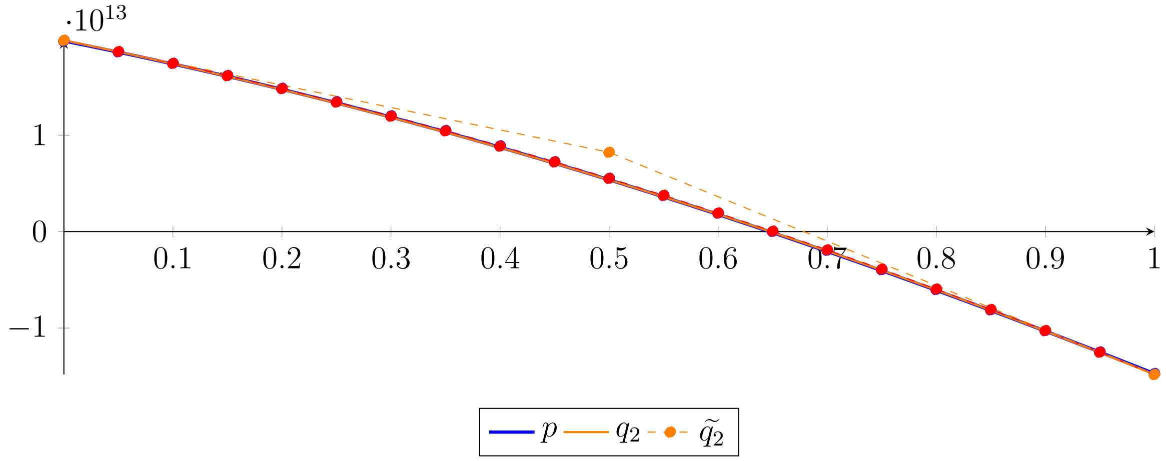
$$\begin{aligned}p = &-7250.63X^{20} + 5133.91X^{19} - 294798X^{18} + 669260X^{17} - 5.3471\cdot10^{6}X^{16} + 4.4987\cdot10^{6}X^{15}\\ &- 2.19751\cdot10^{6}X^{14} - 1.23971\cdot10^{6}X^{13} - 6.09085\cdot10^{6}X^{12} - 1.03794\cdot10^{6}X^{11} - 1.69889\cdot10^{6}X^{10}\\ &- 1.005\cdot10^{7}X^{9} - 2.05903\cdot10^{8}X^{8} - 1.51909\cdot10^{9}X^{7} + 3.08155\cdot10^{9}X^{6} + 1.28686\cdot10^{11}X^{5}\\ &+ 7.09244\cdot10^{11}X^{4} - 2.50257\cdot10^{11}X^{3} - 1.25036\cdot10^{13}X^{2} - 2.25678\cdot10^{13}X + 1.97627\cdot10^{13}\\ = &\ 1.97627\cdot10^{13}B_{0,20}(X) + 1.86343\cdot10^{13}B_{1,20}(X) + 1.74401\cdot10^{13}B_{2,20}(X) + 1.61799\\ &\cdot10^{13}B_{3,20}(X) + 1.48536\cdot10^{13}B_{4,20}(X) + 1.34613\cdot10^{13}B_{5,20}(X) + 1.20031\cdot10^{13}B_{6,20}(X)\\ &+ 1.04797\cdot10^{13}B_{7,20}(X) + 8.89141\cdot10^{12}B_{8,20}(X) + 7.23919\cdot10^{12}B_{9,20}(X) + 5.52397\\ &\cdot10^{12}B_{10,20}(X) + 3.74694\cdot10^{12}B_{11,20}(X) + 1.90951\cdot10^{12}B_{12,20}(X) + 1.32874\cdot10^{10}B_{13,20}(X)\\ &- 1.93987\cdot10^{12}B_{14,20}(X) - 3.94786\cdot10^{12}B_{15,20}(X) - 6.00836\cdot10^{12}B_{16,20}(X) - 8.11878\\ &\cdot10^{12}B_{17,20}(X) - 1.02763\cdot10^{13}B_{18,20}(X) - 1.24777\cdot10^{13}B_{19,20}(X) - 1.47196\cdot10^{13}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= -1.14309\cdot10^{13}X^{2} - 2.32054\cdot10^{13}X + 1.9825\cdot10^{13}\\ &= 1.9825\cdot10^{13}B_{0,2} + 8.22223\cdot10^{12}B_{1,2} - 1.48114\cdot10^{13}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 = &-3.6009\cdot10^{15}X^{20} + 3.60676\cdot10^{16}X^{19} - 1.67207\cdot10^{17}X^{18} + 4.76045\cdot10^{17}X^{17} - 9.31325\cdot10^{17}X^{16}\\ &+ 1.3274\cdot10^{18}X^{15} - 1.42592\cdot10^{18}X^{14} + 1.17814\cdot10^{18}X^{13} - 7.57332\cdot10^{17}X^{12} + 3.80617\cdot10^{17}X^{11}\\ &- 1.49457\cdot10^{17}X^{10} + 4.55834\cdot10^{16}X^{9} - 1.06789\cdot10^{16}X^{8} + 1.89058\cdot10^{15}X^{7} - 2.47505\cdot10^{14}X^{6} + 2.32779\\ &\cdot10^{13}X^{5} - 1.50828\cdot10^{12}X^{4} + 6.23699\cdot10^{10}X^{3} - 1.14323\cdot10^{13}X^{2} - 2.32054\cdot10^{13}X + 1.9825\cdot10^{13}\\ = &\ 1.9825\cdot10^{13}B_{0,20} + 1.86647\cdot10^{13}B_{1,20} + 1.74442\cdot10^{13}B_{2,20} + 1.61637\cdot10^{13}B_{3,20} + 1.48228\\ &\cdot10^{13}B_{4,20} + 1.34224\cdot10^{13}B_{5,20} + 1.19598\cdot10^{13}B_{6,20} + 1.04417\cdot10^{13}B_{7,20} + 8.8549\cdot10^{12}B_{8,20}\\ &+ 7.22095\cdot10^{12}B_{9,20} + 5.50993\cdot10^{12}B_{10,20} + 3.75733\cdot10^{12}B_{11,20} + 1.92771\cdot10^{12}B_{12,20}\\ &+ 5.06773\cdot10^{10}B_{13,20} - 1.89465\cdot10^{12}B_{14,20} - 3.89579\cdot10^{12}B_{15,20} - 5.95903\cdot10^{12}B_{16,20}\\ &- 8.08175\cdot10^{12}B_{17,20} - 1.02648\cdot10^{13}B_{18,20} - 1.2508\cdot10^{13}B_{19,20} - 1.48114\cdot10^{13}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 9.17405 \cdot 10^{10}$.

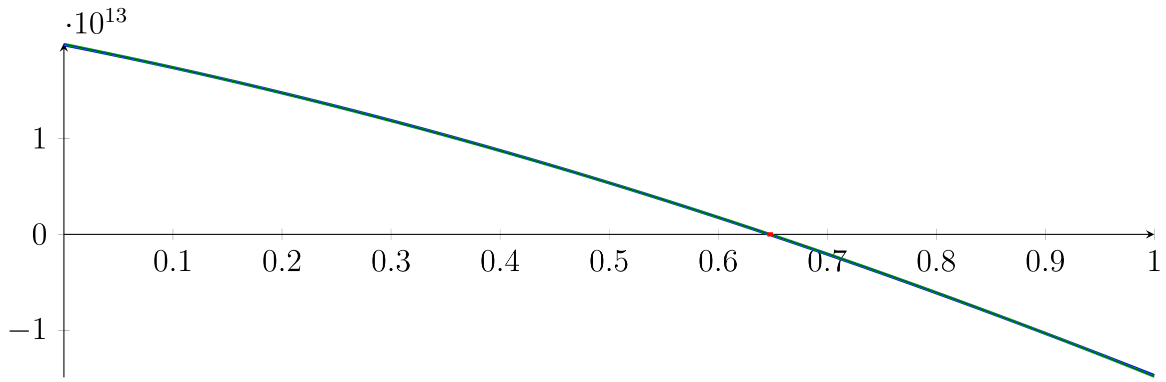
Bounding polynomials M and m:

$$M = -1.14309 \cdot 10^{13} X^2 - 2.32054 \cdot 10^{13} X + 1.99167 \cdot 10^{13}$$
$$m = -1.14309 \cdot 10^{13} X^2 - 2.32054 \cdot 10^{13} X + 1.97332 \cdot 10^{13}$$

Root of M and m:

$$N(M) = \{-2.68016, 0.650095\} \qquad N(m) = \{-2.67534, 0.645268\}$$

Intersection intervals:



$$[0.645268, 0.650095]$$

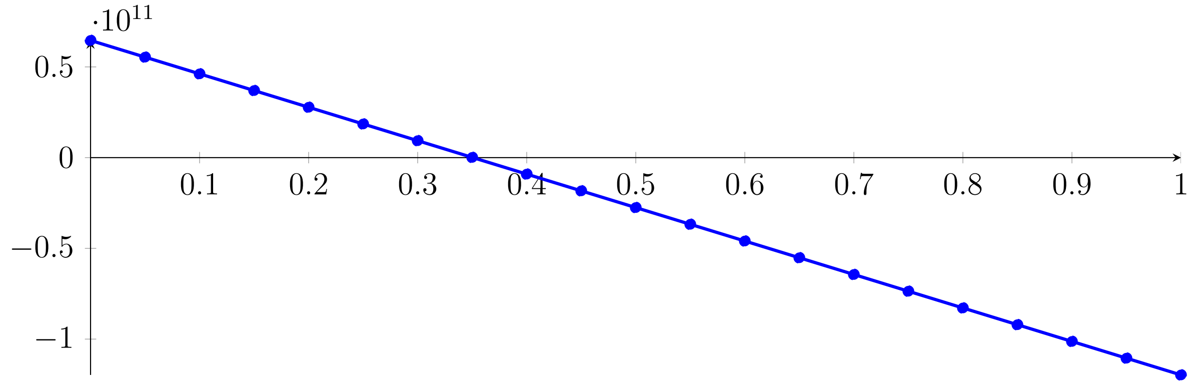
Longest intersection interval: 0.00482685

$\Longrightarrow$ Selective recursion: interval 1: $[17.0969, 17.0981]$,

2.81 Recursion Branch 1 2 1 2 1 2 1 in Interval 1: $[17.0969, 17.0981]$

Normalized monomial und Bézier representations and the Bézier polygon:

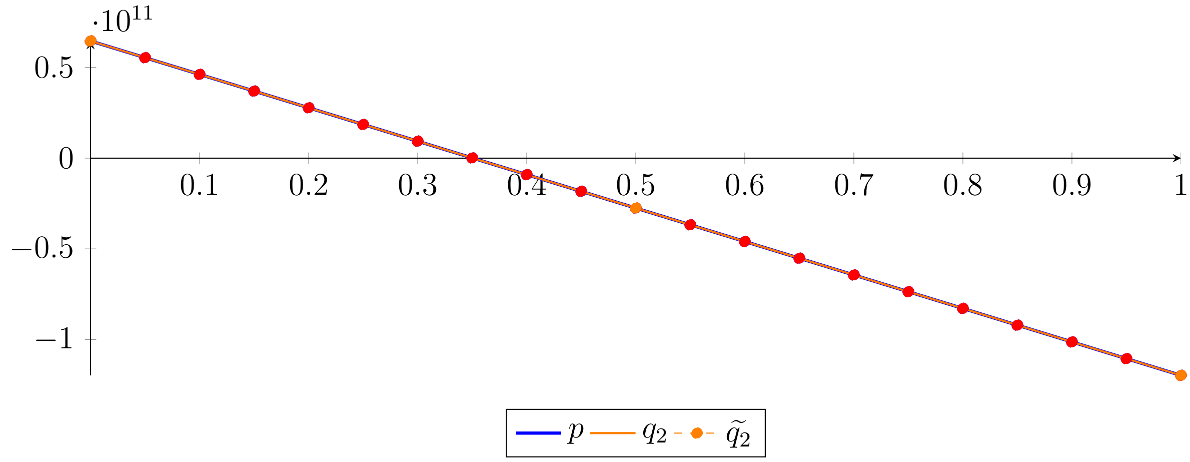
$$\begin{aligned} p = {} & 15.9816 X^{20} - 270.553 X^{19} + 127.32 X^{18} - 3294.83 X^{17} + 7979.88 X^{16} - 3878.84 X^{15} \\ & - 972.312 X^{14} - 2105.49 X^{13} - 4497.83 X^{12} - 3557.26 X^{11} - 2650 X^{10} - 902.129 X^{9} \\ & - 30.7544 X^{6} + 611.096 X^{4} + 238655 X^{3} - 2.53172 \cdot 10^{8} X^{2} - 1.84105 \cdot 10^{11} X + 6.46002 \cdot 10^{10} \\ = {} & 6.46002 \cdot 10^{10} B_{0,20}(X) + 5.53949 \cdot 10^{10} B_{1,20}(X) + 4.61884 \cdot 10^{10} B_{2,20}(X) + 3.69805 \\ & \cdot 10^{10} B_{3,20}(X) + 2.77713 \cdot 10^{10} B_{4,20}(X) + 1.85607 \cdot 10^{10} B_{5,20}(X) + 9.34882 \cdot 10^{9} B_{6,20}(X) \\ & + 1.35601 \cdot 10^{8} B_{7,20}(X) - 9.07895 \cdot 10^{9} B_{8,20}(X) - 1.82948 \cdot 10^{10} B_{9,20}(X) - 2.7512 \\ & \cdot 10^{10} B_{10,20}(X) - 3.67306 \cdot 10^{10} B_{11,20}(X) - 4.59505 \cdot 10^{10} B_{12,20}(X) - 5.51717 \cdot 10^{10} B_{13,20}(X) \\ & - 6.43942 \cdot 10^{10} B_{14,20}(X) - 7.36181 \cdot 10^{10} B_{15,20}(X) - 8.28433 \cdot 10^{10} B_{16,20}(X) - 9.20698 \\ & \cdot 10^{10} B_{17,20}(X) - 1.01298 \cdot 10^{11} B_{18,20}(X) - 1.10527 \cdot 10^{11} B_{19,20}(X) - 1.19757 \cdot 10^{11} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = -2.52813 \cdot 10^8 X^2 - 1.84105 \cdot 10^{11} X + 6.46002 \cdot 10^{10}$$
$$= 6.46002 \cdot 10^{10} B_{0,2} - 2.74522 \cdot 10^{10} B_{1,2} - 1.19757 \cdot 10^{11} B_{2,2}$$

$$\widetilde{q}_2 = -2.08367 \cdot 10^{12} X^{20} + 2.07077 \cdot 10^{13} X^{19} - 9.49128 \cdot 10^{13} X^{18} + 2.66404 \cdot 10^{14} X^{17} - 5.13039 \cdot 10^{14} X^{16} + 7.20199 \cdot 10^{14} X^{15} - 7.64631 \cdot 10^{14} X^{14} + 6.28769 \cdot 10^{14} X^{13} - 4.0651 \cdot 10^{14} X^{12} + 2.08166 \cdot 10^{14} X^{11} - 8.44148 \cdot 10^{13} X^{10} + 2.68883 \cdot 10^{13} X^9 - 6.61801 \cdot 10^{12} X^8 + 1.22641 \cdot 10^{12} X^7 - 1.64325 \cdot 10^{11} X^6 + 1.48741 \cdot 10^{10} X^5 - 7.96335 \cdot 10^8 X^4 + 1.93305 \cdot 10^7 X^3 - 2.52835 \cdot 10^8 X^2 - 1.84105 \cdot 10^{11} X + 6.46002 \cdot 10^{10}$$
$$= 6.46002 \cdot 10^{10} B_{0,20} + 5.53949 \cdot 10^{10} B_{1,20} + 4.61884 \cdot 10^{10} B_{2,20} + 3.69805 \cdot 10^{10} B_{3,20} + 2.77712 \cdot 10^{10} B_{4,20} + 1.8561 \cdot 10^{10} B_{5,20} + 9.3482 \cdot 10^9 B_{6,20} + 1.36723 \cdot 10^8 B_{7,20} - 9.08047 \cdot 10^9 B_{8,20} - 1.82926 \cdot 10^{10} B_{9,20} - 2.75148 \cdot 10^{10} B_{10,20} - 3.67269 \cdot 10^{10} B_{11,20} - 4.59519 \cdot 10^{10} B_{12,20} - 5.51707 \cdot 10^{10} B_{13,20} - 6.43945 \cdot 10^{10} B_{14,20} - 7.3618 \cdot 10^{10} B_{15,20} - 8.28433 \cdot 10^{10} B_{16,20} - 9.20698 \cdot 10^{10} B_{17,20} - 1.01298 \cdot 10^{11} B_{18,20} - 1.10527 \cdot 10^{11} B_{19,20} - 1.19757 \cdot 10^{11} B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 3.67044 \cdot 10^6$.
Bounding polynomials M and m:

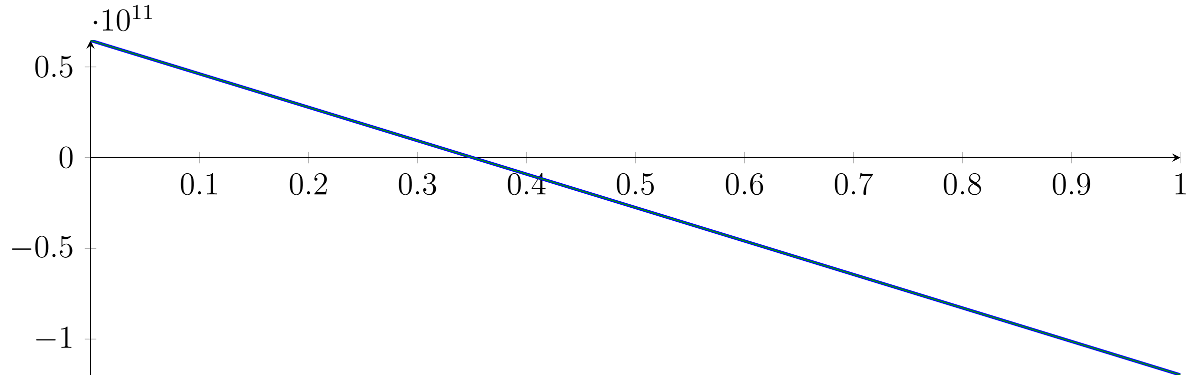
$$M = -2.52813 \cdot 10^8 X^2 - 1.84105 \cdot 10^{11} X + 6.46039 \cdot 10^{10}$$

$$m = -2.52813 \cdot 10^8 X^2 - 1.84105 \cdot 10^{11} X + 6.45965 \cdot 10^{10}$$

Root of M and m:

$$N(M) = \{-728.576, 0.350739\} \qquad N(m) = \{-728.576, 0.350699\}$$

Intersection intervals:



$[0.350699, 0.350739]$

Longest intersection interval: $3.98351 \cdot 10^{-05}$
$\Longrightarrow$ Selective recursion: interval 1: $[17.0973, 17.0973]$,

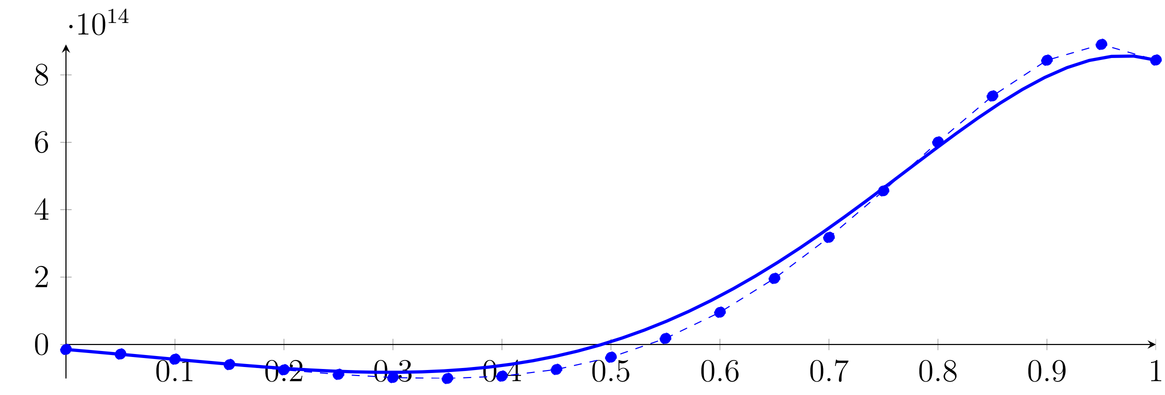
2.82 Recursion Branch 1 2 1 2 1 2 1 1 in Interval 1: $[17.0973, 17.0973]$

Found root in interval $[17.0973, 17.0973]$ at recursion depth 8!

2.83 Recursion Branch 1 2 1 2 2 on the Second Half $[17.1875, 18.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

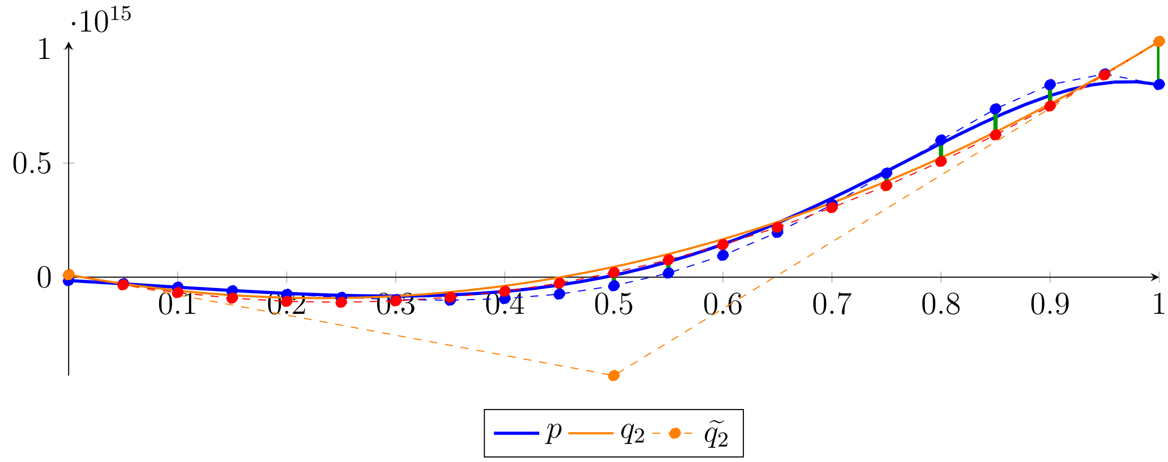
$$p = 51309.5X^{20} + 1.13609 \cdot 10^{6} X^{19} + 2.62513 \cdot 10^{7} X^{18} + 5.89696 \cdot 10^{8} X^{17} + 9.46063 \cdot 10^{9} X^{16} + 1.05848 \cdot 10^{11} X^{15} + 8.64537 \cdot 10^{11} X^{14} + 5.14688 \cdot 10^{12} X^{13} + 2.19482 \cdot 10^{13} X^{12} + 6.31615 \cdot 10^{13} X^{11} + 9.96023 \cdot 10^{13} X^{10} - 2.75387 \cdot 10^{13} X^{9} - 5.24772 \cdot 10^{14} X^{8} - 1.10687 \cdot 10^{15} X^{7} - 7.34813 \cdot 10^{14} X^{6} + 8.78049 \cdot 10^{14} X^{5} + 1.86093 \cdot 10^{15} X^{4} + 8.85216 \cdot 10^{14} X^{3} - 2.8815 \cdot 10^{14} X^{2} - 2.74229 \cdot 10^{14} X - 1.47196 \cdot 10^{13}$$
$$= -1.47196 \cdot 10^{13} B_{0,20}(X) - 2.84311 \cdot 10^{13} B_{1,20}(X) - 4.36591 \cdot 10^{13} B_{2,20}(X) - 5.96272 \cdot 10^{13} B_{3,20}(X) - 7.51748 \cdot 10^{13} B_{4,20}(X) - 8.87006 \cdot 10^{13} B_{5,20}(X) - 9.81247 \cdot 10^{13} B_{6,20}(X) - 1.00885 \cdot 10^{14} B_{7,20}(X) - 9.39824 \cdot 10^{13} B_{8,20}(X) - 7.41057 \cdot 10^{13} B_{9,20}(X) - 3.78514 \cdot 10^{13} B_{10,20}(X) + 1.7921 \cdot 10^{13} B_{11,20}(X) + 9.55764 \cdot 10^{13} B_{12,20}(X) + 1.96007 \cdot 10^{14} B_{13,20}(X) + 3.17738 \cdot 10^{14} B_{14,20}(X) + 4.5586 \cdot 10^{14} B_{15,20}(X) + 6.00841 \cdot 10^{14} B_{16,20}(X) + 7.37367 \cdot 10^{14} B_{17,20}(X) + 8.43467 \cdot 10^{14} B_{18,20}(X) + 8.90392 \cdot 10^{14} B_{19,20}(X) + 8.43944 \cdot 10^{14} B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 1.90548 \cdot 10^{15} X^2 - 8.83759 \cdot 10^{14} X + 1.07132 \cdot 10^{13}$$
$$= 1.07132 \cdot 10^{13} B_{0,2} - 4.31167 \cdot 10^{14} B_{1,2} + 1.03243 \cdot 10^{15} B_{2,2}$$

$$\widetilde{q}_2 = 2.46959{\cdot}10^{17}X^{20} - 2.46848{\cdot}10^{18}X^{19} + 1.14177{\cdot}10^{19}X^{18} - 3.24266{\cdot}10^{19}X^{17} + 6.32685{\cdot}10^{19}X^{16} - 8.99104{\cdot}10^{19}X^{15} + 9.6268{\cdot}10^{19}X^{14} - 7.92479{\cdot}10^{19}X^{13} + 5.07324{\cdot}10^{19}X^{12} - 2.53819{\cdot}10^{19}X^{11} + 9.91999{\cdot}10^{18}X^{10} - 3.01193{\cdot}10^{18}X^{9} + 7.0272{\cdot}10^{17}X^{8} - 1.23844{\cdot}10^{17}X^{7} + 1.60825{\cdot}10^{16}X^{6} - 1.48641{\cdot}10^{15}X^{5} + 9.29388{\cdot}10^{13}X^{4} - 3.61586{\cdot}10^{12}X^{3} + 1.90556{\cdot}10^{15}X^{2} - 8.8376{\cdot}10^{14}X + 1.07132{\cdot}10^{13}$$
$$= 1.07132{\cdot}10^{13}B_{0,20} - 3.34748{\cdot}10^{13}B_{1,20} - 6.76336{\cdot}10^{13}B_{2,20} - 9.17663{\cdot}10^{13}B_{3,20} - 1.05857{\cdot}10^{14}B_{4,20} - 1.09966{\cdot}10^{14}B_{5,20} - 1.03912{\cdot}10^{14}B_{6,20} - 8.81349{\cdot}10^{13}B_{7,20} - 6.17601{\cdot}10^{13}B_{8,20} - 2.62408{\cdot}10^{13}B_{9,20} + 2.04615{\cdot}10^{13}B_{10,20} + 7.59255{\cdot}10^{13}B_{11,20} + 1.42585{\cdot}10^{14}B_{12,20} + 2.18381{\cdot}10^{14}B_{13,20} + 3.04774{\cdot}10^{14}B_{14,20} + 4.00894{\cdot}10^{14}B_{15,20} + 5.07174{\cdot}10^{14}B_{16,20} + 6.23437{\cdot}10^{14}B_{17,20} + 7.49742{\cdot}10^{14}B_{18,20} + 8.86072{\cdot}10^{14}B_{19,20} + 1.03243{\cdot}10^{15}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.88488{\cdot}10^{14}$.

Bounding polynomials M and m:

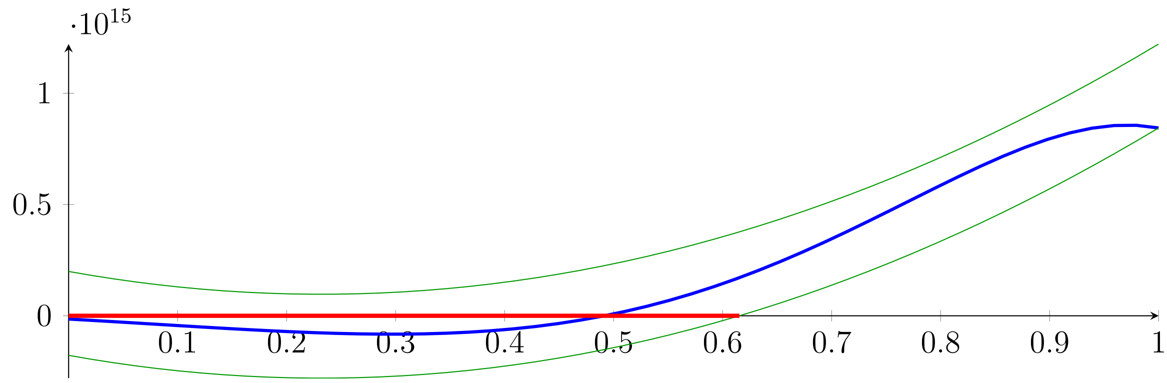
$$M = 1.90548{\cdot}10^{15}X^2 - 8.83759{\cdot}10^{14}X + 1.99201{\cdot}10^{14}$$

$$m = 1.90548{\cdot}10^{15}X^2 - 8.83759{\cdot}10^{14}X - 1.77775{\cdot}10^{14}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{-0.151603, 0.615402\}$$

Intersection intervals:



$[0, 0.615402]$

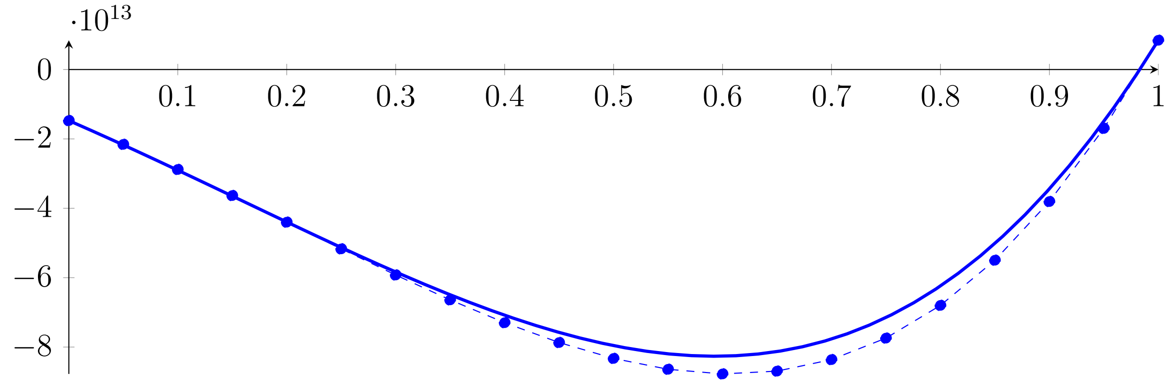
Longest intersection interval: 0.615402

$\Longrightarrow$ Bisection: first half $[17.1875, 17.9688]$ und second half $[17.9688, 18.75]$

2.84 Recursion Branch 1 2 1 2 2 1 on the First Half [17.1875, 17.9688]

Normalized monomial und Bézier representations and the Bézier polygon:

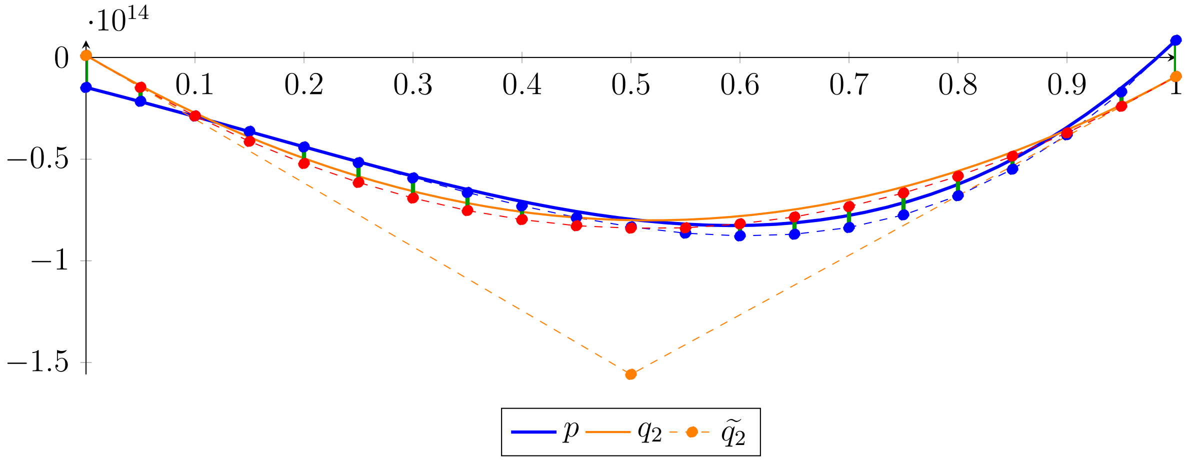
$$p = 77549.2X^{20} - 481986X^{19} + 1.96467{\cdot}10^{6}X^{18} - 9.60214{\cdot}10^{6}X^{17} + 4.74973{\cdot}10^{7}X^{16} - 3.11619{\cdot}10^{7}X^{15} + 6.44235{\cdot}10^{7}X^{14} + 6.32879{\cdot}10^{8}X^{13} + 5.38653{\cdot}10^{9}X^{12} + 3.08428{\cdot}10^{10}X^{11} + 9.72751{\cdot}10^{10}X^{10} - 5.37861{\cdot}10^{10}X^{9} - 2.04989{\cdot}10^{12}X^{8} - 8.64744{\cdot}10^{12}X^{7} - 1.14815{\cdot}10^{13}X^{6} + 2.7439{\cdot}10^{13}X^{5} + 1.16308{\cdot}10^{14}X^{4} + 1.10652{\cdot}10^{14}X^{3} - 7.20374{\cdot}10^{13}X^{2} - 1.37115{\cdot}10^{14}X - 1.47196{\cdot}10^{13}$$
$$= -1.47196{\cdot}10^{13}B_{0,20}(X) - 2.15754{\cdot}10^{13}B_{1,20}(X) - 2.88102{\cdot}10^{13}B_{2,20}(X) - 3.63272{\cdot}10^{13}B_{3,20}(X) - 4.40052{\cdot}10^{13}B_{4,20}(X) - 5.16973{\cdot}10^{13}B_{5,20}(X) - 5.92295{\cdot}10^{13}B_{6,20}(X) - 6.63994{\cdot}10^{13}B_{7,20}(X) - 7.29757{\cdot}10^{13}B_{8,20}(X) - 7.86984{\cdot}10^{13}B_{9,20}(X) - 8.328{\cdot}10^{13}B_{10,20}(X) - 8.6407{\cdot}10^{13}B_{11,20}(X) - 8.77436{\cdot}10^{13}B_{12,20}(X) - 8.69362{\cdot}10^{13}B_{13,20}(X) - 8.36193{\cdot}10^{13}B_{14,20}(X) - 7.74242{\cdot}10^{13}B_{15,20}(X) - 6.79884{\cdot}10^{13}B_{16,20}(X) - 5.49683{\cdot}10^{13}B_{17,20}(X) - 3.80537{\cdot}10^{13}B_{18,20}(X) - 1.69845{\cdot}10^{13}B_{19,20}(X) + 8.42928{\cdot}10^{12}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 3.03331{\cdot}10^{14}X^{2} - 3.13735{\cdot}10^{14}X + 1.02343{\cdot}10^{12}$$
$$= 1.02343{\cdot}10^{12}B_{0,2} - 1.55844{\cdot}10^{14}B_{1,2} - 9.38116{\cdot}10^{12}B_{2,2}$$

$$\widetilde{q}_2 = 4.8565{\cdot}10^{16}X^{20} - 4.86123{\cdot}10^{17}X^{19} + 2.25252{\cdot}10^{18}X^{18} - 6.41048{\cdot}10^{18}X^{17} + 1.2536{\cdot}10^{19}X^{16} - 1.78559{\cdot}10^{19}X^{15} + 1.91598{\cdot}10^{19}X^{14} - 1.58004{\cdot}10^{19}X^{13} + 1.01268{\cdot}10^{19}X^{12} - 5.06807{\cdot}10^{18}X^{11} + 1.97929{\cdot}10^{18}X^{10} - 5.99839{\cdot}10^{17}X^{9} + 1.39567{\cdot}10^{17}X^{8} - 2.45356{\cdot}10^{16}X^{7} + 3.18896{\cdot}10^{15}X^{6} - 2.97829{\cdot}10^{14}X^{5} + 1.92035{\cdot}10^{13}X^{4} - 7.92141{\cdot}10^{11}X^{3} + 3.03349{\cdot}10^{14}X^{2} - 3.13736{\cdot}10^{14}X + 1.02343{\cdot}10^{12}$$
$$= 1.02343{\cdot}10^{12}B_{0,20} - 1.46633{\cdot}10^{13}B_{1,20} - 2.87535{\cdot}10^{13}B_{2,20} - 4.12479{\cdot}10^{13}B_{3,20} - 5.2143{\cdot}10^{13}B_{4,20} - 6.1451{\cdot}10^{13}B_{5,20} - 6.9136{\cdot}10^{13}B_{6,20} - 7.52855{\cdot}10^{13}B_{7,20} - 7.97243{\cdot}10^{13}B_{8,20} - 8.27433{\cdot}10^{13}B_{9,20} - 8.39366{\cdot}10^{13}B_{10,20} - 8.37819{\cdot}10^{13}B_{11,20} - 8.18054{\cdot}10^{13}B_{12,20} - 7.84063{\cdot}10^{13}B_{13,20} - 7.32979{\cdot}10^{13}B_{14,20} - 6.66537{\cdot}10^{13}B_{15,20} - 5.83858{\cdot}10^{13}B_{16,20} - 4.85311{\cdot}10^{13}B_{17,20} - 3.70772{\cdot}10^{13}B_{18,20} - 2.40275{\cdot}10^{13}B_{19,20} - 9.38116{\cdot}10^{12}B_{20,20}$$



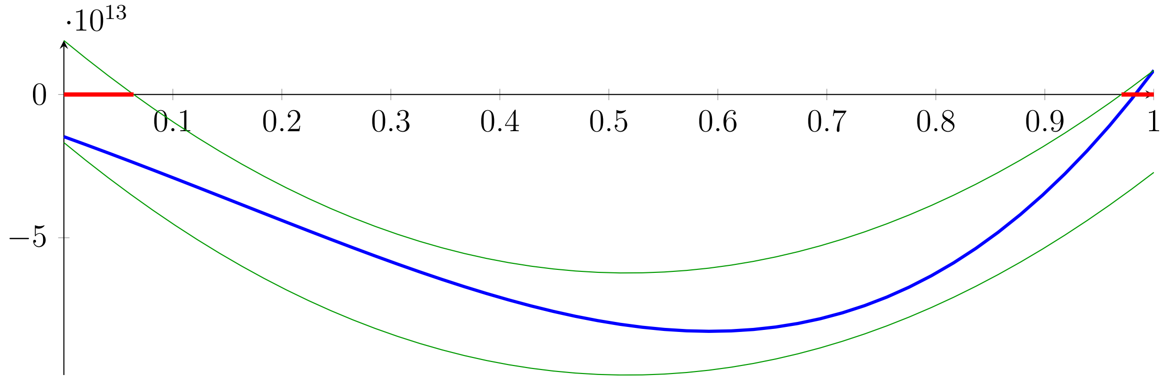
The maximum difference of the Bézier coefficients is $\delta = 1.78104{\cdot}10^{13}$.
Bounding polynomials M and m:

$$M = 3.03331{\cdot}10^{14}X^2 - 3.13735{\cdot}10^{14}X + 1.88339{\cdot}10^{13}$$
$$m = 3.03331{\cdot}10^{14}X^2 - 3.13735{\cdot}10^{14}X - 1.6787{\cdot}10^{13}$$

Root of M and m:

$$N(M) = \{0.06399, 0.970311\} \qquad N(m) = \{-0.0509929, 1.08529\}$$

Intersection intervals:



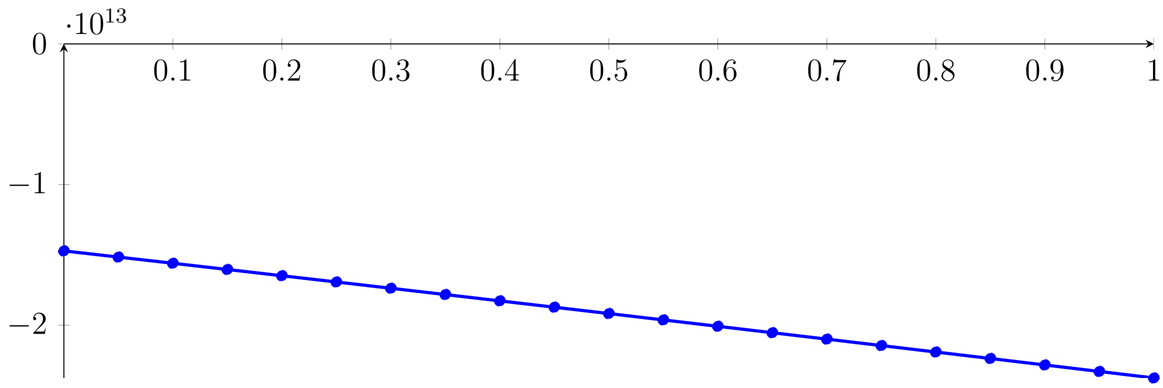
$$[0, 0.06399],\ [0.970311, 1]$$

Longest intersection interval: 0.06399
$\Longrightarrow$ Selective recursion: interval 1: $[17.1875, 17.2375]$, interval 2: $[17.9456, 17.9688]$,

2.85 Recursion Branch 1 2 1 2 2 1 1 in Interval 1: $[17.1875, 17.2375]$

Normalized monomial und Bézier representations and the Bézier polygon:

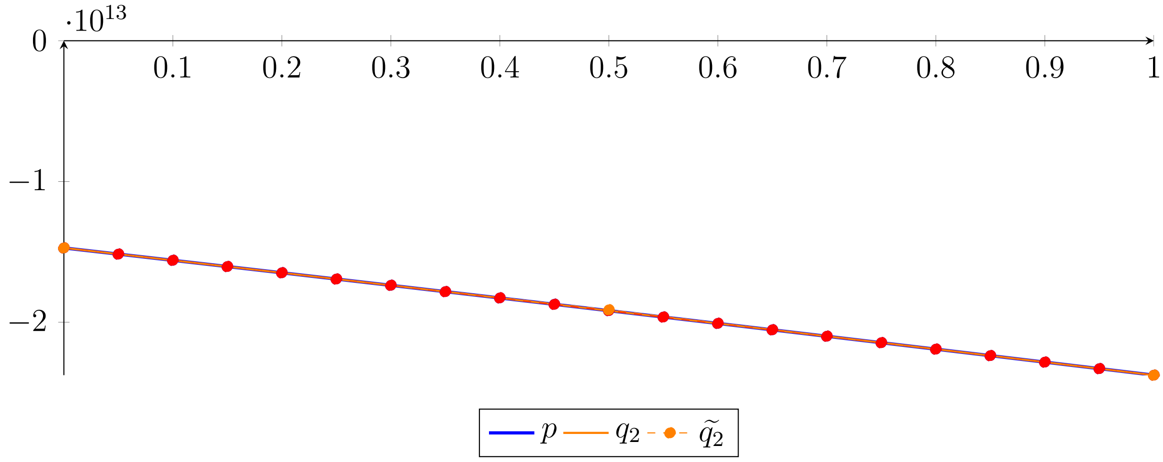
$$\begin{aligned} p = {} & 18072.7X^{20} - 108729X^{19} + 571963X^{18} - 2.31928{\cdot}10^6X^{17} + 1.19489{\cdot}10^7X^{16} - 9.24747 \\ & {\cdot}10^6X^{15} + 3.47947{\cdot}10^6X^{14} + 1.27757{\cdot}10^6X^{13} + 8.44245{\cdot}10^6X^{12} + 1.18294{\cdot}10^6X^{11} \\ & + 2.42276{\cdot}10^6X^{10} + 285401X^9 + 24111.4X^8 - 28161.6X^7 - 777471X^6 + 2.94394{\cdot}10^7X^5 \\ & + 1.95011{\cdot}10^9X^4 + 2.89932{\cdot}10^{10}X^3 - 2.94973{\cdot}10^{11}X^2 - 8.77397{\cdot}10^{12}X - 1.47196{\cdot}10^{13} \\ = {} & -1.47196{\cdot}10^{13}B_{0,20}(X) - 1.51583{\cdot}10^{13}B_{1,20}(X) - 1.55986{\cdot}10^{13}B_{2,20}(X) - 1.60404 \\ & {\cdot}10^{13}B_{3,20}(X) - 1.64836{\cdot}10^{13}B_{4,20}(X) - 1.69284{\cdot}10^{13}B_{5,20}(X) - 1.73746{\cdot}10^{13}B_{6,20}(X) \\ & - 1.78222{\cdot}10^{13}B_{7,20}(X) - 1.82712{\cdot}10^{13}B_{8,20}(X) - 1.87216{\cdot}10^{13}B_{9,20}(X) - 1.91733 \\ & {\cdot}10^{13}B_{10,20}(X) - 1.96264{\cdot}10^{13}B_{11,20}(X) - 2.00807{\cdot}10^{13}B_{12,20}(X) - 2.05362{\cdot}10^{13}B_{13,20}(X) \\ & - 2.0993{\cdot}10^{13}B_{14,20}(X) - 2.1451{\cdot}10^{13}B_{15,20}(X) - 2.19101{\cdot}10^{13}B_{16,20}(X) - 2.23704 \\ & {\cdot}10^{13}B_{17,20}(X) - 2.28317{\cdot}10^{13}B_{18,20}(X) - 2.32941{\cdot}10^{13}B_{19,20}(X) - 2.37576{\cdot}10^{13}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -2.48089\cdot 10^{11}X^2 - 8.79318\cdot 10^{12}X - 1.4718\cdot 10^{13} \\ &= -1.4718\cdot 10^{13}B_{0,2} - 1.91146\cdot 10^{13}B_{1,2} - 2.37593\cdot 10^{13}B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 3.11852\cdot 10^{15}X^{20} - 3.13453\cdot 10^{16}X^{19} + 1.45953\cdot 10^{17}X^{18} - 4.17659\cdot 10^{17}X^{17} + 8.21623\cdot 10^{17}X^{16} \\ &\quad - 1.17755\cdot 10^{18}X^{15} + 1.27135\cdot 10^{18}X^{14} - 1.05459\cdot 10^{18}X^{13} + 6.79431\cdot 10^{17}X^{12} - 3.41456\cdot 10^{17}X^{11} \\ &\quad + 1.33714\cdot 10^{17}X^{10} - 4.05561\cdot 10^{16}X^{9} + 9.4283\cdot 10^{15}X^{8} - 1.65739\cdot 10^{15}X^{7} + 2.17166\cdot 10^{14}X^{6} - 2.08999 \\ &\quad \cdot 10^{13}X^{5} + 1.44579\cdot 10^{12}X^{4} - 6.67176\cdot 10^{10}X^{3} - 2.46322\cdot 10^{11}X^{2} - 8.79319\cdot 10^{12}X - 1.4718\cdot 10^{13} \\ &= -1.4718\cdot 10^{13}B_{0,20} - 1.51577\cdot 10^{13}B_{1,20} - 1.55986\cdot 10^{13}B_{2,20} - 1.60409\cdot 10^{13}B_{3,20} - 1.64844 \\ &\quad \cdot 10^{13}B_{4,20} - 1.69297\cdot 10^{13}B_{5,20} - 1.73746\cdot 10^{13}B_{6,20} - 1.78249\cdot 10^{13}B_{7,20} - 1.82688\cdot 10^{13}B_{8,20} \\ &\quad - 1.87257\cdot 10^{13}B_{9,20} - 1.91689\cdot 10^{13}B_{10,20} - 1.96293\cdot 10^{13}B_{11,20} - 2.00771\cdot 10^{13}B_{12,20} \\ &\quad - 2.05371\cdot 10^{13}B_{13,20} - 2.09911\cdot 10^{13}B_{14,20} - 2.14504\cdot 10^{13}B_{15,20} - 2.19091\cdot 10^{13}B_{16,20} \\ &\quad - 2.23698\cdot 10^{13}B_{17,20} - 2.28316\cdot 10^{13}B_{18,20} - 2.32948\cdot 10^{13}B_{19,20} - 2.37593\cdot 10^{13}B_{20,20} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 4.46747\cdot 10^{9}$.
Bounding polynomials M and m:

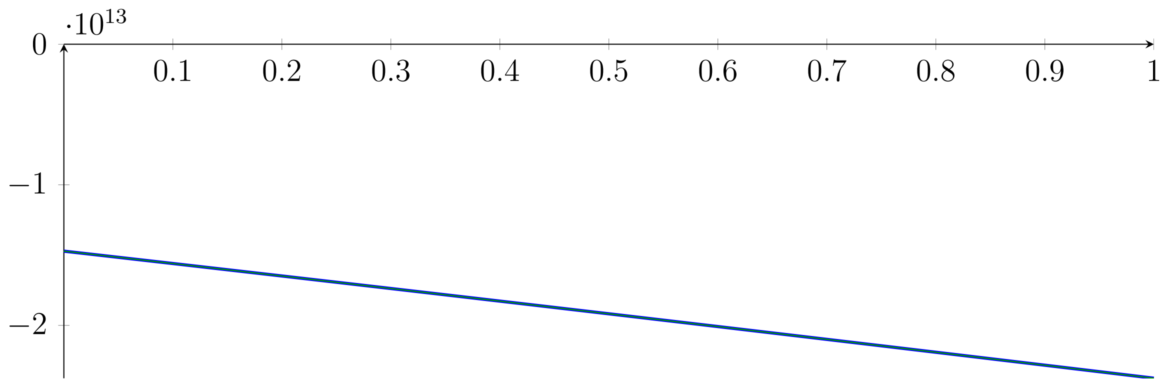
$$M = -2.48089\cdot 10^{11}X^2 - 8.79318\cdot 10^{12}X - 1.47135\cdot 10^{13}$$

$$m = -2.48089\cdot 10^{11}X^2 - 8.79318\cdot 10^{12}X - 1.47225\cdot 10^{13}$$

Root of M and m:

$$N(M) = \{-33.6829, -1.76076\} \qquad N(m) = \{-33.6817, -1.76189\}$$

Intersection intervals:

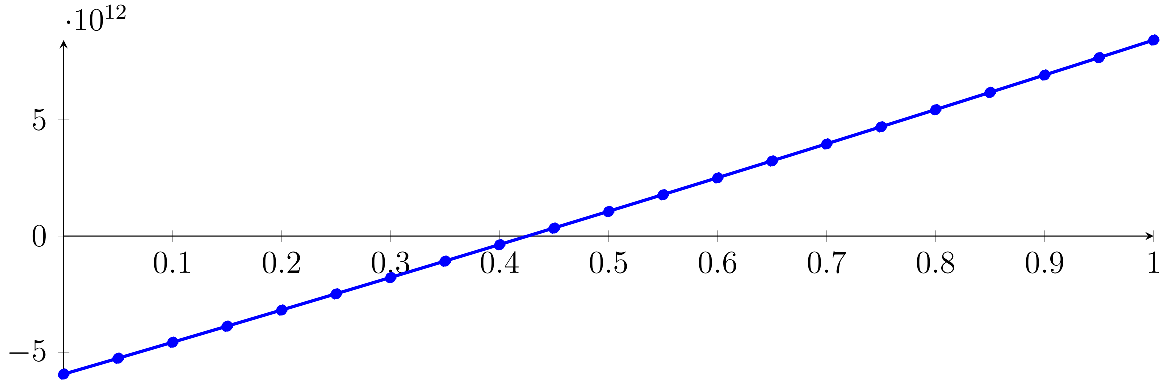


No intersection intervals with the x axis.

2.86 Recursion Branch 1 2 1 2 2 1 2 in Interval 2: $[17.9456, 17.9688]$

Normalized monomial und Bézier representations and the Bézier polygon:

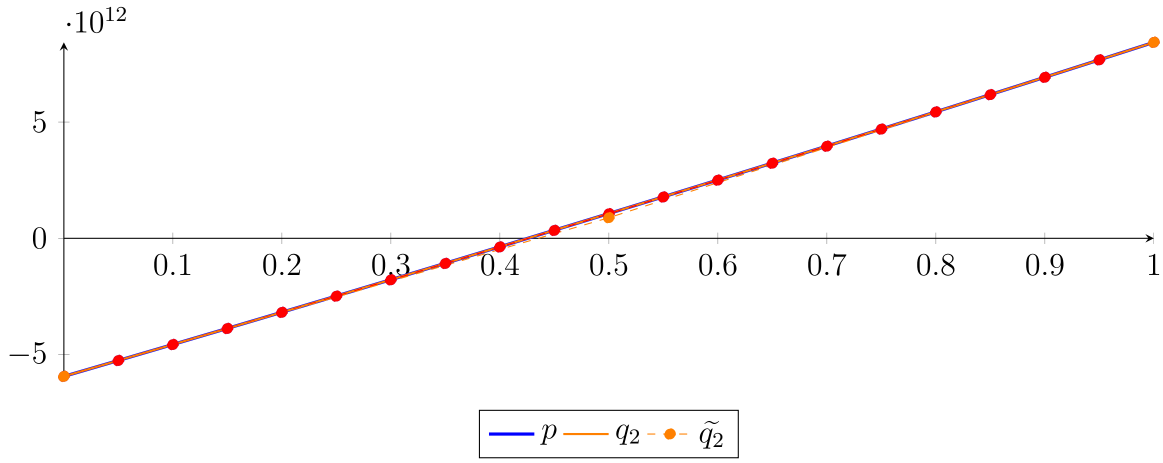
$$\begin{aligned}p &= -330.172X^{20} + 15917.8X^{19} + 20718.9X^{18} + 123619X^{17} + 66013.1X^{16} \\ &\quad - 232076X^{15} + 279799X^{14} + 241039X^{13} + 891631X^{12} + 343793X^{11} + 363738X^{10} \\ &\quad + 74794.7X^{9} + 4920.7X^{8} + 605.625X^{7} - 60108.3X^{6} - 6.55347\cdot 10^{6}X^{5} - 2.28786 \\ &\quad \cdot 10^{8}X^{4} + 6.65569\cdot 10^{9}X^{3} + 7.09082\cdot 10^{11}X^{2} + 1.3653\cdot 10^{13}X - 5.93918\cdot 10^{12} \\ &= -5.93918\cdot 10^{12}B_{0,20}(X) - 5.25654\cdot 10^{12}B_{1,20}(X) - 4.57016\cdot 10^{12}B_{2,20}(X) - 3.88004 \\ &\quad \cdot 10^{12}B_{3,20}(X) - 3.18618\cdot 10^{12}B_{4,20}(X) - 2.48857\cdot 10^{12}B_{5,20}(X) - 1.7872\cdot 10^{12}B_{6,20}(X) \\ &\quad - 1.08207\cdot 10^{12}B_{7,20}(X) - 3.7318\cdot 10^{11}B_{8,20}(X) + 3.39485\cdot 10^{11}B_{9,20}(X) + 1.05593 \\ &\quad \cdot 10^{12}B_{10,20}(X) + 1.77615\cdot 10^{12}B_{11,20}(X) + 2.50017\cdot 10^{12}B_{12,20}(X) + 3.22797\cdot 10^{12}B_{13,20}(X) \\ &\quad + 3.95958\cdot 10^{12}B_{14,20}(X) + 4.69499\cdot 10^{12}B_{15,20}(X) + 5.43421\cdot 10^{12}B_{16,20}(X) + 6.17724 \\ &\quad \cdot 10^{12}B_{17,20}(X) + 6.92409\cdot 10^{12}B_{18,20}(X) + 7.67477\cdot 10^{12}B_{19,20}(X) + 8.42928\cdot 10^{12}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 7.18661\cdot 10^{11}X^{2} + 1.36492\cdot 10^{13}X - 5.93887\cdot 10^{12} \\ &= -5.93887\cdot 10^{12}B_{0,2} + 8.8572\cdot 10^{11}B_{1,2} + 8.42897\cdot 10^{12}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 4.38187\cdot 10^{14}X^{20} - 4.38063\cdot 10^{15}X^{19} + 2.02502\cdot 10^{16}X^{18} - 5.74466\cdot 10^{16}X^{17} + 1.11946\cdot 10^{17}X^{16} \\ &\quad - 1.58966\cdot 10^{17}X^{15} + 1.70309\cdot 10^{17}X^{14} - 1.40613\cdot 10^{17}X^{13} + 9.05818\cdot 10^{16}X^{12} - 4.57848\cdot 10^{16}X^{11} \\ &\quad + 1.815\cdot 10^{16}X^{10} - 5.60731\cdot 10^{15}X^{9} + 1.33341\cdot 10^{15}X^{8} - 2.39477\cdot 10^{14}X^{7} + 3.16101\cdot 10^{13}X^{6} - 2.94514 \\ &\quad \cdot 10^{12}X^{5} + 1.81724\cdot 10^{11}X^{4} - 6.76296\cdot 10^{9}X^{3} + 7.18795\cdot 10^{11}X^{2} + 1.36492\cdot 10^{13}X - 5.93887\cdot 10^{12} \\ &= -5.93887\cdot 10^{12}B_{0,20} - 5.25641\cdot 10^{12}B_{1,20} - 4.57017\cdot 10^{12}B_{2,20} - 3.88015\cdot 10^{12}B_{3,20} - 3.18632 \\ &\quad \cdot 10^{12}B_{4,20} - 2.48881\cdot 10^{12}B_{5,20} - 1.78725\cdot 10^{12}B_{6,20} - 1.08248\cdot 10^{12}B_{7,20} - 3.72909\cdot 10^{11}B_{8,20} \\ &\quad + 3.3892\cdot 10^{11}B_{9,20} + 1.05653\cdot 10^{12}B_{10,20} + 1.77562\cdot 10^{12}B_{11,20} + 2.50065\cdot 10^{12}B_{12,20} \\ &\quad + 3.2279\cdot 10^{12}B_{13,20} + 3.95987\cdot 10^{12}B_{14,20} + 4.69513\cdot 10^{12}B_{15,20} + 5.43438\cdot 10^{12}B_{16,20} \\ &\quad + 6.17734\cdot 10^{12}B_{17,20} + 6.92411\cdot 10^{12}B_{18,20} + 7.67465\cdot 10^{12}B_{19,20} + 8.42897\cdot 10^{12}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 6.02288 \cdot 10^{8}$.

Bounding polynomials M and m:

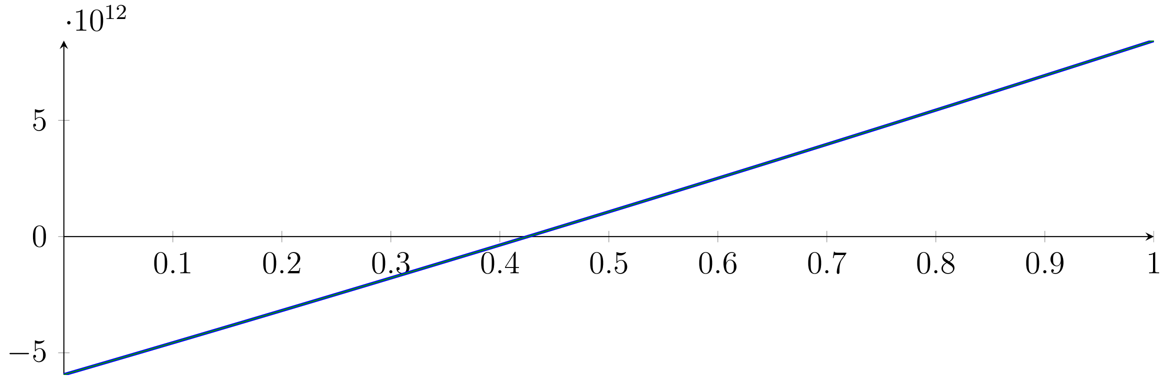
$$M = 7.18661 \cdot 10^{11} X^2 + 1.36492 \cdot 10^{13} X - 5.93827 \cdot 10^{12}$$

$$m = 7.18661 \cdot 10^{11} X^2 + 1.36492 \cdot 10^{13} X - 5.93947 \cdot 10^{12}$$

Root of M and m:

$$N(M) = \{-19.418, 0.42553\} \qquad N(m) = \{-19.4181, 0.425614\}$$

Intersection intervals:



$$[0.42553, 0.425614]$$

Longest intersection interval: $8.44673 \cdot 10^{-05}$

$\Longrightarrow$ Selective recursion: interval 1: $[17.9554, 17.9554]$,

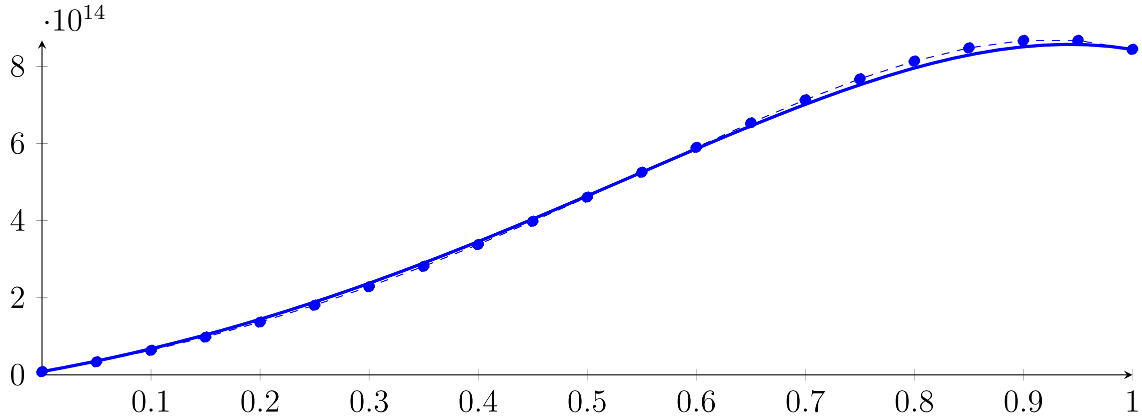
2.87 Recursion Branch 1 2 1 2 2 1 2 1 in Interval 1: $[17.9554, 17.9554]$

Found root in interval $[17.9554, 17.9554]$ at recursion depth 8!

2.88 Recursion Branch 1 2 1 2 2 2 on the Second Half $[17.9688, 18.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

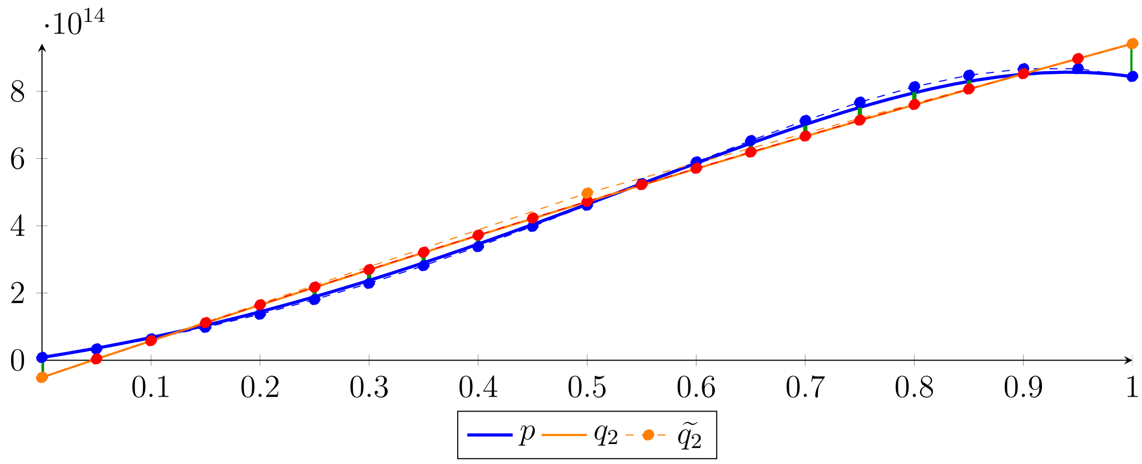
$$\begin{aligned} p = & -406735 X^{20} + 3.23454 \cdot 10^{6} X^{19} - 9.27253 \cdot 10^{6} X^{18} + 5.54286 \cdot 10^{7} X^{17} - 2.32388 \cdot 10^{8} X^{16} + 1.68297 \\ & \cdot 10^{8} X^{15} + 6.61085 \cdot 10^{7} X^{14} + 1.79234 \cdot 10^{9} X^{13} + 1.9977 \cdot 10^{10} X^{12} + 1.68452 \cdot 10^{11} X^{11} + 1.0336 \\ & \cdot 10^{12} X^{10} + 4.36677 \cdot 10^{12} X^{9} + 1.0574 \cdot 10^{13} X^{8} + 3.92297 \cdot 10^{11} X^{7} - 9.30488 \cdot 10^{13} X^{6} - 3.00689 \\ & \cdot 10^{14} X^{5} - 3.37885 \cdot 10^{14} X^{4} + 2.16815 \cdot 10^{14} X^{3} + 8.25489 \cdot 10^{14} X^{2} + 5.08276 \cdot 10^{14} X + 8.42928 \cdot 10^{12} \\ = & \; 8.42928 \cdot 10^{12} B_{0,20}(X) + 3.38431 \cdot 10^{13} B_{1,20}(X) + 6.36016 \cdot 10^{13} B_{2,20}(X) + 9.7895 \\ & \cdot 10^{13} B_{3,20}(X) + 1.36844 \cdot 10^{14} B_{4,20}(X) + 1.80479 \cdot 10^{14} B_{5,20}(X) + 2.28721 \cdot 10^{14} B_{6,20}(X) \\ & + 2.81356 \cdot 10^{14} B_{7,20}(X) + 3.38006 \cdot 10^{14} B_{8,20}(X) + 3.98106 \cdot 10^{14} B_{9,20}(X) + 4.60869 \\ & \cdot 10^{14} B_{10,20}(X) + 5.25254 \cdot 10^{14} B_{11,20}(X) + 5.89938 \cdot 10^{14} B_{12,20}(X) + 6.53281 \cdot 10^{14} B_{13,20}(X) \\ & + 7.13299 \cdot 10^{14} B_{14,20}(X) + 7.67635 \cdot 10^{14} B_{15,20}(X) + 8.13539 \cdot 10^{14} B_{16,20}(X) + 8.47861 \\ & \cdot 10^{14} B_{17,20}(X) + 8.67049 \cdot 10^{14} B_{18,20}(X) + 8.67168 \cdot 10^{14} B_{19,20}(X) + 8.43944 \cdot 10^{14} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = -1.03934 \cdot 10^{14} X^2 + 1.09649 \cdot 10^{15} X - 5.08813 \cdot 10^{13}$$
$$= -5.08813 \cdot 10^{13} B_{0,2} + 4.97365 \cdot 10^{14} B_{1,2} + 9.41676 \cdot 10^{14} B_{2,2}$$

$$\widetilde{q}_2 = -5.96013 \cdot 10^{16} X^{20} + 5.9942 \cdot 10^{17} X^{19} - 2.79398 \cdot 10^{18} X^{18} + 8.0062 \cdot 10^{18} X^{17} - 1.57732 \cdot 10^{19} X^{16} + 2.2635 \cdot 10^{19} X^{15} - 2.44542 \cdot 10^{19} X^{14} + 2.02765 \cdot 10^{19} X^{13} - 1.30379 \cdot 10^{19} X^{12} + 6.52709 \cdot 10^{18} X^{11} - 2.54089 \cdot 10^{18} X^{10} + 7.64639 \cdot 10^{17} X^9 - 1.76131 \cdot 10^{17} X^8 + 3.06754 \cdot 10^{16} X^7 - 3.99351 \cdot 10^{15} X^6 + 3.85347 \cdot 10^{14} X^5 - 2.72122 \cdot 10^{13} X^4 + 1.30118 \cdot 10^{12} X^3 - 1.0397 \cdot 10^{14} X^2 + 1.09649 \cdot 10^{15} X - 5.08813 \cdot 10^{13}$$
$$= -5.08813 \cdot 10^{13} B_{0,20} + 3.94336 \cdot 10^{12} B_{1,20} + 5.82208 \cdot 10^{13} B_{2,20} + 1.11952 \cdot 10^{14} B_{3,20} + 1.65133 \cdot 10^{14} B_{4,20} + 2.17778 \cdot 10^{14} B_{5,20} + 2.69843 \cdot 10^{14} B_{6,20} + 3.21439 \cdot 10^{14} B_{7,20} + 3.72339 \cdot 10^{14} B_{8,20} + 4.2292 \cdot 10^{14} B_{9,20} + 4.72663 \cdot 10^{14} B_{10,20} + 5.2216 \cdot 10^{14} B_{11,20} + 5.70852 \cdot 10^{14} B_{12,20} + 6.19203 \cdot 10^{14} B_{13,20} + 6.66865 \cdot 10^{14} B_{14,20} + 7.14059 \cdot 10^{14} B_{15,20} + 7.60667 \cdot 10^{14} B_{16,20} + 8.06742 \cdot 10^{14} B_{17,20} + 8.52267 \cdot 10^{14} B_{18,20} + 8.97245 \cdot 10^{14} B_{19,20} + 9.41676 \cdot 10^{14} B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 9.77323 \cdot 10^{13}$.

Bounding polynomials M and m:

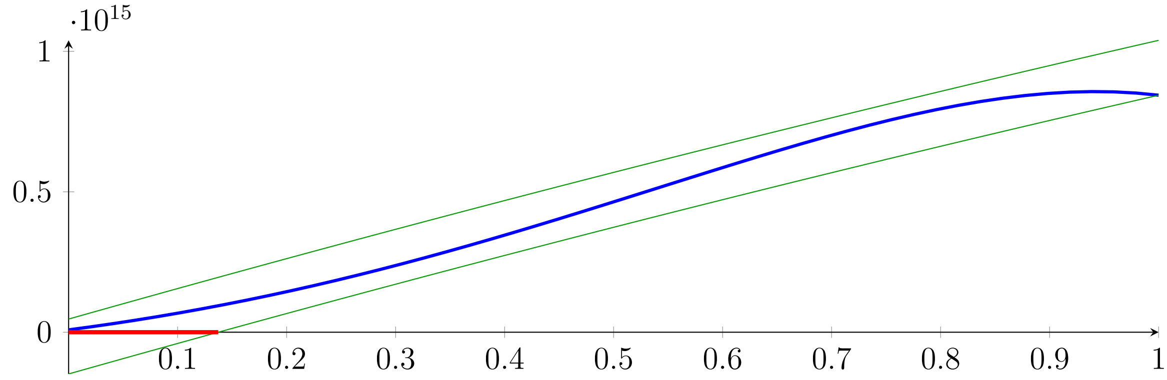
$$M = -1.03934 \cdot 10^{14} X^2 + 1.09649 \cdot 10^{15} X + 4.68511 \cdot 10^{13}$$

$$m = -1.03934 \cdot 10^{14} X^2 + 1.09649 \cdot 10^{15} X - 1.48614 \cdot 10^{14}$$

Root of M and m:

$$N(M) = \{-0.0425565, 10.5924\} \qquad N(m) = \{0.137323, 10.4125\}$$

Intersection intervals:



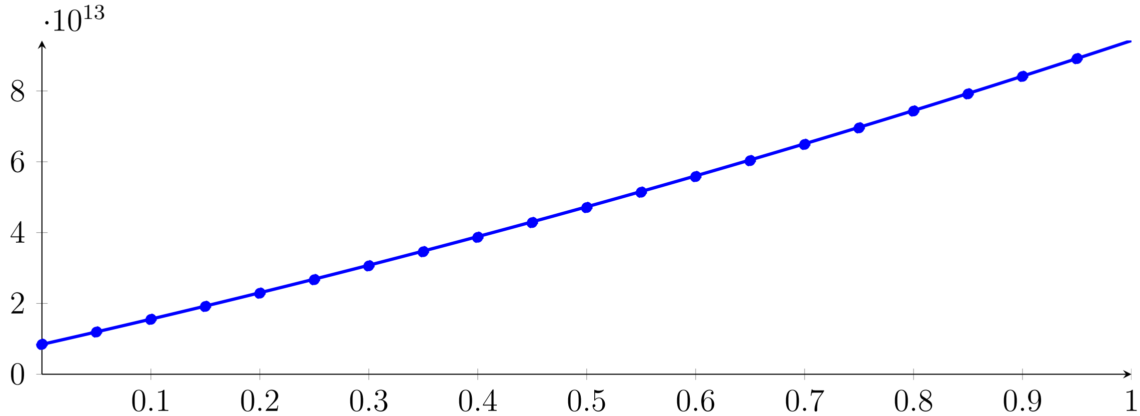
$[0, 0.137323]$

Longest intersection interval: 0.137323
$\Longrightarrow$ Selective recursion: interval 1: $[17.9688, 18.076]$,

2.89 Recursion Branch 1 2 1 2 2 2 1 in Interval 1: $[17.9688, 18.076]$

Normalized monomial und Bézier representations and the Bézier polygon:

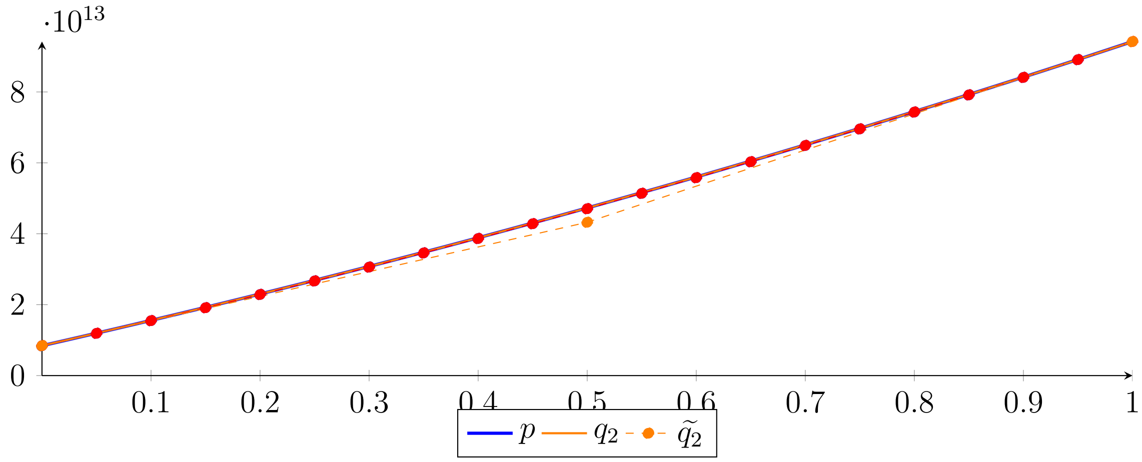
$$\begin{aligned}p = &-42376.7X^{20} + 305021X^{19} - 1.13146\cdot 10^{6}X^{18} + 5.57019\cdot 10^{6}X^{17} - 2.58383\cdot 10^{7}X^{16} + 1.82323\\ &\cdot 10^{7}X^{15} - 5.76945\cdot 10^{6}X^{14} - 3.13176\cdot 10^{6}X^{13} - 1.49559\cdot 10^{7}X^{12} - 906886X^{11} - 3.57802\\ &\cdot 10^{6}X^{10} - 174685X^{9} + 1.27459\cdot 10^{6}X^{8} + 366327X^{7} - 6.23993\cdot 10^{8}X^{6} - 1.46836\cdot 10^{10}X^{5}\\ &- 1.20155\cdot 10^{11}X^{4} + 5.61459\cdot 10^{11}X^{3} + 1.55667\cdot 10^{13}X^{2} + 6.9798\cdot 10^{13}X + 8.42928\cdot 10^{12}\\ = &\ 8.42928\cdot 10^{12}B_{0,20}(X) + 1.19192\cdot 10^{13}B_{1,20}(X) + 1.5491\cdot 10^{13}B_{2,20}(X) + 1.91453\\ &\cdot 10^{13}B_{3,20}(X) + 2.28824\cdot 10^{13}B_{4,20}(X) + 2.67029\cdot 10^{13}B_{5,20}(X) + 3.06071\cdot 10^{13}B_{6,20}(X)\\ &+ 3.45955\cdot 10^{13}B_{7,20}(X) + 3.86683\cdot 10^{13}B_{8,20}(X) + 4.2826\cdot 10^{13}B_{9,20}(X) + 4.70688\\ &\cdot 10^{13}B_{10,20}(X) + 5.1397\cdot 10^{13}B_{11,20}(X) + 5.58108\cdot 10^{13}B_{12,20}(X) + 6.03104\cdot 10^{13}B_{13,20}(X)\\ &+ 6.4896\cdot 10^{13}B_{14,20}(X) + 6.95678\cdot 10^{13}B_{15,20}(X) + 7.43257\cdot 10^{13}B_{16,20}(X) + 7.91699\\ &\cdot 10^{13}B_{17,20}(X) + 8.41004\cdot 10^{13}B_{18,20}(X) + 8.91171\cdot 10^{13}B_{19,20}(X) + 9.422\cdot 10^{13}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 1.61756\cdot 10^{13}X^{2} + 6.95874\cdot 10^{13}X + 8.44541\cdot 10^{12}\\ &= 8.44541\cdot 10^{12}B_{0,2} + 4.32391\cdot 10^{13}B_{1,2} + 9.42085\cdot 10^{13}B_{2,2}\end{aligned}$$

$$
\begin{aligned}
\widetilde{q}_2 = & -3.4916 \cdot 10^{15} X^{20} + 3.52843 \cdot 10^{16} X^{19} - 1.65406 \cdot 10^{17} X^{18} + 4.77018 \cdot 10^{17} X^{17} - 9.46223 \cdot 10^{17} X^{16} \\
& + 1.36727 \cdot 10^{18} X^{15} - 1.48693 \cdot 10^{18} X^{14} + 1.24015 \cdot 10^{18} X^{13} - 8.01182 \cdot 10^{17} X^{12} + 4.02331 \cdot 10^{17} X^{11} \\
& - 1.56778 \cdot 10^{17} X^{10} + 4.71115 \cdot 10^{16} X^{9} - 1.08132 \cdot 10^{16} X^{8} + 1.87793 \cdot 10^{15} X^{7} - 2.46077 \cdot 10^{14} X^{6} + 2.4488 \\
& \cdot 10^{13} X^{5} - 1.84998 \cdot 10^{12} X^{4} + 9.6497 \cdot 10^{10} X^{3} + 1.61728 \cdot 10^{13} X^{2} + 6.95875 \cdot 10^{13} X + 8.44541 \cdot 10^{12} \\
= & \; 8.44541 \cdot 10^{12} B_{0,20} + 1.19248 \cdot 10^{13} B_{1,20} + 1.54893 \cdot 10^{13} B_{2,20} + 1.9139 \cdot 10^{13} B_{3,20} + 2.28736 \\
& \cdot 10^{13} B_{4,20} + 2.6694 \cdot 10^{13} B_{5,20} + 3.05975 \cdot 10^{13} B_{6,20} + 3.45911 \cdot 10^{13} B_{7,20} + 3.86604 \cdot 10^{13} B_{8,20} \\
& + 4.28288 \cdot 10^{13} B_{9,20} + 4.70649 \cdot 10^{13} B_{10,20} + 5.14036 \cdot 10^{13} B_{11,20} + 5.58132 \cdot 10^{13} B_{12,20} \\
& + 6.03196 \cdot 10^{13} B_{13,20} + 6.49026 \cdot 10^{13} B_{14,20} + 6.95757 \cdot 10^{13} B_{15,20} + 7.43314 \cdot 10^{13} B_{16,20} \\
& + 7.91731 \cdot 10^{13} B_{17,20} + 8.40997 \cdot 10^{13} B_{18,20} + 8.91115 \cdot 10^{13} B_{19,20} + 9.42085 \cdot 10^{13} B_{20,20}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 1.61266 \cdot 10^{10}$.

Bounding polynomials M and m:

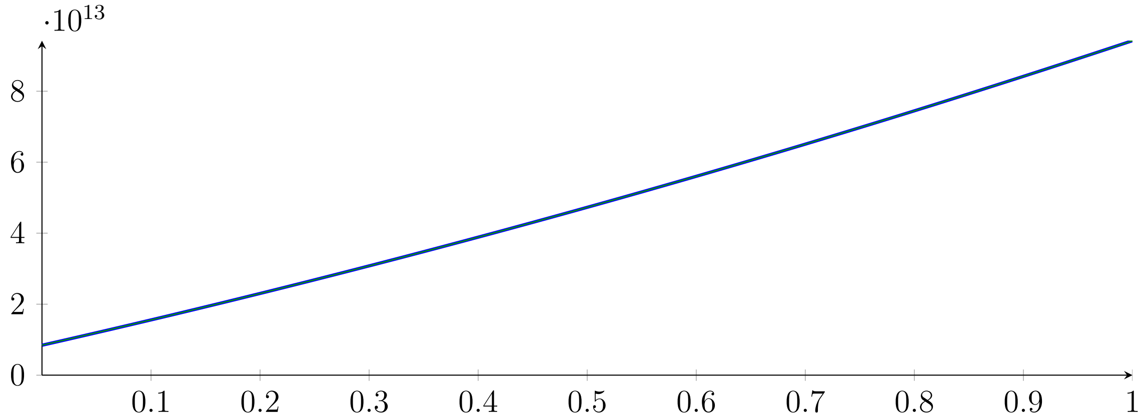
$$M = 1.61756 \cdot 10^{13} X^2 + 6.95874 \cdot 10^{13} X + 8.46153 \cdot 10^{12}$$

$$m = 1.61756 \cdot 10^{13} X^2 + 6.95874 \cdot 10^{13} X + 8.42928 \cdot 10^{12}$$

Root of M and m:

$$N(M) = \{-4.17675, -0.125242\} \qquad N(m) = \{-4.17725, -0.12475\}$$

Intersection intervals:

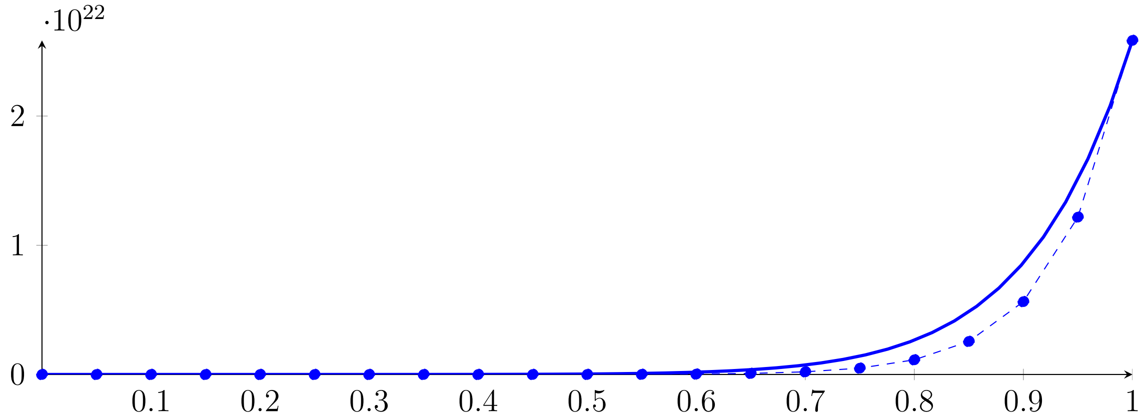


No intersection intervals with the x axis.

2.90 Recursion Branch 1 2 2 on the Second Half $[18.75, 25]$

Normalized monomial und Bézier representations and the Bézier polygon:

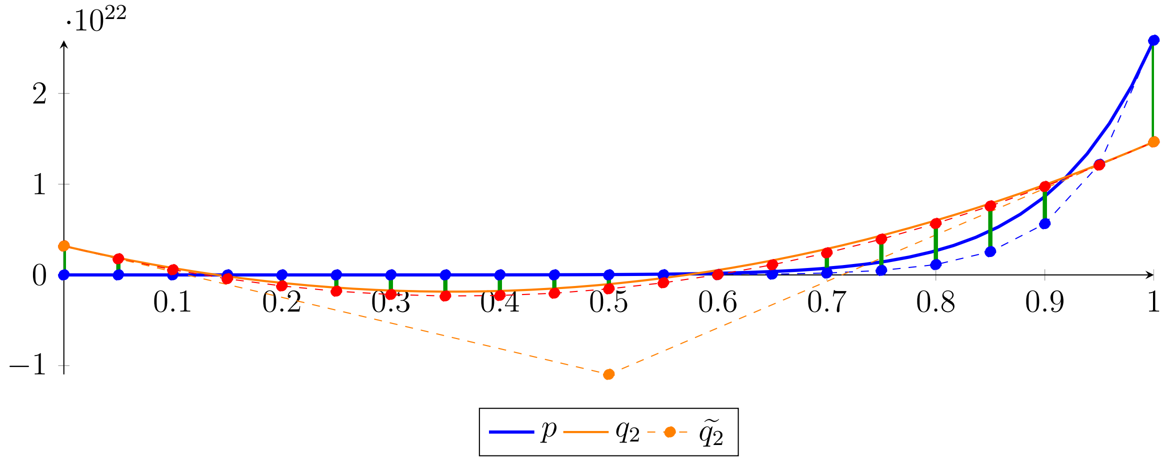
$$p = 8.27177{\cdot}10^{15}X^{20} + 2.18376{\cdot}10^{17}X^{19} + 2.66802{\cdot}10^{18}X^{18} + 2.00154{\cdot}10^{19}X^{17} + 1.03147{\cdot}10^{20}X^{16} + 3.86992{\cdot}10^{20}X^{15} + 1.09286{\cdot}10^{21}X^{14} + 2.36814{\cdot}10^{21}X^{13} + 3.97654{\cdot}10^{21}X^{12} + 5.18646{\cdot}10^{21}X^{11} + 5.22867{\cdot}10^{21}X^{10} + 4.02002{\cdot}10^{21}X^{9} + 2.29598{\cdot}10^{21}X^{8} + 9.25412{\cdot}10^{20}X^{7} + 2.3318{\cdot}10^{20}X^{6} + 2.12469{\cdot}10^{19}X^{5} - 6.75399{\cdot}10^{18}X^{4} - 2.49502{\cdot}10^{18}X^{3} - 2.83854{\cdot}10^{17}X^{2} - 3.71586{\cdot}10^{15}X + 8.43944{\cdot}10^{14}$$
$$= 8.43944{\cdot}10^{14}B_{0,20}(X) + 6.58151{\cdot}10^{14}B_{1,20}(X) - 1.02161{\cdot}10^{15}B_{2,20}(X) - 6.38396{\cdot}10^{15}B_{3,20}(X) - 1.90115{\cdot}10^{16}B_{4,20}(X) - 4.25105{\cdot}10^{16}B_{5,20}(X) - 7.31244{\cdot}10^{16}B_{6,20}(X) - 7.43935{\cdot}10^{16}B_{7,20}(X) + 9.63026{\cdot}10^{16}B_{8,20}(X) + 8.81646{\cdot}10^{17}B_{9,20}(X) + 3.50544{\cdot}10^{18}B_{10,20}(X) + 1.11134{\cdot}10^{19}B_{11,20}(X) + 3.13849{\cdot}10^{19}B_{12,20}(X) + 8.23454{\cdot}10^{19}B_{13,20}(X) + 2.04998{\cdot}10^{20}B_{14,20}(X) + 4.9022{\cdot}10^{20}B_{15,20}(X) + 1.13504{\cdot}10^{21}B_{16,20}(X) + 2.55855{\cdot}10^{21}B_{17,20}(X) + 5.63734{\cdot}10^{21}B_{18,20}(X) + 1.21777{\cdot}10^{22}B_{19,20}(X) + 2.5852{\cdot}10^{22}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 3.97783{\cdot}10^{22}X^2 - 2.831{\cdot}10^{22}X + 3.19008{\cdot}10^{21}$$
$$= 3.19008{\cdot}10^{21}B_{0,2} - 1.09649{\cdot}10^{22}B_{1,2} + 1.46584{\cdot}10^{22}B_{2,2}$$

$$\widetilde{q}_2 = 5.1352{\cdot}10^{24}X^{20} - 5.13399{\cdot}10^{25}X^{19} + 2.3754{\cdot}10^{26}X^{18} - 6.74871{\cdot}10^{26}X^{17} + 1.3173{\cdot}10^{27}X^{16} - 1.87274{\cdot}10^{27}X^{15} + 2.00579{\cdot}10^{27}X^{14} - 1.65143{\cdot}10^{27}X^{13} + 1.05711{\cdot}10^{27}X^{12} - 5.28674{\cdot}10^{26}X^{11} + 2.06469{\cdot}10^{26}X^{10} - 6.26217{\cdot}10^{25}X^{9} + 1.45915{\cdot}10^{25}X^{8} - 2.56829{\cdot}10^{24}X^{7} + 3.33309{\cdot}10^{23}X^{6} - 3.08449{\cdot}10^{22}X^{5} + 1.93941{\cdot}10^{21}X^{4} - 7.63509{\cdot}10^{19}X^{3} + 3.97799{\cdot}10^{22}X^{2} - 2.831{\cdot}10^{22}X + 3.19008{\cdot}10^{21}$$
$$= 3.19008{\cdot}10^{21}B_{0,20} + 1.77458{\cdot}10^{21}B_{1,20} + 5.68448{\cdot}10^{20}B_{2,20} - 4.28382{\cdot}10^{20}B_{3,20} - 1.21558{\cdot}10^{21}B_{4,20} - 1.79439{\cdot}10^{21}B_{5,20} - 2.16107{\cdot}10^{21}B_{6,20} - 2.32473{\cdot}10^{21}B_{7,20} - 2.26718{\cdot}10^{21}B_{8,20} - 2.01872{\cdot}10^{21}B_{9,20} - 1.53685{\cdot}10^{21}B_{10,20} - 8.71923{\cdot}10^{20}B_{11,20} + 2.65166{\cdot}10^{19}B_{12,20} + 1.11576{\cdot}10^{21}B_{13,20} + 2.42624{\cdot}10^{21}B_{14,20} + 3.93976{\cdot}10^{21}B_{15,20} + 5.66542{\cdot}10^{21}B_{16,20} + 7.59944{\cdot}10^{21}B_{17,20} + 9.7431{\cdot}10^{21}B_{18,20} + 1.20961{\cdot}10^{22}B_{19,20} + 1.46584{\cdot}10^{22}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.11936 \cdot 10^{22}$.

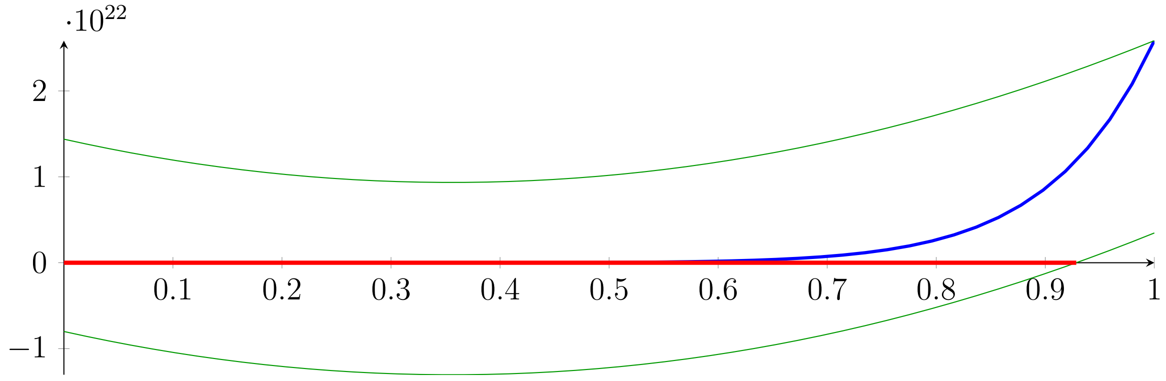
Bounding polynomials M and m:

$$M = 3.97783 \cdot 10^{22} X^2 - 2.831 \cdot 10^{22} X + 1.43837 \cdot 10^{22}$$
$$m = 3.97783 \cdot 10^{22} X^2 - 2.831 \cdot 10^{22} X - 8.00354 \cdot 10^{21}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{-0.216718, 0.928412\}$$

Intersection intervals:



$[0, 0.928412]$

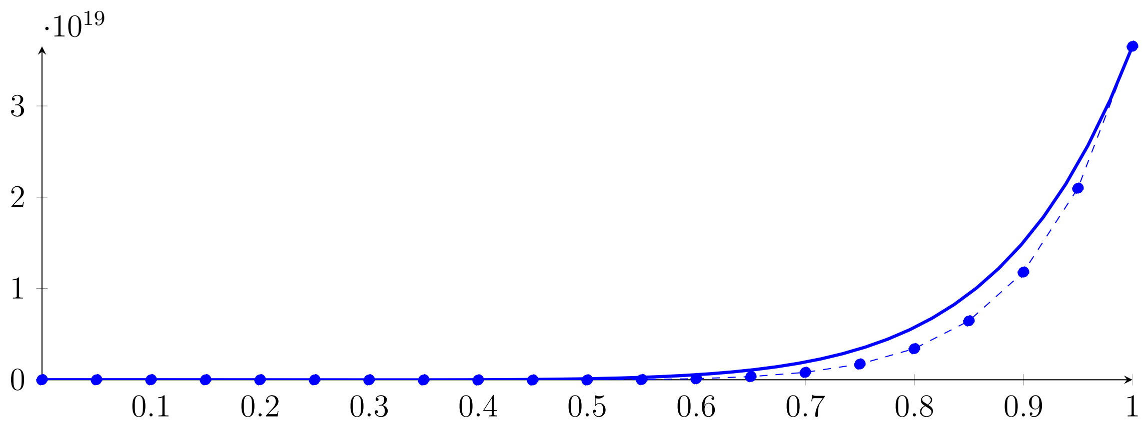
Longest intersection interval: 0.928412

$\Longrightarrow$ Bisection: first half $[18.75, 21.875]$ und second half $[21.875, 25]$

2.91 Recursion Branch 1 2 2 1 on the First Half $[18.75, 21.875]$

Normalized monomial und Bézier representations and the Bézier polygon:

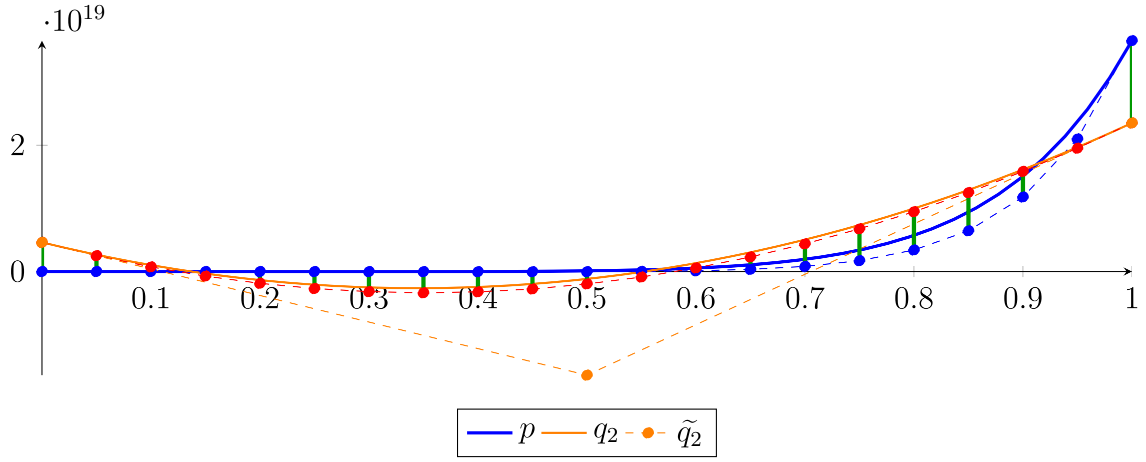
$$\begin{aligned} p = {} & 7.80391 \cdot 10^{9} X^{20} + 4.16786 \cdot 10^{11} X^{19} + 1.01746 \cdot 10^{13} X^{18} + 1.52705 \cdot 10^{14} X^{17} + 1.57391 \cdot 10^{15} X^{16} \\ & + 1.18101 \cdot 10^{16} X^{15} + 6.67027 \cdot 10^{16} X^{14} + 2.8908 \cdot 10^{17} X^{13} + 9.70834 \cdot 10^{17} X^{12} + 2.53245 \cdot 10^{18} X^{11} \\ & + 5.10612 \cdot 10^{18} X^{10} + 7.8516 \cdot 10^{18} X^{9} + 8.96866 \cdot 10^{18} X^{8} + 7.22978 \cdot 10^{18} X^{7} + 3.64345 \cdot 10^{18} X^{6} + 6.63965 \\ & \cdot 10^{17} X^{5} - 4.22124 \cdot 10^{17} X^{4} - 3.11878 \cdot 10^{17} X^{3} - 7.09636 \cdot 10^{16} X^{2} - 1.85793 \cdot 10^{15} X + 8.43944 \cdot 10^{14} \\ = {} & 8.43944 \cdot 10^{14} B_{0,20}(X) + 7.51047 \cdot 10^{14} B_{1,20}(X) + 2.84658 \cdot 10^{14} B_{2,20}(X) - 8.288 \\ & \cdot 10^{14} B_{3,20}(X) - 2.95003 \cdot 10^{15} B_{4,20}(X) - 6.48404 \cdot 10^{15} B_{5,20}(X) - 1.17433 \cdot 10^{16} B_{6,20}(X) \\ & - 1.86237 \cdot 10^{16} B_{7,20}(X) - 2.59286 \cdot 10^{16} B_{8,20}(X) - 3.00592 \cdot 10^{16} B_{9,20}(X) - 2.25952 \\ & \cdot 10^{16} B_{10,20}(X) + 1.40163 \cdot 10^{16} B_{11,20}(X) + 1.13942 \cdot 10^{17} B_{12,20}(X) + 3.40665 \cdot 10^{17} B_{13,20}(X) \\ & + 8.08159 \cdot 10^{17} B_{14,20}(X) + 1.71567 \cdot 10^{18} B_{15,20}(X) + 3.40411 \cdot 10^{18} B_{16,20}(X) + 6.44636 \\ & \cdot 10^{18} B_{17,20}(X) + 1.17905 \cdot 10^{19} B_{18,20}(X) + 2.09852 \cdot 10^{19} B_{19,20}(X) + 3.65302 \cdot 10^{19} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = 6.09272\cdot 10^{19}X^2 - 4.20353\cdot 10^{19}X + 4.61482\cdot 10^{18}$$
$$= 4.61482\cdot 10^{18}B_{0,2} - 1.64028\cdot 10^{19}B_{1,2} + 2.35068\cdot 10^{19}B_{2,2}$$

$$\widetilde{q}_2 = 7.84491\cdot 10^{21}X^{20} - 7.84279\cdot 10^{22}X^{19} + 3.62855\cdot 10^{23}X^{18} - 1.03085\cdot 10^{24}X^{17} + 2.01202\cdot 10^{24}X^{16} - 2.86023\cdot 10^{24}X^{15} + 3.06329\cdot 10^{24}X^{14} - 2.522\cdot 10^{24}X^{13} + 1.61436\cdot 10^{24}X^{12} - 8.07377\cdot 10^{23}X^{11} + 3.15331\cdot 10^{23}X^{10} - 9.56479\cdot 10^{22}X^{9} + 2.22895\cdot 10^{22}X^{8} - 3.92367\cdot 10^{21}X^{7} + 5.0922\cdot 10^{20}X^{6} - 4.71131\cdot 10^{19}X^{5} + 2.95994\cdot 10^{18}X^{4} - 1.16341\cdot 10^{17}X^{3} + 6.09298\cdot 10^{19}X^{2} - 4.20353\cdot 10^{19}X + 4.61482\cdot 10^{18}$$
$$= 4.61482\cdot 10^{18}B_{0,20} + 2.51306\cdot 10^{18}B_{1,20} + 7.31974\cdot 10^{17}B_{2,20} - 7.28526\cdot 10^{17}B_{3,20} - 1.86794\cdot 10^{18}B_{4,20} - 2.68818\cdot 10^{18}B_{5,20} - 3.18349\cdot 10^{18}B_{6,20} - 3.36784\cdot 10^{18}B_{7,20} - 3.21341\cdot 10^{18}B_{8,20} - 2.7665\cdot 10^{18}B_{9,20} - 1.96217\cdot 10^{18}B_{10,20} - 8.77363\cdot 10^{17}B_{11,20} + 5.65029\cdot 10^{17}B_{12,20} + 2.29974\cdot 10^{18}B_{13,20} + 4.37326\cdot 10^{18}B_{14,20} + 6.75779\cdot 10^{18}B_{15,20} + 9.46724\cdot 10^{18}B_{16,20} + 1.24959\cdot 10^{19}B_{17,20} + 1.58455\cdot 10^{19}B_{18,20} + 1.95158\cdot 10^{19}B_{19,20} + 2.35068\cdot 10^{19}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.30234\cdot 10^{19}$.

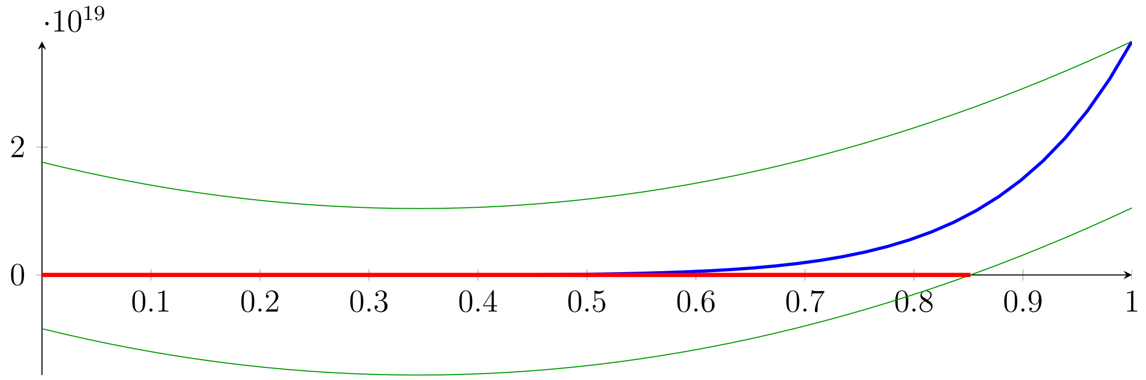
Bounding polynomials M and m:

$$M = 6.09272\cdot 10^{19}X^2 - 4.20353\cdot 10^{19}X + 1.76382\cdot 10^{19}$$
$$m = 6.09272\cdot 10^{19}X^2 - 4.20353\cdot 10^{19}X - 8.40861\cdot 10^{18}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{-0.161999, 0.851924\}$$

Intersection intervals:



$[0, 0.851924]$

Longest intersection interval: 0.851924

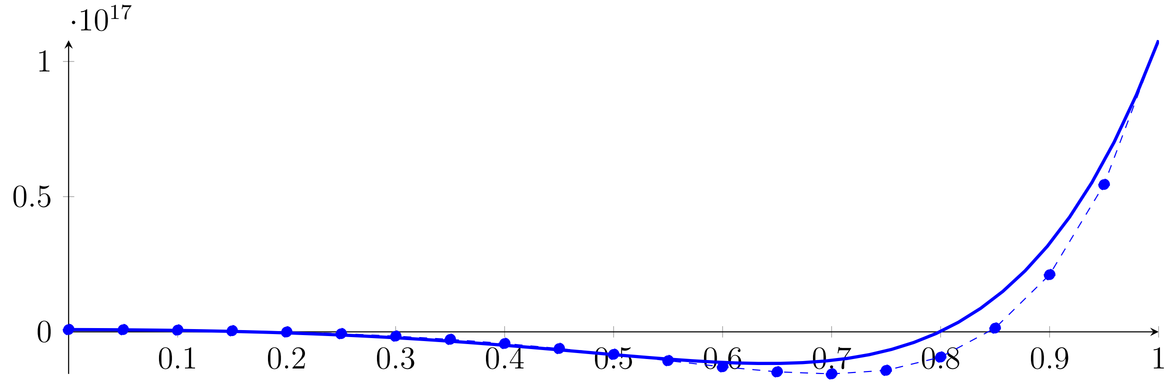
$\Longrightarrow$ Bisection: first half $[18.75, 20.3125]$ und second half $[20.3125, 21.875]$

2.92 Recursion Branch 1 2 2 1 1 on the First Half [18.75, 20.3125]

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 6.05184\cdot10^{6}X^{20} - 7.45545\cdot10^{7}X^{19} + 1.58964\cdot10^{8}X^{18} + 2.46862\cdot10^{8}X^{17} + 2.74811\cdot10^{10}X^{16} + 3.58417\cdot10^{11}X^{15} + 4.07176\cdot10^{12}X^{14} + 3.52881\cdot10^{13}X^{13} + 2.37021\cdot10^{14}X^{12} + 1.23655\cdot10^{15}X^{11} + 4.98645\cdot10^{15}X^{10} + 1.53352\cdot10^{16}X^{9} + 3.50338\cdot10^{16}X^{8} + 5.64827\cdot10^{16}X^{7} + 5.69288\cdot10^{16}X^{6} + 2.07489\cdot10^{16}X^{5} - 2.63828\cdot10^{16}X^{4} - 3.89847\cdot10^{16}X^{3} - 1.77409\cdot10^{16}X^{2} - 9.28966\cdot10^{14}X + 8.43944\cdot10^{14}$$

$$= 8.43944\cdot10^{14}B_{0,20}(X) + 7.97496\cdot10^{14}B_{1,20}(X) + 6.57674\cdot10^{14}B_{2,20}(X) + 3.90283\cdot10^{14}B_{3,20}(X) - 4.43219\cdot10^{13}B_{4,20}(X) - 6.89889\cdot10^{14}B_{5,20}(X) - 1.59147\cdot10^{15}B_{6,20}(X) - 2.7904\cdot10^{15}B_{7,20}(X) - 4.31613\cdot10^{15}B_{8,20}(X) - 6.17337\cdot10^{15}B_{9,20}(X) - 8.32293\cdot10^{15}B_{10,20}(X) - 1.06535\cdot10^{16}B_{11,20}(X) - 1.29411\cdot10^{16}B_{12,20}(X) - 1.47922\cdot10^{16}B_{13,20}(X) - 1.55635\cdot10^{16}B_{14,20}(X) - 1.42523\cdot10^{16}B_{15,20}(X) - 9.34631\cdot10^{15}B_{16,20}(X) + 1.37971\cdot10^{15}B_{17,20}(X) + 2.11374\cdot10^{16}B_{18,20}(X) + 5.44898\cdot10^{16}B_{19,20}(X) + 1.07836\cdot10^{17}B_{20,20}(X)$$

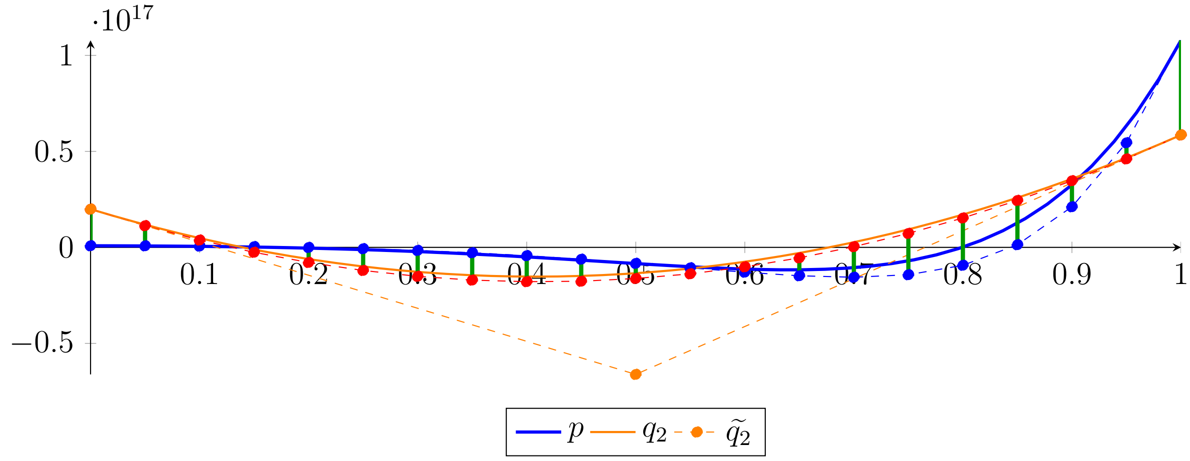


Degree reduction and raising:

$$q_2 = 2.10768\cdot10^{17}X^{2} - 1.72153\cdot10^{17}X + 1.99185\cdot10^{16}$$
$$= 1.99185\cdot10^{16}B_{0,2} - 6.61581\cdot10^{16}B_{1,2} + 5.85331\cdot10^{16}B_{2,2}$$

$$\tilde{q}_2 = 2.78282\cdot10^{19}X^{20} - 2.78275\cdot10^{20}X^{19} + 1.28787\cdot10^{21}X^{18} - 3.66008\cdot10^{21}X^{17} + 7.14666\cdot10^{21}X^{16} - 1.01636\cdot10^{22}X^{15} + 1.08891\cdot10^{22}X^{14} - 8.96747\cdot10^{21}X^{13} + 5.74098\cdot10^{21}X^{12} - 2.87106\cdot10^{21}X^{11} + 1.12103\cdot10^{21}X^{10} - 3.39868\cdot10^{20}X^{9} + 7.91492\cdot10^{19}X^{8} - 1.39241\cdot10^{19}X^{7} + 1.80705\cdot10^{18}X^{6} - 1.67476\cdot10^{17}X^{5} + 1.05802\cdot10^{16}X^{4} - 4.20423\cdot10^{14}X^{3} + 2.10777\cdot10^{17}X^{2} - 1.72153\cdot10^{17}X + 1.99185\cdot10^{16}$$

$$= 1.99185\cdot10^{16}B_{0,20} + 1.13108\cdot10^{16}B_{1,20} + 3.81249\cdot10^{15}B_{2,20} - 2.57682\cdot10^{15}B_{3,20} - 7.85535\cdot10^{15}B_{4,20} - 1.20299\cdot10^{16}B_{5,20} - 1.508\cdot10^{16}B_{6,20} - 1.70553\cdot10^{16}B_{7,20} - 1.78569\cdot10^{16}B_{8,20} - 1.76494\cdot10^{16}B_{9,20} - 1.6202\cdot10^{16}B_{10,20} - 1.37878\cdot10^{16}B_{11,20} - 1.01338\cdot10^{16}B_{12,20} - 5.47091\cdot10^{15}B_{13,20} + 3.65789\cdot10^{14}B_{14,20} + 7.27738\cdot10^{15}B_{15,20} + 1.53135\cdot10^{16}B_{16,20} + 2.44535\cdot10^{16}B_{17,20} + 3.47042\cdot10^{16}B_{18,20} + 4.6064\cdot10^{16}B_{19,20} + 5.85331\cdot10^{16}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 4.93026{\cdot}10^{16}$.

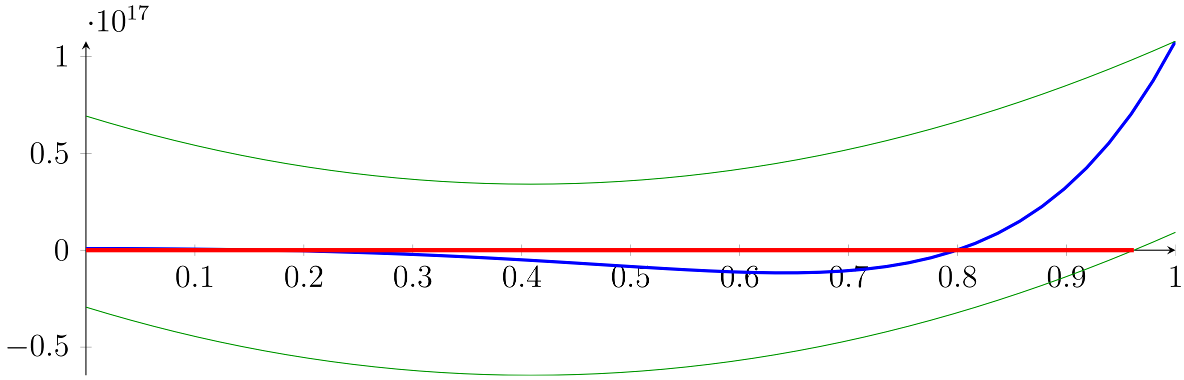
Bounding polynomials M and m:

$$M = 2.10768{\cdot}10^{17}X^2 - 1.72153{\cdot}10^{17}X + 6.92211{\cdot}10^{16}$$
$$m = 2.10768{\cdot}10^{17}X^2 - 1.72153{\cdot}10^{17}X - 2.93842{\cdot}10^{16}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{-0.14496, 0.96175\}$$

Intersection intervals:



$[0, 0.96175]$

Longest intersection interval: 0.96175

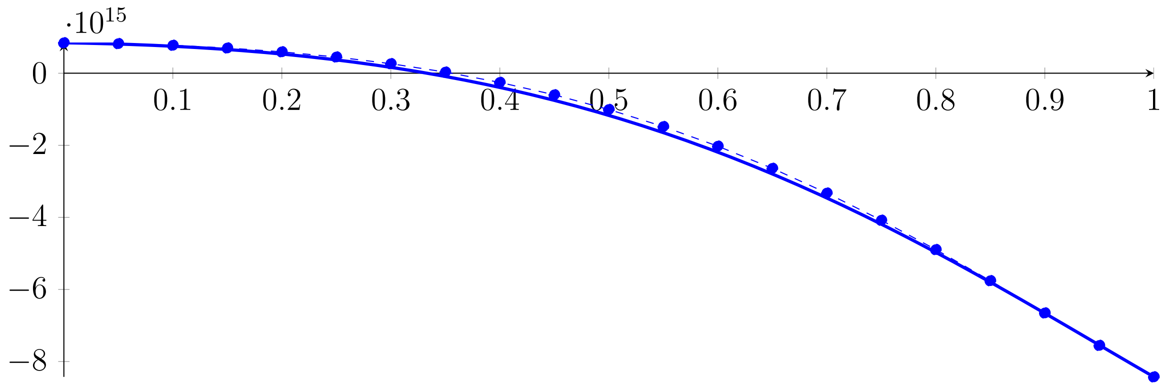
$\Longrightarrow$ Bisection: first half $[18.75, 19.5312]$ und second half $[19.5312, 20.3125]$

Bisection point is very near to a root?!?

2.93 Recursion Branch 1 2 2 1 1 1 on the First Half $[18.75, 19.5312]$

Normalized monomial und Bézier representations and the Bézier polygon:

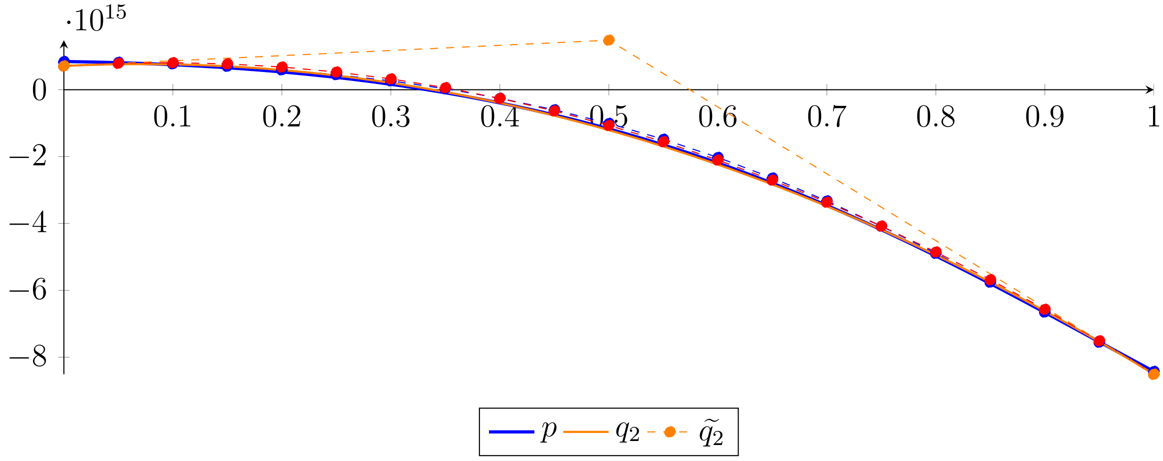
$$\begin{aligned} p = {} & 671857X^{20} - 1.12793{\cdot}10^{7}X^{19} + 1.00397{\cdot}10^{7}X^{18} - 1.19008{\cdot}10^{8}X^{17} + 2.98863{\cdot}10^{8}X^{16} - 1.11857 \\ & {\cdot}10^{8}X^{15} + 2.53287{\cdot}10^{8}X^{14} + 4.24137{\cdot}10^{9}X^{13} + 5.7751{\cdot}10^{10}X^{12} + 6.037{\cdot}10^{11}X^{11} + 4.86952 \\ & {\cdot}10^{12}X^{10} + 2.99515{\cdot}10^{13}X^{9} + 1.36851{\cdot}10^{14}X^{8} + 4.41271{\cdot}10^{14}X^{7} + 8.89513{\cdot}10^{14}X^{6} + 6.48403 \\ & {\cdot}10^{14}X^{5} - 1.64892{\cdot}10^{15}X^{4} - 4.87309{\cdot}10^{15}X^{3} - 4.43522{\cdot}10^{15}X^{2} - 4.64483{\cdot}10^{14}X + 8.43944{\cdot}10^{14} \\ = {} & 8.43944{\cdot}10^{14}B_{0,20}(X) + 8.2072{\cdot}10^{14}B_{1,20}(X) + 7.74152{\cdot}10^{14}B_{2,20}(X) + 6.99967 \\ & {\cdot}10^{14}B_{3,20}(X) + 5.93549{\cdot}10^{14}B_{4,20}(X) + 4.49984{\cdot}10^{14}B_{5,20}(X) + 2.64126{\cdot}10^{14}B_{6,20}(X) \\ & + 3.06864{\cdot}10^{13}B_{7,20}(X) - 2.55633{\cdot}10^{14}B_{8,20}(X) - 5.99971{\cdot}10^{14}B_{9,20}(X) - 1.00708 \\ & {\cdot}10^{15}B_{10,20}(X) - 1.48104{\cdot}10^{15}B_{11,20}(X) - 2.02487{\cdot}10^{15}B_{12,20}(X) - 2.64013{\cdot}10^{15}B_{13,20}(X) \\ & - 3.32625{\cdot}10^{15}B_{14,20}(X) - 4.07993{\cdot}10^{15}B_{15,20}(X) - 4.89422{\cdot}10^{15}B_{16,20}(X) - 5.75748 \\ & {\cdot}10^{15}B_{17,20}(X) - 6.65215{\cdot}10^{15}B_{18,20}(X) - 7.55318{\cdot}10^{15}B_{19,20}(X) - 8.42625{\cdot}10^{15}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = -1.07632{\cdot}10^{16}X^2 + 1.54111{\cdot}10^{15}X + 7.11403{\cdot}10^{14}$$
$$= 7.11403{\cdot}10^{14}B_{0,2} + 1.48196{\cdot}10^{15}B_{1,2} - 8.51066{\cdot}10^{15}B_{2,2}$$

$$\widetilde{q}_2 = -1.35355{\cdot}10^{18}X^{20} + 1.3523{\cdot}10^{19}X^{19} - 6.25105{\cdot}10^{19}X^{18} + 1.77399{\cdot}10^{20}X^{17} - 3.45845{\cdot}10^{20}X^{16} + 4.91078{\cdot}10^{20}X^{15} - 5.2543{\cdot}10^{20}X^{14} + 4.32326{\cdot}10^{20}X^{13} - 2.76728{\cdot}10^{20}X^{12} + 1.38496{\cdot}10^{20}X^{11} - 5.41753{\cdot}10^{19}X^{10} + 1.64719{\cdot}10^{19}X^{9} - 3.84999{\cdot}10^{18}X^{8} + 6.79661{\cdot}10^{17}X^{7} - 8.83026{\cdot}10^{16}X^{6} + 8.13543{\cdot}10^{15}X^{5} - 5.02915{\cdot}10^{14}X^{4} + 1.91048{\cdot}10^{13}X^{3} - 1.07636{\cdot}10^{16}X^{2} + 1.54112{\cdot}10^{15}X + 7.11403{\cdot}10^{14}$$
$$= 7.11403{\cdot}10^{14}B_{0,20} + 7.88459{\cdot}10^{14}B_{1,20} + 8.08864{\cdot}10^{14}B_{2,20} + 7.72636{\cdot}10^{14}B_{3,20} + 6.79687{\cdot}10^{14}B_{4,20} + 5.30352{\cdot}10^{14}B_{5,20} + 3.23631{\cdot}10^{14}B_{6,20} + 6.19296{\cdot}10^{13}B_{7,20} - 2.59514{\cdot}10^{14}B_{8,20} - 6.32792{\cdot}10^{14}B_{9,20} - 1.06902{\cdot}10^{15}B_{10,20} - 1.55493{\cdot}10^{15}B_{11,20} - 2.10394{\cdot}10^{15}B_{12,20} - 2.70468{\cdot}10^{15}B_{13,20} - 3.36518{\cdot}10^{15}B_{14,20} - 4.08068{\cdot}10^{15}B_{15,20} - 4.85355{\cdot}10^{15}B_{16,20} - 5.68281{\cdot}10^{15}B_{17,20} - 6.56878{\cdot}10^{15}B_{18,20} - 7.51139{\cdot}10^{15}B_{19,20} - 8.51066{\cdot}10^{15}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.32541{\cdot}10^{14}$.

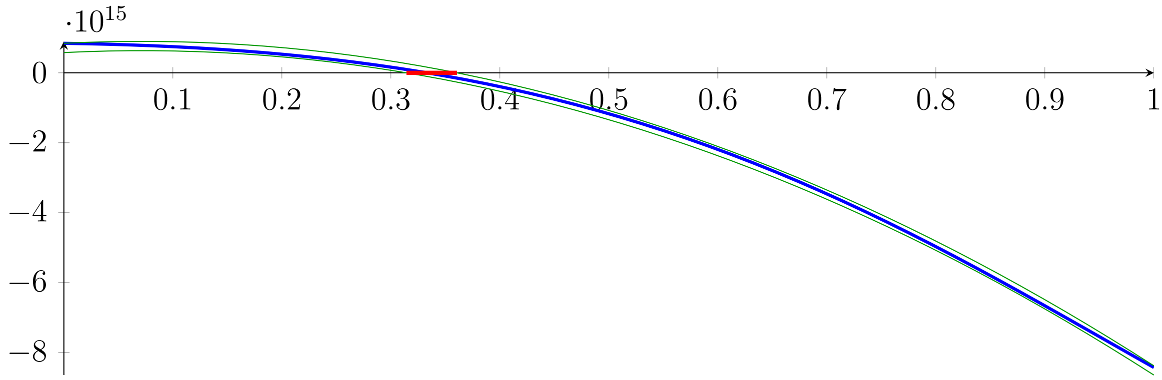
Bounding polynomials M and m:

$$M = -1.07632{\cdot}10^{16}X^2 + 1.54111{\cdot}10^{15}X + 8.43944{\cdot}10^{14}$$
$$m = -1.07632{\cdot}10^{16}X^2 + 1.54111{\cdot}10^{15}X + 5.78862{\cdot}10^{14}$$

Root of M and m:

$$N(M) = \{-0.217434, 0.360617\} \qquad N(m) = \{-0.171116, 0.3143\}$$

Intersection intervals:



$$[0.3143, 0.360617]$$

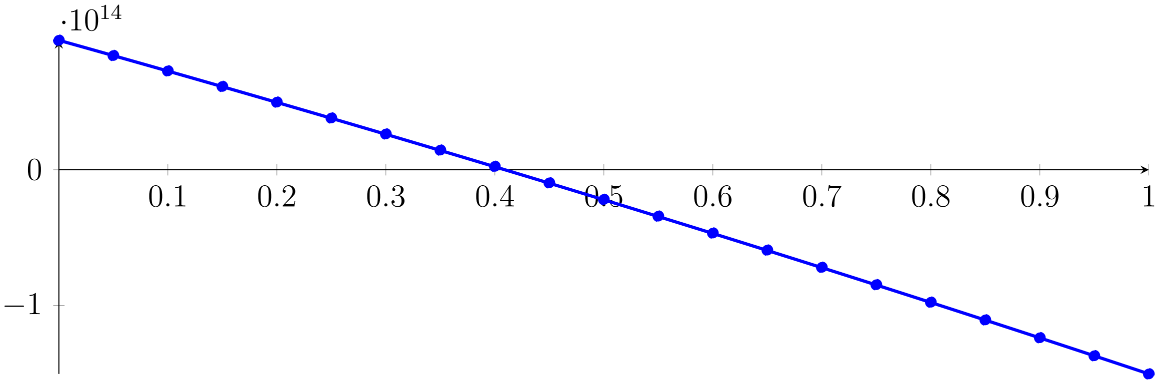
Longest intersection interval: 0.0463177

$\Longrightarrow$ Selective recursion: interval 1: [18.9955, 19.0317],

2.94 Recursion Branch 1 2 2 1 1 1 1 in Interval 1: $[18.9955, 19.0317]$

Normalized monomial und Bézier representations and the Bézier polygon:

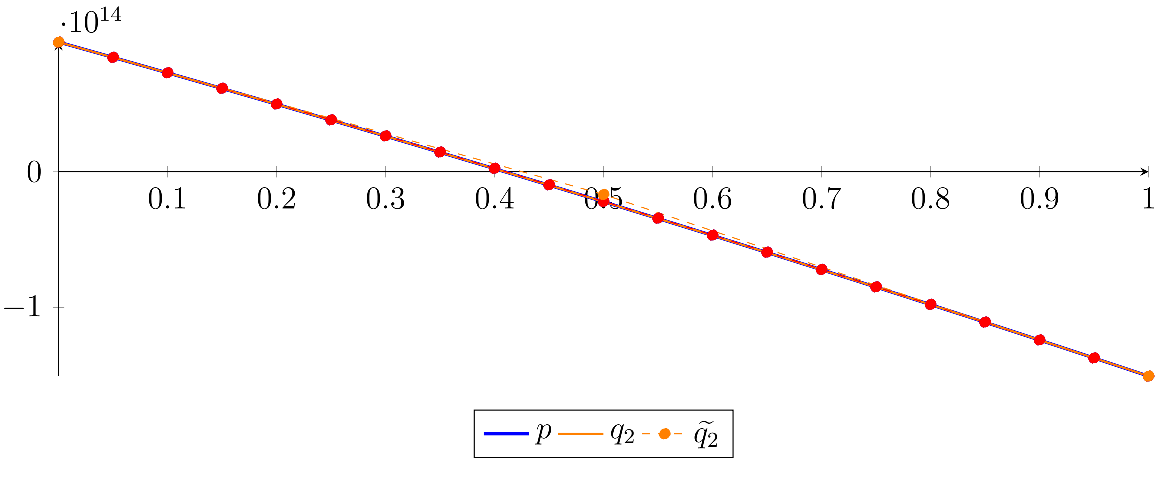
$$p = 10030.4X^{20} - 278477X^{19} - 225079X^{18} - 2.64608\cdot 10^6 X^{17} + 1.33419\cdot 10^6 X^{16} + 1.85854 \cdot 10^6 X^{15} - 4.52039\cdot 10^6 X^{14} - 4.1667\cdot 10^6 X^{13} - 1.29749\cdot 10^7 X^{12} - 5.64765\cdot 10^6 X^{11} - 5.61196 \cdot 10^6 X^{10} - 1.34368\cdot 10^6 X^9 - 15746.2X^8 + 397290X^7 + 2.29314\cdot 10^7 X^6 + 7.50382\cdot 10^8 X^5 + 5.86218\cdot 10^9 X^4 - 5.54108\cdot 10^{11} X^3 - 2.06882\cdot 10^{13} X^2 - 2.24643\cdot 10^{14} X + 9.543\cdot 10^{13}$$
$$= 9.543\cdot 10^{13} B_{0,20}(X) + 8.41978\cdot 10^{13} B_{1,20}(X) + 7.28567\cdot 10^{13} B_{2,20}(X) + 6.14063 \cdot 10^{13} B_{3,20}(X) + 4.9846\cdot 10^{13} B_{4,20}(X) + 3.81754\cdot 10^{13} B_{5,20}(X) + 2.6394\cdot 10^{13} B_{6,20}(X) + 1.45012\cdot 10^{13} B_{7,20}(X) + 2.49668\cdot 10^{12} B_{8,20}(X) - 9.62011\cdot 10^{12} B_{9,20}(X) - 2.18496 \cdot 10^{13} B_{10,20}(X) - 3.41924\cdot 10^{13} B_{11,20}(X) - 4.66488\cdot 10^{13} B_{12,20}(X) - 5.92194\cdot 10^{13} B_{13,20}(X) - 7.19046\cdot 10^{13} B_{14,20}(X) - 8.47049\cdot 10^{13} B_{15,20}(X) - 9.76208\cdot 10^{13} B_{16,20}(X) - 1.10653 \cdot 10^{14} B_{17,20}(X) - 1.23801\cdot 10^{14} B_{18,20}(X) - 1.37066\cdot 10^{14} B_{19,20}(X) - 1.50449\cdot 10^{14} B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = -2.1508\cdot 10^{13} X^2 - 2.24317\cdot 10^{14} X + 9.54028\cdot 10^{13}$$
$$= 9.54028\cdot 10^{13} B_{0,2} - 1.67557\cdot 10^{13} B_{1,2} - 1.50422\cdot 10^{14} B_{2,2}$$

$$\widetilde{q}_2 = -7.83023\cdot 10^{15} X^{20} + 7.8241\cdot 10^{16} X^{19} - 3.61485\cdot 10^{17} X^{18} + 1.02486\cdot 10^{18} X^{17} - 1.99585\cdot 10^{18} X^{16} + 2.83212\cdot 10^{18} X^{15} - 3.03178\cdot 10^{18} X^{14} + 2.50088\cdot 10^{18} X^{13} - 1.60945\cdot 10^{18} X^{12} + 8.12626\cdot 10^{17} X^{11} - 3.21794\cdot 10^{17} X^{10} + 9.93177\cdot 10^{16} X^9 - 2.35974\cdot 10^{16} X^8 + 4.23413\cdot 10^{15} X^7 - 5.5792\cdot 10^{14} X^6 + 5.17758 \cdot 10^{13} X^5 - 3.16659\cdot 10^{12} X^4 + 1.158\cdot 10^{11} X^3 - 2.15102\cdot 10^{13} X^2 - 2.24317\cdot 10^{14} X + 9.54028\cdot 10^{13}$$
$$= 9.54028\cdot 10^{13} B_{0,20} + 8.4187\cdot 10^{13} B_{1,20} + 7.28579\cdot 10^{13} B_{2,20} + 6.14157\cdot 10^{13} B_{3,20} + 4.98599 \cdot 10^{13} B_{4,20} + 3.81925\cdot 10^{13} B_{5,20} + 2.64074\cdot 10^{13} B_{6,20} + 1.45191\cdot 10^{13} B_{7,20} + 2.49962\cdot 10^{12} B_{8,20} - 9.60592\cdot 10^{12} B_{9,20} - 2.18603\cdot 10^{13} B_{10,20} - 3.41869\cdot 10^{13} B_{11,20} - 4.66652\cdot 10^{13} B_{12,20} - 5.92288\cdot 10^{13} B_{13,20} - 7.19223\cdot 10^{13} B_{14,20} - 8.47202\cdot 10^{13} B_{15,20} - 9.76351\cdot 10^{13} B_{16,20} - 1.10662\cdot 10^{14} B_{17,20} - 1.23802\cdot 10^{14} B_{18,20} - 1.37056\cdot 10^{14} B_{19,20} - 1.50422\cdot 10^{14} B_{20,20}$$



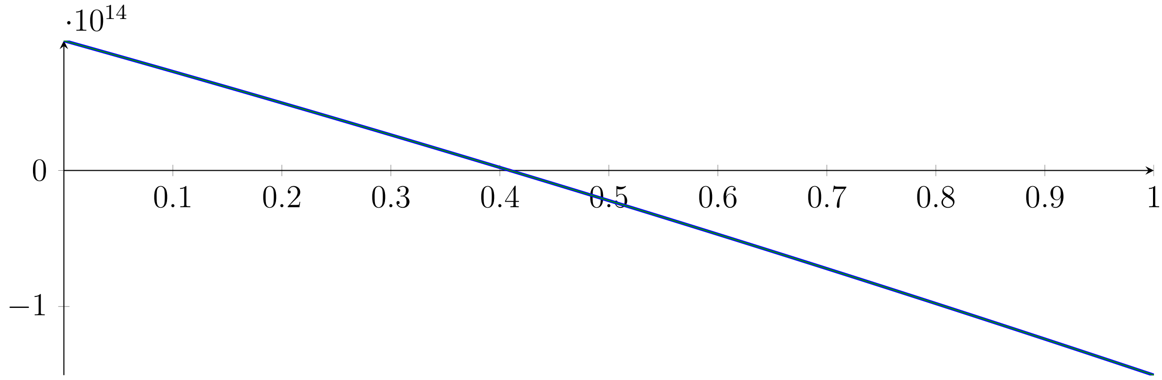
The maximum difference of the Bézier coefficients is $\delta = 2.71198{\cdot}10^{10}$.
Bounding polynomials M and m:

$$M = -2.1508{\cdot}10^{13}X^2 - 2.24317{\cdot}10^{14}X + 9.543{\cdot}10^{13}$$
$$m = -2.1508{\cdot}10^{13}X^2 - 2.24317{\cdot}10^{14}X + 9.53757{\cdot}10^{13}$$

Root of M and m:

$$N(M) = \{-10.8388, 0.409357\} \qquad N(m) = \{-10.8386, 0.409133\}$$

Intersection intervals:



$$[0.409133, 0.409357]$$

Longest intersection interval: 0.000224204
$\Longrightarrow$ Selective recursion: interval 1: $[19.0104, 19.0104]$,

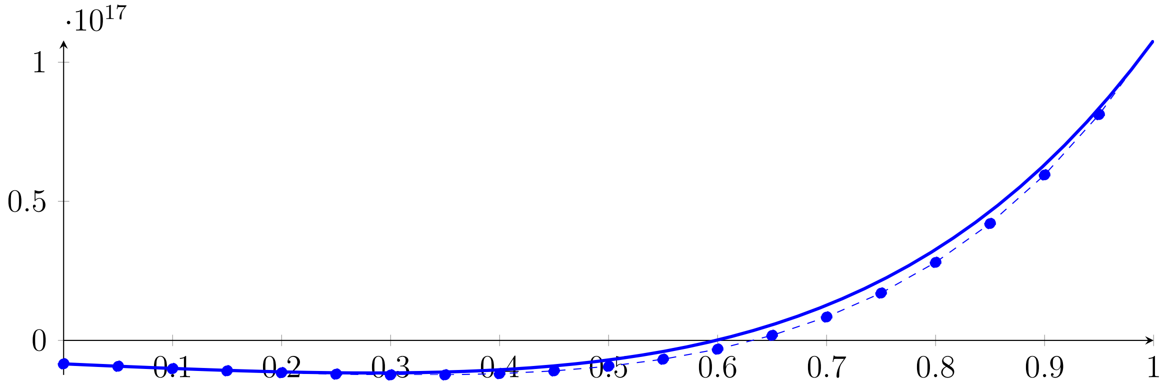
2.95 Recursion Branch 1 2 2 1 1 1 1 1 in Interval 1: $[19.0104, 19.0104]$

Found root in interval $[19.0104, 19.0104]$ at recursion depth 8!

2.96 Recursion Branch 1 2 2 1 1 2 on the Second Half $[19.5312, 20.3125]$

Normalized monomial und Bézier representations and the Bézier polygon:

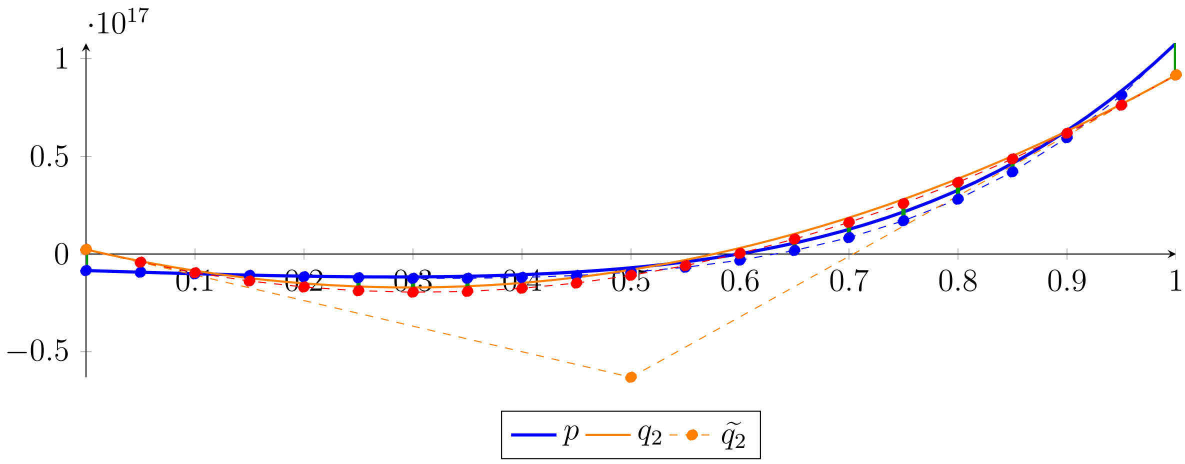
$$\begin{aligned}p &= 9.60014{\cdot}10^{6}X^{20} - 1.47184{\cdot}10^{7}X^{19} + 2.89235{\cdot}10^{8}X^{18} - 1.01982{\cdot}10^{9}X^{17} + 6.86881{\cdot}10^{9}X^{16} - 5.88208\\&\quad{\cdot}10^{9}X^{15} + 2.56494{\cdot}10^{9}X^{14} + 1.03059{\cdot}10^{10}X^{13} + 1.4806{\cdot}10^{11}X^{12} + 1.7421{\cdot}10^{12}X^{11} + 1.68488\\&\quad{\cdot}10^{13}X^{10} + 1.28223{\cdot}10^{14}X^{9} + 7.60179{\cdot}10^{14}X^{8} + 3.45208{\cdot}10^{15}X^{7} + 1.16894{\cdot}10^{16}X^{6} + 2.82477\\&\quad{\cdot}10^{16}X^{5} + 4.49876{\cdot}10^{16}X^{4} + 3.91276{\cdot}10^{16}X^{3} + 5.31198{\cdot}10^{15}X^{2} - 1.74614{\cdot}10^{16}X - 8.42625{\cdot}10^{15}\\&= -8.42625{\cdot}10^{15}B_{0,20}(X) - 9.29932{\cdot}10^{15}B_{1,20}(X) - 1.01444{\cdot}10^{16}B_{2,20}(X) - 1.09273\\&\quad{\cdot}10^{16}B_{3,20}(X) - 1.16042{\cdot}10^{16}B_{4,20}(X) - 1.21206{\cdot}10^{16}B_{5,20}(X) - 1.24084{\cdot}10^{16}B_{6,20}(X)\\&\quad - 1.2384{\cdot}10^{16}B_{7,20}(X) - 1.19451{\cdot}10^{16}B_{8,20}(X) - 1.09678{\cdot}10^{16}B_{9,20}(X) - 9.30216\\&\quad{\cdot}10^{15}B_{10,20}(X) - 6.76818{\cdot}10^{15}B_{11,20}(X) - 3.15062{\cdot}10^{15}B_{12,20}(X) + 1.80691{\cdot}10^{15}B_{13,20}(X)\\&\quad + 8.40872{\cdot}10^{15}B_{14,20}(X) + 1.70147{\cdot}10^{16}B_{15,20}(X) + 2.80495{\cdot}10^{16}B_{16,20}(X) + 4.20121\\&\quad{\cdot}10^{16}B_{17,20}(X) + 5.94882{\cdot}10^{16}B_{18,20}(X) + 8.11628{\cdot}10^{16}B_{19,20}(X) + 1.07836{\cdot}10^{17}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$q_2 = 2.20011\cdot10^{17}X^2 - 1.30774\cdot10^{17}X + 2.35127\cdot10^{15}$$
$$= 2.35127\cdot10^{15}B_{0,2} - 6.30357\cdot10^{16}B_{1,2} + 9.15888\cdot10^{16}B_{2,2}$$

$$\widetilde{q}_2 = 2.98271\cdot10^{19}X^{20} - 2.98239\cdot10^{20}X^{19} + 1.38008\cdot10^{21}X^{18} - 3.92146\cdot10^{21}X^{17} + 7.65555\cdot10^{21}X^{16} - 1.08854\cdot10^{22}X^{15} + 1.16613\cdot10^{22}X^{14} - 9.60367\cdot10^{21}X^{13} + 6.14966\cdot10^{21}X^{12} - 3.07687\cdot10^{21}X^{11} + 1.20226\cdot10^{21}X^{10} - 3.64846\cdot10^{20}X^9 + 8.50606\cdot10^{19}X^8 - 1.49806\cdot10^{19}X^7 + 1.9457\cdot10^{18}X^6 - 1.80287\cdot10^{17}X^5 + 1.136\cdot10^{16}X^4 - 4.48764\cdot10^{14}X^3 + 2.20021\cdot10^{17}X^2 - 1.30774\cdot10^{17}X + 2.35127\cdot10^{15}$$
$$= 2.35127\cdot10^{15}B_{0,20} - 4.18743\cdot10^{15}B_{1,20} - 9.56813\cdot10^{15}B_{2,20} - 1.37912\cdot10^{16}B_{3,20} - 1.68547\cdot10^{16}B_{4,20} - 1.8766\cdot10^{16}B_{5,20} - 1.95031\cdot10^{16}B_{6,20} - 1.91193\cdot10^{16}B_{7,20} - 1.75084\cdot10^{16}B_{8,20} - 1.48469\cdot10^{16}B_{9,20} - 1.08876\cdot10^{16}B_{10,20} - 5.92336\cdot10^{15}B_{11,20} + 3.39175\cdot10^{14}B_{12,20} + 7.65204\cdot10^{15}B_{13,20} + 1.61917\cdot10^{16}B_{14,20} + 2.58527\cdot10^{16}B_{15,20} + 3.66878\cdot10^{16}B_{16,20} + 4.8675\cdot10^{16}B_{17,20} + 6.18219\cdot10^{16}B_{18,20} + 7.61263\cdot10^{16}B_{19,20} + 9.15888\cdot10^{16}B_{20,20}$$



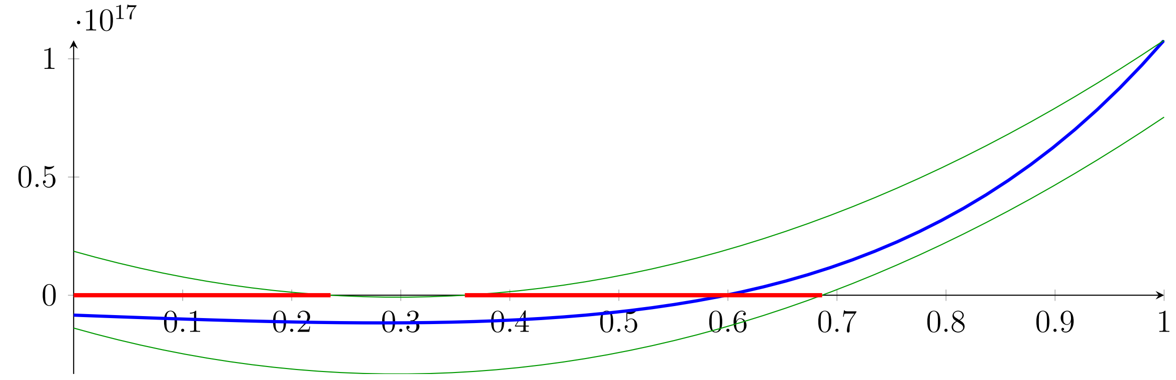
The maximum difference of the Bézier coefficients is $\delta = 1.6247\cdot10^{16}$.
Bounding polynomials M and m:

$$M = 2.20011\cdot10^{17}X^2 - 1.30774\cdot10^{17}X + 1.85983\cdot10^{16}$$
$$m = 2.20011\cdot10^{17}X^2 - 1.30774\cdot10^{17}X - 1.38957\cdot10^{16}$$

Root of M and m:

$$N(M) = \{0.235606, 0.35879\} \qquad N(m) = \{-0.0920136, 0.68641\}$$

Intersection intervals:



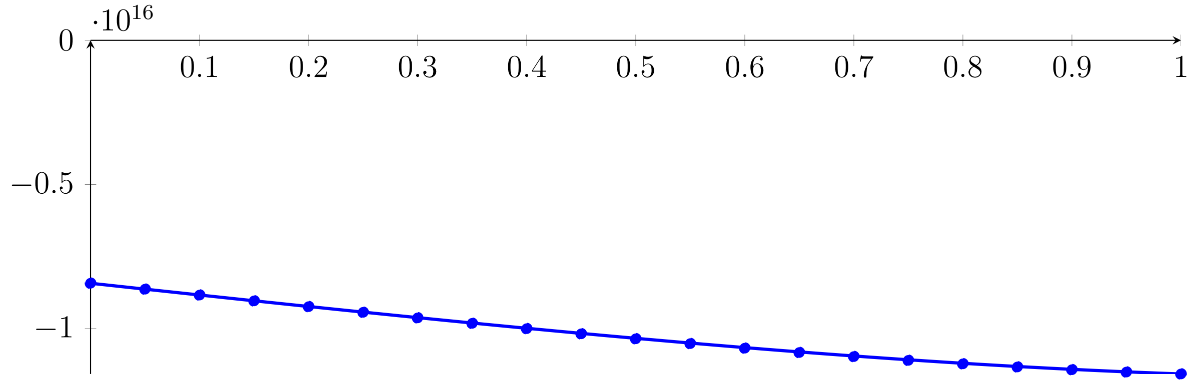
$[0, 0.235606]$, $[0.35879, 0.68641]$

Longest intersection interval: 0.32762
$\Longrightarrow$ Selective recursion: interval 1: $[19.5312, 19.7153]$, interval 2: $[19.8116, 20.0675]$,

2.97 Recursion Branch 1 2 2 1 1 2 1 in Interval 1: $[19.5312, 19.7153]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 1.0745{\cdot}10^{7}X^{20} - 5.94121{\cdot}10^{7}X^{19} + 3.13147{\cdot}10^{8}X^{18} - 1.25197{\cdot}10^{9}X^{17} + 6.55617{\cdot}10^{9}X^{16} - 4.90644{\cdot}10^{9}X^{15} + 2.09176{\cdot}10^{9}X^{14} + 7.40549{\cdot}10^{8}X^{13} + 4.68193{\cdot}10^{9}X^{12} + 5.56451{\cdot}10^{8}X^{11} + 1.39509{\cdot}10^{9}X^{10} + 4.70456{\cdot}10^{8}X^{9} + 7.23257{\cdot}10^{9}X^{8} + 1.3912{\cdot}10^{11}X^{7} + 1.99946{\cdot}10^{12}X^{6} + 2.05076{\cdot}10^{13}X^{5} + 1.38624{\cdot}10^{14}X^{4} + 5.11732{\cdot}10^{14}X^{3} + 2.94869{\cdot}10^{14}X^{2} - 4.11402{\cdot}10^{15}X - 8.42625{\cdot}10^{15}$$
$$= -8.42625{\cdot}10^{15}B_{0,20}(X) - 8.63195{\cdot}10^{15}B_{1,20}(X) - 8.8361{\cdot}10^{15}B_{2,20}(X) - 9.03825{\cdot}10^{15}B_{3,20}(X) - 9.23792{\cdot}10^{15}B_{4,20}(X) - 9.4346{\cdot}10^{15}B_{5,20}(X) - 9.62776{\cdot}10^{15}B_{6,20}(X) - 9.81683{\cdot}10^{15}B_{7,20}(X) - 1.00012{\cdot}10^{16}B_{8,20}(X) - 1.01802{\cdot}10^{16}B_{9,20}(X) - 1.03532{\cdot}10^{16}B_{10,20}(X) - 1.05195{\cdot}10^{16}B_{11,20}(X) - 1.06782{\cdot}10^{16}B_{12,20}(X) - 1.08287{\cdot}10^{16}B_{13,20}(X) - 1.097{\cdot}10^{16}B_{14,20}(X) - 1.11013{\cdot}10^{16}B_{15,20}(X) - 1.12216{\cdot}10^{16}B_{16,20}(X) - 1.13299{\cdot}10^{16}B_{17,20}(X) - 1.14252{\cdot}10^{16}B_{18,20}(X) - 1.15064{\cdot}10^{16}B_{19,20}(X) - 1.15724{\cdot}10^{16}B_{20,20}(X)$$

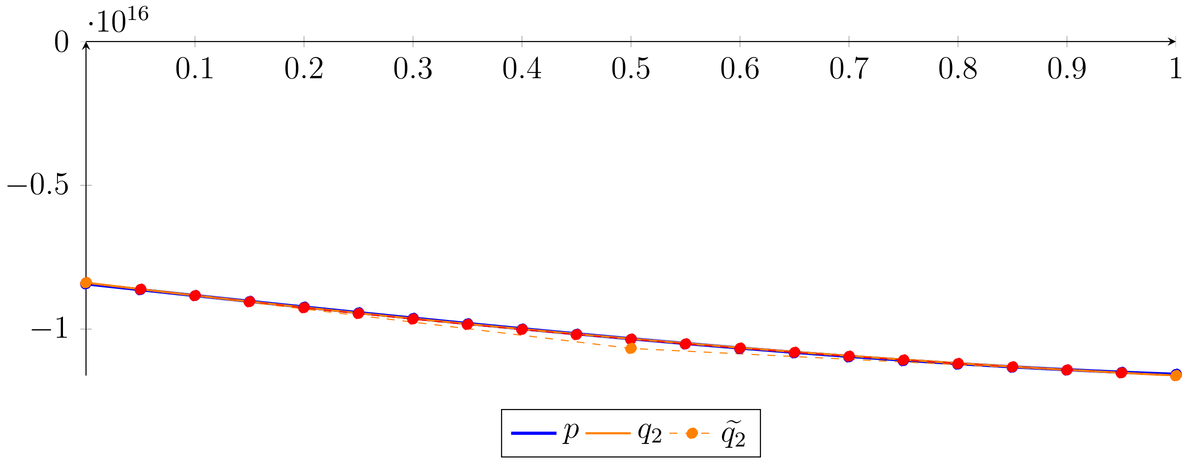


Degree reduction and raising:

$$q_2 = 1.34056{\cdot}10^{15}X^2 - 4.57223{\cdot}10^{15}X - 8.38633{\cdot}10^{15}$$
$$= -8.38633{\cdot}10^{15}B_{0,2} - 1.06724{\cdot}10^{16}B_{1,2} - 1.1618{\cdot}10^{16}B_{2,2}$$

$$\widetilde{q}_2 = 1.91115{\cdot}10^{18}X^{20} - 1.91977{\cdot}10^{19}X^{19} + 8.93238{\cdot}10^{19}X^{18} - 2.55392{\cdot}10^{20}X^{17} + 5.0195{\cdot}10^{20}X^{16} - 7.18727{\cdot}10^{20}X^{15} + 7.75287{\cdot}10^{20}X^{14} - 6.42606{\cdot}10^{20}X^{13} + 4.13763{\cdot}10^{20}X^{12} - 2.07873{\cdot}10^{20}X^{11} + 8.14024{\cdot}10^{19}X^{10} - 2.46987{\cdot}10^{19}X^{9} + 5.74565{\cdot}10^{18}X^{8} - 1.01058{\cdot}10^{18}X^{7} + 1.32322{\cdot}10^{17}X^{6} - 1.26823{\cdot}10^{16}X^{5} + 8.68489{\cdot}10^{14}X^{4} - 3.94683{\cdot}10^{13}X^{3} + 1.34158{\cdot}10^{15}X^{2} - 4.57224{\cdot}10^{15}X - 8.38633{\cdot}10^{15}$$

$$= -8.38633{\cdot}10^{15}B_{0,20} - 8.61494{\cdot}10^{15}B_{1,20} - 8.83649{\cdot}10^{15}B_{2,20} - 9.05102{\cdot}10^{15}B_{3,20} - 9.25837{\cdot}10^{15}B_{4,20} - 9.45905{\cdot}10^{15}B_{5,20} - 9.65158{\cdot}10^{15}B_{6,20} - 9.83959{\cdot}10^{15}B_{7,20} - 1.00158{\cdot}10^{16}B_{8,20} - 1.01921{\cdot}10^{16}B_{9,20} - 1.03522{\cdot}10^{16}B_{10,20} - 1.0515{\cdot}10^{16}B_{11,20} - 1.06622{\cdot}10^{16}B_{12,20} - 1.0809{\cdot}10^{16}B_{13,20} - 1.09443{\cdot}10^{16}B_{14,20} - 1.10749{\cdot}10^{16}B_{15,20} - 1.11974{\cdot}10^{16}B_{16,20} - 1.13132{\cdot}10^{16}B_{17,20} - 1.14218{\cdot}10^{16}B_{18,20} - 1.15234{\cdot}10^{16}B_{19,20} - 1.1618{\cdot}10^{16}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 4.56106{\cdot}10^{13}$.

Bounding polynomials M and m:

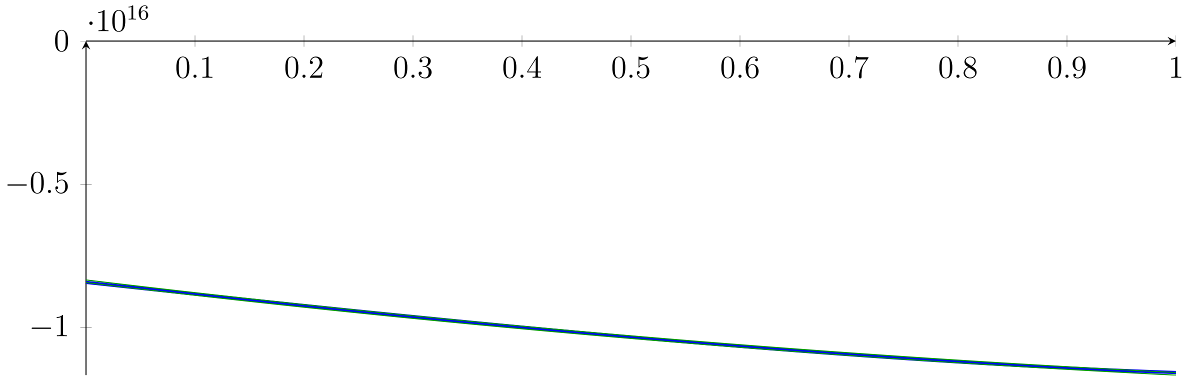
$$M = 1.34056{\cdot}10^{15}X^2 - 4.57223{\cdot}10^{15}X - 8.34072{\cdot}10^{15}$$

$$m = 1.34056{\cdot}10^{15}X^2 - 4.57223{\cdot}10^{15}X - 8.43194{\cdot}10^{15}$$

Root of M and m:

$$N(M) = \{-1.31625, 4.72695\} \qquad N(m) = \{-1.32749, 4.73819\}$$

Intersection intervals:



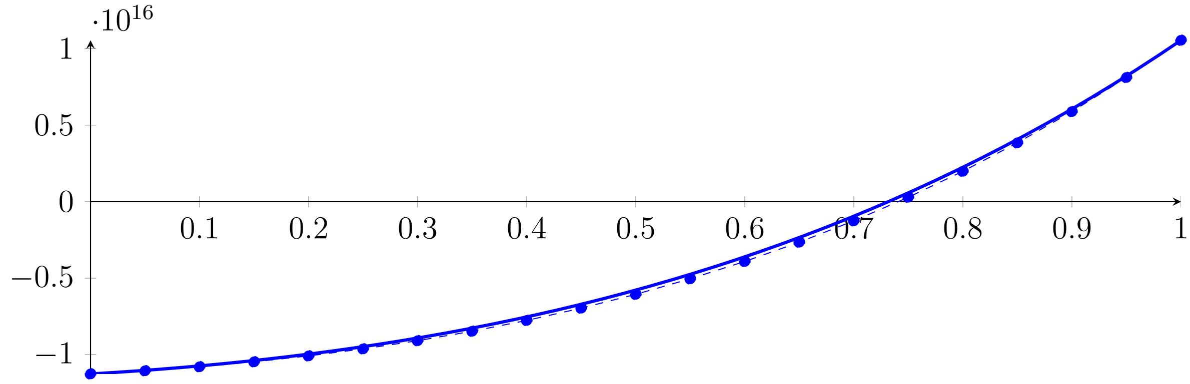
No intersection intervals with the x axis.

2.98 Recursion Branch 1 2 2 1 1 2 2 in Interval 2: $[19.8116, 20.0675]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 6.68466 \cdot 10^{6} X^{20} - 1.73546 \cdot 10^{7} X^{19} + 2.32412 \cdot 10^{8} X^{18} - 7.40311 \cdot 10^{8} X^{17} + 4.69827 \cdot 10^{9} X^{16} - 3.89343 \cdot 10^{9} X^{15} + 1.46559 \cdot 10^{9} X^{14} + 8.67914 \cdot 10^{8} X^{13} + 4.32581 \cdot 10^{9} X^{12} + 7.24915 \cdot 10^{8} X^{11} + 1.76257 \cdot 10^{9} X^{10} + 9.01811 \cdot 10^{9} X^{9} + 1.70735 \cdot 10^{11} X^{8} + 2.5652 \cdot 10^{12} X^{7} + 2.9258 \cdot 10^{13} X^{6} + 2.45356 \cdot 10^{14} X^{5} + 1.43779 \cdot 10^{15} X^{4} + 5.38492 \cdot 10^{15} X^{3} + 1.05841 \cdot 10^{16} X^{2} + 4.12359 \cdot 10^{15} X - 1.1259 \cdot 10^{16}$$

$$= -1.1259 \cdot 10^{16} B_{0,20}(X) - 1.10528 \cdot 10^{16} B_{1,20}(X) - 1.07909 \cdot 10^{16} B_{2,20}(X) - 1.04686 \cdot 10^{16} B_{3,20}(X) - 1.00808 \cdot 10^{16} B_{4,20}(X) - 9.62228 \cdot 10^{15} B_{5,20}(X) - 9.08729 \cdot 10^{15} B_{6,20}(X) - 8.46985 \cdot 10^{15} B_{7,20}(X) - 7.76358 \cdot 10^{15} B_{8,20}(X) - 6.96172 \cdot 10^{15} B_{9,20}(X) - 6.05712 \cdot 10^{15} B_{10,20}(X) - 5.0422 \cdot 10^{15} B_{11,20}(X) - 3.9089 \cdot 10^{15} B_{12,20}(X) - 2.64875 \cdot 10^{15} B_{13,20}(X) - 1.25274 \cdot 10^{15} B_{14,20}(X) + 2.88644 \cdot 10^{14} B_{15,20}(X) + 1.98545 \cdot 10^{15} B_{16,20}(X) + 3.84832 \cdot 10^{15} B_{17,20}(X) + 5.88847 \cdot 10^{15} B_{18,20}(X) + 8.11777 \cdot 10^{15} B_{19,20}(X) + 1.05487 \cdot 10^{16} B_{20,20}(X)$$

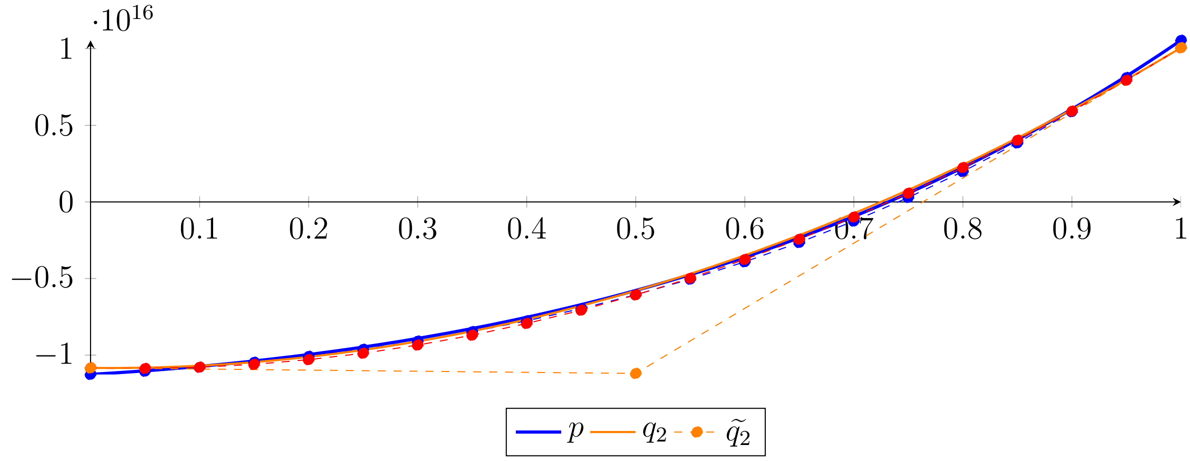


Degree reduction and raising:

$$q_2 = 2.16214 \cdot 10^{16} X^{2} - 7.21434 \cdot 10^{14} X - 1.08364 \cdot 10^{16}$$
$$= -1.08364 \cdot 10^{16} B_{0,2} - 1.11971 \cdot 10^{16} B_{1,2} + 1.00636 \cdot 10^{16} B_{2,2}$$

$$\widetilde{q_2} = 4.30746 \cdot 10^{18} X^{20} - 4.31217 \cdot 10^{19} X^{19} + 1.99811 \cdot 10^{20} X^{18} - 5.68602 \cdot 10^{20} X^{17} + 1.11183 \cdot 10^{21} X^{16} - 1.58367 \cdot 10^{21} X^{15} + 1.69973 \cdot 10^{21} X^{14} - 1.40263 \cdot 10^{21} X^{13} + 9.00071 \cdot 10^{20} X^{12} - 4.51316 \cdot 10^{20} X^{11} + 1.76719 \cdot 10^{20} X^{10} - 5.37273 \cdot 10^{19} X^{9} + 1.25452 \cdot 10^{19} X^{8} - 2.21338 \cdot 10^{18} X^{7} + 2.88627 \cdot 10^{17} X^{6} - 2.70111 \cdot 10^{16} X^{5} + 1.7393 \cdot 10^{15} X^{4} - 7.1352 \cdot 10^{13} X^{3} + 2.1623 \cdot 10^{16} X^{2} - 7.2145 \cdot 10^{14} X - 1.08364 \cdot 10^{16}$$

$$= -1.08364 \cdot 10^{16} B_{0,20} - 1.08724 \cdot 10^{16} B_{1,20} - 1.07947 \cdot 10^{16} B_{2,20} - 1.06032 \cdot 10^{16} B_{3,20} - 1.02977 \cdot 10^{16} B_{4,20} - 9.87925 \cdot 10^{15} B_{5,20} - 9.3446 \cdot 10^{15} B_{6,20} - 8.7016 \cdot 10^{15} B_{7,20} - 7.93467 \cdot 10^{15} B_{8,20} - 7.06953 \cdot 10^{15} B_{9,20} - 6.07034 \cdot 10^{15} B_{10,20} - 4.97948 \cdot 10^{15} B_{11,20} - 3.75471 \cdot 10^{15} B_{12,20} - 2.43152 \cdot 10^{15} B_{13,20} - 9.84652 \cdot 10^{14} B_{14,20} + 5.70719 \cdot 10^{14} B_{15,20} + 2.24225 \cdot 10^{15} B_{16,20} + 4.02674 \cdot 10^{15} B_{17,20} + 5.92525 \cdot 10^{15} B_{18,20} + 7.93752 \cdot 10^{15} B_{19,20} + 1.00636 \cdot 10^{16} B_{20,20}$$



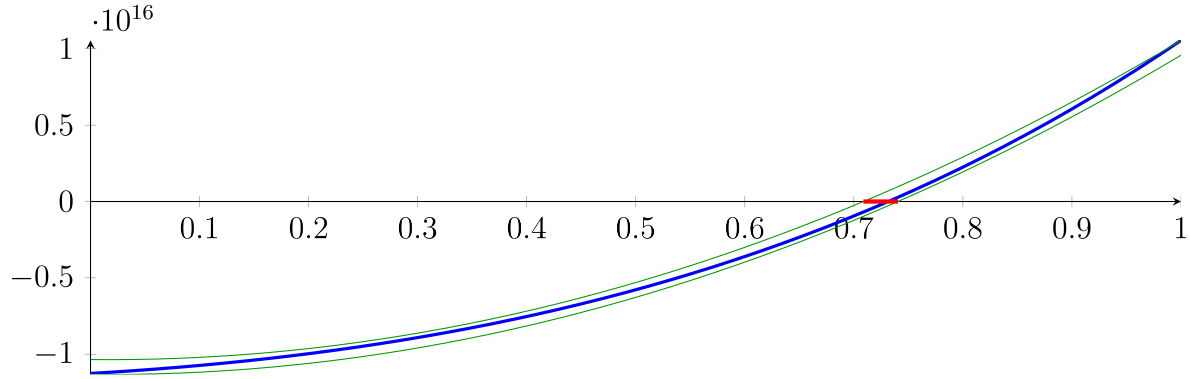
The maximum difference of the Bézier coefficients is $\delta = 4.85154 \cdot 10^{14}$.
Bounding polynomials M and m:

$$M = 2.16214 \cdot 10^{16} X^2 - 7.21434 \cdot 10^{14} X - 1.03512 \cdot 10^{16}$$
$$m = 2.16214 \cdot 10^{16} X^2 - 7.21434 \cdot 10^{14} X - 1.13215 \cdot 10^{16}$$

Root of M and m:

$$N(M) = \{-0.675435, 0.708801\} \qquad N(m) = \{-0.707129, 0.740496\}$$

Intersection intervals:



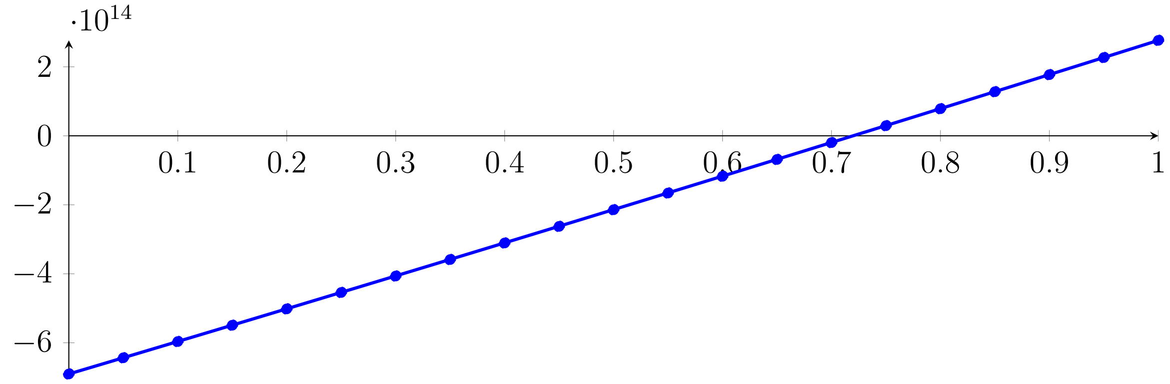
$$[0.708801, 0.740496]$$

Longest intersection interval: 0.0316945
$\Longrightarrow$ Selective recursion: interval 1: $[19.993, 20.0011]$,

2.99 Recursion Branch 1 2 2 1 1 2 2 1 in Interval 1: $[19.993, 20.0011]$

Normalized monomial und Bézier representations and the Bézier polygon:

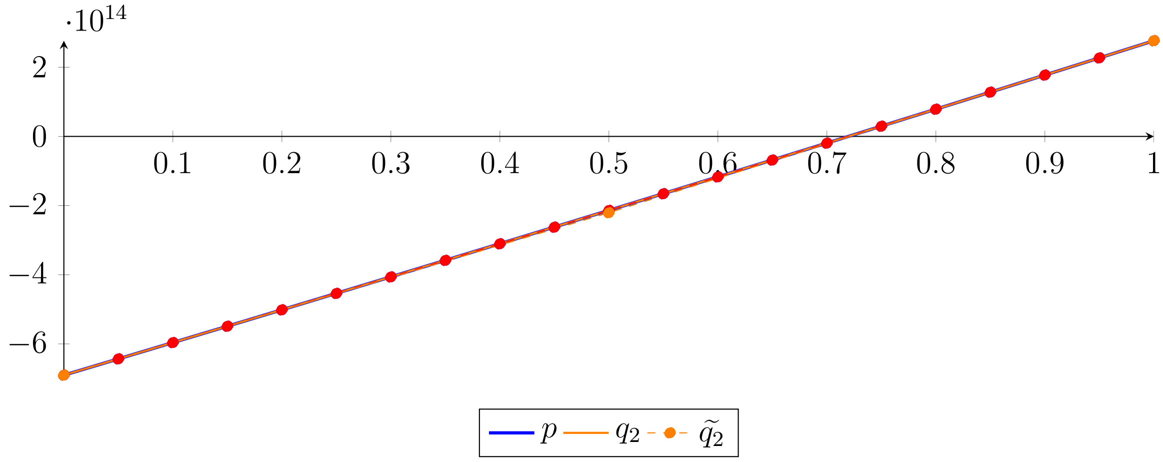
$$\begin{aligned} p = {} & 262064X^{20} - 450047X^{19} + 1.03422 \cdot 10^{7} X^{18} - 2.37899 \cdot 10^{7} X^{17} + 1.87581 \cdot 10^{8} X^{16} - 1.54736 \\ & \cdot 10^{8} X^{15} + 7.65268 \cdot 10^{7} X^{14} + 3.53976 \cdot 10^{7} X^{13} + 2.04497 \cdot 10^{8} X^{12} + 3.7896 \cdot 10^{7} X^{11} + 6.52882 \\ & \cdot 10^{7} X^{10} + 1.03505 \cdot 10^{7} X^{9} + 677089X^{8} - 339150X^{7} + 479655X^{6} + 1.28082 \cdot 10^{7} X^{5} \\ & + 2.58637 \cdot 10^{9} X^{4} + 3.47896 \cdot 10^{11} X^{3} + 2.74877 \cdot 10^{13} X^{2} + 9.39273 \cdot 10^{14} X - 6.90417 \cdot 10^{14} \\ = {} & -6.90417 \cdot 10^{14} B_{0,20}(X) - 6.43453 \cdot 10^{14} B_{1,20}(X) - 5.96345 \cdot 10^{14} B_{2,20}(X) - 5.49091 \\ & \cdot 10^{14} B_{3,20}(X) - 5.01693 \cdot 10^{14} B_{4,20}(X) - 4.54149 \cdot 10^{14} B_{5,20}(X) - 4.06459 \cdot 10^{14} B_{6,20}(X) \\ & - 3.58622 \cdot 10^{14} B_{7,20}(X) - 3.1064 \cdot 10^{14} B_{8,20}(X) - 2.6251 \cdot 10^{14} B_{9,20}(X) - 2.14233 \\ & \cdot 10^{14} B_{10,20}(X) - 1.65809 \cdot 10^{14} B_{11,20}(X) - 1.17237 \cdot 10^{14} B_{12,20}(X) - 6.85172 \cdot 10^{13} B_{13,20}(X) \\ & - 1.96489 \cdot 10^{13} B_{14,20}(X) + 2.93682 \cdot 10^{13} B_{15,20}(X) + 7.85342 \cdot 10^{13} B_{16,20}(X) + 1.27849 \\ & \cdot 10^{14} B_{17,20}(X) + 1.77314 \cdot 10^{14} B_{18,20}(X) + 2.26929 \cdot 10^{14} B_{19,20}(X) + 2.76694 \cdot 10^{14} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = 2.8014 \cdot 10^{13} X^2 + 9.39062 \cdot 10^{14} X - 6.90399 \cdot 10^{14}$$
$$= -6.90399 \cdot 10^{14} B_{0,2} - 2.20868 \cdot 10^{14} B_{1,2} + 2.76677 \cdot 10^{14} B_{2,2}$$

$$\widetilde{q_2} = 7.92061 \cdot 10^{16} X^{20} - 7.94452 \cdot 10^{17} X^{19} + 3.6887 \cdot 10^{18} X^{18} - 1.05196 \cdot 10^{19} X^{17} + 2.0618 \cdot 10^{19} X^{16} - 2.94446 \cdot 10^{19} X^{15} + 3.16973 \cdot 10^{19} X^{14} - 2.62493 \cdot 10^{19} X^{13} + 1.69148 \cdot 10^{19} X^{12} - 8.52259 \cdot 10^{18} X^{11} + 3.3549 \cdot 10^{18} X^{10} - 1.02549 \cdot 10^{18} X^{9} + 2.40695 \cdot 10^{17} X^{8} - 4.27033 \cdot 10^{16} X^{7} + 5.61526 \cdot 10^{15} X^{6} - 5.33627 \cdot 10^{14} X^{5} + 3.53246 \cdot 10^{13} X^{4} - 1.51215 \cdot 10^{12} X^{3} + 2.80508 \cdot 10^{13} X^{2} + 9.39061 \cdot 10^{14} X - 6.90399 \cdot 10^{14}$$
$$= -6.90399 \cdot 10^{14} B_{0,20} - 6.43446 \cdot 10^{14} B_{1,20} - 5.96345 \cdot 10^{14} B_{2,20} - 5.49098 \cdot 10^{14} B_{3,20} - 5.01699 \cdot 10^{14} B_{4,20} - 4.54169 \cdot 10^{14} B_{5,20} - 4.06445 \cdot 10^{14} B_{6,20} - 3.58678 \cdot 10^{14} B_{7,20} - 3.10572 \cdot 10^{14} B_{8,20} - 2.62607 \cdot 10^{14} B_{9,20} - 2.14121 \cdot 10^{14} B_{10,20} - 1.65898 \cdot 10^{14} B_{11,20} - 1.17159 \cdot 10^{14} B_{12,20} - 6.85501 \cdot 10^{13} B_{13,20} - 1.96161 \cdot 10^{13} B_{14,20} + 2.93696 \cdot 10^{13} B_{15,20} + 7.85462 \cdot 10^{13} B_{16,20} + 1.27855 \cdot 10^{14} B_{17,20} + 1.77315 \cdot 10^{14} B_{18,20} + 2.26922 \cdot 10^{14} B_{19,20} + 2.76677 \cdot 10^{14} B_{20,20}$$



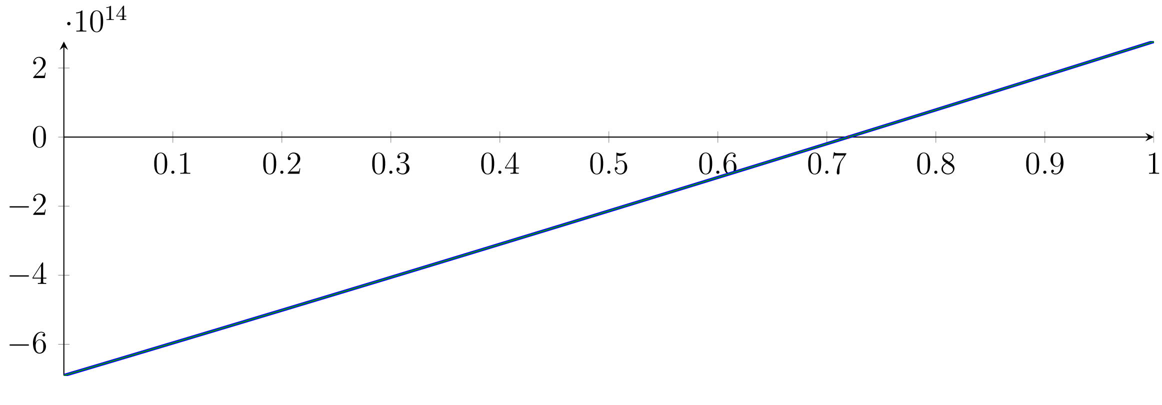
The maximum difference of the Bézier coefficients is $\delta = 1.11863 \cdot 10^{11}$.
Bounding polynomials M and m:

$$M = 2.8014 \cdot 10^{13} X^2 + 9.39062 \cdot 10^{14} X - 6.90287 \cdot 10^{14}$$
$$m = 2.8014 \cdot 10^{13} X^2 + 9.39062 \cdot 10^{14} X - 6.90511 \cdot 10^{14}$$

Root of M and m:

$$N(M) = \{-34.2408, 0.719633\} \qquad N(m) = \{-34.241, 0.719861\}$$

Intersection intervals:



$$[0.719633, 0.719861]$$

Longest intersection interval: 0.000228435
$\implies$ Selective recursion: interval 1: $[19.9988, 19.9988]$,

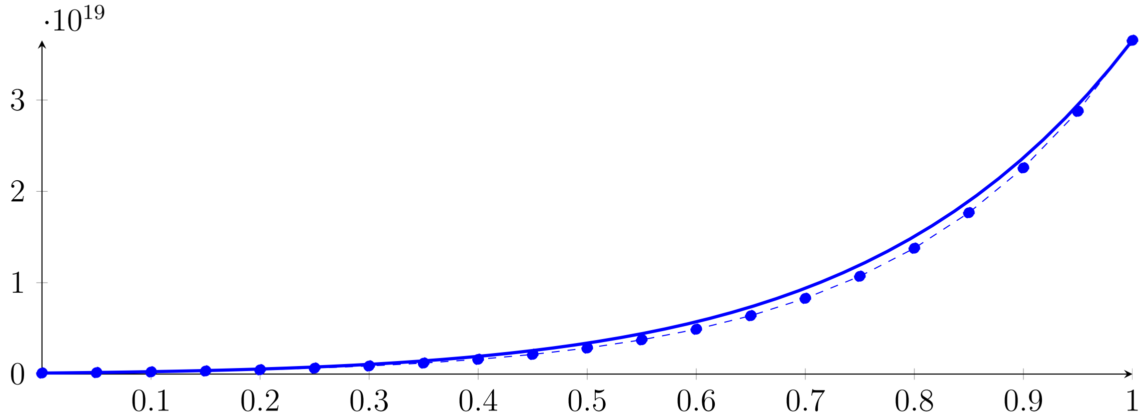
2.100 Recursion Branch 1 2 2 1 1 2 2 1 1 in Interval 1: $[19.9988, 19.9988]$

Found root in interval $[19.9988, 19.9988]$ at recursion depth 9!

2.101 Recursion Branch 1 2 2 1 2 on the Second Half $[20.3125, 21.875]$

Normalized monomial und Bézier representations and the Bézier polygon:

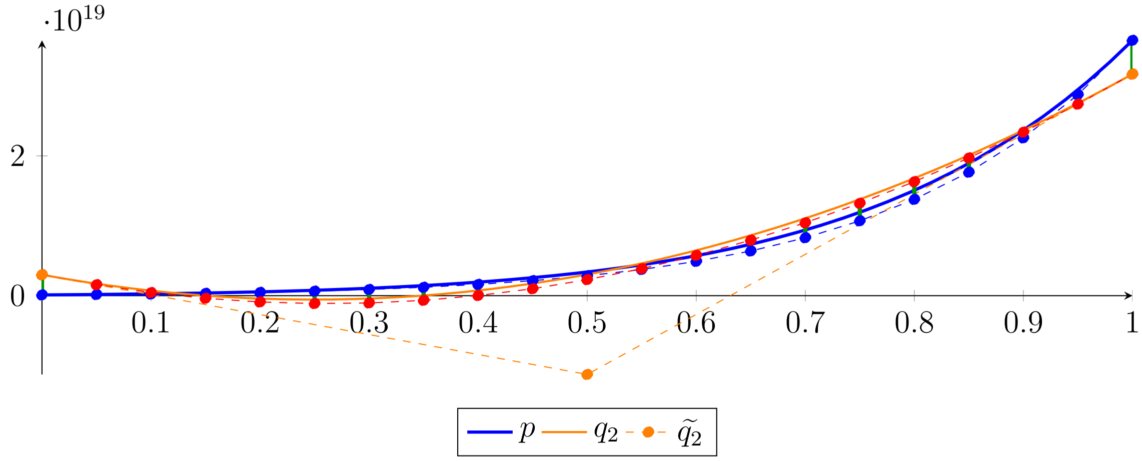
$$p = -2.3982\cdot10^{9}X^{20} + 2.73788\cdot10^{10}X^{19} - 6.19725\cdot10^{10}X^{18} + 3.56643\cdot10^{11}X^{17} - 1.25055\cdot10^{12}X^{16} + 1.77598\cdot10^{12}X^{15} + 1.30077\cdot10^{13}X^{14} + 1.46709\cdot10^{14}X^{13} + 1.28145\cdot10^{15}X^{12} + 8.9278\cdot10^{15}X^{11} + 4.96984\cdot10^{16}X^{10} + 2.20845\cdot10^{17}X^{9} + 7.79096\cdot10^{17}X^{8} + 2.16012\cdot10^{18}X^{7} + 4.63365\cdot10^{18}X^{6} + 7.51293\cdot10^{18}X^{5} + 8.89517\cdot10^{18}X^{4} + 7.29479\cdot10^{18}X^{3} + 3.79879\cdot10^{18}X^{2} + 1.06692\cdot10^{18}X + 1.07836\cdot10^{17}$$
$$= 1.07836\cdot10^{17}B_{0,20}(X) + 1.61182\cdot10^{17}B_{1,20}(X) + 2.34521\cdot10^{17}B_{2,20}(X) + 3.34253\cdot10^{17}B_{3,20}(X) + 4.68613\cdot10^{17}B_{4,20}(X) + 6.48155\cdot10^{17}B_{5,20}(X) + 8.86361\cdot10^{17}B_{6,20}(X) + 1.20039\cdot10^{18}B_{7,20}(X) + 1.61199\cdot10^{18}B_{8,20}(X) + 2.14872\cdot10^{18}B_{9,20}(X) + 2.84528\cdot10^{18}B_{10,20}(X) + 3.74538\cdot10^{18}B_{11,20}(X) + 4.90381\cdot10^{18}B_{12,20}(X) + 6.38923\cdot10^{18}B_{13,20}(X) + 8.28736\cdot10^{18}B_{14,20}(X) + 1.0705\cdot10^{19}B_{15,20}(X) + 1.37752\cdot10^{19}B_{16,20}(X) + 1.7663\cdot10^{19}B_{17,20}(X) + 2.25728\cdot10^{19}B_{18,20}(X) + 2.87577\cdot10^{19}B_{19,20}(X) + 3.65302\cdot10^{19}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 5.72376\cdot10^{19}X^{2} - 2.85344\cdot10^{19}X + 2.99641\cdot10^{18}$$
$$= 2.99641\cdot10^{18}B_{0,2} - 1.12708\cdot10^{19}B_{1,2} + 3.16997\cdot10^{19}B_{2,2}$$

$$\widetilde{q_2} = 7.03589\cdot10^{21}X^{20} - 7.03098\cdot10^{22}X^{19} + 3.25118\cdot10^{23}X^{18} - 9.23046\cdot10^{23}X^{17} + 1.80035\cdot10^{24}X^{16} - 2.5575\cdot10^{24}X^{15} + 2.73728\cdot10^{24}X^{14} - 2.25246\cdot10^{24}X^{13} + 1.44142\cdot10^{24}X^{12} - 7.20909\cdot10^{23}X^{11} + 2.81674\cdot10^{23}X^{10} - 8.55067\cdot10^{22}X^{9} + 1.9948\cdot10^{22}X^{8} - 3.51503\cdot10^{21}X^{7} + 4.56169\cdot10^{20}X^{6} - 4.20741\cdot10^{19}X^{5} + 2.61747\cdot10^{18}X^{4} - 1.00863\cdot10^{17}X^{3} + 5.72398\cdot10^{19}X^{2} - 2.85344\cdot10^{19}X + 2.99641\cdot10^{18}$$
$$= 2.99641\cdot10^{18}B_{0,20} + 1.56969\cdot10^{18}B_{1,20} + 4.44236\cdot10^{17}B_{2,20} - 3.80049\cdot10^{17}B_{3,20} - 9.02708\cdot10^{17}B_{4,20} - 1.12546\cdot10^{18}B_{5,20} - 1.04316\cdot10^{18}B_{6,20} - 6.68264\cdot10^{17}B_{7,20} + 2.40185\cdot10^{16}B_{8,20} + 9.92404\cdot10^{17}B_{9,20} + 2.29489\cdot10^{18}B_{10,20} + 3.86252\cdot10^{18}B_{11,20} + 5.76472\cdot10^{18}B_{12,20} + 7.94267\cdot10^{18}B_{13,20} + 1.04381\cdot10^{19}B_{14,20} + 1.32262\cdot10^{19}B_{15,20} + 1.63192\cdot10^{19}B_{16,20} + 1.97122\cdot10^{19}B_{17,20} + 2.34068\cdot10^{19}B_{18,20} + 2.74026\cdot10^{19}B_{19,20} + 3.16997\cdot10^{19}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 4.83055 \cdot 10^{18}$.

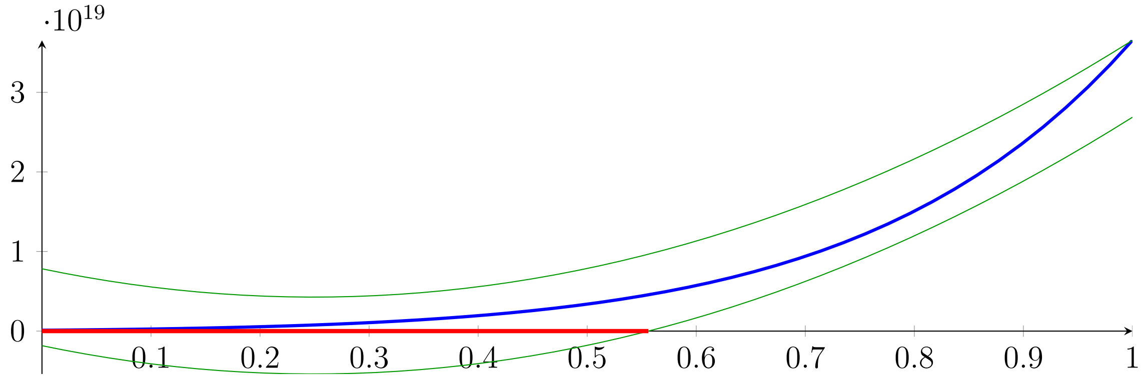
Bounding polynomials M and m:

$$M = 5.72376 \cdot 10^{19} X^2 - 2.85344 \cdot 10^{19} X + 7.82697 \cdot 10^{18}$$
$$m = 5.72376 \cdot 10^{19} X^2 - 2.85344 \cdot 10^{19} X - 1.83414 \cdot 10^{18}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{-0.0576188, 0.556144\}$$

Intersection intervals:



$[0, 0.556144]$

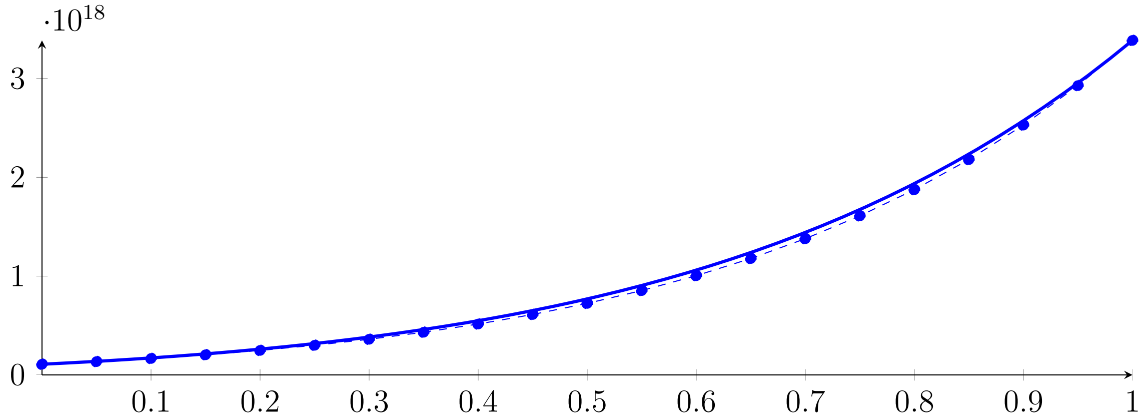
Longest intersection interval: 0.556144

$\Longrightarrow$ Bisection: first half $[20.3125, 21.0938]$ und second half $[21.0938, 21.875]$

2.102 Recursion Branch 1 2 2 1 2 1 on the First Half $[20.3125, 21.0938]$

Normalized monomial und Bézier representations and the Bézier polygon:

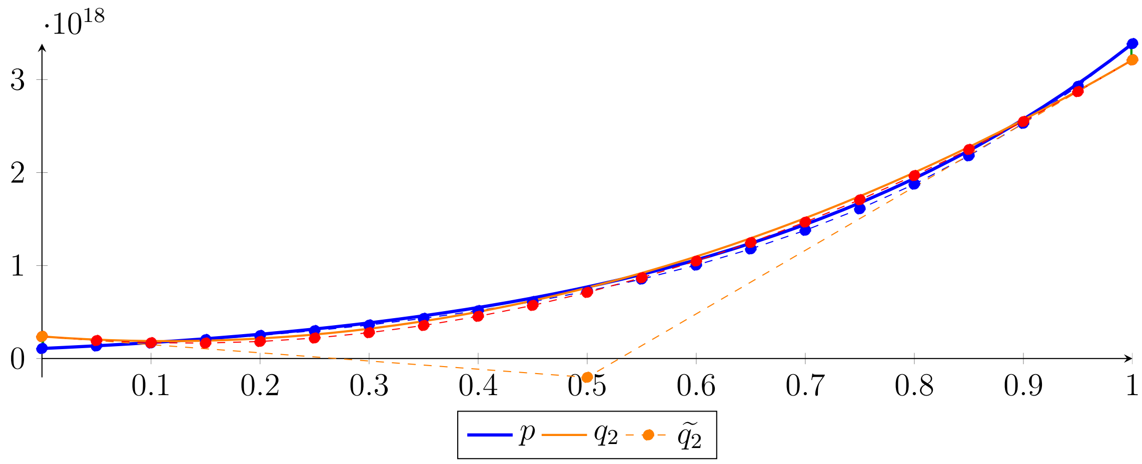
$$p = -6.39473 \cdot 10^{8} X^{20} + 5.48963 \cdot 10^{9} X^{19} - 1.70506 \cdot 10^{10} X^{18} + 8.72253 \cdot 10^{10} X^{17} - 3.63131 \cdot 10^{11} X^{16} + 2.52683 \cdot 10^{11} X^{15} - 9.26339 \cdot 10^{10} X^{14} + 1.43815 \cdot 10^{10} X^{13} + 1.19135 \cdot 10^{11} X^{12} + 4.35957 \cdot 10^{12} X^{11} + 4.84859 \cdot 10^{13} X^{10} + 4.31335 \cdot 10^{14} X^{9} + 3.04334 \cdot 10^{15} X^{8} + 1.68759 \cdot 10^{16} X^{7} + 7.24007 \cdot 10^{16} X^{6} + 2.34779 \cdot 10^{17} X^{5} + 5.55948 \cdot 10^{17} X^{4} + 9.11849 \cdot 10^{17} X^{3} + 9.49697 \cdot 10^{17} X^{2} + 5.3346 \cdot 10^{17} X + 1.07836 \cdot 10^{17}$$
$$= 1.07836 \cdot 10^{17} B_{0,20}(X) + 1.34509 \cdot 10^{17} B_{1,20}(X) + 1.6618 \cdot 10^{17} B_{2,20}(X) + 2.0365 \cdot 10^{17} B_{3,20}(X) + 2.47832 \cdot 10^{17} B_{4,20}(X) + 2.99772 \cdot 10^{17} B_{5,20}(X) + 3.60661 \cdot 10^{17} B_{6,20}(X) + 4.31856 \cdot 10^{17} B_{7,20}(X) + 5.14902 \cdot 10^{17} B_{8,20}(X) + 6.11555 \cdot 10^{17} B_{9,20}(X) + 7.2381 \cdot 10^{17} B_{10,20}(X) + 8.5393 \cdot 10^{17} B_{11,20}(X) + 1.00448 \cdot 10^{18} B_{12,20}(X) + 1.17837 \cdot 10^{18} B_{13,20}(X) + 1.37888 \cdot 10^{18} B_{14,20}(X) + 1.60973 \cdot 10^{18} B_{15,20}(X) + 1.87511 \cdot 10^{18} B_{16,20}(X) + 2.17978 \cdot 10^{18} B_{17,20}(X) + 2.52907 \cdot 10^{18} B_{18,20}(X) + 2.929 \cdot 10^{18} B_{19,20}(X) + 3.38637 \cdot 10^{18} B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 3.85455{\cdot}10^{18}X^2 - 8.80021{\cdot}10^{17}X + 2.37413{\cdot}10^{17}$$
$$= 2.37413{\cdot}10^{17}B_{0,2} - 2.02597{\cdot}10^{17}B_{1,2} + 3.21194{\cdot}10^{18}B_{2,2}$$

$$\widetilde{q}_2 = 4.12389{\cdot}10^{20}X^{20} - 4.1164{\cdot}10^{21}X^{19} + 1.9008{\cdot}10^{22}X^{18} - 5.38781{\cdot}10^{22}X^{17} + 1.04899{\cdot}10^{23}X^{16} - 1.48742{\cdot}10^{23}X^{15} + 1.58922{\cdot}10^{23}X^{14} - 1.30582{\cdot}10^{23}X^{13} + 8.34801{\cdot}10^{22}X^{12} - 4.17367{\cdot}10^{22}X^{11} + 1.63147{\cdot}10^{22}X^{10} - 4.95914{\cdot}10^{21}X^9 + 1.15925{\cdot}10^{21}X^8 - 2.04635{\cdot}10^{20}X^7 + 2.65337{\cdot}10^{19}X^6 - 2.42646{\cdot}10^{18}X^5 + 1.4709{\cdot}10^{17}X^4 - 5.36877{\cdot}10^{15}X^3 + 3.85465{\cdot}10^{18}X^2 - 8.80022{\cdot}10^{17}X + 2.37413{\cdot}10^{17}$$
$$= 2.37413{\cdot}10^{17}B_{0,20} + 1.93412{\cdot}10^{17}B_{1,20} + 1.69699{\cdot}10^{17}B_{2,20} + 1.66268{\cdot}10^{17}B_{3,20} + 1.83146{\cdot}10^{17}B_{4,20} + 2.20232{\cdot}10^{17}B_{5,20} + 2.77828{\cdot}10^{17}B_{6,20} + 3.55209{\cdot}10^{17}B_{7,20} + 4.53807{\cdot}10^{17}B_{8,20} + 5.71237{\cdot}10^{17}B_{9,20} + 7.10864{\cdot}10^{17}B_{10,20} + 8.68658{\cdot}10^{17}B_{11,20} + 1.04872{\cdot}10^{18}B_{12,20} + 1.24756{\cdot}10^{18}B_{13,20} + 1.46763{\cdot}10^{18}B_{14,20} + 1.7075{\cdot}10^{18}B_{15,20} + 1.96786{\cdot}10^{18}B_{16,20} + 2.24844{\cdot}10^{18}B_{17,20} + 2.54932{\cdot}10^{18}B_{18,20} + 2.87048{\cdot}10^{18}B_{19,20} + 3.21194{\cdot}10^{18}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.74435{\cdot}10^{17}$.

Bounding polynomials M and m:

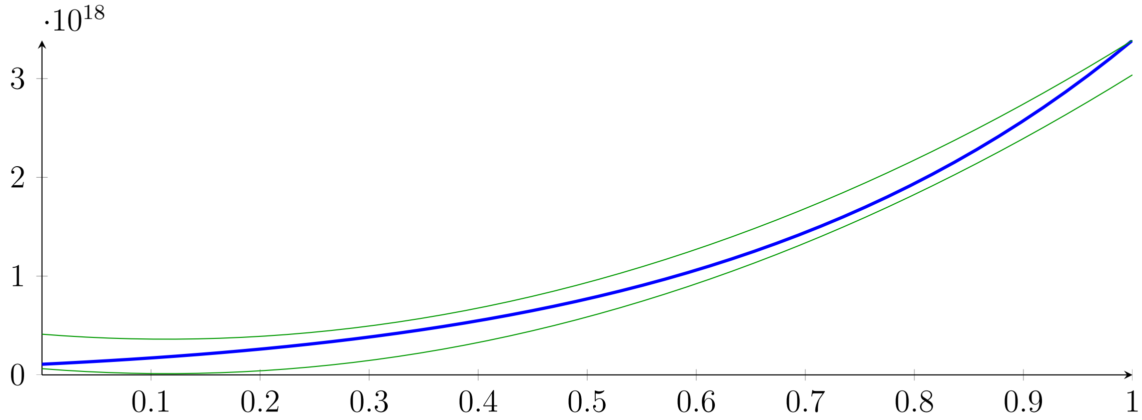
$$M = 3.85455{\cdot}10^{18}X^2 - 8.80021{\cdot}10^{17}X + 4.11848{\cdot}10^{17}$$

$$m = 3.85455{\cdot}10^{18}X^2 - 8.80021{\cdot}10^{17}X + 6.29786{\cdot}10^{16}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{\}$$

Intersection intervals:

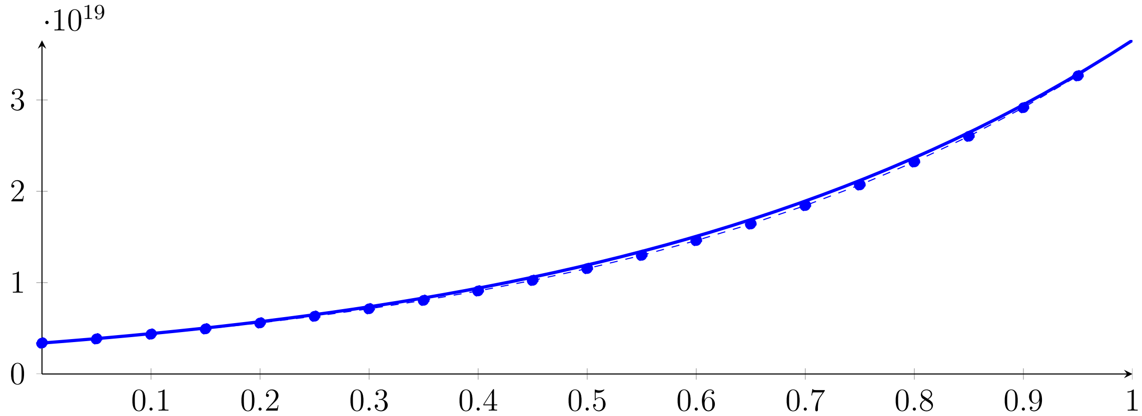


No intersection intervals with the x axis.

2.103 Recursion Branch 1 2 2 1 2 2 on the Second Half $[21.0938, 21.875]$

Normalized monomial und Bézier representations and the Bézier polygon:

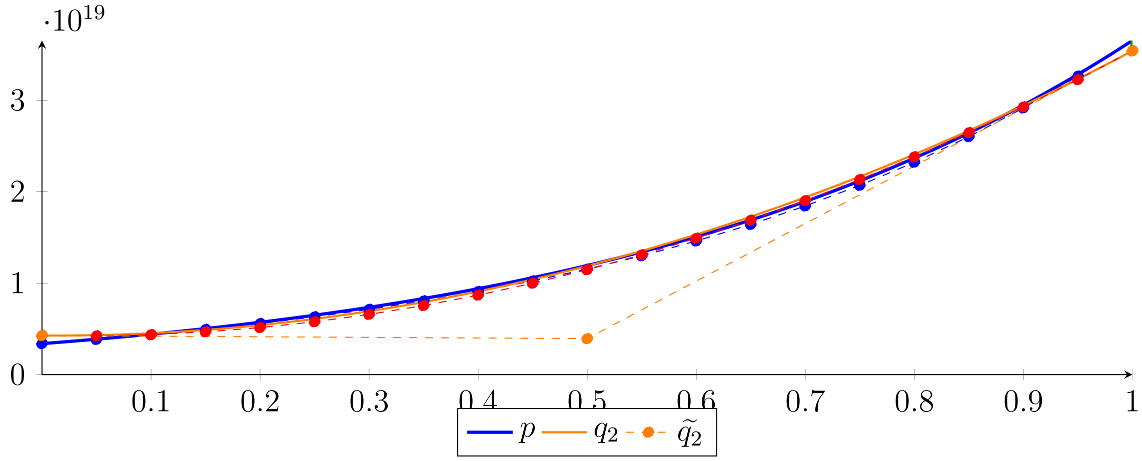
$$
\begin{aligned}
p = & -1.00367\cdot 10^{10}X^{20} + 7.3925\cdot 10^{10}X^{19} - 2.71133\cdot 10^{11}X^{18} + 1.37448\cdot 10^{12}X^{17} - 6.21856\cdot 10^{12}X^{16} \\
& + 4.41079\cdot 10^{12}X^{15} - 1.51307\cdot 10^{12}X^{14} - 5.07241\cdot 10^{11}X^{13} - 2.98361\cdot 10^{12}X^{12} + 9.7488\cdot 10^{12}X^{11} \\
& + 1.22268\cdot 10^{14}X^{10} + 1.2398\cdot 10^{15}X^{9} + 1.00092\cdot 10^{16}X^{8} + 6.4295\cdot 10^{16}X^{7} + 3.24507\cdot 10^{17}X^{6} + 1.26287 \\
& \cdot 10^{18}X^{5} + 3.68569\cdot 10^{18}X^{4} + 7.73538\cdot 10^{18}X^{3} + 1.09123\cdot 10^{19}X^{2} + 9.14739\cdot 10^{18}X + 3.38637\cdot 10^{18} \\
= & \, 3.38637\cdot 10^{18}B_{0,20}(X) + 3.84374\cdot 10^{18}B_{1,20}(X) + 4.35855\cdot 10^{18}B_{2,20}(X) + 4.93757 \\
& \cdot 10^{18}B_{3,20}(X) + 5.58835\cdot 10^{18}B_{4,20}(X) + 6.31929\cdot 10^{18}B_{5,20}(X) + 7.13971\cdot 10^{18}B_{6,20}(X) \\
& + 8.05994\cdot 10^{18}B_{7,20}(X) + 9.0915\cdot 10^{18}B_{8,20}(X) + 1.02471\cdot 10^{19}B_{9,20}(X) + 1.1541 \\
& \cdot 10^{19}B_{10,20}(X) + 1.29887\cdot 10^{19}B_{11,20}(X) + 1.46077\cdot 10^{19}B_{12,20}(X) + 1.64172\cdot 10^{19}B_{13,20}(X) \\
& + 1.84387\cdot 10^{19}B_{14,20}(X) + 2.06956\cdot 10^{19}B_{15,20}(X) + 2.32141\cdot 10^{19}B_{16,20}(X) + 2.60231 \\
& \cdot 10^{19}B_{17,20}(X) + 2.91546\cdot 10^{19}B_{18,20}(X) + 3.2644\cdot 10^{19}B_{19,20}(X) + 3.65302\cdot 10^{19}B_{20,20}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 &= 3.18001\cdot 10^{19}X^2 - 6.75784\cdot 10^{17}X + 4.27248\cdot 10^{18} \\
&= 4.27248\cdot 10^{18}B_{0,2} + 3.93459\cdot 10^{18}B_{1,2} + 3.53968\cdot 10^{19}B_{2,2}
\end{aligned}
$$

$$\widetilde{q}_2 = 2.63342 \cdot 10^{21} X^{20} - 2.6228 \cdot 10^{22} X^{19} + 1.2078 \cdot 10^{23} X^{18} - 3.41262 \cdot 10^{23} X^{17} + 6.62097 \cdot 10^{23} X^{16} - 9.35404 \cdot 10^{23} X^{15} + 9.95883 \cdot 10^{23} X^{14} - 8.15721 \cdot 10^{23} X^{13} + 5.20227 \cdot 10^{23} X^{12} - 2.59748 \cdot 10^{23} X^{11} + 1.01543 \cdot 10^{23} X^{10} - 3.09174 \cdot 10^{22} X^{9} + 7.2485 \cdot 10^{21} X^{8} - 1.28269 \cdot 10^{21} X^{7} + 1.65854 \cdot 10^{20} X^{6} - 1.48933 \cdot 10^{19} X^{5} + 8.53929 \cdot 10^{17} X^{4} - 2.73203 \cdot 10^{16} X^{3} + 3.18005 \cdot 10^{19} X^{2} - 6.75787 \cdot 10^{17} X + 4.27248 \cdot 10^{18}$$
$$= 4.27248 \cdot 10^{18} B_{0,20} + 4.23869 \cdot 10^{18} B_{1,20} + 4.37228 \cdot 10^{18} B_{2,20} + 4.6732 \cdot 10^{18} B_{3,20} + 5.14163 \cdot 10^{18} B_{4,20} + 5.77693 \cdot 10^{18} B_{5,20} + 6.58099 \cdot 10^{18} B_{6,20} + 7.54931 \cdot 10^{18} B_{7,20} + 8.69073 \cdot 10^{18} B_{8,20} + 9.99044 \cdot 10^{18} B_{9,20} + 1.14695 \cdot 10^{19} B_{10,20} + 1.31024 \cdot 10^{19} B_{11,20} + 1.49157 \cdot 10^{19} B_{12,20} + 1.68865 \cdot 10^{19} B_{13,20} + 1.90307 \cdot 10^{19} B_{14,20} + 2.13391 \cdot 10^{19} B_{15,20} + 2.38162 \cdot 10^{19} B_{16,20} + 2.64602 \cdot 10^{19} B_{17,20} + 2.92717 \cdot 10^{19} B_{18,20} + 3.22505 \cdot 10^{19} B_{19,20} + 3.53968 \cdot 10^{19} B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.13345 \cdot 10^{18}$.

Bounding polynomials M and m:

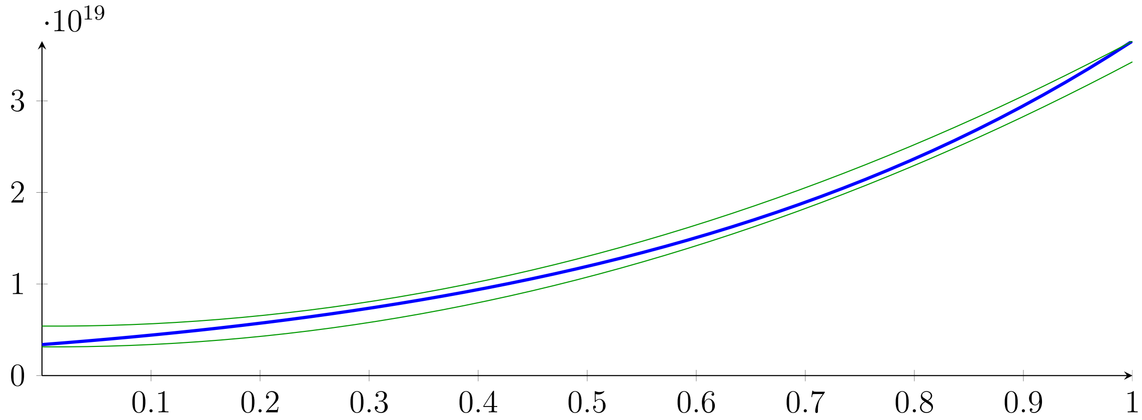
$$M = 3.18001 \cdot 10^{19} X^2 - 6.75784 \cdot 10^{17} X + 5.40593 \cdot 10^{18}$$

$$m = 3.18001 \cdot 10^{19} X^2 - 6.75784 \cdot 10^{17} X + 3.13904 \cdot 10^{18}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{\}$$

Intersection intervals:

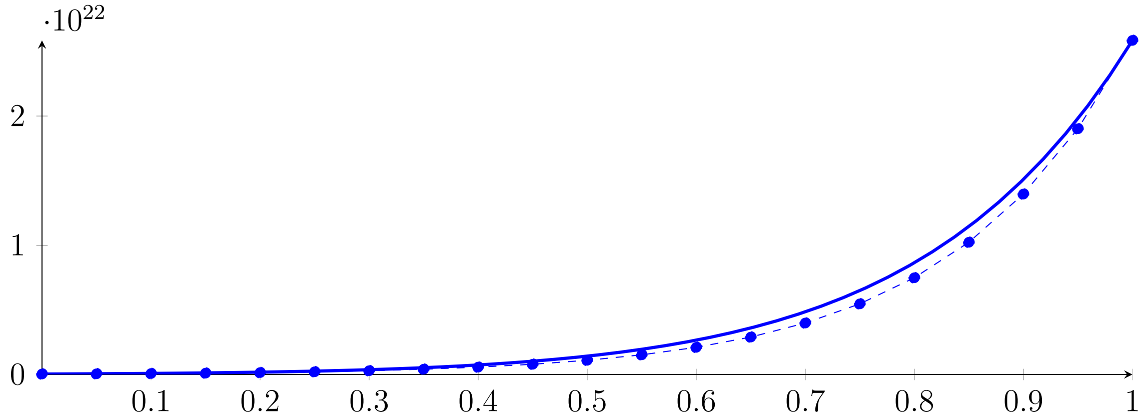


No intersection intervals with the x axis.

2.104 Recursion Branch 1 2 2 2 on the Second Half $[21.875, 25]$

Normalized monomial und Bézier representations and the Bézier polygon:

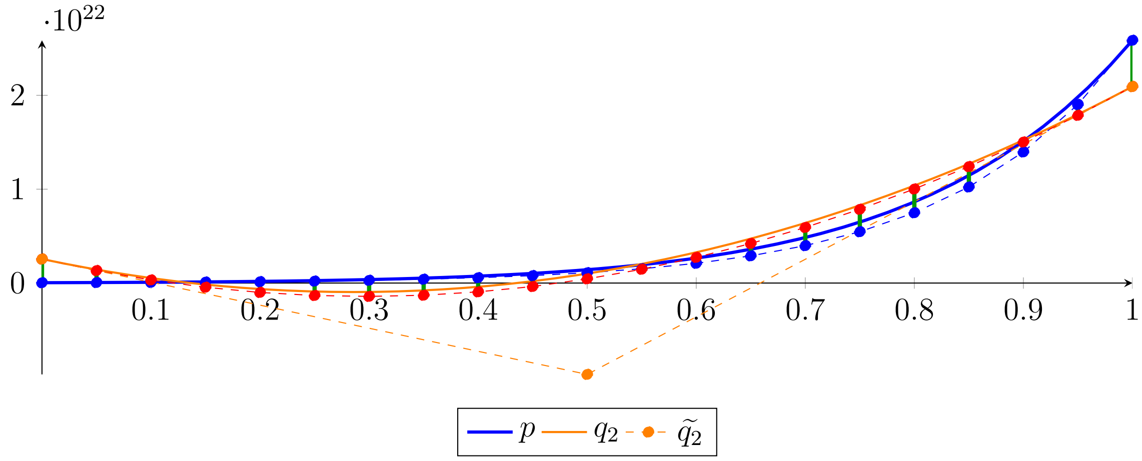
$$p = -9.43719\cdot 10^{11}X^{20} + 1.17887\cdot 10^{13}X^{19} - 7.98914\cdot 10^{12}X^{18} + 5.47338\cdot 10^{14}X^{17} + 5.6893\cdot 10^{15}X^{16} + 6.80909\cdot 10^{16}X^{15} + 5.72765\cdot 10^{17}X^{14} + 3.80691\cdot 10^{18}X^{13} + 2.01926\cdot 10^{19}X^{12} + 8.62526\cdot 10^{19}X^{11} + 2.98009\cdot 10^{20}X^{10} + 8.33374\cdot 10^{20}X^{9} + 1.88062\cdot 10^{21}X^{8} + 3.40128\cdot 10^{21}X^{7} + 4.87441\cdot 10^{21}X^{6} + 5.4405\cdot 10^{21}X^{5} + 4.6091\cdot 10^{21}X^{4} + 2.84983\cdot 10^{21}X^{3} + 1.20656\cdot 10^{21}X^{2} + 3.109\cdot 10^{20}X + 3.65302\cdot 10^{19}$$
$$= 3.65302\cdot 10^{19}B_{0,20}(X) + 5.20752\cdot 10^{19}B_{1,20}(X) + 7.39705\cdot 10^{19}B_{2,20}(X) + 1.04716\cdot 10^{20}B_{3,20}(X) + 1.47763\cdot 10^{20}B_{4,20}(X) + 2.07864\cdot 10^{20}B_{5,20}(X) + 2.91553\cdot 10^{20}B_{6,20}(X) + 4.07786\cdot 10^{20}B_{7,20}(X) + 5.68821\cdot 10^{20}B_{8,20}(X) + 7.91397\cdot 10^{20}B_{9,20}(X) + 1.09833\cdot 10^{21}B_{10,20}(X) + 1.52065\cdot 10^{21}B_{11,20}(X) + 2.10052\cdot 10^{21}B_{12,20}(X) + 2.89506\cdot 10^{21}B_{13,20}(X) + 3.98159\cdot 10^{21}B_{14,20}(X) + 5.46453\cdot 10^{21}B_{15,20}(X) + 7.48476\cdot 10^{21}B_{16,20}(X) + 1.0232\cdot 10^{22}B_{17,20}(X) + 1.39612\cdot 10^{22}B_{18,20}(X) + 1.90149\cdot 10^{22}B_{19,20}(X) + 2.5852\cdot 10^{22}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 4.29441\cdot 10^{22}X^2 - 2.45771\cdot 10^{22}X + 2.559\cdot 10^{21}$$
$$= 2.559\cdot 10^{21}B_{0,2} - 9.72955\cdot 10^{21}B_{1,2} + 2.0926\cdot 10^{22}B_{2,2}$$

$$\widetilde{q}_2 = 5.38936\cdot 10^{24}X^{20} - 5.38659\cdot 10^{25}X^{19} + 2.49138\cdot 10^{26}X^{18} - 7.07524\cdot 10^{26}X^{17} + 1.3804\cdot 10^{27}X^{16} - 1.96153\cdot 10^{27}X^{15} + 2.10001\cdot 10^{27}X^{14} - 1.72844\cdot 10^{27}X^{13} + 1.10623\cdot 10^{27}X^{12} - 5.53267\cdot 10^{26}X^{11} + 2.1614\cdot 10^{26}X^{10} - 6.55923\cdot 10^{25}X^{9} + 1.52954\cdot 10^{25}X^{8} - 2.69413\cdot 10^{24}X^{7} + 3.4965\cdot 10^{23}X^{6} - 3.22928\cdot 10^{22}X^{5} + 2.01744\cdot 10^{21}X^{4} - 7.84054\cdot 10^{19}X^{3} + 4.29458\cdot 10^{22}X^{2} - 2.45771\cdot 10^{22}X + 2.559\cdot 10^{21}$$
$$= 2.559\cdot 10^{21}B_{0,20} + 1.33015\cdot 10^{21}B_{1,20} + 3.27319\cdot 10^{20}B_{2,20} - 4.49545\cdot 10^{20}B_{3,20} - 1.0001\cdot 10^{21}B_{4,20} - 1.32567\cdot 10^{21}B_{5,20} - 1.42229\cdot 10^{21}B_{6,20} - 1.29954\cdot 10^{21}B_{7,20} - 9.38369\cdot 10^{20}B_{8,20} - 3.70484\cdot 10^{20}B_{9,20} + 4.48617\cdot 10^{20}B_{10,20} + 1.4661\cdot 10^{21}B_{11,20} + 2.73507\cdot 10^{21}B_{12,20} + 4.21054\cdot 10^{21}B_{13,20} + 5.92447\cdot 10^{21}B_{14,20} + 7.85782\cdot 10^{21}B_{15,20} + 1.00201\cdot 10^{22}B_{16,20} + 1.24073\cdot 10^{22}B_{17,20} + 1.50209\cdot 10^{22}B_{18,20} + 1.78604\cdot 10^{22}B_{19,20} + 2.0926\cdot 10^{22}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 4.92604{\cdot}10^{21}$.

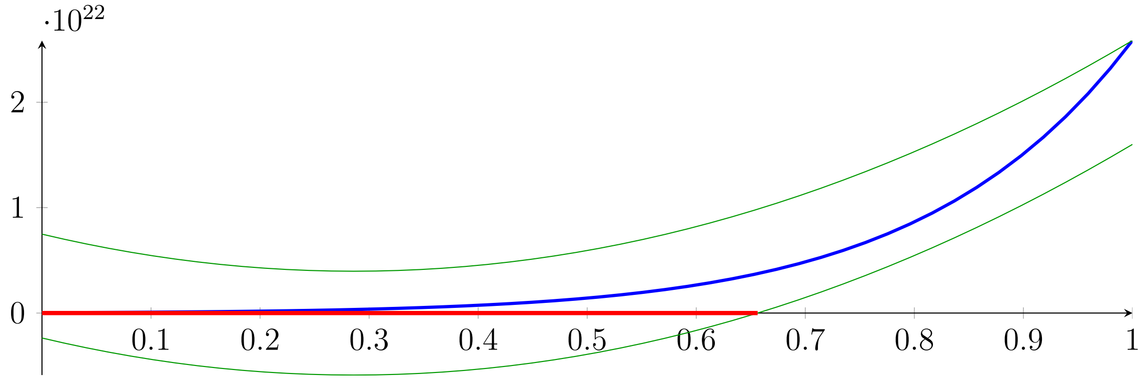
Bounding polynomials M and m:

$$M = 4.29441{\cdot}10^{22}X^2 - 2.45771{\cdot}10^{22}X + 7.48504{\cdot}10^{21}$$
$$m = 4.29441{\cdot}10^{22}X^2 - 2.45771{\cdot}10^{22}X - 2.36704{\cdot}10^{21}$$

Root of M and m:

$$N(M) = \{\} \qquad\qquad N(m) = \{-0.0839858, 0.656291\}$$

Intersection intervals:



$$[0, 0.656291]$$

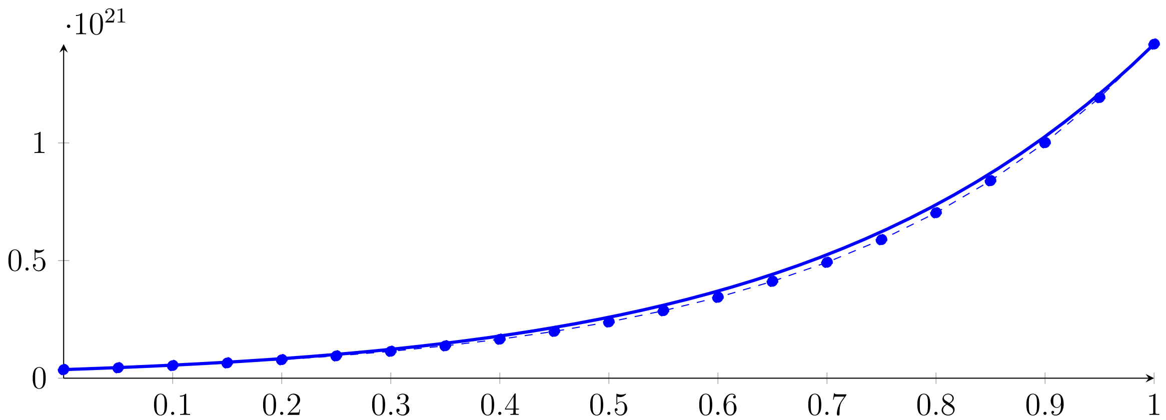
Longest intersection interval: 0.656291

$\Longrightarrow$ Bisection: first half $[21.875, 23.4375]$ und second half $[23.4375, 25]$

2.105 Recursion Branch 1 2 2 2 1 on the First Half $[21.875, 23.4375]$

Normalized monomial und Bézier representations and the Bézier polygon:

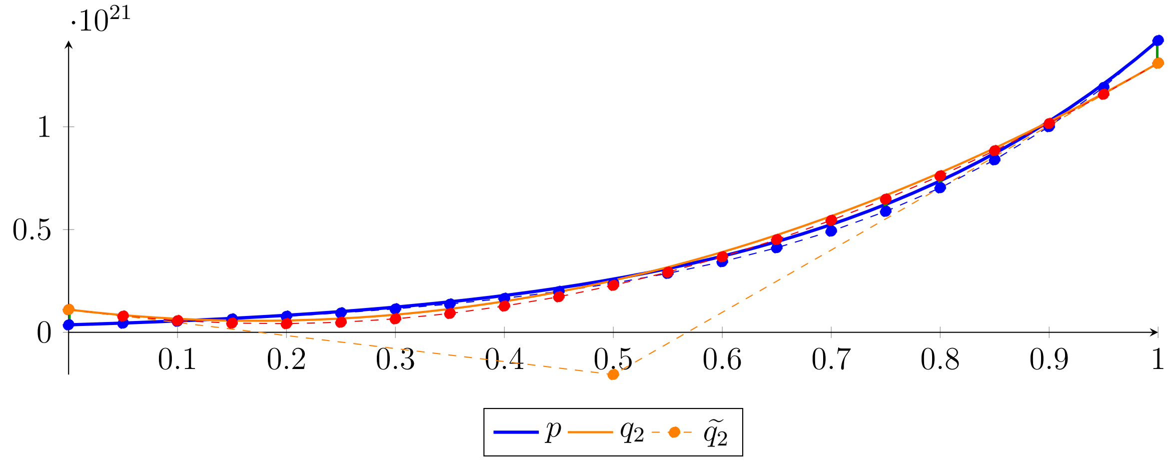
$$\begin{aligned} p = &-2.01933{\cdot}10^{11}X^{20} + 1.74162{\cdot}10^{12}X^{19} - 5.05078{\cdot}10^{12}X^{18} + 2.75484{\cdot}10^{13}X^{17} - 1.18885{\cdot}10^{14}X^{16} \\ &+ 8.19896{\cdot}10^{13}X^{15} + 7.11535{\cdot}10^{12}X^{14} + 4.61935{\cdot}10^{14}X^{13} + 4.87343{\cdot}10^{15}X^{12} + 4.21153{\cdot}10^{16}X^{11} \\ &+ 2.91009{\cdot}10^{17}X^{10} + 1.62768{\cdot}10^{18}X^{9} + 7.34619{\cdot}10^{18}X^{8} + 2.65725{\cdot}10^{19}X^{7} + 7.61627{\cdot}10^{19}X^{6} + 1.70015 \\ &{\cdot}10^{20}X^{5} + 2.88069{\cdot}10^{20}X^{4} + 3.56228{\cdot}10^{20}X^{3} + 3.01641{\cdot}10^{20}X^{2} + 1.5545{\cdot}10^{20}X + 3.65302{\cdot}10^{19} \\ = &\ 3.65302{\cdot}10^{19}B_{0,20}(X) + 4.43027{\cdot}10^{19}B_{1,20}(X) + 5.36628{\cdot}10^{19}B_{2,20}(X) + 6.49229 \\ &{\cdot}10^{19}B_{3,20}(X) + 7.84551{\cdot}10^{19}B_{4,20}(X) + 9.47016{\cdot}10^{19}B_{5,20}(X) + 1.14188{\cdot}10^{20}B_{6,20}(X) \\ &+ 1.37539{\cdot}10^{20}B_{7,20}(X) + 1.65495{\cdot}10^{20}B_{8,20}(X) + 1.98935{\cdot}10^{20}B_{9,20}(X) + 2.389 \\ &{\cdot}10^{20}B_{10,20}(X) + 2.86622{\cdot}10^{20}B_{11,20}(X) + 3.43561{\cdot}10^{20}B_{12,20}(X) + 4.11441{\cdot}10^{20}B_{13,20}(X) \\ &+ 4.92302{\cdot}10^{20}B_{14,20}(X) + 5.88551{\cdot}10^{20}B_{15,20}(X) + 7.03032{\cdot}10^{20}B_{16,20}(X) + 8.39098 \\ &{\cdot}10^{20}B_{17,20}(X) + 1.0007{\cdot}10^{21}B_{18,20}(X) + 1.1925{\cdot}10^{21}B_{19,20}(X) + 1.41998{\cdot}10^{21}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = 1.83158{\cdot}10^{21}X^2 - 6.32658{\cdot}10^{20}X + 1.10822{\cdot}10^{20}$$
$$= 1.10822{\cdot}10^{20}B_{0,2} - 2.05507{\cdot}10^{20}B_{1,2} + 1.30974{\cdot}10^{21}B_{2,2}$$

$$\widetilde{q}_2 = 2.07633{\cdot}10^{23}X^{20} - 2.07358{\cdot}10^{24}X^{19} + 9.58098{\cdot}10^{24}X^{18} - 2.71769{\cdot}10^{25}X^{17} + 5.29545{\cdot}10^{25}X^{16} - 7.51485{\cdot}10^{25}X^{15} + 8.03536{\cdot}10^{25}X^{14} - 6.60674{\cdot}10^{25}X^{13} + 4.22547{\cdot}10^{25}X^{12} - 2.11286{\cdot}10^{25}X^{11} + 8.25721{\cdot}10^{24}X^{10} - 2.50837{\cdot}10^{24}X^9 + 5.8581{\cdot}10^{23}X^8 - 1.03324{\cdot}10^{23}X^7 + 1.34021{\cdot}10^{22}X^6 - 1.23026{\cdot}10^{21}X^5 + 7.54539{\cdot}10^{19}X^4 - 2.82386{\cdot}10^{18}X^3 + 1.83164{\cdot}10^{21}X^2 - 6.32658{\cdot}10^{20}X + 1.10822{\cdot}10^{20}$$
$$= 1.10822{\cdot}10^{20}B_{0,20} + 7.91894{\cdot}10^{19}B_{1,20} + 5.71967{\cdot}10^{19}B_{2,20} + 4.48417{\cdot}10^{19}B_{3,20} + 4.21375{\cdot}10^{19}B_{4,20} + 4.90335{\cdot}10^{19}B_{5,20} + 6.56815{\cdot}10^{19}B_{6,20} + 9.17156{\cdot}10^{19}B_{7,20} + 1.27861{\cdot}10^{20}B_{8,20} + 1.7291{\cdot}10^{20}B_{9,20} + 2.28563{\cdot}10^{20}B_{10,20} + 2.92791{\cdot}10^{20}B_{11,20} + 3.67648{\cdot}10^{20}B_{12,20} + 4.5139{\cdot}10^{20}B_{13,20} + 5.45249{\cdot}10^{20}B_{14,20} + 6.48496{\cdot}10^{20}B_{15,20} + 7.61491{\cdot}10^{20}B_{16,20} + 8.84087{\cdot}10^{20}B_{17,20} + 1.01633{\cdot}10^{21}B_{18,20} + 1.15822{\cdot}10^{21}B_{19,20} + 1.30974{\cdot}10^{21}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.10236{\cdot}10^{20}$.

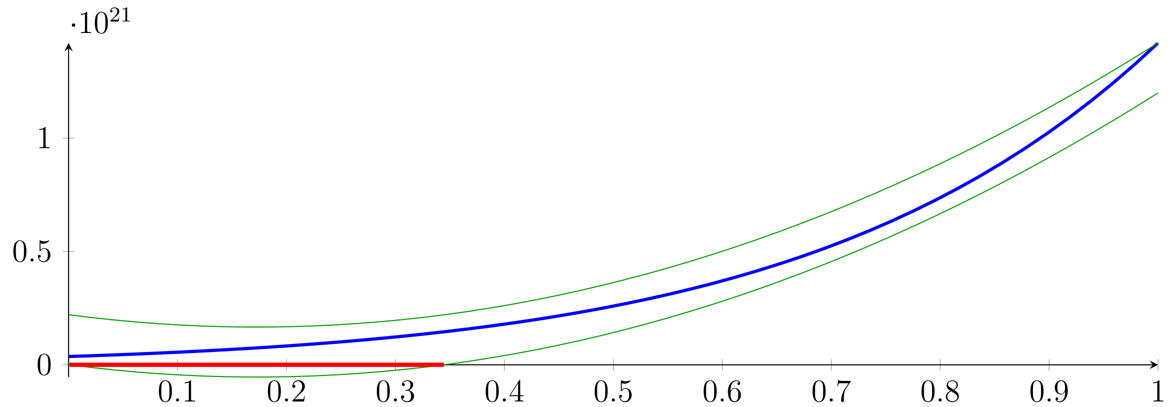
Bounding polynomials M and m:

$$M = 1.83158{\cdot}10^{21}X^2 - 6.32658{\cdot}10^{20}X + 2.21059{\cdot}10^{20}$$
$$m = 1.83158{\cdot}10^{21}X^2 - 6.32658{\cdot}10^{20}X + 5.86027{\cdot}10^{17}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.00092879, 0.344488\}$$

Intersection intervals:



$[0.00092879, 0.344488]$

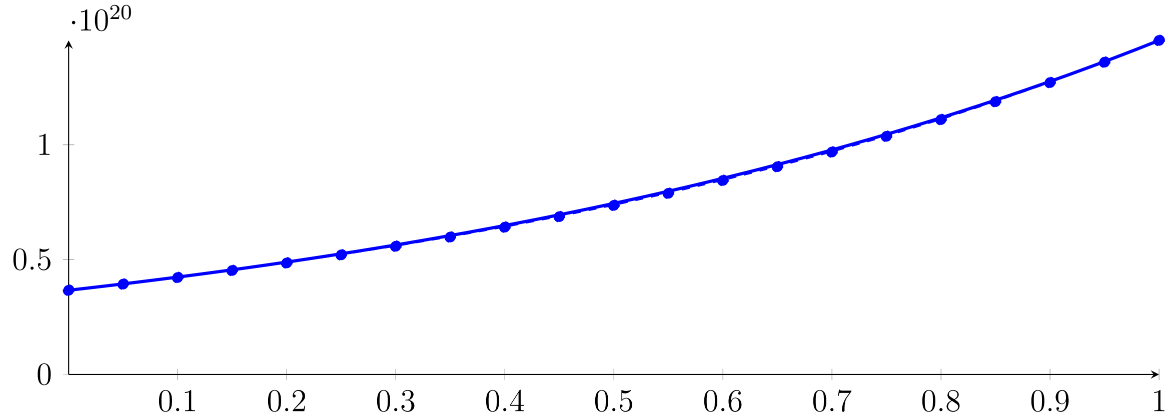
Longest intersection interval: 0.343559

$\Longrightarrow$ Selective recursion: interval 1: $[21.8765, 22.4133]$,

2.106 Recursion Branch 1 2 2 2 1 1 in Interval 1: $[21.8765, 22.4133]$

Normalized monomial und Bézier representations and the Bézier polygon:

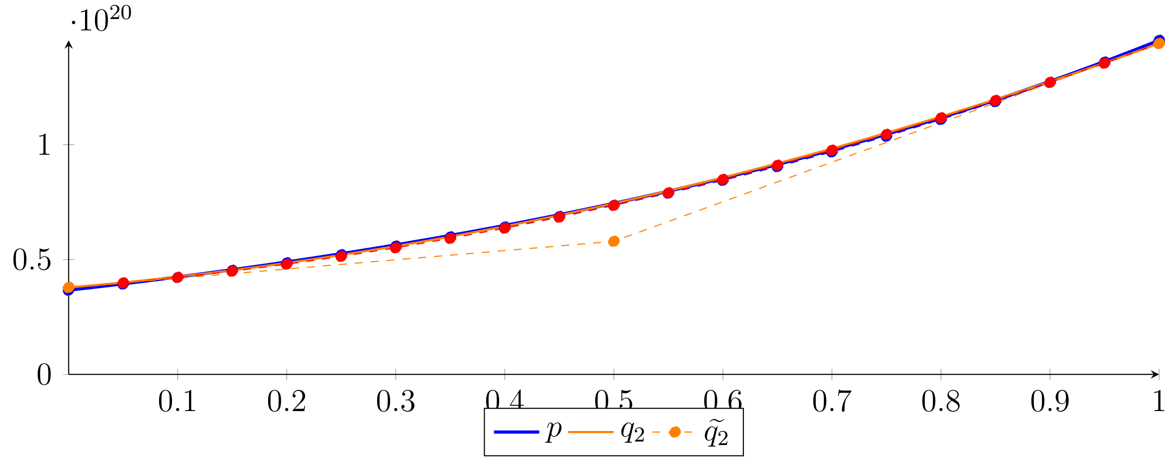
$$p = -6.86839\cdot10^{10}X^{20} + 4.76089\cdot10^{11}X^{19} - 1.91937\cdot10^{12}X^{18} + 8.73876\cdot10^{12}X^{17} - 4.28558\cdot10^{13}X^{16} + 3.24202\cdot10^{13}X^{15} - 1.42134\cdot10^{13}X^{14} - 2.49223\cdot10^{12}X^{13} - 2.78084\cdot10^{13}X^{12} - 1.28305\cdot10^{12}X^{11} - 1.3493\cdot10^{12}X^{10} + 1.08296\cdot10^{14}X^{9} + 1.42828\cdot10^{15}X^{8} + 1.5043\cdot10^{16}X^{7} + 1.25526\cdot10^{17}X^{6} + 8.15789\cdot10^{17}X^{5} + 4.0243\cdot10^{18}X^{4} + 1.44889\cdot10^{19}X^{3} + 3.57208\cdot10^{19}X^{2} + 5.3599\cdot10^{19}X + 3.66749\cdot10^{19}$$
$$= 3.66749\cdot10^{19}B_{0,20}(X) + 3.93548\cdot10^{19}B_{1,20}(X) + 4.22228\cdot10^{19}B_{2,20}(X) + 4.52914\cdot10^{19}B_{3,20}(X) + 4.85744\cdot10^{19}B_{4,20}(X) + 5.2086\cdot10^{19}B_{5,20}(X) + 5.58416\cdot10^{19}B_{6,20}(X) + 5.98576\cdot10^{19}B_{7,20}(X) + 6.41515\cdot10^{19}B_{8,20}(X) + 6.87417\cdot10^{19}B_{9,20}(X) + 7.36481\cdot10^{19}B_{10,20}(X) + 7.88916\cdot10^{19}B_{11,20}(X) + 8.44946\cdot10^{19}B_{12,20}(X) + 9.0481\cdot10^{19}B_{13,20}(X) + 9.68761\cdot10^{19}B_{14,20}(X) + 1.03707\cdot10^{20}B_{15,20}(X) + 1.11002\cdot10^{20}B_{16,20}(X) + 1.18792\cdot10^{20}B_{17,20}(X) + 1.27109\cdot10^{20}B_{18,20}(X) + 1.35988\cdot10^{20}B_{19,20}(X) + 1.45466\cdot10^{20}B_{20,20}(X)$$



Degree reduction and raising:

$$q_2 = 6.60628\cdot10^{19}X^{2} + 4.01894\cdot10^{19}X + 3.78487\cdot10^{19}$$
$$= 3.78487\cdot10^{19}B_{0,2} + 5.79434\cdot10^{19}B_{1,2} + 1.44101\cdot10^{20}B_{2,2}$$

$$\widetilde{q}_2 = -2.00082\cdot10^{21}X^{20} + 2.06333\cdot10^{22}X^{19} - 9.90071\cdot10^{22}X^{18} + 2.92859\cdot10^{23}X^{17} - 5.96321\cdot10^{23}X^{16} + 8.84094\cdot10^{23}X^{15} - 9.84782\cdot10^{23}X^{14} + 8.38808\cdot10^{23}X^{13} - 5.51189\cdot10^{23}X^{12} + 2.80079\cdot10^{23}X^{11} - 1.09723\cdot10^{23}X^{10} + 3.28949\cdot10^{22}X^{9} - 7.48191\cdot10^{21}X^{8} + 1.29091\cdot10^{21}X^{7} - 1.7331\cdot10^{20}X^{6} + 1.89796\cdot10^{19}X^{5} - 1.70107\cdot10^{18}X^{4} + 1.05257\cdot10^{17}X^{3} + 6.60593\cdot10^{19}X^{2} + 4.01895\cdot10^{19}X + 3.78487\cdot10^{19}$$
$$= 3.78487\cdot10^{19}B_{0,20} + 3.98581\cdot10^{19}B_{1,20} + 4.22153\cdot10^{19}B_{2,20} + 4.49202\cdot10^{19}B_{3,20} + 4.79727\cdot10^{19}B_{4,20} + 5.13732\cdot10^{19}B_{5,20} + 5.51202\cdot10^{19}B_{6,20} + 5.92183\cdot10^{19}B_{7,20} + 6.36573\cdot10^{19}B_{8,20} + 6.84534\cdot10^{19}B_{9,20} + 7.35861\cdot10^{19}B_{10,20} + 7.90764\cdot10^{19}B_{11,20} + 8.49082\cdot10^{19}B_{12,20} + 9.10932\cdot10^{19}B_{13,20} + 9.7621\cdot10^{19}B_{14,20} + 1.045\cdot10^{20}B_{15,20} + 1.11724\cdot10^{20}B_{16,20} + 1.19297\cdot10^{20}B_{17,20} + 1.27217\cdot10^{20}B_{18,20} + 1.35485\cdot10^{20}B_{19,20} + 1.44101\cdot10^{20}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.36485 \cdot 10^{18}$.

Bounding polynomials M and m:

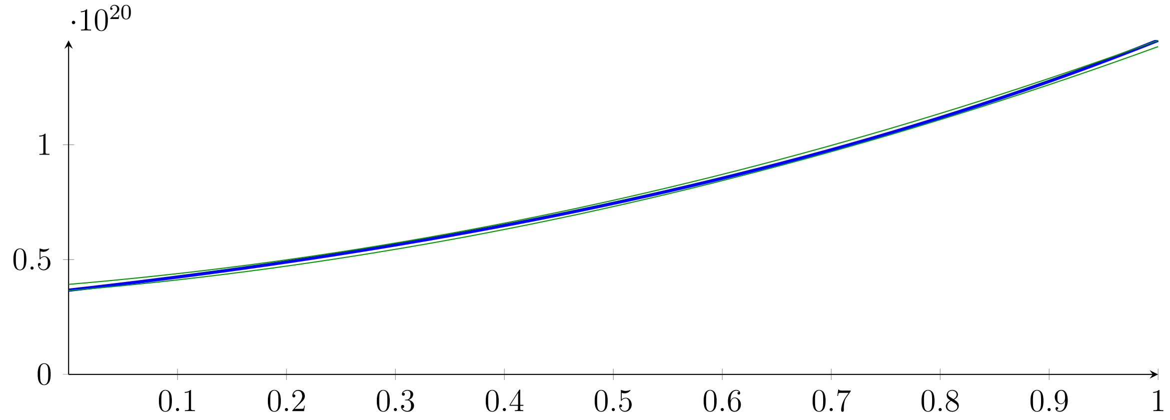
$$M = 6.60628 \cdot 10^{19} X^2 + 4.01894 \cdot 10^{19} X + 3.92135 \cdot 10^{19}$$

$$m = 6.60628 \cdot 10^{19} X^2 + 4.01894 \cdot 10^{19} X + 3.64838 \cdot 10^{19}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{\}$$

Intersection intervals:

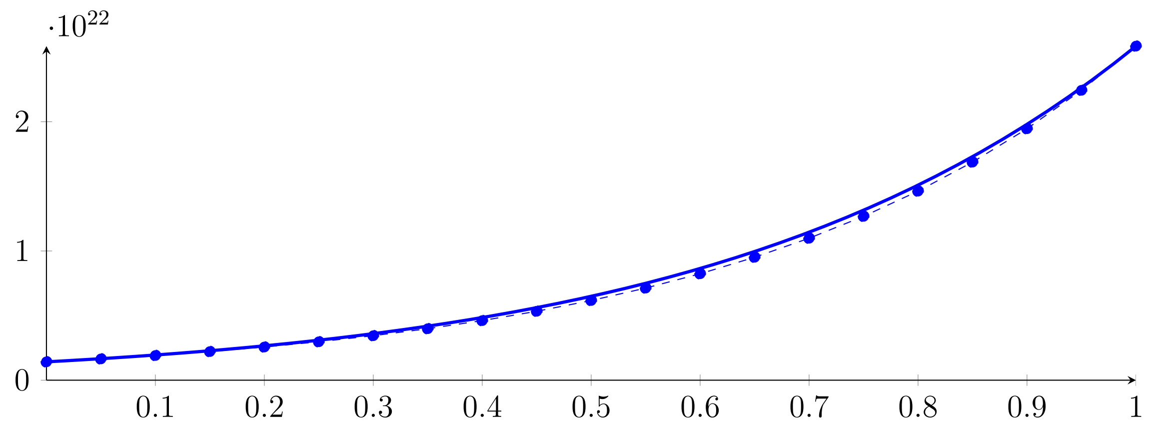


No intersection intervals with the x axis.

2.107 Recursion Branch 1 2 2 2 2 on the Second Half $[23.4375, 25]$

Normalized monomial und Bézier representations and the Bézier polygon:

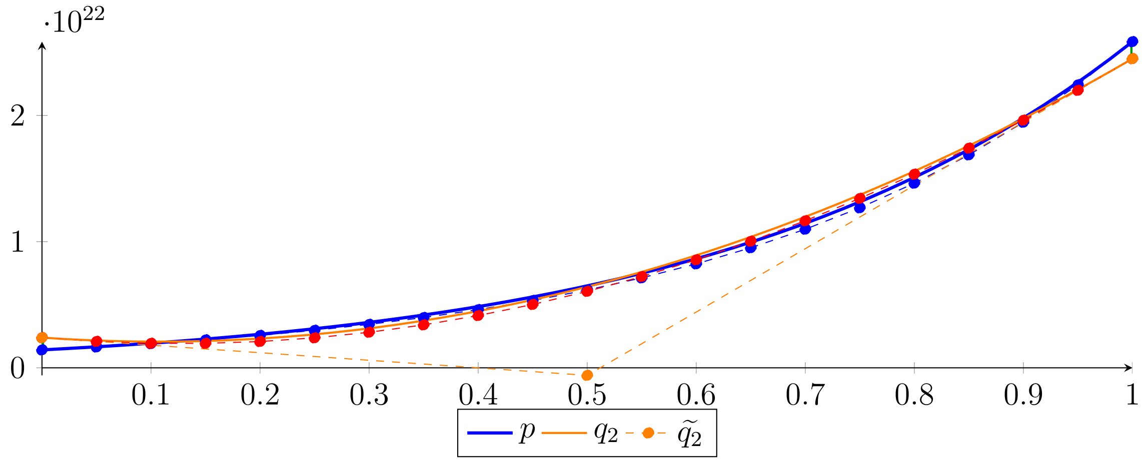
$$\begin{aligned} p = & -5.54586 \cdot 10^{12} X^{20} + 4.52218 \cdot 10^{13} X^{19} - 1.44038 \cdot 10^{14} X^{18} + 7.39598 \cdot 10^{14} X^{17} - 3.2688 \cdot 10^{15} X^{16} \\ & + 2.3812 \cdot 10^{15} X^{15} - 7.8844 \cdot 10^{14} X^{14} + 1.10835 \cdot 10^{15} X^{13} + 1.34875 \cdot 10^{16} X^{12} + 1.53483 \cdot 10^{17} X^{11} \\ & + 1.25414 \cdot 10^{18} X^{10} + 8.35262 \cdot 10^{18} X^{9} + 4.51985 \cdot 10^{19} X^{8} + 1.97596 \cdot 10^{20} X^{7} + 6.9063 \cdot 10^{20} X^{6} + 1.89886 \\ & \cdot 10^{21} X^{5} + 4.00777 \cdot 10^{21} X^{4} + 6.25317 \cdot 10^{21} X^{3} + 6.77942 \cdot 10^{21} X^{2} + 4.54961 \cdot 10^{21} X + 1.41998 \cdot 10^{21} \\ = & \ 1.41998 \cdot 10^{21} B_{0,20}(X) + 1.64746 \cdot 10^{21} B_{1,20}(X) + 1.91062 \cdot 10^{21} B_{2,20}(X) + 2.21495 \\ & \cdot 10^{21} B_{3,20}(X) + 2.56676 \cdot 10^{21} B_{4,20}(X) + 2.97331 \cdot 10^{21} B_{5,20}(X) + 3.44295 \cdot 10^{21} B_{6,20}(X) \\ & + 3.98528 \cdot 10^{21} B_{7,20}(X) + 4.61135 \cdot 10^{21} B_{8,20}(X) + 5.33384 \cdot 10^{21} B_{9,20}(X) + 6.16731 \\ & \cdot 10^{21} B_{10,20}(X) + 7.12849 \cdot 10^{21} B_{11,20}(X) + 8.23659 \cdot 10^{21} B_{12,20}(X) + 9.51366 \cdot 10^{21} B_{13,20}(X) \\ & + 1.0985 \cdot 10^{22} B_{14,20}(X) + 1.26796 \cdot 10^{22} B_{15,20}(X) + 1.46307 \cdot 10^{22} B_{16,20}(X) + 1.68765 \\ & \cdot 10^{22} B_{17,20}(X) + 1.94607 \cdot 10^{22} B_{18,20}(X) + 2.24334 \cdot 10^{22} B_{19,20}(X) + 2.5852 \cdot 10^{22} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = 2.80921\cdot 10^{22}X^2 - 5.9846\cdot 10^{21}X + 2.39352\cdot 10^{21}$$
$$= 2.39352\cdot 10^{21}B_{0,2} - 5.98778\cdot 10^{20}B_{1,2} + 2.4501\cdot 10^{22}B_{2,2}$$

$$\widetilde{q}_2 = 2.86206\cdot 10^{24}X^{20} - 2.85587\cdot 10^{25}X^{19} + 1.31818\cdot 10^{26}X^{18} - 3.73457\cdot 10^{26}X^{17} + 7.26719\cdot 10^{26}X^{16} - 1.02988\cdot 10^{27}X^{15} + 1.09977\cdot 10^{27}X^{14} - 9.03197\cdot 10^{26}X^{13} + 5.77167\cdot 10^{26}X^{12} - 2.8848\cdot 10^{26}X^{11} + 1.12755\cdot 10^{26}X^{10} - 3.42781\cdot 10^{25}X^{9} + 8.01525\cdot 10^{24}X^{8} - 1.41521\cdot 10^{24}X^{7} + 1.83399\cdot 10^{23}X^{6} - 1.67239\cdot 10^{22}X^{5} + 1.0056\cdot 10^{21}X^{4} - 3.60657\cdot 10^{19}X^{3} + 2.80928\cdot 10^{22}X^{2} - 5.98461\cdot 10^{21}X + 2.39352\cdot 10^{21}$$
$$= 2.39352\cdot 10^{21}B_{0,20} + 2.09429\cdot 10^{21}B_{1,20} + 1.94292\cdot 10^{21}B_{2,20} + 1.93937\cdot 10^{21}B_{3,20} + 2.08382\cdot 10^{21}B_{4,20} + 2.37558\cdot 10^{21}B_{5,20} + 2.81673\cdot 10^{21}B_{6,20} + 3.40227\cdot 10^{21}B_{7,20} + 4.14209\cdot 10^{21}B_{8,20} + 5.01968\cdot 10^{21}B_{9,20} + 6.05837\cdot 10^{21}B_{10,20} + 7.23018\cdot 10^{21}B_{11,20} + 8.56364\cdot 10^{21}B_{12,20} + 1.00345\cdot 10^{22}B_{13,20} + 1.16597\cdot 10^{22}B_{14,20} + 1.34294\cdot 10^{22}B_{15,20} + 1.53483\cdot 10^{22}B_{16,20} + 1.74146\cdot 10^{22}B_{17,20} + 1.96289\cdot 10^{22}B_{18,20} + 2.19911\cdot 10^{22}B_{19,20} + 2.4501\cdot 10^{22}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.35097\cdot 10^{21}$.

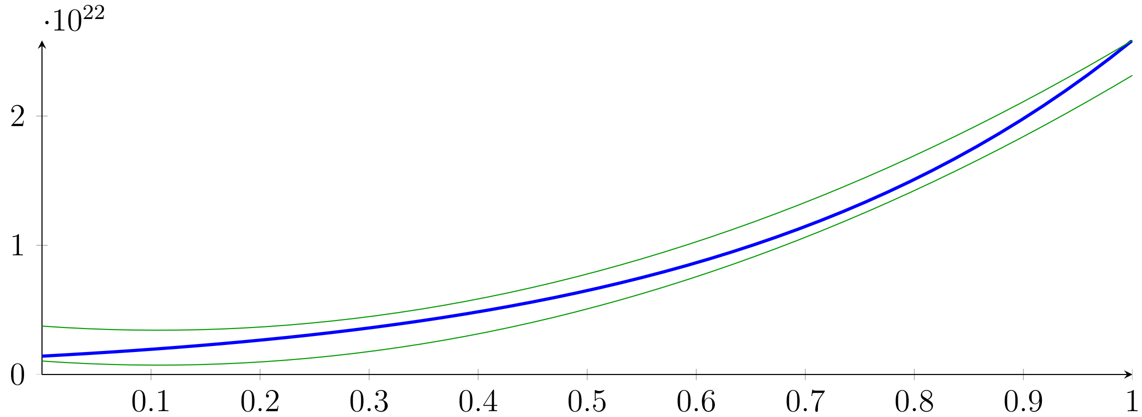
Bounding polynomials M and m:

$$M = 2.80921\cdot 10^{22}X^2 - 5.9846\cdot 10^{21}X + 3.7445\cdot 10^{21}$$
$$m = 2.80921\cdot 10^{22}X^2 - 5.9846\cdot 10^{21}X + 1.04255\cdot 10^{21}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{\}$$

Intersection intervals:

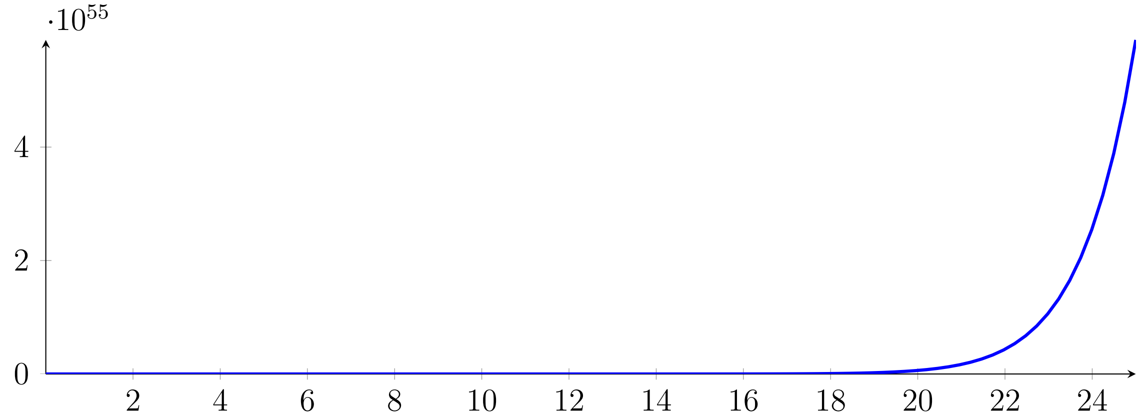


No intersection intervals with the x axis.

2.108 Result: 20 Root Intervals

Input Polynomial on Interval $[0, 25]$

$$p = 9.09495\cdot 10^{27} X^{20} - 7.63976\cdot 10^{28} X^{19} + 2.99988\cdot 10^{29} X^{18} - 7.31583\cdot 10^{29} X^{17} + 1.24164\cdot 10^{30} X^{16} - 1.55743\cdot 10^{30} X^{15} + 1.49652\cdot 10^{30} X^{14} - 1.12669\cdot 10^{30} X^{13} + 6.74145\cdot 10^{29} X^{12} - 3.2326\cdot 10^{29} X^{11} + 1.24696\cdot 10^{29} X^{10} - 3.86898\cdot 10^{28} X^{9} + 9.61774\cdot 10^{27} X^{8} - 1.90023\cdot 10^{27} X^{7} + 2.94592\cdot 10^{26} X^{6} - 3.5156\cdot 10^{25} X^{5} + 3.13977\cdot 10^{24} X^{4} - 2.01108\cdot 10^{23} X^{3} + 8.62735\cdot 10^{21} X^{2} - 2.18824\cdot 10^{20} X + 2.4329\cdot 10^{18}$$



Result: Root Intervals

$[1, 1]$, $[1.99998, 2.00002]$, $[2.99994, 3.0001]$, $[4, 4]$, $[5, 5]$, $[6, 6]$, $[7.00004, 7.00007]$, $[7.99948, 7.99948]$, $[9.00307, 9.00307]$, $[9.98806, 9.98808]$, $[11.0363, 11.0363]$, $[11.9255, 11.9255]$, $[13.1501, 13.1501]$, $[13.8088, 13.8088]$, $[15.2176, 15.2176]$, $[15.8091, 15.8091]$, $[17.0973, 17.0973]$, $[17.9554, 17.9554]$, $[19.0104, 19.0104]$, $[19.9988, 19.9988]$

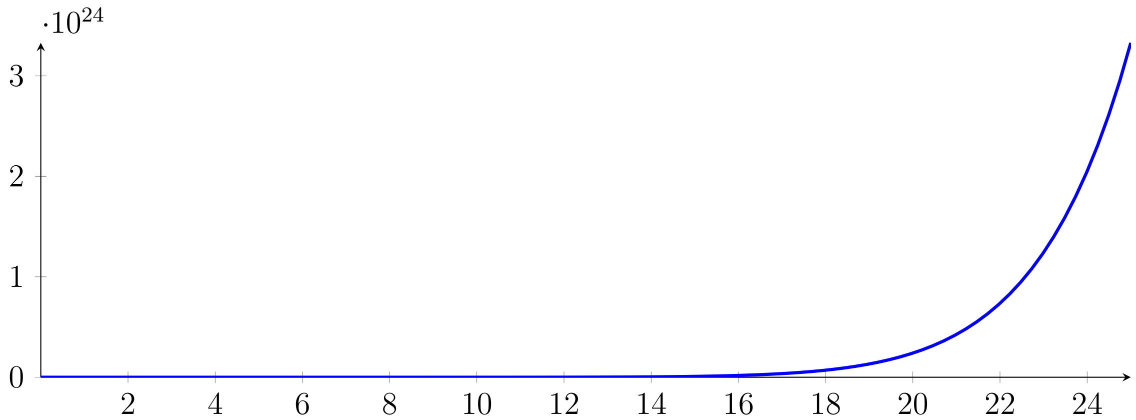
with precision $\varepsilon = 0.001$.

3 CubeClip Applied to the Wilkinson Polynomial

$$1X^{20} - 210X^{19} + 20615X^{18} - 1.25685\cdot 10^{6}X^{17} + 5.33279\cdot 10^{7}X^{16} - 1.67228\cdot 10^{9}X^{15} + 4.01718\cdot 10^{10}X^{14} - 7.56111\cdot 10^{11}X^{13} + 1.13103\cdot 10^{13}X^{12} - 1.35585\cdot 10^{14}X^{11} + 1.30754\cdot 10^{15}X^{10} - 1.01423\cdot 10^{16}X^{9} + 6.30308\cdot 10^{16}X^{8} - 3.11334\cdot 10^{17}X^{7} + 1.20665\cdot 10^{18}X^{6} - 3.59998\cdot 10^{18}X^{5} + 8.03781\cdot 10^{18}X^{4} - 1.28709\cdot 10^{19}X^{3} + 1.38038\cdot 10^{19}X^{2} - 8.75295\cdot 10^{18}X + 2.4329\cdot 10^{18}$$

Called CubeClip with input polynomial on interval $[0, 25]$:

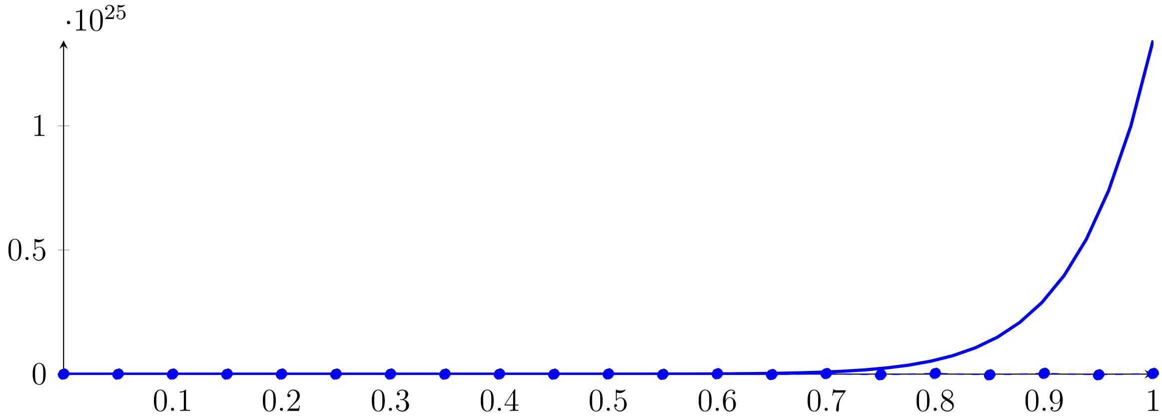
$$p = 1X^{20} - 210X^{19} + 20615X^{18} - 1.25685\cdot 10^{6}X^{17} + 5.33279\cdot 10^{7}X^{16} - 1.67228\cdot 10^{9}X^{15} + 4.01718\cdot 10^{10}X^{14} - 7.56111\cdot 10^{11}X^{13} + 1.13103\cdot 10^{13}X^{12} - 1.35585\cdot 10^{14}X^{11} + 1.30754\cdot 10^{15}X^{10} - 1.01423\cdot 10^{16}X^{9} + 6.30308\cdot 10^{16}X^{8} - 3.11334\cdot 10^{17}X^{7} + 1.20665\cdot 10^{18}X^{6} - 3.59998\cdot 10^{18}X^{5} + 8.03781\cdot 10^{18}X^{4} - 1.28709\cdot 10^{19}X^{3} + 1.38038\cdot 10^{19}X^{2} - 8.75295\cdot 10^{18}X + 2.4329\cdot 10^{18}$$



3.1 Recursion Branch 1 for Input Interval $[0, 25]$

Normalized monomial und Bézier representations and the Bézier polygon:

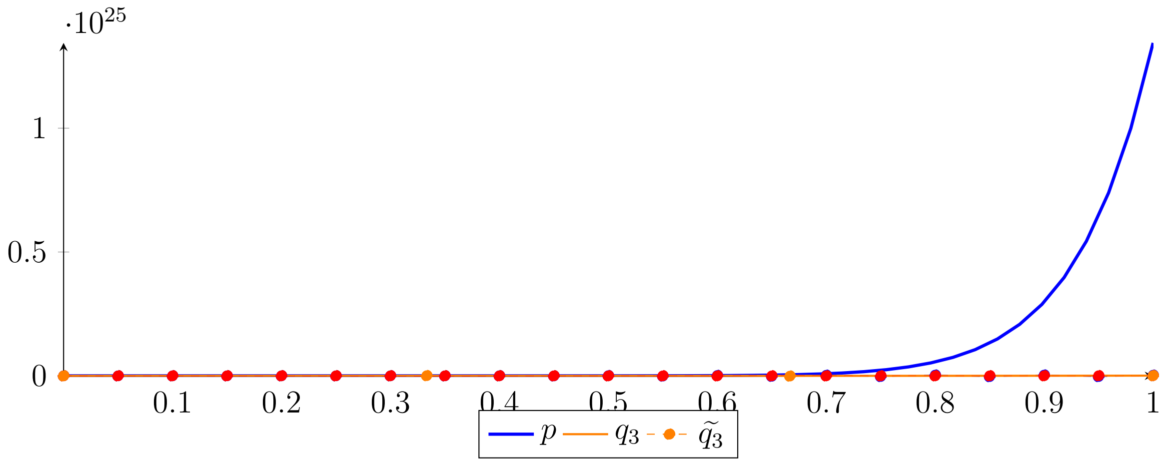
$$\begin{aligned} p &= 9.09495\cdot 10^{27}X^{20} - 7.63976\cdot 10^{28}X^{19} + 2.99988\cdot 10^{29}X^{18} - 7.31583\cdot 10^{29}X^{17} + 1.24164\cdot 10^{30}X^{16} - 1.55743\cdot 10^{30}X^{15} + 1.49652\cdot 10^{30}X^{14} - 1.12669\cdot 10^{30}X^{13} + 6.74145\cdot 10^{29}X^{12} - 3.2326\cdot 10^{29}X^{11} + 1.24696\cdot 10^{29}X^{10} - 3.86898\cdot 10^{28}X^{9} + 9.61774\cdot 10^{27}X^{8} - 1.90023\cdot 10^{27}X^{7} + 2.94592\cdot 10^{26}X^{6} - 3.5156\cdot 10^{25}X^{5} + 3.13977\cdot 10^{24}X^{4} - 2.01108\cdot 10^{23}X^{3} + 8.62735\cdot 10^{21}X^{2} - 2.18824\cdot 10^{20}X + 2.4329\cdot 10^{18} \\ &= 2.4329\cdot 10^{18}B_{0,20}(X) - 8.50828\cdot 10^{18}B_{1,20}(X) + 2.59576\cdot 10^{19}B_{2,20}(X) - 7.05801\cdot 10^{19}B_{3,20}(X) + 1.73511\cdot 10^{20}B_{4,20}(X) - 3.8964\cdot 10^{20}B_{5,20}(X) + 8.05451\cdot 10^{20}B_{6,20}(X) - 1.54188\cdot 10^{21}B_{7,20}(X) + 2.74637\cdot 10^{21}B_{8,20}(X) - 4.56922\cdot 10^{21}B_{9,20}(X) + 7.12322\cdot 10^{21}B_{10,20}(X) - 1.04331\cdot 10^{22}B_{11,20}(X) + 1.43886\cdot 10^{22}B_{12,20}(X) - 1.87204\cdot 10^{22}B_{13,20}(X) + 2.30149\cdot 10^{22}B_{14,20}(X) - 2.67735\cdot 10^{22}B_{15,20}(X) + 2.95071\cdot 10^{22}B_{16,20}(X) - 3.08413\cdot 10^{22}B_{17,20}(X) + 3.06005\cdot 10^{22}B_{18,20}(X) - 2.88452\cdot 10^{22}B_{19,20}(X) + 2.5852\cdot 10^{22}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 6.20951\cdot 10^{22}X^3 - 7.80035\cdot 10^{22}X^2 + 2.54158\cdot 10^{22}X - 1.65689\cdot 10^{21} \\ &= -1.65689\cdot 10^{21}B_{0,3} + 6.81506\cdot 10^{21}B_{1,3} - 1.07142\cdot 10^{22}B_{2,3} + 7.8506\cdot 10^{21}B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= 1.10183\cdot 10^{24}X^{20} - 1.10871\cdot 10^{25}X^{19} + 5.16436\cdot 10^{25}X^{18} - 1.47739\cdot 10^{26}X^{17} + 2.90385\cdot 10^{26}X^{16} \\ &\quad - 4.15665\cdot 10^{26}X^{15} + 4.48154\cdot 10^{26}X^{14} - 3.713\cdot 10^{26}X^{13} + 2.39088\cdot 10^{26}X^{12} - 1.20262\cdot 10^{26}X^{11} \\ &\quad + 4.72542\cdot 10^{25}X^{10} - 1.44354\cdot 10^{25}X^{9} + 3.39411\cdot 10^{24}X^{8} - 6.03985\cdot 10^{23}X^{7} + 7.92851\cdot 10^{22}X^{6} - 7.41094 \\ &\quad \cdot 10^{21}X^{5} + 4.7236\cdot 10^{20}X^{4} + 6.20759\cdot 10^{22}X^{3} - 7.8003\cdot 10^{22}X^{2} + 2.54158\cdot 10^{22}X - 1.65689\cdot 10^{21} \\ &= -1.65689\cdot 10^{21}B_{0,20} - 3.86094\cdot 10^{20}B_{1,20} + 4.74155\cdot 10^{20}B_{2,20} + 9.78314\cdot 10^{20}B_{3,20} + 1.18093 \\ &\quad \cdot 10^{21}B_{4,20} + 1.13618\cdot 10^{21}B_{5,20} + 8.99419\cdot 10^{20}B_{6,20} + 5.23007\cdot 10^{20}B_{7,20} + 6.55485\cdot 10^{19}B_{8,20} \\ &\quad - 4.2533\cdot 10^{20}B_{9,20} - 8.85666\cdot 10^{20}B_{10,20} - 1.27196\cdot 10^{21}B_{11,20} - 1.51909\cdot 10^{21}B_{12,20} \\ &\quad - 1.58138\cdot 10^{21}B_{13,20} - 1.3982\cdot 10^{21}B_{14,20} - 9.18725\cdot 10^{20}B_{15,20} - 8.66673\cdot 10^{19}B_{16,20} \\ &\quad + 1.1517\cdot 10^{21}B_{17,20} + 2.85108\cdot 10^{21}B_{18,20} + 5.06589\cdot 10^{21}B_{19,20} + 7.8506\cdot 10^{21}B_{20,20} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.39111\cdot 10^{22}$.

Bounding polynomials M and m:

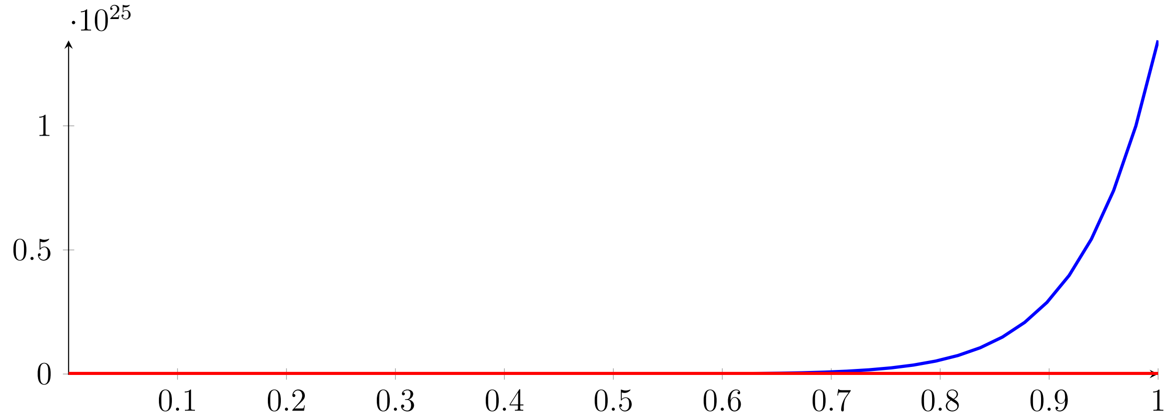
$$M = 6.20951\cdot 10^{22}X^3 - 7.80035\cdot 10^{22}X^2 + 2.54158\cdot 10^{22}X + 3.22542\cdot 10^{22}$$

$$m = 6.20951\cdot 10^{22}X^3 - 7.80035\cdot 10^{22}X^2 + 2.54158\cdot 10^{22}X - 3.5568\cdot 10^{22}$$

Root of M and m:

$$N(M) = \{-0.445194\} \qquad N(m) = \{1.28466\}$$

Intersection intervals:



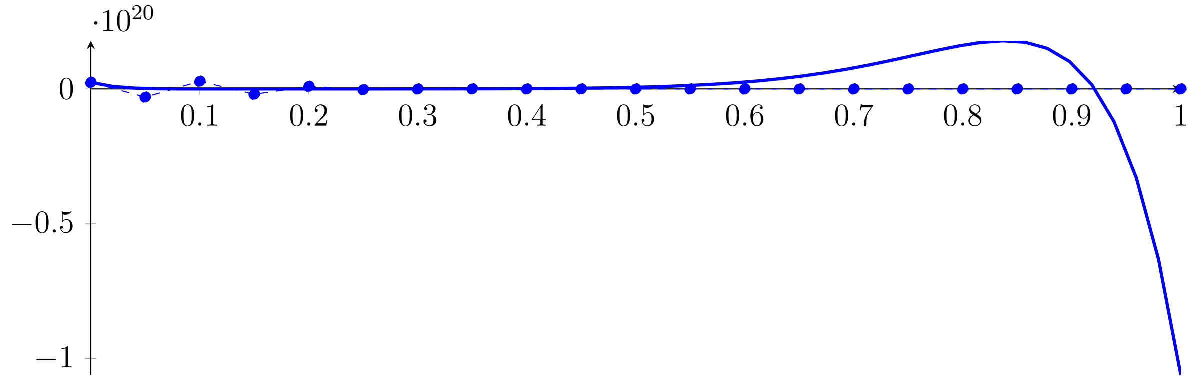
$[0, 1]$

Longest intersection interval: 1
$\Longrightarrow$ Bisection: first half $[0, 12.5]$ und second half $[12.5, 25]$

3.2 Recursion Branch 1 1 on the First Half $[0, 12.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

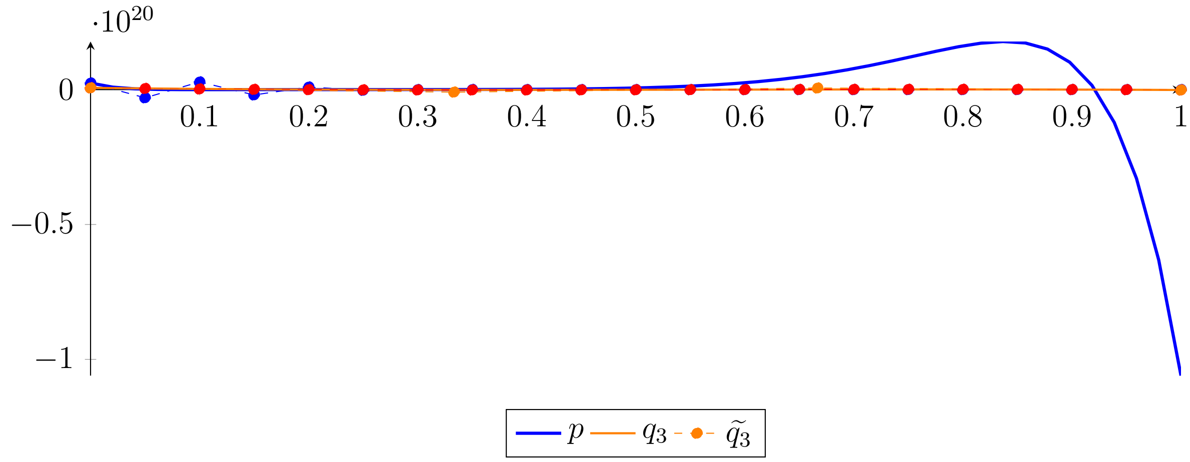
$$\begin{aligned}p &= 8.67362{\cdot}10^{21}X^{20} - 1.45717{\cdot}10^{23}X^{19} + 1.14436{\cdot}10^{24}X^{18} - 5.58154{\cdot}10^{24}X^{17} + 1.89459{\cdot}10^{25}X^{16} \\ &\quad - 4.75291{\cdot}10^{25}X^{15} + 9.134{\cdot}10^{25}X^{14} - 1.37536{\cdot}10^{26}X^{13} + 1.64586{\cdot}10^{26}X^{12} - 1.57842{\cdot}10^{26}X^{11} \\ &\quad + 1.21774{\cdot}10^{26}X^{10} - 7.5566{\cdot}10^{25}X^{9} + 3.75693{\cdot}10^{25}X^{8} - 1.48455{\cdot}10^{25}X^{7} + 4.603{\cdot}10^{24}X^{6} - 1.09863 \\ &\quad {\cdot}10^{24}X^{5} + 1.96236{\cdot}10^{23}X^{4} - 2.51385{\cdot}10^{22}X^{3} + 2.15684{\cdot}10^{21}X^{2} - 1.09412{\cdot}10^{20}X + 2.4329{\cdot}10^{18} \\ &= 2.4329{\cdot}10^{18}B_{0,20}(X) - 3.03769{\cdot}10^{18}B_{1,20}(X) + 2.84349{\cdot}10^{18}B_{2,20}(X) - 1.9749 \\ &\quad {\cdot}10^{18}B_{3,20}(X) + 9.58506{\cdot}10^{17}B_{4,20}(X) - 2.63073{\cdot}10^{17}B_{5,20}(X) - 9.0343{\cdot}10^{15}B_{6,20}(X) \\ &\quad + 3.44399{\cdot}10^{16}B_{7,20}(X) - 5.41351{\cdot}10^{15}B_{8,20}(X) - 4.28958{\cdot}10^{15}B_{9,20}(X) + 1.09675 \\ &\quad {\cdot}10^{15}B_{10,20}(X) + 6.89924{\cdot}10^{14}B_{11,20}(X) - 1.57583{\cdot}10^{14}B_{12,20}(X) - 1.3719{\cdot}10^{14}B_{13,20}(X) \\ &\quad + 1.13888{\cdot}10^{13}B_{14,20}(X) + 2.83586{\cdot}10^{13}B_{15,20}(X) + 3.54186{\cdot}10^{12}B_{16,20}(X) - 4.9643 \\ &\quad {\cdot}10^{12}B_{17,20}(X) - 2.0514{\cdot}10^{12}B_{18,20}(X) + 5.37337{\cdot}10^{11}B_{19,20}(X) + 8.1419{\cdot}10^{11}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= -5.34664{\cdot}10^{18}X^3 + 9.2856{\cdot}10^{18}X^2 - 4.74379{\cdot}10^{18}X + 6.59851{\cdot}10^{17} \\ &= 6.59851{\cdot}10^{17}B_{0,3} - 9.21412{\cdot}10^{17}B_{1,3} + 5.92527{\cdot}10^{17}B_{2,3} - 1.44971{\cdot}10^{17}B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= -1.29076\cdot10^{20}X^{20} + 1.2974\cdot10^{21}X^{19} - 6.03603\cdot10^{21}X^{18} + 1.72453\cdot10^{22}X^{17} - 3.38509\cdot10^{22}X^{16} \\ &\quad + 4.83901\cdot10^{22}X^{15} - 5.21037\cdot10^{22}X^{14} + 4.31139\cdot10^{22}X^{13} - 2.77279\cdot10^{22}X^{12} + 1.39295\cdot10^{22}X^{11} \\ &\quad - 5.46512\cdot10^{21}X^{10} + 1.66631\cdot10^{21}X^{9} - 3.90825\cdot10^{20}X^{8} + 6.93475\cdot10^{19}X^{7} - 9.07468\cdot10^{18}X^{6} + 8.43869 \\ &\quad \cdot10^{17}X^{5} - 5.29511\cdot10^{16}X^{4} - 5.34456\cdot10^{18}X^{3} + 9.28556\cdot10^{18}X^{2} - 4.74379\cdot10^{18}X + 6.59851\cdot10^{17} \\ &= 6.59851\cdot10^{17}B_{0,20} + 4.22661\cdot10^{17}B_{1,20} + 2.34343\cdot10^{17}B_{2,20} + 9.02086\cdot10^{16}B_{3,20} - 1.44423 \\ &\quad \cdot10^{16}B_{4,20} - 8.42647\cdot10^{16}B_{5,20} - 1.24051\cdot10^{17}B_{6,20} - 1.38247\cdot10^{17}B_{7,20} - 1.32024\cdot10^{17}B_{8,20} \\ &\quad - 1.09277\cdot10^{17}B_{9,20} - 7.57979\cdot10^{16}B_{10,20} - 3.4996\cdot10^{16}B_{11,20} + 7.18373\cdot10^{15}B_{12,20} \\ &\quad + 4.70916\cdot10^{16}B_{13,20} + 7.93089\cdot10^{16}B_{14,20} + 9.95759\cdot10^{16}B_{15,20} + 1.0299\cdot10^{17}B_{16,20} \\ &\quad + 8.4947\cdot10^{16}B_{17,20} + 4.07292\cdot10^{16}B_{18,20} - 3.43463\cdot10^{16}B_{19,20} - 1.44971\cdot10^{17}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.46035\cdot10^{18}$.

Bounding polynomials M and m:

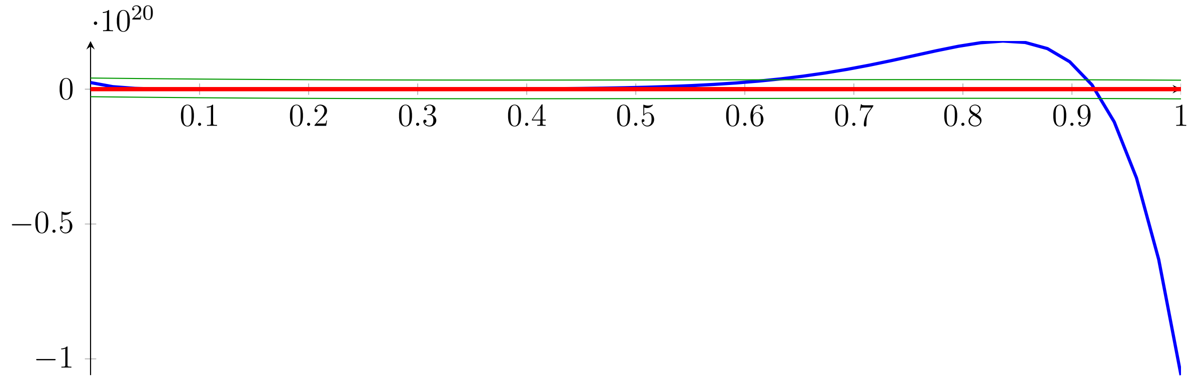
$$M = -5.34664\cdot10^{18}X^3 + 9.2856\cdot10^{18}X^2 - 4.74379\cdot10^{18}X + 4.1202\cdot10^{18}$$

$$m = -5.34664\cdot10^{18}X^3 + 9.2856\cdot10^{18}X^2 - 4.74379\cdot10^{18}X - 2.8005\cdot10^{18}$$

Root of M and m:

$$N(M) = \{1.48846\} \qquad N(m) = \{-0.332505\}$$

Intersection intervals:



$$[0, 1]$$

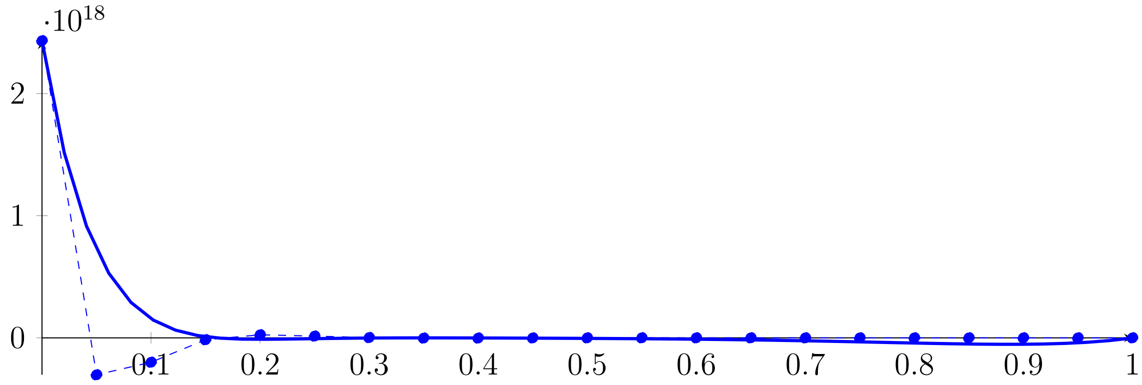
Longest intersection interval: 1

$\Longrightarrow$ Bisection: first half $[0, 6.25]$ und second half $[6.25, 12.5]$

3.3 Recursion Branch 1 1 1 on the First Half $[0, 6.25]$

Normalized monomial und Bézier representations and the Bézier polygon:

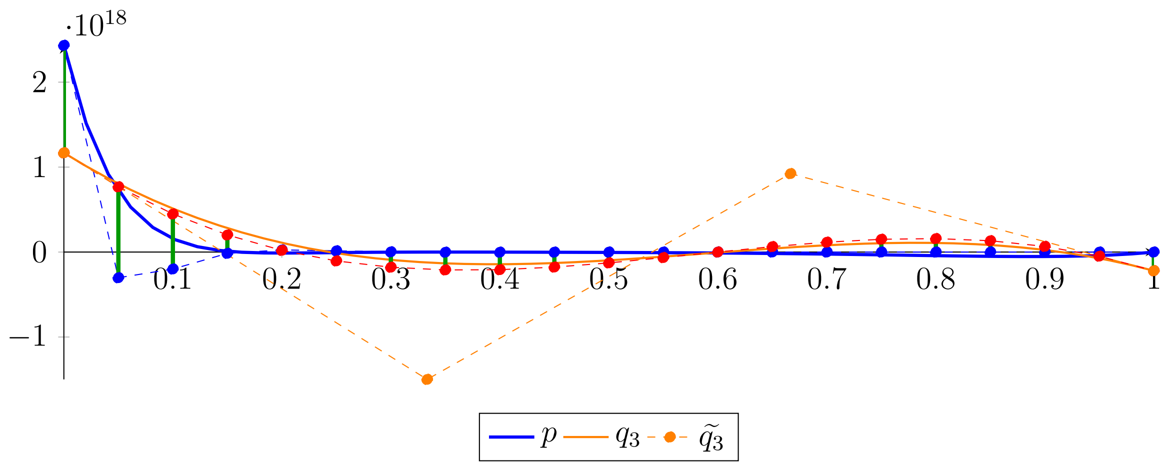
$$p = 8.27181{\cdot}10^{15}X^{20} - 2.77933{\cdot}10^{17}X^{19} + 4.3654{\cdot}10^{18}X^{18} - 4.25837{\cdot}10^{19}X^{17} + 2.89091{\cdot}10^{20}X^{16} - 1.45047{\cdot}10^{21}X^{15} + 5.57495{\cdot}10^{21}X^{14} - 1.6789{\cdot}10^{22}X^{13} + 4.01822{\cdot}10^{22}X^{12} - 7.70713{\cdot}10^{22}X^{11} + 1.1892{\cdot}10^{23}X^{10} - 1.4759{\cdot}10^{23}X^{9} + 1.46755{\cdot}10^{23}X^{8} - 1.15981{\cdot}10^{23}X^{7} + 7.19218{\cdot}10^{22}X^{6} - 3.43321{\cdot}10^{22}X^{5} + 1.22647{\cdot}10^{22}X^{4} - 3.14232{\cdot}10^{21}X^{3} + 5.39209{\cdot}10^{20}X^{2} - 5.47059{\cdot}10^{19}X + 2.4329{\cdot}10^{18}$$
$$= 2.4329{\cdot}10^{18}B_{0,20}(X) - 3.02394{\cdot}10^{17}B_{1,20}(X) - 1.99746{\cdot}10^{17}B_{2,20}(X) - 1.55733{\cdot}10^{16}B_{3,20}(X) + 2.51263{\cdot}10^{16}B_{4,20}(X) + 1.43711{\cdot}10^{16}B_{5,20}(X) + 2.36483{\cdot}10^{15}B_{6,20}(X) - 1.91069{\cdot}10^{15}B_{7,20}(X) - 1.81457{\cdot}10^{15}B_{8,20}(X) - 7.4091{\cdot}10^{14}B_{9,20}(X) - 3.15634{\cdot}10^{13}B_{10,20}(X) + 1.92739{\cdot}10^{14}B_{11,20}(X) + 1.62719{\cdot}10^{14}B_{12,20}(X) + 7.31276{\cdot}10^{13}B_{13,20}(X) + 9.11723{\cdot}10^{12}B_{14,20}(X) - 1.65546{\cdot}10^{13}B_{15,20}(X) - 1.79828{\cdot}10^{13}B_{16,20}(X) - 1.06656{\cdot}10^{13}B_{17,20}(X) - 3.51597{\cdot}10^{12}B_{18,20}(X) + 5.61716{\cdot}10^{11}B_{19,20}(X) + 1.86285{\cdot}10^{12}B_{20,20}(X)$$



Degree reduction and raising:

$$q_3 = -8.6532{\cdot}10^{18}X^3 + 1.52653{\cdot}10^{19}X^2 - 8.00002{\cdot}10^{18}X + 1.16789{\cdot}10^{18}$$
$$= 1.16789{\cdot}10^{18}B_{0,3} - 1.49878{\cdot}10^{18}B_{1,3} + 9.2296{\cdot}10^{17}B_{2,3} - 2.20078{\cdot}10^{17}B_{3,3}$$

$$\tilde{q}_3 = -2.16694{\cdot}10^{20}X^{20} + 2.17787{\cdot}10^{21}X^{19} - 1.01313{\cdot}10^{22}X^{18} + 2.89425{\cdot}10^{22}X^{17} - 5.68052{\cdot}10^{22}X^{16} + 8.11946{\cdot}10^{22}X^{15} - 8.74171{\cdot}10^{22}X^{14} + 7.23284{\cdot}10^{22}X^{13} - 4.65135{\cdot}10^{22}X^{12} + 2.33654{\cdot}10^{22}X^{11} - 9.16673{\cdot}10^{21}X^{10} + 2.7947{\cdot}10^{21}X^{9} - 6.55402{\cdot}10^{20}X^{8} + 1.16276{\cdot}10^{20}X^{7} - 1.52146{\cdot}10^{19}X^{6} + 1.41497{\cdot}10^{18}X^{5} - 8.87931{\cdot}10^{16}X^{4} - 8.64971{\cdot}10^{18}X^{3} + 1.52652{\cdot}10^{19}X^{2} - 8.00002{\cdot}10^{18}X + 1.16789{\cdot}10^{18}$$
$$= 1.16789{\cdot}10^{18}B_{0,20} + 7.67889{\cdot}10^{17}B_{1,20} + 4.48231{\cdot}10^{17}B_{2,20} + 2.01329{\cdot}10^{17}B_{3,20} + 1.95761{\cdot}10^{16}B_{4,20} - 1.0456{\cdot}10^{17}B_{5,20} - 1.7884{\cdot}10^{17}B_{6,20} - 2.10447{\cdot}10^{17}B_{7,20} - 2.07777{\cdot}10^{17}B_{8,20} - 1.77088{\cdot}10^{17}B_{9,20} - 1.27819{\cdot}10^{17}B_{10,20} - 6.54107{\cdot}10^{16}B_{11,20} + 4.39255{\cdot}10^{14}B_{12,20} + 6.38876{\cdot}10^{16}B_{13,20} + 1.1612{\cdot}10^{17}B_{14,20} + 1.50268{\cdot}10^{17}B_{15,20} + 1.58384{\cdot}10^{17}B_{16,20} + 1.33023{\cdot}10^{17}B_{17,20} + 6.65474{\cdot}10^{16}B_{18,20} - 4.8622{\cdot}10^{16}B_{19,20} - 2.20078{\cdot}10^{17}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.26501 \cdot 10^{18}$.

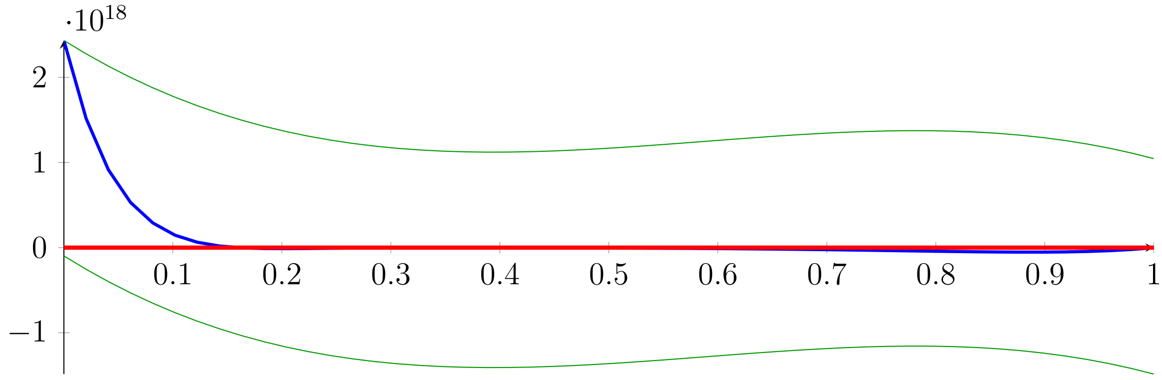
Bounding polynomials M and m:

$$M = -8.6532 \cdot 10^{18} X^3 + 1.52653 \cdot 10^{19} X^2 - 8.00002 \cdot 10^{18} X + 2.4329 \cdot 10^{18}$$
$$m = -8.6532 \cdot 10^{18} X^3 + 1.52653 \cdot 10^{19} X^2 - 8.00002 \cdot 10^{18} X - 9.7121 \cdot 10^{16}$$

Root of M and m:

$$N(M) = \{1.18376\} \qquad N(m) = \{-0.0118695\}$$

Intersection intervals:



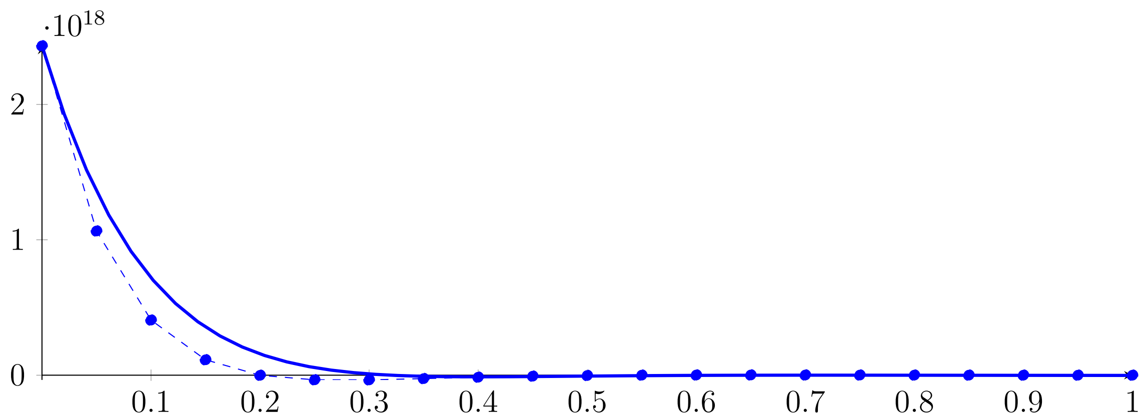
$[0, 1]$

Longest intersection interval: 1
$\Longrightarrow$ Bisection: first half $[0, 3.125]$ und second half $[3.125, 6.25]$
Bisection point is very near to a root?!?

3.4 Recursion Branch 1 1 1 1 on the First Half $[0, 3.125]$

Normalized monomial und Bézier representations and the Bézier polygon:

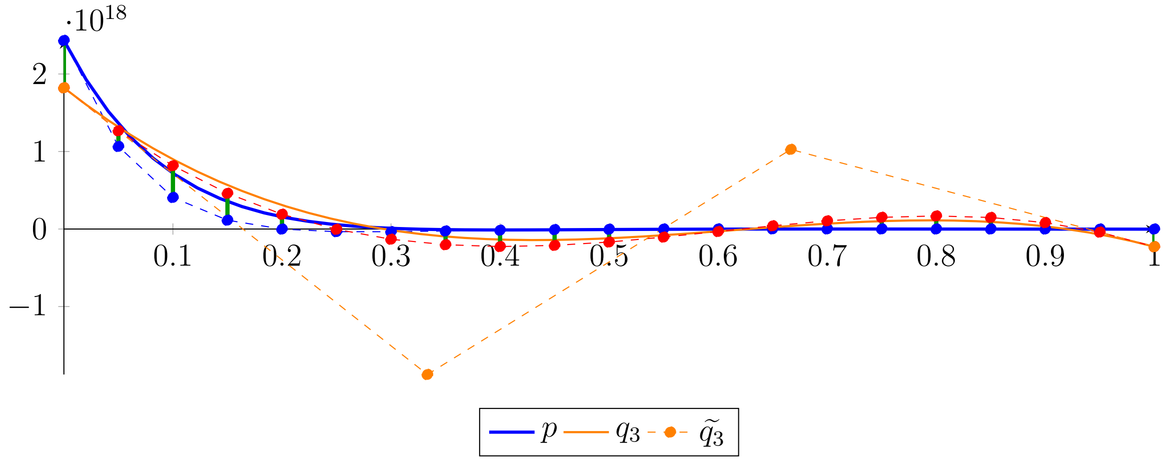
$$\begin{aligned} p = {} & 7.89961 \cdot 10^9 X^{20} - 5.30084 \cdot 10^{11} X^{19} + 1.66534 \cdot 10^{13} X^{18} - 3.24889 \cdot 10^{14} X^{17} + 4.41119 \cdot 10^{15} X^{16} \\ & - 4.42649 \cdot 10^{16} X^{15} + 3.40268 \cdot 10^{17} X^{14} - 2.04944 \cdot 10^{18} X^{13} + 9.8101 \cdot 10^{18} X^{12} - 3.76324 \cdot 10^{19} X^{11} \\ & + 1.16132 \cdot 10^{20} X^{10} - 2.88261 \cdot 10^{20} X^9 + 5.73262 \cdot 10^{20} X^8 - 9.061 \cdot 10^{20} X^7 + 1.12378 \cdot 10^{21} X^6 - 1.07288 \\ & \cdot 10^{21} X^5 + 7.66545 \cdot 10^{20} X^4 - 3.9279 \cdot 10^{20} X^3 + 1.34802 \cdot 10^{20} X^2 - 2.7353 \cdot 10^{19} X + 2.4329 \cdot 10^{18} \\ = {} & 2.4329 \cdot 10^{18} B_{0,20}(X) + 1.06525 \cdot 10^{18} B_{1,20}(X) + 4.07092 \cdot 10^{17} B_{2,20}(X) + 1.13863 \\ & \cdot 10^{17} B_{3,20}(X) - 7.70051 \cdot 10^{14} B_{4,20}(X) - 3.41333 \cdot 10^{16} B_{5,20}(X) - 3.47444 \cdot 10^{16} B_{6,20}(X) \\ & - 2.52167 \cdot 10^{16} B_{7,20}(X) - 1.49942 \cdot 10^{16} B_{8,20}(X) - 7.22308 \cdot 10^{15} B_{9,20}(X) - 2.31656 \\ & \cdot 10^{15} B_{10,20}(X) + 2.94801 \cdot 10^{14} B_{11,20}(X) + 1.37334 \cdot 10^{15} B_{12,20}(X) + 1.56871 \cdot 10^{15} B_{13,20}(X) \\ & + 1.33924 \cdot 10^{15} B_{14,20}(X) + 9.67327 \cdot 10^{14} B_{15,20}(X) + 6.03998 \cdot 10^{14} B_{16,20}(X) + 3.14379 \\ & \cdot 10^{14} B_{17,20}(X) + 1.13755 \cdot 10^{14} B_{18,20}(X) - 7.46015 \cdot 10^{12} B_{19,20}(X) - 6.82353 \cdot 10^{13} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_3 = -1.07626\cdot10^{19}X^3 + 1.98088\cdot10^{19}X^2 - 1.1097\cdot10^{19}X + 1.82222\cdot10^{18}$$
$$= 1.82222\cdot10^{18}B_{0,3} - 1.87678\cdot10^{18}B_{1,3} + 1.02716\cdot10^{18}B_{2,3} - 2.2856\cdot10^{17}B_{3,3}$$

$$\tilde{q}_3 = -2.99006\cdot10^{20}X^{20} + 3.00434\cdot10^{21}X^{19} - 1.39721\cdot10^{22}X^{18} + 3.99033\cdot10^{22}X^{17} - 7.82957\cdot10^{22}X^{16} + 1.11882\cdot10^{23}X^{15} - 1.20425\cdot10^{23}X^{14} + 9.96178\cdot10^{22}X^{13} - 6.40518\cdot10^{22}X^{12} + 3.21709\cdot10^{22}X^{11} - 1.26194\cdot10^{22}X^{10} + 3.8465\cdot10^{21}X^9 - 9.01761\cdot10^{20}X^8 + 1.59921\cdot10^{20}X^7 - 2.09224\cdot10^{19}X^6 + 1.94664\cdot10^{18}X^5 - 1.22242\cdot10^{17}X^4 - 1.07578\cdot10^{19}X^3 + 1.98087\cdot10^{19}X^2 - 1.1097\cdot10^{19}X + 1.82222\cdot10^{18}$$
$$= 1.82222\cdot10^{18}B_{0,20} + 1.26737\cdot10^{18}B_{1,20} + 8.16777\cdot10^{17}B_{2,20} + 4.61003\cdot10^{17}B_{3,20} + 1.90587\cdot10^{17}B_{4,20} - 3.8331\cdot10^{15}B_{5,20} - 1.31932\cdot10^{17}B_{6,20} - 2.02591\cdot10^{17}B_{7,20} - 2.26358\cdot10^{17}B_{8,20} - 2.10839\cdot10^{17}B_{9,20} - 1.68024\cdot10^{17}B_{10,20} - 1.04386\cdot10^{17}B_{11,20} - 3.22758\cdot10^{16}B_{12,20} + 4.12787\cdot10^{16}B_{13,20} + 1.05144\cdot10^{17}B_{14,20} + 1.50881\cdot10^{17}B_{15,20} + 1.68553\cdot10^{17}B_{16,20} + 1.48921\cdot10^{17}B_{17,20} + 8.2477\cdot10^{16}B_{18,20} - 4.02019\cdot10^{16}B_{19,20} - 2.2856\cdot10^{17}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 6.10681\cdot10^{17}$.

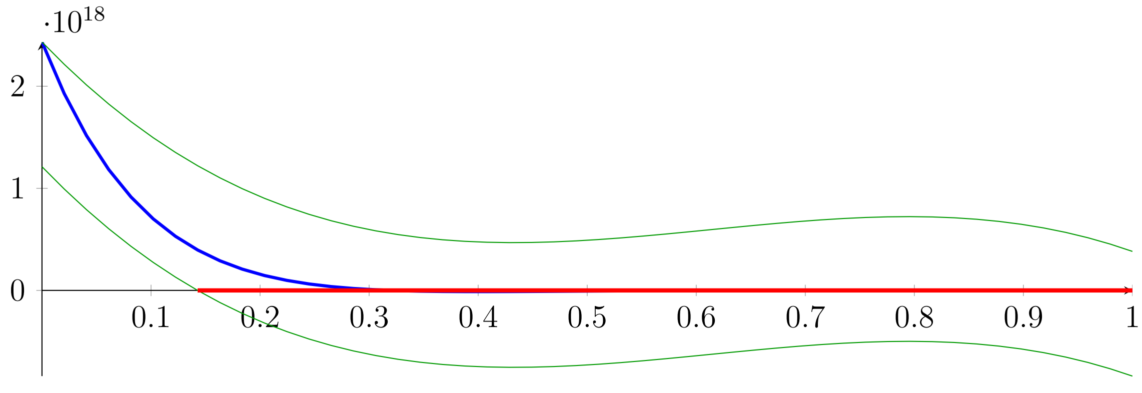
Bounding polynomials M and m:

$$M = -1.07626\cdot10^{19}X^3 + 1.98088\cdot10^{19}X^2 - 1.1097\cdot10^{19}X + 2.4329\cdot10^{18}$$
$$m = -1.07626\cdot10^{19}X^3 + 1.98088\cdot10^{19}X^2 - 1.1097\cdot10^{19}X + 1.21154\cdot10^{18}$$

Root of M and m:

$$N(M) = \{1.07922\} \qquad N(m) = \{0.142716\}$$

Intersection intervals:



$[0.142716, 1]$

Longest intersection interval: 0.857284

$\Longrightarrow$ Bisection: first half $[0, 1.5625]$ und second half $[1.5625, 3.125]$

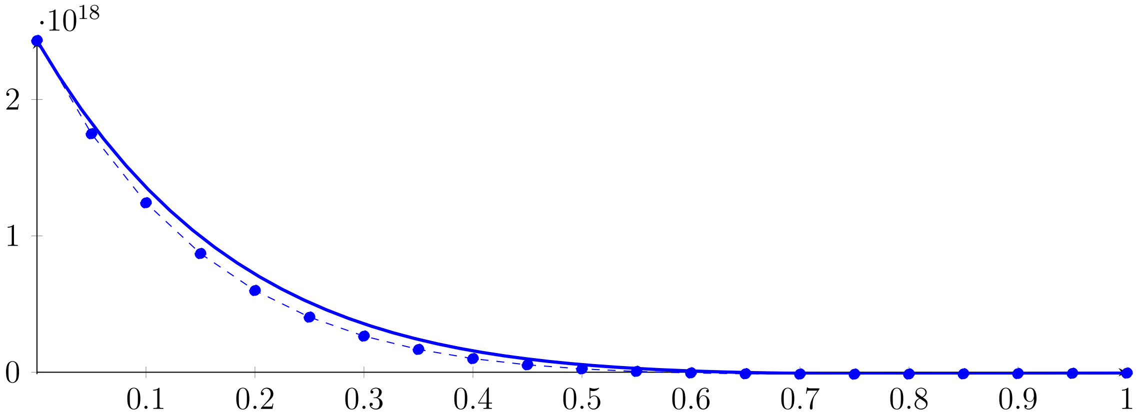
Bisection point is very near to a root?!?

3.5 Recursion Branch 1 1 1 1 1 on the First Half $[0, 1.5625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -8.46356\cdot10^{7}X^{20} - 1.83419\cdot10^{8}X^{19} - 5.89672\cdot10^{9}X^{18} - 1.44753\cdot10^{8}X^{17} - 9.10891\cdot10^{9}X^{16} - 1.29397\cdot10^{12}X^{15} + 2.06942\cdot10^{13}X^{14} - 2.50213\cdot10^{14}X^{13} + 2.39489\cdot10^{15}X^{12} - 1.83753\cdot10^{16}X^{11} + 1.13411\cdot10^{17}X^{10} - 5.63011\cdot10^{17}X^{9} + 2.2393\cdot10^{18}X^{8} - 7.07891\cdot10^{18}X^{7} + 1.7559\cdot10^{19}X^{6} - 3.35274\cdot10^{19}X^{5} + 4.79091\cdot10^{19}X^{4} - 4.90987\cdot10^{19}X^{3} + 3.37006\cdot10^{19}X^{2} - 1.36765\cdot10^{19}X + 2.4329\cdot10^{18}$$

$$= 2.4329\cdot10^{18}B_{0,20}(X) + 1.74908\cdot10^{18}B_{1,20}(X) + 1.24263\cdot10^{18}B_{2,20}(X) + 8.70475\cdot10^{17}B_{3,20}(X) + 5.99447\cdot10^{17}B_{4,20}(X) + 4.04086\cdot10^{17}B_{5,20}(X) + 2.64953\cdot10^{17}B_{6,20}(X) + 1.67278\cdot10^{17}B_{7,20}(X) + 9.9902\cdot10^{16}B_{8,20}(X) + 5.44408\cdot10^{16}B_{9,20}(X) + 2.46418\cdot10^{16}B_{10,20}(X) + 5.87625\cdot10^{15}B_{11,20}(X) - 5.2528\cdot10^{15}B_{12,20}(X) - 1.12129\cdot10^{16}B_{13,20}(X) - 1.37757\cdot10^{16}B_{14,20}(X) - 1.41949\cdot10^{16}B_{15,20}(X) - 1.33428\cdot10^{16}B_{16,20}(X) - 1.1813\cdot10^{16}B_{17,20}(X) - 9.99781\cdot10^{15}B_{18,20}(X) - 8.1465\cdot10^{15}B_{19,20}(X) - 6.40794\cdot10^{15}B_{20,20}(X)$$

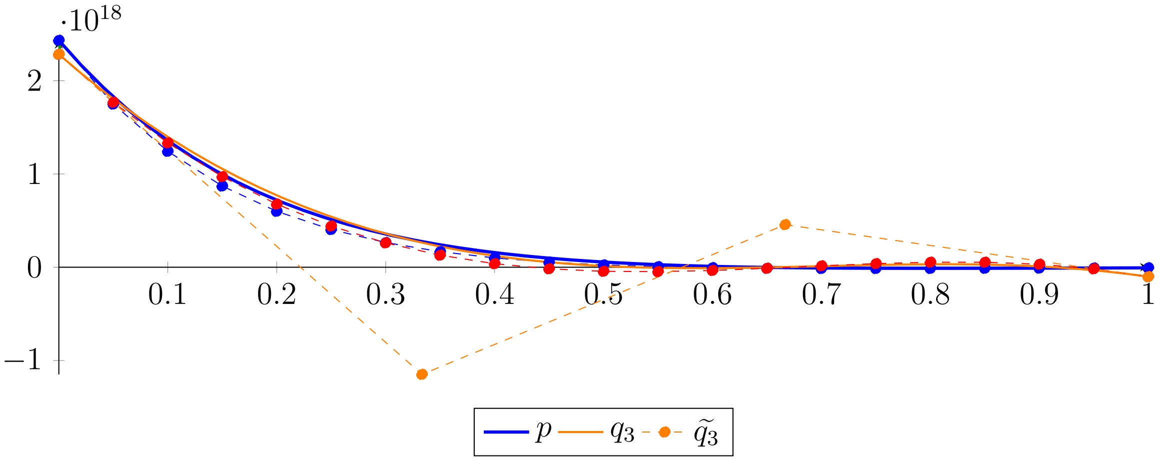


Degree reduction and raising:

$$q_3 = -7.20092\cdot10^{18}X^{3} + 1.51133\cdot10^{19}X^{2} - 1.02969\cdot10^{19}X + 2.28339\cdot10^{18}$$
$$= 2.28339\cdot10^{18}B_{0,3} - 1.14892\cdot10^{18}B_{1,3} + 4.56528\cdot10^{17}B_{2,3} - 1.0118\cdot10^{17}B_{3,3}$$

$$\widetilde{q}_3 = -2.91328\cdot10^{20}X^{20} + 2.92501\cdot10^{21}X^{19} - 1.3593\cdot10^{22}X^{18} + 3.87915\cdot10^{22}X^{17} - 7.60581\cdot10^{22}X^{16} + 1.08608\cdot10^{23}X^{15} - 1.16827\cdot10^{23}X^{14} + 9.65872\cdot10^{22}X^{13} - 6.20747\cdot10^{22}X^{12} + 3.11655\cdot10^{22}X^{11} - 1.22195\cdot10^{22}X^{10} + 3.72201\cdot10^{21}X^{9} - 8.71653\cdot10^{20}X^{8} + 1.5441\cdot10^{20}X^{7} - 2.0202\cdot10^{19}X^{6} + 1.88516\cdot10^{18}X^{5} - 1.19185\cdot10^{17}X^{4} - 7.19617\cdot10^{18}X^{3} + 1.51132\cdot10^{19}X^{2} - 1.02969\cdot10^{19}X + 2.28339\cdot10^{18}$$

$$= 2.28339\cdot10^{18}B_{0,20} + 1.76855\cdot10^{18}B_{1,20} + 1.33324\cdot10^{18}B_{2,20} + 9.71169\cdot10^{17}B_{3,20} + 6.75989\cdot10^{17}B_{4,20} + 4.41463\cdot10^{17}B_{5,20} + 2.61048\cdot10^{17}B_{6,20} + 1.28969\cdot10^{17}B_{7,20} + 3.78356\cdot10^{16}B_{8,20} - 1.68925\cdot10^{16}B_{9,20} - 4.4001\cdot10^{16}B_{10,20} - 4.69171\cdot10^{16}B_{11,20} - 3.48005\cdot10^{16}B_{12,20} - 1.16076\cdot10^{16}B_{13,20} + 1.46849\cdot10^{16}B_{14,20} + 3.87466\cdot10^{16}B_{15,20} + 5.37715\cdot10^{16}B_{16,20} + 5.36422\cdot10^{16}B_{17,20} + 3.1977\cdot10^{16}B_{18,20} - 1.75237\cdot10^{16}B_{19,20} - 1.0118\cdot10^{17}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.49509{\cdot}10^{17}$.

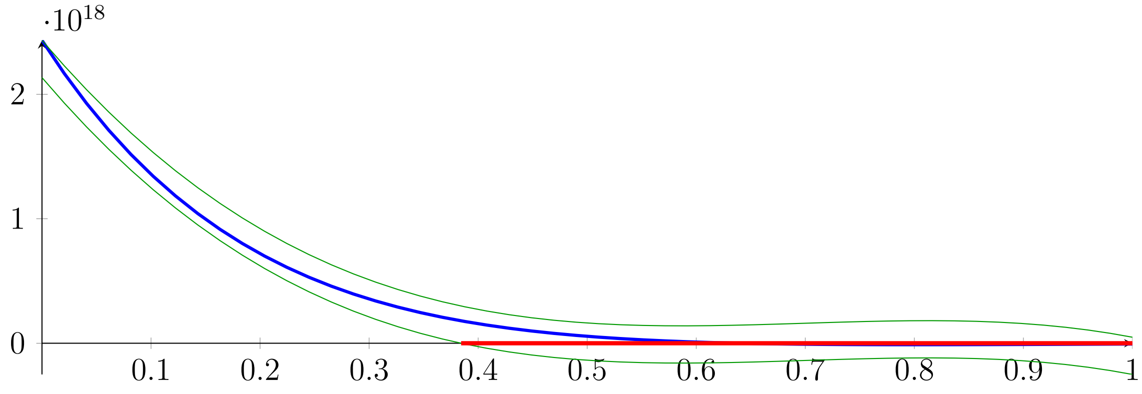
Bounding polynomials M and m:

$$M = -7.20092{\cdot}10^{18}X^3 + 1.51133{\cdot}10^{19}X^2 - 1.02969{\cdot}10^{19}X + 2.4329{\cdot}10^{18}$$
$$m = -7.20092{\cdot}10^{18}X^3 + 1.51133{\cdot}10^{19}X^2 - 1.02969{\cdot}10^{19}X + 2.13388{\cdot}10^{18}$$

Root of M and m:

$$N(M) = \{1.02616\} \qquad N(m) = \{0.384355\}$$

Intersection intervals:



$[0.384355, 1]$

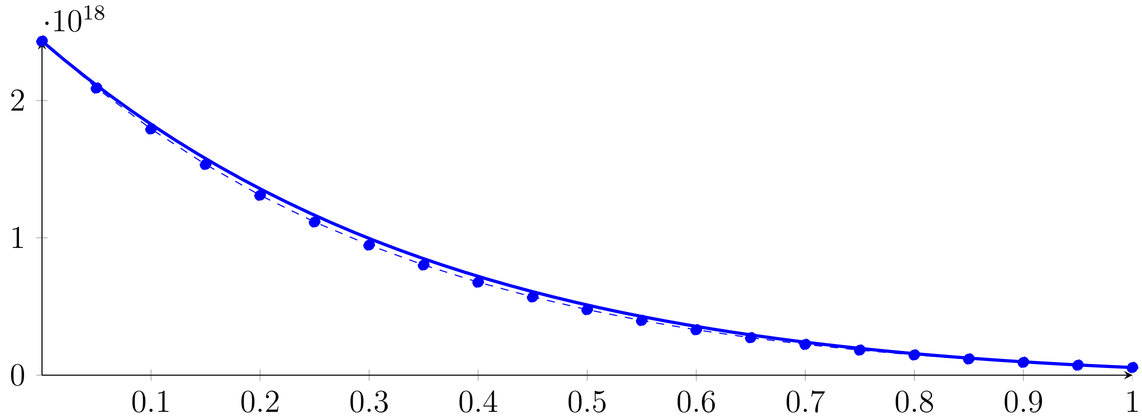
Longest intersection interval: 0.615645

$\Longrightarrow$ Bisection: first half $[0, 0.78125]$ und second half $[0.78125, 1.5625]$

3.6 Recursion Branch 1 1 1 1 1 1 on the First Half $[0, 0.78125]$

Normalized monomial und Bézier representations and the Bézier polygon:

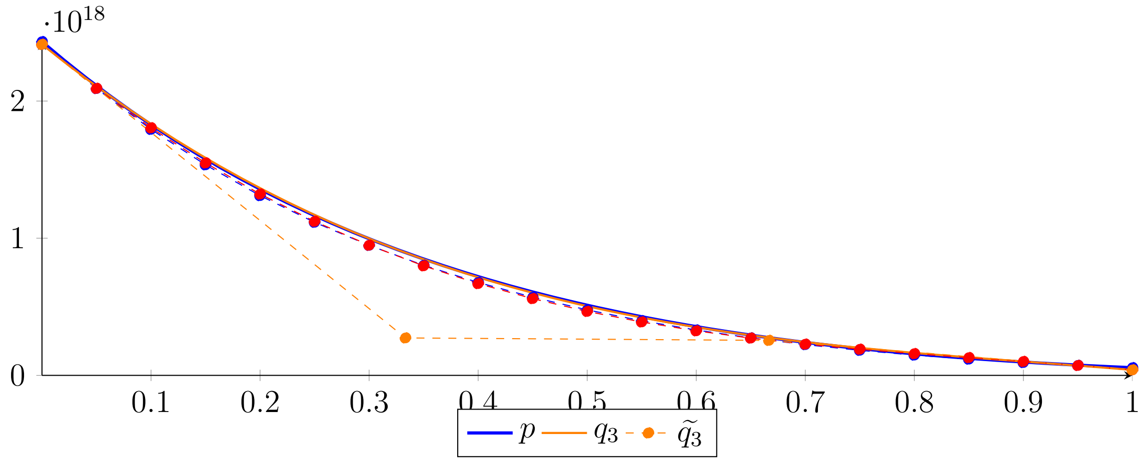
$$p = -5.99458{\cdot}10^{8}X^{20} + 1.22241{\cdot}10^{9}X^{19} - 2.5064{\cdot}10^{10}X^{18} + 5.69111{\cdot}10^{10}X^{17} - 4.32167{\cdot}10^{11}X^{16} + 3.59181{\cdot}10^{11}X^{15} - 1.88449{\cdot}10^{11}X^{14} - 1.23516{\cdot}10^{11}X^{13} + 9.39071{\cdot}10^{10}X^{12} - 9.07218{\cdot}10^{12}X^{11} + 1.10587{\cdot}10^{14}X^{10} - 1.09966{\cdot}10^{15}X^{9} + 8.74728{\cdot}10^{15}X^{8} - 5.5304{\cdot}10^{16}X^{7} + 2.7436{\cdot}10^{17}X^{6} - 1.04773{\cdot}10^{18}X^{5} + 2.99432{\cdot}10^{18}X^{4} - 6.13734{\cdot}10^{18}X^{3} + 8.42515{\cdot}10^{18}X^{2} - 6.83824{\cdot}10^{18}X + 2.4329{\cdot}10^{18}$$
$$= 2.4329{\cdot}10^{18}B_{0,20}(X) + 2.09099{\cdot}10^{18}B_{1,20}(X) + 1.79342{\cdot}10^{18}B_{2,20}(X) + 1.53481{\cdot}10^{18}B_{3,20}(X) + 1.31039{\cdot}10^{18}B_{4,20}(X) + 1.11596{\cdot}10^{18}B_{5,20}(X) + 9.47772{\cdot}10^{17}B_{6,20}(X) + 8.02552{\cdot}10^{17}B_{7,20}(X) + 6.77393{\cdot}10^{17}B_{8,20}(X) + 5.69738{\cdot}10^{17}B_{9,20}(X) + 4.77334{\cdot}10^{17}B_{10,20}(X) + 3.98201{\cdot}10^{17}B_{11,20}(X) + 3.30596{\cdot}10^{17}B_{12,20}(X) + 2.72992{\cdot}10^{17}B_{13,20}(X) + 2.24047{\cdot}10^{17}B_{14,20}(X) + 1.82588{\cdot}10^{17}B_{15,20}(X) + 1.47587{\cdot}10^{17}B_{16,20}(X) + 1.18148{\cdot}10^{17}B_{17,20}(X) + 9.34869{\cdot}10^{16}B_{18,20}(X) + 7.29227{\cdot}10^{16}B_{19,20}(X) + 5.58617{\cdot}10^{16}B_{20,20}(X)$$



Degree reduction and raising:

$$q_3 = -2.31929 \cdot 10^{18} X^3 + 6.36659 \cdot 10^{18} X^2 - 6.42057 \cdot 10^{18} X + 2.41313 \cdot 10^{18}$$
$$= 2.41313 \cdot 10^{18} B_{0,3} + 2.72944 \cdot 10^{17} B_{1,3} + 2.5495 \cdot 10^{17} B_{2,3} + 3.98664 \cdot 10^{16} B_{3,3}$$

$$\widetilde{q}_3 = -2.64856 \cdot 10^{20} X^{20} + 2.65665 \cdot 10^{21} X^{19} - 1.23351 \cdot 10^{22} X^{18} + 3.51744 \cdot 10^{22} X^{17} - 6.89177 \cdot 10^{22} X^{16} + 9.8349 \cdot 10^{22} X^{15} - 1.05729 \cdot 10^{23} X^{14} + 8.73625 \cdot 10^{22} X^{13} - 5.61127 \cdot 10^{22} X^{12} + 2.81516 \cdot 10^{22} X^{11} - 1.1026 \cdot 10^{22} X^{10} + 3.35285 \cdot 10^{21} X^9 - 7.83378 \cdot 10^{20} X^8 + 1.38501 \cdot 10^{20} X^7 - 1.81514 \cdot 10^{19} X^6 + 1.71295 \cdot 10^{18} X^5 - 1.11472 \cdot 10^{17} X^4 - 2.3146 \cdot 10^{18} X^3 + 6.36648 \cdot 10^{18} X^2 - 6.42057 \cdot 10^{18} X + 2.41313 \cdot 10^{18}$$
$$= 2.41313 \cdot 10^{18} B_{0,20} + 2.09211 \cdot 10^{18} B_{1,20} + 1.80458 \cdot 10^{18} B_{2,20} + 1.54854 \cdot 10^{18} B_{3,20} + 1.32192 \cdot 10^{18} B_{4,20} + 1.12276 \cdot 10^{18} B_{5,20} + 9.48822 \cdot 10^{17} B_{6,20} + 7.98559 \cdot 10^{17} B_{7,20} + 6.68963 \cdot 10^{17} B_{8,20} + 5.59606 \cdot 10^{17} B_{9,20} + 4.66222 \cdot 10^{17} B_{10,20} + 3.89399 \cdot 10^{17} B_{11,20} + 3.24521 \cdot 10^{17} B_{12,20} + 2.71702 \cdot 10^{17} B_{13,20} + 2.27378 \cdot 10^{17} B_{14,20} + 1.90434 \cdot 10^{17} B_{15,20} + 1.58374 \cdot 10^{17} B_{16,20} + 1.29355 \cdot 10^{17} B_{17,20} + 1.01279 \cdot 10^{17} B_{18,20} + 7.2129 \cdot 10^{16} B_{19,20} + 3.98664 \cdot 10^{16} B_{20,20}$$



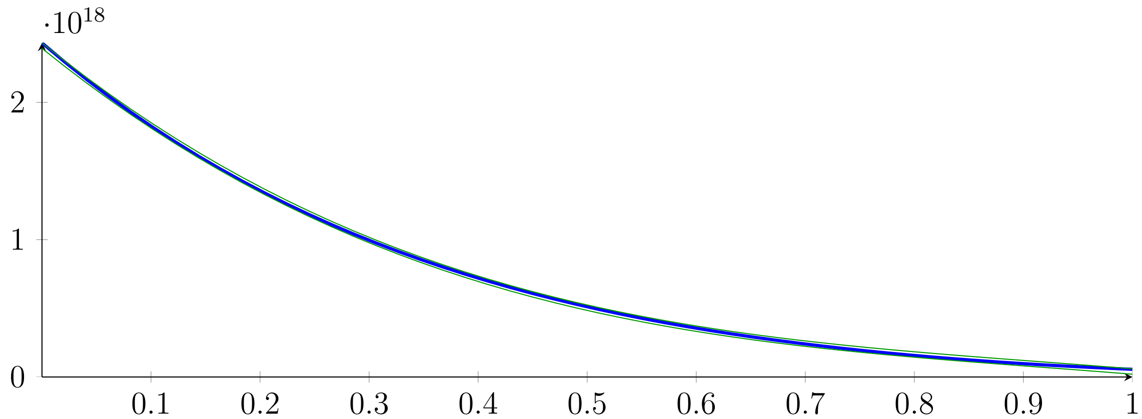
The maximum difference of the Bézier coefficients is $\delta = 1.97686 \cdot 10^{16}$.
Bounding polynomials M and m:

$$M = -2.31929 \cdot 10^{18} X^3 + 6.36659 \cdot 10^{18} X^2 - 6.42057 \cdot 10^{18} X + 2.4329 \cdot 10^{18}$$
$$m = -2.31929 \cdot 10^{18} X^3 + 6.36659 \cdot 10^{18} X^2 - 6.42057 \cdot 10^{18} X + 2.39336 \cdot 10^{18}$$

Root of M and m:

$$N(M) = \{1.08386\} \qquad N(m) = \{1.03021\}$$

Intersection intervals:

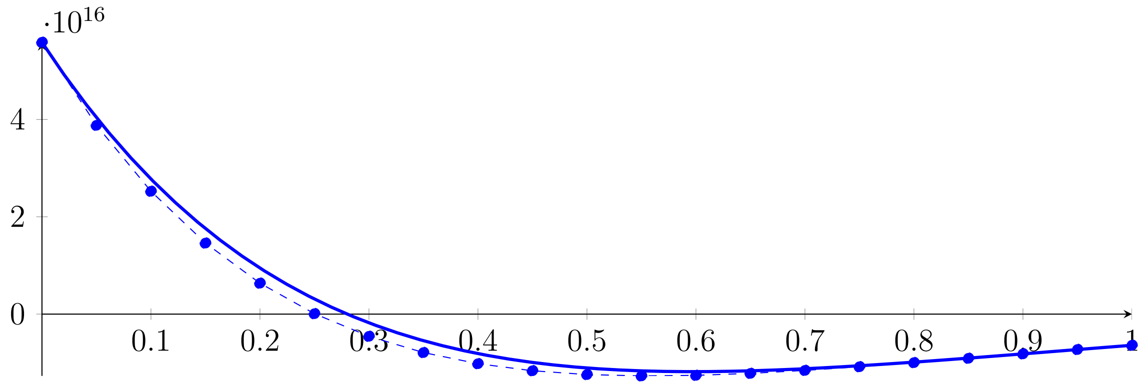


No intersection intervals with the x axis.

3.7 Recursion Branch 1 1 1 1 1 2 on the Second Half $[0.78125, 1.5625]$

Normalized monomial und Bézier representations and the Bézier polygon:

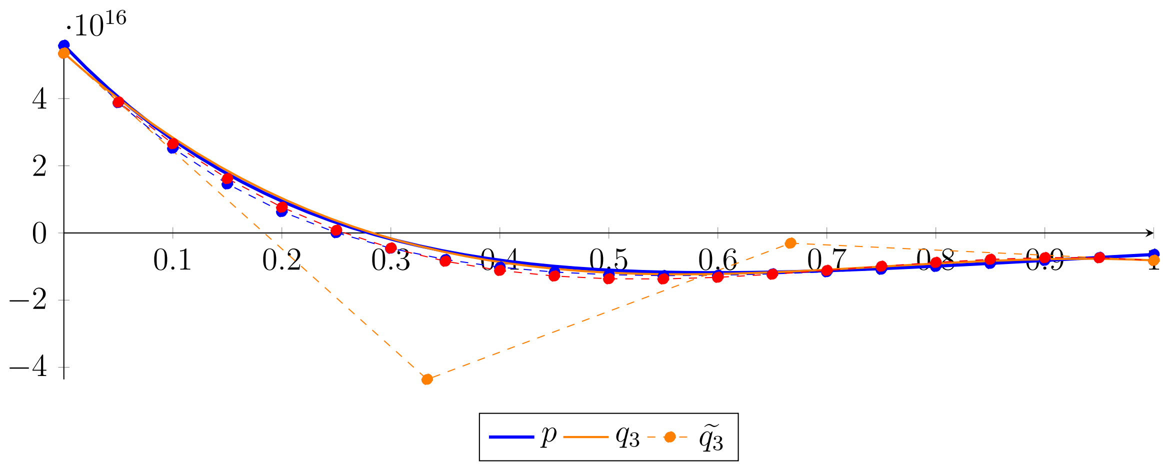
$$p = 1.04049{\cdot}10^{7}X^{20} - 6.97254{\cdot}10^{7}X^{19} + 2.1021{\cdot}10^{8}X^{18} - 1.44877{\cdot}10^{9}X^{17} + 6.15974{\cdot}10^{9}X^{16} - 4.81145{\cdot}10^{9}X^{15} + 1.28745{\cdot}10^{9}X^{14} - 1.6789{\cdot}10^{10}X^{13} + 2.87929{\cdot}10^{11}X^{12} - 3.92923{\cdot}10^{12}X^{11} + 4.3068{\cdot}10^{13}X^{10} - 3.76433{\cdot}10^{14}X^{9} + 2.60774{\cdot}10^{15}X^{8} - 1.41682{\cdot}10^{16}X^{7} + 5.93915{\cdot}10^{16}X^{6} - 1.8747{\cdot}10^{17}X^{5} + 4.29741{\cdot}10^{17}X^{4} - 6.76415{\cdot}10^{17}X^{3} + 6.65601{\cdot}10^{17}X^{2} - 3.41221{\cdot}10^{17}X + 5.58617{\cdot}10^{16}$$
$$= 5.58617{\cdot}10^{16}B_{0,20}(X) + 3.88007{\cdot}10^{16}B_{1,20}(X) + 2.52428{\cdot}10^{16}B_{2,20}(X) + 1.45947{\cdot}10^{16}B_{3,20}(X) + 6.35188{\cdot}10^{15}B_{4,20}(X) + 8.61285{\cdot}10^{13}B_{5,20}(X) - 4.56449{\cdot}10^{15}B_{6,20}(X) - 7.90513{\cdot}10^{15}B_{7,20}(X) - 1.01922{\cdot}10^{16}B_{8,20}(X) - 1.16401{\cdot}10^{16}B_{9,20}(X) - 1.24276{\cdot}10^{16}B_{10,20}(X) - 1.27029{\cdot}10^{16}B_{11,20}(X) - 1.25884{\cdot}10^{16}B_{12,20}(X) - 1.21842{\cdot}10^{16}B_{13,20}(X) - 1.15719{\cdot}10^{16}B_{14,20}(X) - 1.08174{\cdot}10^{16}B_{15,20}(X) - 9.97347{\cdot}10^{15}B_{16,20}(X) - 9.08173{\cdot}10^{15}B_{17,20}(X) - 8.17469{\cdot}10^{15}B_{18,20}(X) - 7.27722{\cdot}10^{15}B_{19,20}(X) - 6.40794{\cdot}10^{15}B_{20,20}(X)$$



Degree reduction and raising:

$$q_3 = -1.83363{\cdot}10^{17}X^{3} + 4.13088{\cdot}10^{17}X^{2} - 2.91428{\cdot}10^{17}X + 5.35463{\cdot}10^{16}$$
$$= 5.35463{\cdot}10^{16}B_{0,3} - 4.35965{\cdot}10^{16}B_{1,3} - 3.04348{\cdot}10^{15}B_{2,3} - 8.15796{\cdot}10^{15}B_{3,3}$$

$$\widetilde{q}_3 = -6.09575{\cdot}10^{18}X^{20} + 6.12175{\cdot}10^{19}X^{19} - 2.84507{\cdot}10^{20}X^{18} + 8.11867{\cdot}10^{20}X^{17} - 1.59156{\cdot}10^{21}X^{16} + 2.27223{\cdot}10^{21}X^{15} - 2.44381{\cdot}10^{21}X^{14} + 2.02041{\cdot}10^{21}X^{13} - 1.29878{\cdot}10^{21}X^{12} + 6.52455{\cdot}10^{20}X^{11} - 2.56093{\cdot}10^{20}X^{10} + 7.81329{\cdot}10^{19}X^{9} - 1.83349{\cdot}10^{19}X^{8} + 3.2524{\cdot}10^{18}X^{7} - 4.24606{\cdot}10^{17}X^{6} + 3.91694{\cdot}10^{16}X^{5} - 2.39685{\cdot}10^{15}X^{4} - 1.83274{\cdot}10^{17}X^{3} + 4.13086{\cdot}10^{17}X^{2} - 2.91428{\cdot}10^{17}X + 5.35463{\cdot}10^{16}$$
$$= 5.35463{\cdot}10^{16}B_{0,20} + 3.89749{\cdot}10^{16}B_{1,20} + 2.65776{\cdot}10^{16}B_{2,20} + 1.61937{\cdot}10^{16}B_{3,20} + 7.66185{\cdot}10^{15}B_{4,20} + 8.22917{\cdot}10^{14}B_{5,20} - 4.48877{\cdot}10^{15}B_{6,20} - 8.42265{\cdot}10^{15}B_{7,20} - 1.1162{\cdot}10^{16}B_{8,20} - 1.28306{\cdot}10^{16}B_{9,20} - 1.36409{\cdot}10^{16}B_{10,20} - 1.36933{\cdot}10^{16}B_{11,20} - 1.32083{\cdot}10^{16}B_{12,20} - 1.22973{\cdot}10^{16}B_{13,20} - 1.11556{\cdot}10^{16}B_{14,20} - 9.92371{\cdot}10^{15}B_{15,20} - 8.77247{\cdot}10^{15}B_{16,20} - 7.85875{\cdot}10^{15}B_{17,20} - 7.34469{\cdot}10^{15}B_{18,20} - 7.39079{\cdot}10^{15}B_{19,20} - 8.15796{\cdot}10^{15}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 2.3154 \cdot 10^{15}$.

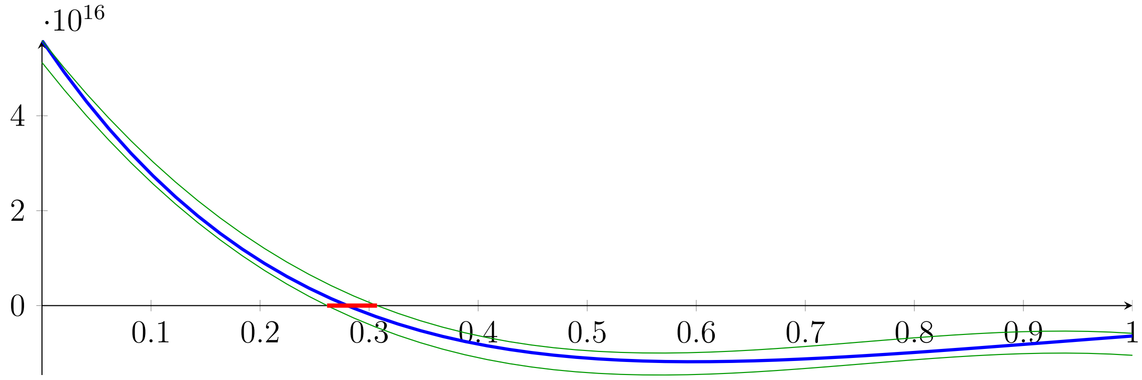
Bounding polynomials M and m:

$$M = -1.83363 \cdot 10^{17} X^3 + 4.13088 \cdot 10^{17} X^2 - 2.91428 \cdot 10^{17} X + 5.58617 \cdot 10^{16}$$
$$m = -1.83363 \cdot 10^{17} X^3 + 4.13088 \cdot 10^{17} X^2 - 2.91428 \cdot 10^{17} X + 5.12309 \cdot 10^{16}$$

Root of M and m:

$$N(M) = \{0.307231\} \qquad N(m) = \{0.261423\}$$

Intersection intervals:



$$[0.261423, 0.307231]$$

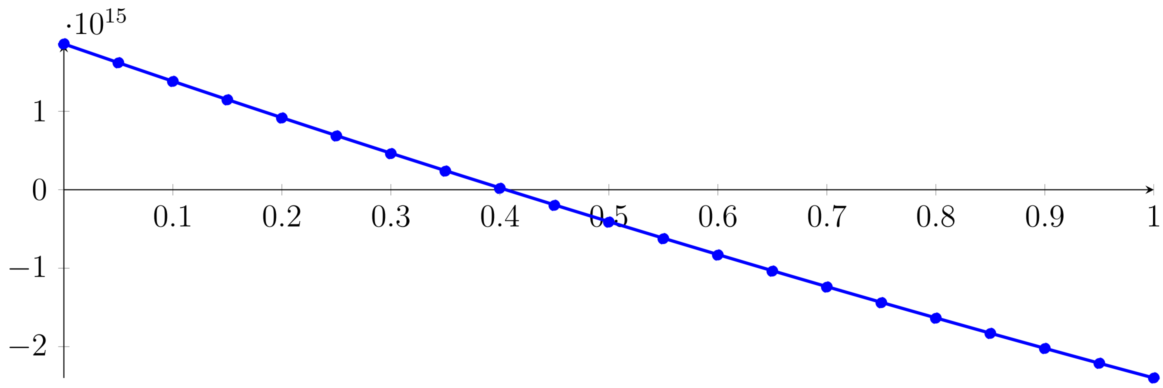
Longest intersection interval: 0.0458084

$\Longrightarrow$ Selective recursion: interval 1: $[0.985487, 1.02127]$,

3.8 Recursion Branch 1 1 1 1 1 2 1 in Interval 1: $[0.985487, 1.02127]$

Normalized monomial und Bézier representations and the Bézier polygon:

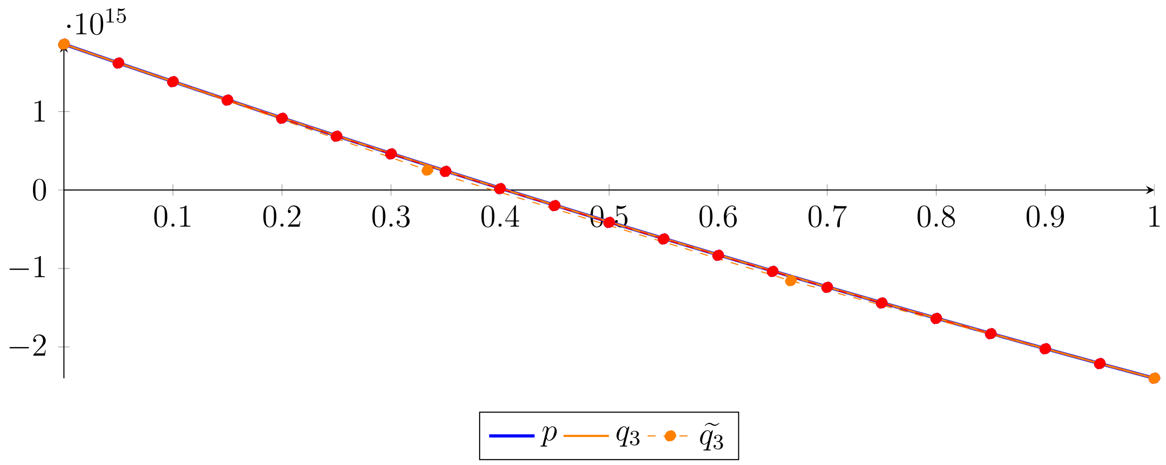
$$\begin{aligned}
p = {} & 178529 X^{20} - 4.99642 \cdot 10^6 X^{19} - 4.39261 \cdot 10^6 X^{18} - 5.0836 \cdot 10^7 X^{17} + 3.57222 \cdot 10^7 X^{16} + 2.47909 \\
& \cdot 10^7 X^{15} - 7.68611 \cdot 10^7 X^{14} - 6.36439 \cdot 10^7 X^{13} - 2.13519 \cdot 10^8 X^{12} - 1.07326 \cdot 10^8 X^{11} - 1.02909 \\
& \cdot 10^8 X^{10} - 2.36824 \cdot 10^7 X^9 + 629850 X^8 - 4.57368 \cdot 10^6 X^7 + 3.49654 \cdot 10^8 X^6 - 2.2641 \cdot 10^{10} X^5 \\
& + 1.04586 \cdot 10^{12} X^4 - 3.2306 \cdot 10^{13} X^3 + 5.91021 \cdot 10^{14} X^2 - 4.81711 \cdot 10^{15} X + 1.85844 \cdot 10^{15} \\
= {} & 1.85844 \cdot 10^{15} B_{0,20}(X) + 1.61758 \cdot 10^{15} B_{1,20}(X) + 1.37984 \cdot 10^{15} B_{2,20}(X) + 1.14518 \\
& \cdot 10^{15} B_{3,20}(X) + 9.13567 \cdot 10^{14} B_{4,20}(X) + 6.84985 \cdot 10^{14} B_{5,20}(X) + 4.59402 \cdot 10^{14} B_{6,20}(X) \\
& + 2.36789 \cdot 10^{14} B_{7,20}(X) + 1.71208 \cdot 10^{13} B_{8,20}(X) - 1.99631 \cdot 10^{14} B_{9,20}(X) - 4.13493 \\
& \cdot 10^{14} B_{10,20}(X) - 6.24491 \cdot 10^{14} B_{11,20}(X) - 8.32653 \cdot 10^{14} B_{12,20}(X) - 1.038 \cdot 10^{15} B_{13,20}(X) \\
& - 1.24057 \cdot 10^{15} B_{14,20}(X) - 1.44038 \cdot 10^{15} B_{15,20}(X) - 1.63745 \cdot 10^{15} B_{16,20}(X) - 1.83182 \\
& \cdot 10^{15} B_{17,20}(X) - 2.02351 \cdot 10^{15} B_{18,20}(X) - 2.21253 \cdot 10^{15} B_{19,20}(X) - 2.39893 \cdot 10^{15} B_{20,20}(X)
\end{aligned}$$



Degree reduction and raising:

$$q_3 = -3.0276\cdot10^{13}X^3 + 5.89729\cdot10^{14}X^2 - 4.81682\cdot10^{15}X + 1.85842\cdot10^{15}$$
$$= 1.85842\cdot10^{15}B_{0,3} + 2.52816\cdot10^{14}B_{1,3} - 1.15621\cdot10^{15}B_{2,3} - 2.39894\cdot10^{15}B_{3,3}$$

$$\widetilde{q}_3 = -1.04321\cdot10^{17}X^{20} + 1.04539\cdot10^{18}X^{19} - 4.84531\cdot10^{18}X^{18} + 1.37837\cdot10^{19}X^{17} - 2.69325\cdot10^{19}X^{16} + 3.83326\cdot10^{19}X^{15} - 4.11301\cdot10^{19}X^{14} + 3.39707\cdot10^{19}X^{13} - 2.1861\cdot10^{19}X^{12} + 1.10225\cdot10^{19}X^{11} - 4.35401\cdot10^{18}X^{10} + 1.33954\cdot10^{18}X^9 - 3.17082\cdot10^{17}X^8 + 5.66059\cdot10^{16}X^7 - 7.39506\cdot10^{15}X^6 + 6.74912\cdot10^{14}X^5 - 3.96204\cdot10^{13}X^4 - 2.89471\cdot10^{13}X^3 + 5.89705\cdot10^{14}X^2 - 4.81682\cdot10^{15}X + 1.85842\cdot10^{15}$$
$$= 1.85842\cdot10^{15}B_{0,20} + 1.61758\cdot10^{15}B_{1,20} + 1.37984\cdot10^{15}B_{2,20} + 1.14519\cdot10^{15}B_{3,20} + 9.13572\cdot10^{14}B_{4,20} + 6.85004\cdot10^{14}B_{5,20} + 4.59373\cdot10^{14}B_{6,20} + 2.36848\cdot10^{14}B_{7,20} + 1.70247\cdot10^{13}B_{8,20} - 1.99508\cdot10^{14}B_{9,20} - 4.13638\cdot10^{14}B_{10,20} - 6.24359\cdot10^{14}B_{11,20} - 8.32742\cdot10^{14}B_{12,20} - 1.03795\cdot10^{15}B_{13,20} - 1.2406\cdot10^{15}B_{14,20} - 1.44036\cdot10^{15}B_{15,20} - 1.63745\cdot10^{15}B_{16,20} - 1.83181\cdot10^{15}B_{17,20} - 2.0235\cdot10^{15}B_{18,20} - 2.21254\cdot10^{15}B_{19,20} - 2.39894\cdot10^{15}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.45525\cdot10^{11}$.

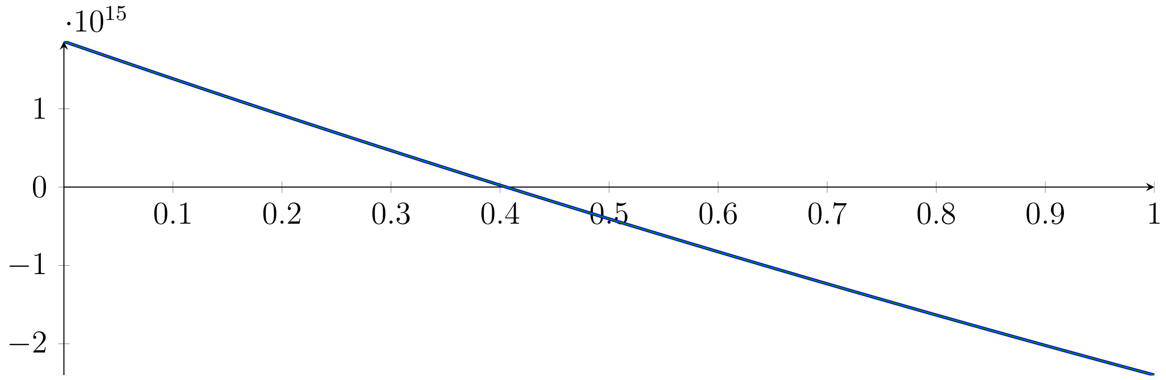
Bounding polynomials M and m:

$$M = -3.0276\cdot10^{13}X^3 + 5.89729\cdot10^{14}X^2 - 4.81682\cdot10^{15}X + 1.85857\cdot10^{15}$$
$$m = -3.0276\cdot10^{13}X^3 + 5.89729\cdot10^{14}X^2 - 4.81682\cdot10^{15}X + 1.85828\cdot10^{15}$$

Root of M and m:

$$N(M) = \{0.405569\} \qquad N(m) = \{0.405502\}$$

Intersection intervals:



$$[0.405502, 0.405569]$$

Longest intersection interval: $6.6855\cdot10^{-05}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.999999, 1]$,

3.9 Recursion Branch 1 1 1 1 1 2 1 1 in Interval 1: $[0.999999, 1]$

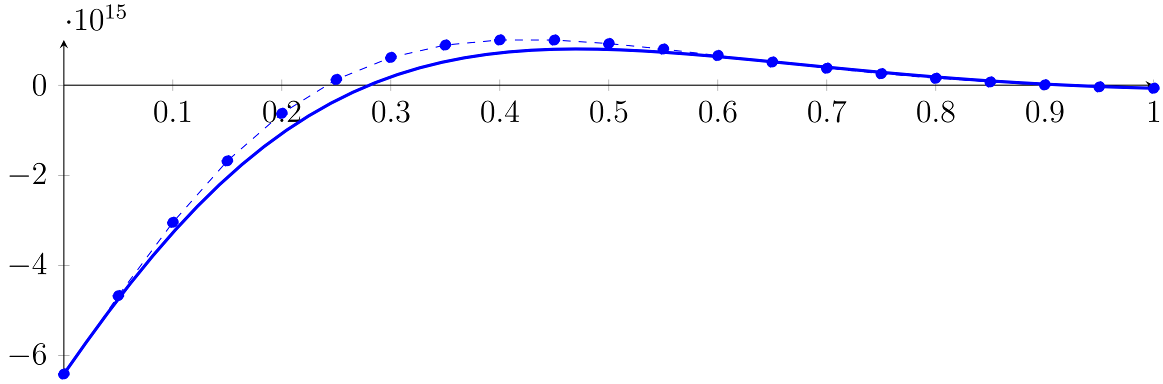
Found root in interval $[0.999999, 1]$ at recursion depth 8!

3.10 Recursion Branch 1 1 1 1 2 on the Second Half $[1.5625, 3.125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -854779X^{20} + 2.70678{\cdot}10^{6}X^{19} + 2.57285{\cdot}10^{7}X^{18} - 1.38387{\cdot}10^{9}X^{17} + 3.34218{\cdot}10^{10}X^{16} - 5.62474{\cdot}10^{11}X^{15} + 7.0799{\cdot}10^{12}X^{14} - 6.89484{\cdot}10^{13}X^{13} + 5.26324{\cdot}10^{14}X^{12} - 3.16741{\cdot}10^{15}X^{11} + 1.50317{\cdot}10^{16}X^{10} - 5.59783{\cdot}10^{16}X^{9} + 1.61826{\cdot}10^{17}X^{8} - 3.56531{\cdot}10^{17}X^{7} + 5.81008{\cdot}10^{17}X^{6} - 6.65758{\cdot}10^{17}X^{5} + 4.85849{\cdot}10^{17}X^{4} - 1.69752{\cdot}10^{17}X^{3} - 2.14228{\cdot}10^{16}X^{2} + 3.47712{\cdot}10^{16}X - 6.40794{\cdot}10^{15}$$

$$= -6.40794{\cdot}10^{15}B_{0,20}(X) - 4.66938{\cdot}10^{15}B_{1,20}(X) - 3.04357{\cdot}10^{15}B_{2,20}(X) - 1.67942{\cdot}10^{15}B_{3,20}(X) - 6.25553{\cdot}10^{14}B_{4,20}(X) + 1.26743{\cdot}10^{14}B_{5,20}(X) + 6.15563{\cdot}10^{14}B_{6,20}(X) + 8.9083{\cdot}10^{14}B_{7,20}(X) + 1.00381{\cdot}10^{15}B_{8,20}(X) + 1.00133{\cdot}10^{15}B_{9,20}(X) + 9.23073{\cdot}10^{14}B_{10,20}(X) + 8.00741{\cdot}10^{14}B_{11,20}(X) + 6.58338{\cdot}10^{14}B_{12,20}(X) + 5.13038{\cdot}10^{14}B_{13,20}(X) + 3.76314{\cdot}10^{14}B_{14,20}(X) + 2.55097{\cdot}10^{14}B_{15,20}(X) + 1.52873{\cdot}10^{14}B_{16,20}(X) + 7.06284{\cdot}10^{13}B_{17,20}(X) + 7.64979{\cdot}10^{12}B_{18,20}(X) - 3.78477{\cdot}10^{13}B_{19,20}(X) - 6.82353{\cdot}10^{13}B_{20,20}(X)$$

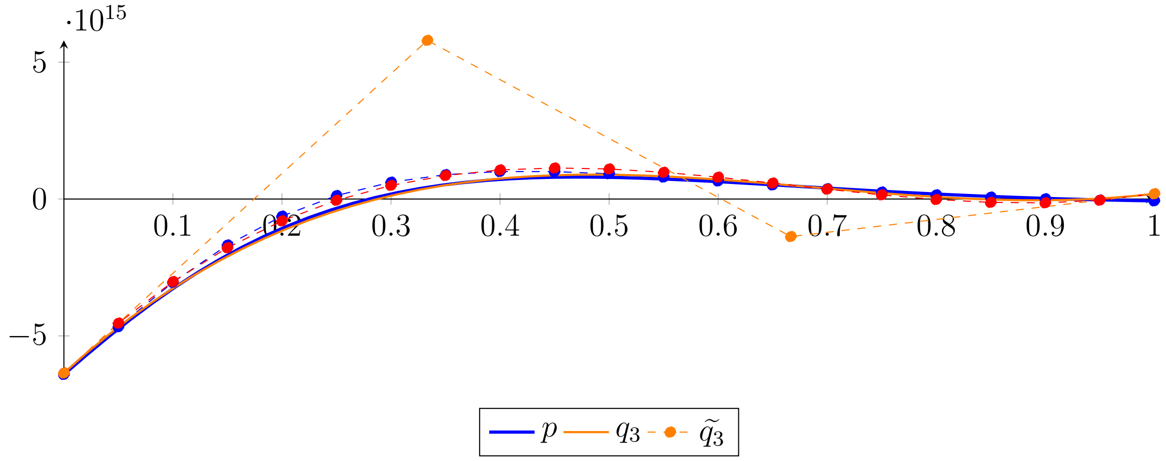


Degree reduction and raising:

$$q_3 = 2.80456{\cdot}10^{16}X^{3} - 5.79504{\cdot}10^{16}X^{2} + 3.64514{\cdot}10^{16}X - 6.35537{\cdot}10^{15}$$
$$= -6.35537{\cdot}10^{15}B_{0,3} + 5.7951{\cdot}10^{15}B_{1,3} - 1.37121{\cdot}10^{15}B_{2,3} + 1.91304{\cdot}10^{14}B_{3,3}$$

$$\widetilde{q}_3 = 8.6338{\cdot}10^{17}X^{20} - 8.67258{\cdot}10^{18}X^{19} + 4.03188{\cdot}10^{19}X^{18} - 1.15101{\cdot}10^{20}X^{17} + 2.25745{\cdot}10^{20}X^{16} - 3.2244{\cdot}10^{20}X^{15} + 3.46923{\cdot}10^{20}X^{14} - 2.86886{\cdot}10^{20}X^{13} + 1.84421{\cdot}10^{20}X^{12} - 9.26195{\cdot}10^{19}X^{11} + 3.63318{\cdot}10^{19}X^{10} - 1.1075{\cdot}10^{19}X^{9} + 2.59644{\cdot}10^{18}X^{8} - 4.6034{\cdot}10^{17}X^{7} + 6.01614{\cdot}10^{16}X^{6} - 5.57839{\cdot}10^{15}X^{5} + 3.46728{\cdot}10^{14}X^{4} + 2.80323{\cdot}10^{16}X^{3} - 5.79501{\cdot}10^{16}X^{2} + 3.64514{\cdot}10^{16}X - 6.35537{\cdot}10^{15}$$

$$= -6.35537{\cdot}10^{15}B_{0,20} - 4.5328{\cdot}10^{15}B_{1,20} - 3.01523{\cdot}10^{15}B_{2,20} - 1.77807{\cdot}10^{15}B_{3,20} - 7.9666{\cdot}10^{14}B_{4,20} - 4.66257{\cdot}10^{13}B_{5,20} + 4.97309{\cdot}10^{14}B_{6,20} + 8.58133{\cdot}10^{14}B_{7,20} + 1.06363{\cdot}10^{15}B_{8,20} + 1.13314{\cdot}10^{15}B_{9,20} + 1.09858{\cdot}10^{15}B_{10,20} + 9.75994{\cdot}10^{14}B_{11,20} + 7.98415{\cdot}10^{14}B_{12,20} + 5.83451{\cdot}10^{14}B_{13,20} + 3.60591{\cdot}10^{14}B_{14,20} + 1.51552{\cdot}10^{14}B_{15,20} - 1.76433{\cdot}10^{13}B_{16,20} - 1.22965{\cdot}10^{14}B_{17,20} - 1.39626{\cdot}10^{14}B_{18,20} - 4.30737{\cdot}10^{13}B_{19,20} + 1.91304{\cdot}10^{14}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 2.59539 \cdot 10^{14}$.

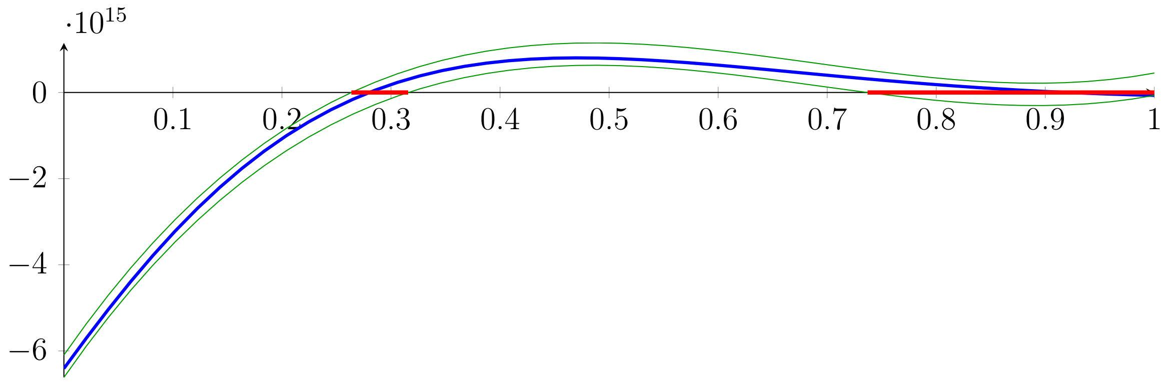
Bounding polynomials M and m:

$$M = 2.80456 \cdot 10^{16} X^3 - 5.79504 \cdot 10^{16} X^2 + 3.64514 \cdot 10^{16} X - 6.09583 \cdot 10^{15}$$
$$m = 2.80456 \cdot 10^{16} X^3 - 5.79504 \cdot 10^{16} X^2 + 3.64514 \cdot 10^{16} X - 6.61491 \cdot 10^{15}$$

Root of M and m:

$$N(M) = \{0.263619\} \qquad N(m) = \{0.315756, 0.737013, 1.01352\}$$

Intersection intervals:



$$[0.263619, 0.315756],\ [0.737013, 1]$$

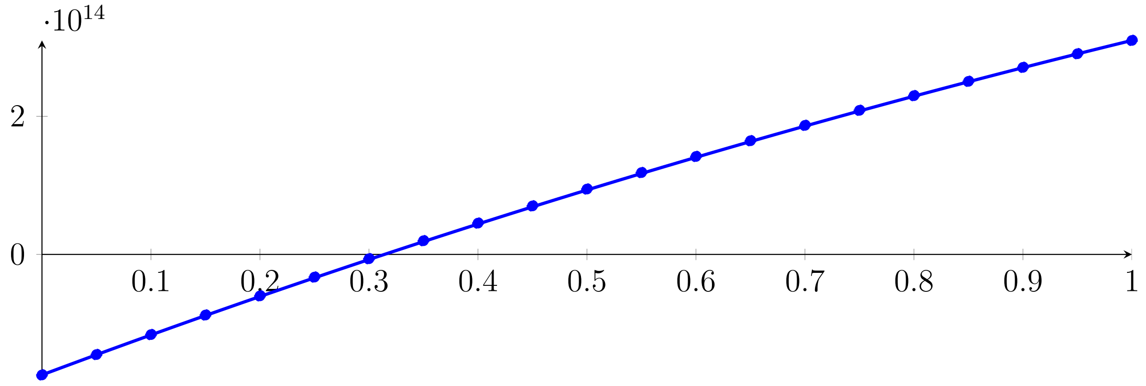
Longest intersection interval: 0.262987

$\Longrightarrow$ Selective recursion: interval 1: $[1.97441, 2.05587]$, interval 2: $[2.71408, 3.125]$,

3.11 Recursion Branch 1 1 1 1 2 1 in Interval 1: $[1.97441, 2.05587]$

Normalized monomial und Bézier representations and the Bézier polygon:

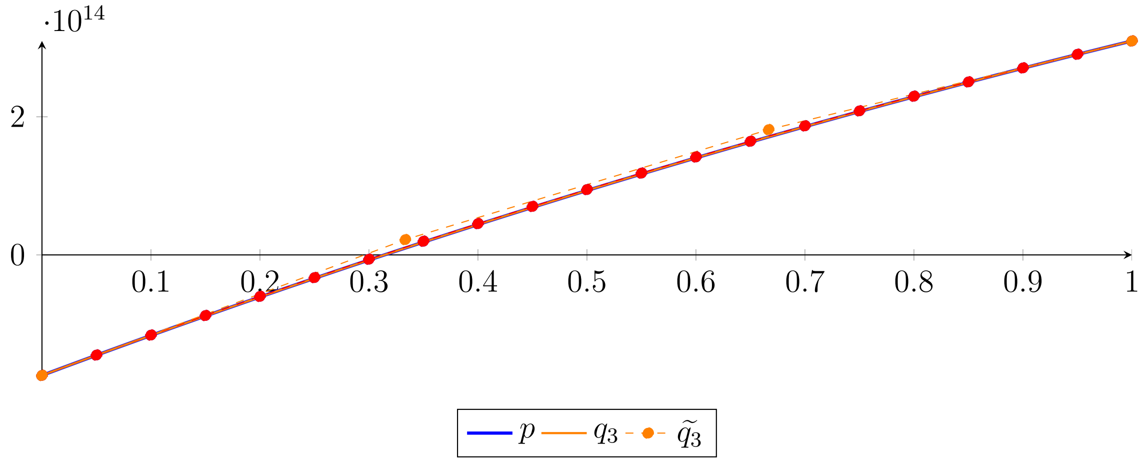
$$\begin{aligned} p = & -65706.9X^{20} + 824057X^{19} - 862196X^{18} + 1.0933 \cdot 10^7 X^{17} - 3.41784 \cdot 10^7 X^{16} + 2.09672 \\ & \cdot 10^7 X^{15} - 644385X^{14} + 5.90121 \cdot 10^6 X^{13} + 2.40918 \cdot 10^6 X^{12} + 8.7969 \cdot 10^6 X^{11} + 5.19626 \cdot 10^6 X^{10} \\ & + 2.16248 \cdot 10^6 X^9 + 3.79485 \cdot 10^6 X^8 - 1.32782 \cdot 10^8 X^7 + 3.32132 \cdot 10^9 X^6 - 5.03726 \cdot 10^{10} X^5 \\ & + 2.36906 \cdot 10^{11} X^4 + 6.04009 \cdot 10^{12} X^3 - 1.1183 \cdot 10^{14} X^2 + 5.90013 \cdot 10^{14} X - 1.74526 \cdot 10^{14} \\ = & -1.74526 \cdot 10^{14} B_{0,20}(X) - 1.45026 \cdot 10^{14} B_{1,20}(X) - 1.16114 \cdot 10^{14} B_{2,20}(X) - 8.77848 \\ & \cdot 10^{13} B_{3,20}(X) - 6.0034 \cdot 10^{13} B_{4,20}(X) - 3.28556 \cdot 10^{13} B_{5,20}(X) - 6.24444 \cdot 10^{12} B_{6,20}(X) \\ & + 1.98051 \cdot 10^{13} B_{7,20}(X) + 4.52986 \cdot 10^{13} B_{8,20}(X) + 7.02415 \cdot 10^{13} B_{9,20}(X) + 9.46394 \\ & \cdot 10^{13} B_{10,20}(X) + 1.18498 \cdot 10^{14} B_{11,20}(X) + 1.41823 \cdot 10^{14} B_{12,20}(X) + 1.64619 \cdot 10^{14} B_{13,20}(X) \\ & + 1.86893 \cdot 10^{14} B_{14,20}(X) + 2.08651 \cdot 10^{14} B_{15,20}(X) + 2.29897 \cdot 10^{14} B_{16,20}(X) + 2.50638 \\ & \cdot 10^{14} B_{17,20}(X) + 2.7088 \cdot 10^{14} B_{18,20}(X) + 2.90627 \cdot 10^{14} B_{19,20}(X) + 3.09887 \cdot 10^{14} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_3 = 6.38457 \cdot 10^{12} X^3 - 1.12024 \cdot 10^{14} X^2 + 5.90053 \cdot 10^{14} X - 1.74528 \cdot 10^{14}$$
$$= -1.74528 \cdot 10^{14} B_{0,3} + 2.21562 \cdot 10^{13} B_{1,3} + 1.81499 \cdot 10^{14} B_{2,3} + 3.09885 \cdot 10^{14} B_{3,3}$$

$$\widetilde{q}_3 = 5.2133 \cdot 10^{15} X^{20} - 5.21968 \cdot 10^{16} X^{19} + 2.41324 \cdot 10^{17} X^{18} - 6.83867 \cdot 10^{17} X^{17} + 1.33002 \cdot 10^{18} X^{16} - 1.88423 \cdot 10^{18} X^{15} + 2.01478 \cdot 10^{18} X^{14} - 1.66278 \cdot 10^{18} X^{13} + 1.07375 \cdot 10^{18} X^{12} - 5.46338 \cdot 10^{17} X^{11} + 2.19158 \cdot 10^{17} X^{10} - 6.88546 \cdot 10^{16} X^9 + 1.66786 \cdot 10^{16} X^8 - 3.0261 \cdot 10^{15} X^7 + 3.91471 \cdot 10^{14} X^6 - 3.30384 \cdot 10^{13} X^5 + 1.4329 \cdot 10^{12} X^4 + 6.38075 \cdot 10^{12} X^3 - 1.12026 \cdot 10^{14} X^2 + 5.90053 \cdot 10^{14} X - 1.74528 \cdot 10^{14}$$
$$= -1.74528 \cdot 10^{14} B_{0,20} - 1.45026 \cdot 10^{14} B_{1,20} - 1.16112 \cdot 10^{14} B_{2,20} - 8.77835 \cdot 10^{13} B_{3,20} - 6.00325 \cdot 10^{13} B_{4,20} - 3.28557 \cdot 10^{13} B_{5,20} - 6.24271 \cdot 10^{12} B_{6,20} + 1.98018 \cdot 10^{13} B_{7,20} + 4.53017 \cdot 10^{13} B_{8,20} + 7.02334 \cdot 10^{13} B_{9,20} + 9.46441 \cdot 10^{13} B_{10,20} + 1.18488 \cdot 10^{14} B_{11,20} + 1.41825 \cdot 10^{14} B_{12,20} + 1.64616 \cdot 10^{14} B_{13,20} + 1.86895 \cdot 10^{14} B_{14,20} + 2.08651 \cdot 10^{14} B_{15,20} + 2.29898 \cdot 10^{14} B_{16,20} + 2.50639 \cdot 10^{14} B_{17,20} + 2.7088 \cdot 10^{14} B_{18,20} + 2.90627 \cdot 10^{14} B_{19,20} + 3.09885 \cdot 10^{14} B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.00791 \cdot 10^{10}$.
Bounding polynomials M and m:

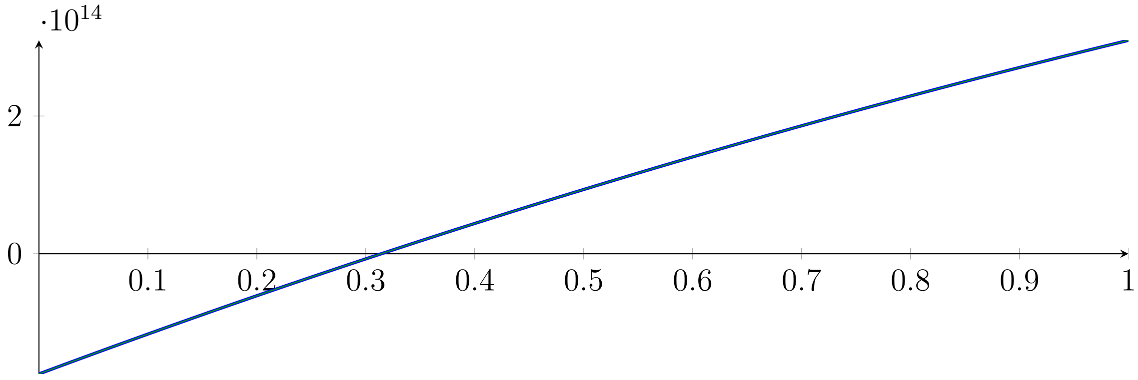
$$M = 6.38457 \cdot 10^{12} X^3 - 1.12024 \cdot 10^{14} X^2 + 5.90053 \cdot 10^{14} X - 1.74518 \cdot 10^{14}$$

$$m = 6.38457 \cdot 10^{12} X^3 - 1.12024 \cdot 10^{14} X^2 + 5.90053 \cdot 10^{14} X - 1.74538 \cdot 10^{14}$$

Root of M and m:

$$N(M) = \{0.314171\} \qquad N(m) = \{0.314209\}$$

Intersection intervals:



$[0.314171, 0.314209]$

Longest intersection interval: $3.86505 \cdot 10^{-05}$
$\Longrightarrow$ Selective recursion: interval 1: $[2, 2]$,

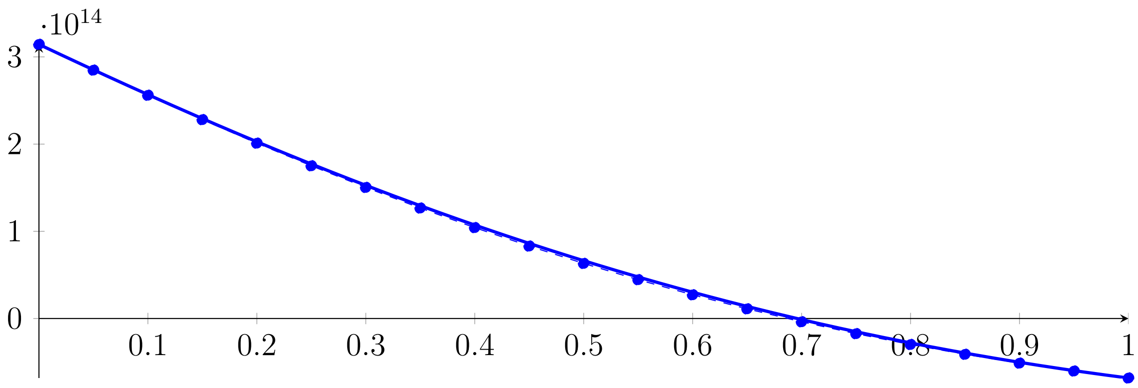
3.12 Recursion Branch 1 1 1 1 2 1 1 in Interval 1: $[2, 2]$

Found root in interval $[2, 2]$ at recursion depth 7!

3.13 Recursion Branch 1 1 1 1 2 2 in Interval 2: $[2.71408, 3.125]$

Normalized monomial und Bézier representations and the Bézier polygon:

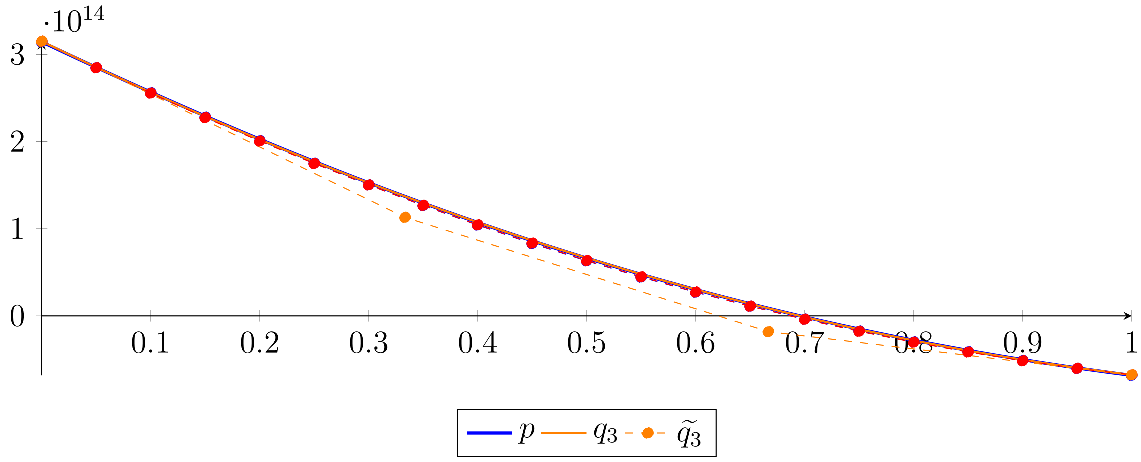
$$\begin{aligned} p = & -86000X^{20} + 112700X^{19} - 3.81311 \cdot 10^{6} X^{18} + 8.34822 \cdot 10^{6} X^{17} - 6.45015 \cdot 10^{7} X^{16} + 5.58299 \\ & \cdot 10^{7} X^{15} - 3.13181 \cdot 10^{7} X^{14} - 1.32947 \cdot 10^{7} X^{13} - 6.07175 \cdot 10^{7} X^{12} - 2.64285 \cdot 10^{8} X^{11} + 3.28339 \\ & \cdot 10^{9} X^{10} - 3.21178 \cdot 10^{10} X^{9} + 2.12933 \cdot 10^{11} X^{8} - 7.86718 \cdot 10^{11} X^{7} - 2.48779 \cdot 10^{11} X^{6} + 1.88463 \\ & \cdot 10^{13} X^{5} - 8.80338 \cdot 10^{13} X^{4} + 1.37431 \cdot 10^{14} X^{3} + 1.41313 \cdot 10^{14} X^{2} - 5.91292 \cdot 10^{14} X + 3.14352 \cdot 10^{14} \\ = & \ 3.14352 \cdot 10^{14} B_{0,20}(X) + 2.84787 \cdot 10^{14} B_{1,20}(X) + 2.55966 \cdot 10^{14} B_{2,20}(X) + 2.2801 \\ & \cdot 10^{14} B_{3,20}(X) + 2.0102 \cdot 10^{14} B_{4,20}(X) + 1.75082 \cdot 10^{14} B_{5,20}(X) + 1.50266 \cdot 10^{14} B_{6,20}(X) \\ & + 1.26627 \cdot 10^{14} B_{7,20}(X) + 1.04207 \cdot 10^{14} B_{8,20}(X) + 8.30346 \cdot 10^{13} B_{9,20}(X) + 6.3129 \\ & \cdot 10^{13} B_{10,20}(X) + 4.44981 \cdot 10^{13} B_{11,20}(X) + 2.71412 \cdot 10^{13} B_{12,20}(X) + 1.10493 \cdot 10^{13} B_{13,20}(X) \\ & - 3.79383 \cdot 10^{12} B_{14,20}(X) - 1.74107 \cdot 10^{13} B_{15,20}(X) - 2.98293 \cdot 10^{13} B_{16,20}(X) - 4.10825 \\ & \cdot 10^{13} B_{17,20}(X) - 5.12072 \cdot 10^{13} B_{18,20}(X) - 6.02438 \cdot 10^{13} B_{19,20}(X) - 6.82353 \cdot 10^{13} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = & \ 1.06887 \cdot 10^{13} X^{3} + 2.12659 \cdot 10^{14} X^{2} - 6.06055 \cdot 10^{14} X + 3.15058 \cdot 10^{14} \\ = & \ 3.15058 \cdot 10^{14} B_{0,3} + 1.1304 \cdot 10^{14} B_{1,3} - 1.80924 \cdot 10^{13} B_{2,3} - 6.76494 \cdot 10^{13} B_{3,3} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 = &-2.94587{\cdot}10^{16}X^{20} + 2.95309{\cdot}10^{17}X^{19} - 1.37027{\cdot}10^{18}X^{18} + 3.90486{\cdot}10^{18}X^{17} - 7.64584{\cdot}10^{18}X^{16} \\ &+ 1.09043{\cdot}10^{19}X^{15} - 1.17167{\cdot}10^{19}X^{14} + 9.67819{\cdot}10^{18}X^{13} - 6.21577{\cdot}10^{18}X^{12} + 3.11903{\cdot}10^{18}X^{11} \\ &- 1.22212{\cdot}10^{18}X^{10} + 3.71804{\cdot}10^{17}X^{9} - 8.68963{\cdot}10^{16}X^{8} + 1.53653{\cdot}10^{16}X^{7} - 2.01492{\cdot}10^{15}X^{6} + 1.90578 \\ &\cdot 10^{14}X^{5} - 1.24562{\cdot}10^{13}X^{4} + 1.12169{\cdot}10^{13}X^{3} + 2.12646{\cdot}10^{14}X^{2} - 6.06054{\cdot}10^{14}X + 3.15058{\cdot}10^{14} \\ = &\, 3.15058{\cdot}10^{14}B_{0,20} + 2.84755{\cdot}10^{14}B_{1,20} + 2.55571{\cdot}10^{14}B_{2,20} + 2.27517{\cdot}10^{14}B_{3,20} + 2.00599 \\ &\cdot 10^{14}B_{4,20} + 1.74834{\cdot}10^{14}B_{5,20} + 1.50209{\cdot}10^{14}B_{6,20} + 1.26788{\cdot}10^{14}B_{7,20} + 1.04473{\cdot}10^{14}B_{8,20} \\ &+ 8.34496{\cdot}10^{13}B_{9,20} + 6.34809{\cdot}10^{13}B_{10,20} + 4.48678{\cdot}10^{13}B_{11,20} + 2.73318{\cdot}10^{13}B_{12,20} \\ &+ 1.11217{\cdot}10^{13}B_{13,20} - 3.92358{\cdot}10^{12}B_{14,20} - 1.76919{\cdot}10^{13}B_{15,20} - 3.02256{\cdot}10^{13}B_{16,20} \\ &- 4.14938{\cdot}10^{13}B_{17,20} - 5.14943{\cdot}10^{13}B_{18,20} - 6.02159{\cdot}10^{13}B_{19,20} - 6.76494{\cdot}10^{13}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 7.06059{\cdot}10^{11}$.
Bounding polynomials M and m:

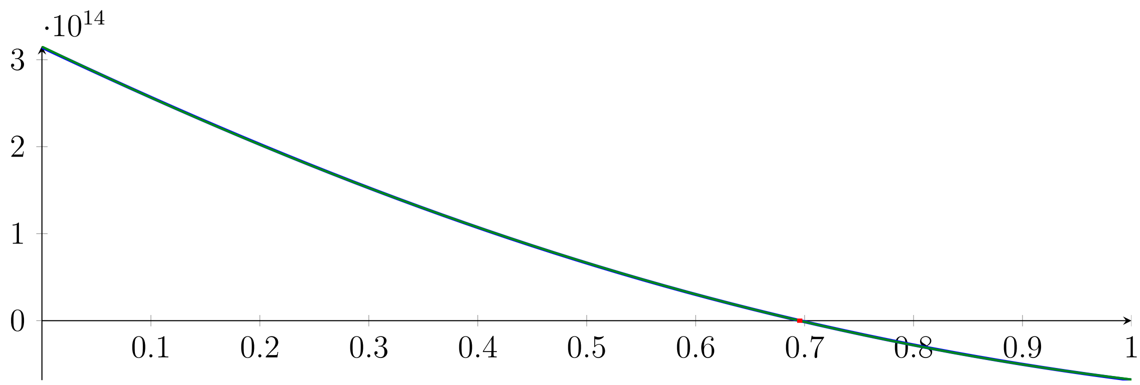
$$M = 1.06887{\cdot}10^{13}X^3 + 2.12659{\cdot}10^{14}X^2 - 6.06055{\cdot}10^{14}X + 3.15764{\cdot}10^{14}$$

$$m = 1.06887{\cdot}10^{13}X^3 + 2.12659{\cdot}10^{14}X^2 - 6.06055{\cdot}10^{14}X + 3.14352{\cdot}10^{14}$$

Root of M and m:

$$N(M) = \{-22.4767, 0.697934, 1.88317\} \qquad N(m) = \{-22.4765, 0.693143, 1.88773\}$$

Intersection intervals:



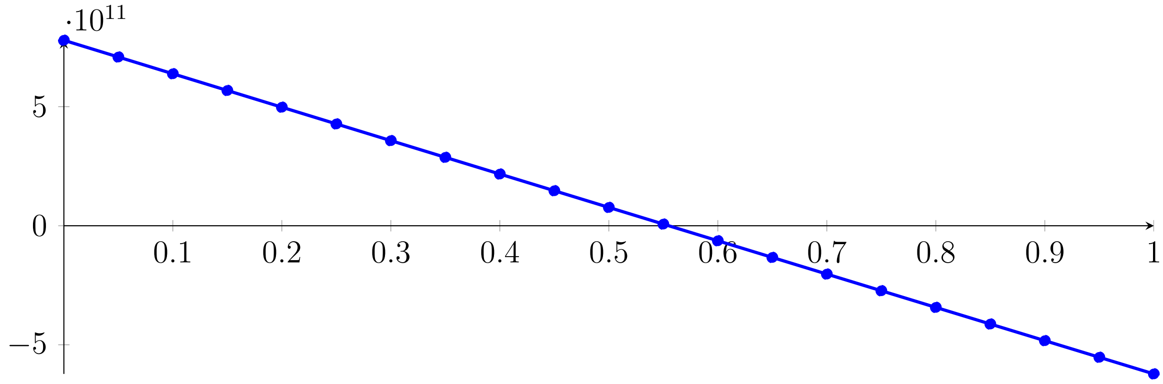
$$[0.693143, 0.697934]$$

Longest intersection interval: 0.00479144
$\Longrightarrow$ Selective recursion: interval 1: $[2.99891, 3.00088]$,

3.14 Recursion Branch 1 1 1 1 2 2 1 in Interval 1: $[2.99891, 3.00088]$

Normalized monomial und Bézier representations and the Bézier polygon:

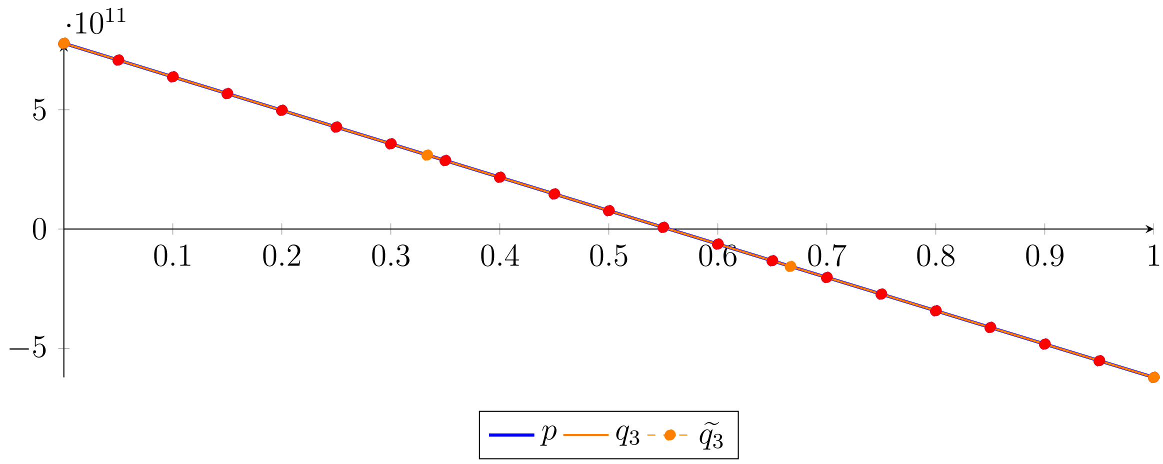
$$\begin{aligned}p &= -144.399X^{20} - 606.201X^{19} - 7847.61X^{18} + 8991.42X^{17} - 122499X^{16} \\ &\quad + 108021X^{15} - 65123.6X^{14} - 47352.3X^{13} - 172778X^{12} - 47648.8X^{11} \\ &\quad - 61074.1X^{10} - 11727.7X^{9} - 676.597X^{8} - 264.961X^{7} - 359.59X^{6} + 22.7109X^{5} \\ &\quad - 16245.4X^{4} - 2.47262\cdot10^{6}X^{3} + 5.35274\cdot10^{9}X^{2} - 1.40655\cdot10^{12}X + 7.78966\cdot10^{11} \\ &= 7.78966\cdot10^{11}B_{0,20}(X) + 7.08639\cdot10^{11}B_{1,20}(X) + 6.38339\cdot10^{11}B_{2,20}(X) + 5.68068 \\ &\quad \cdot10^{11}B_{3,20}(X) + 4.97825\cdot10^{11}B_{4,20}(X) + 4.2761\cdot10^{11}B_{5,20}(X) + 3.57423\cdot10^{11}B_{6,20}(X) \\ &\quad + 2.87264\cdot10^{11}B_{7,20}(X) + 2.17134\cdot10^{11}B_{8,20}(X) + 1.47032\cdot10^{11}B_{9,20}(X) + 7.69576 \\ &\quad \cdot10^{10}B_{10,20}(X) + 6.91157\cdot10^{9}B_{11,20}(X) - 6.31063\cdot10^{10}B_{12,20}(X) - 1.33096\cdot10^{11}B_{13,20}(X) \\ &\quad - 2.03057\cdot10^{11}B_{14,20}(X) - 2.72991\cdot10^{11}B_{15,20}(X) - 3.42896\cdot10^{11}B_{16,20}(X) - 4.12773 \\ &\quad \cdot10^{11}B_{17,20}(X) - 4.82622\cdot10^{11}B_{18,20}(X) - 5.52443\cdot10^{11}B_{19,20}(X) - 6.22236\cdot10^{11}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= -2.50501\cdot10^{6}X^{3} + 5.35276\cdot10^{9}X^{2} - 1.40655\cdot10^{12}X + 7.78966\cdot10^{11} \\ &= 7.78966\cdot10^{11}B_{0,3} + 3.10115\cdot10^{11}B_{1,3} - 1.56951\cdot10^{11}B_{2,3} - 6.22236\cdot10^{11}B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= -6.4359\cdot10^{13}X^{20} + 6.45138\cdot10^{14}X^{19} - 2.9929\cdot10^{15}X^{18} + 8.52592\cdot10^{15}X^{17} - 1.66872\cdot10^{16}X^{16} \\ &\quad + 2.37901\cdot10^{16}X^{15} - 2.55575\cdot10^{16}X^{14} + 2.11146\cdot10^{16}X^{13} - 1.3571\cdot10^{16}X^{12} + 6.82039\cdot10^{15}X^{11} \\ &\quad - 2.67903\cdot10^{15}X^{10} + 8.17787\cdot10^{14}X^{9} - 1.91865\cdot10^{14}X^{8} + 3.4033\cdot10^{13}X^{7} - 4.46339\cdot10^{12}X^{6} + 4.19429 \\ &\quad \cdot10^{11}X^{5} - 2.68975\cdot10^{10}X^{4} + 1.09838\cdot10^{9}X^{3} + 5.32719\cdot10^{9}X^{2} - 1.40655\cdot10^{12}X + 7.78966\cdot10^{11} \\ &= 7.78966\cdot10^{11}B_{0,20} + 7.08639\cdot10^{11}B_{1,20} + 6.38339\cdot10^{11}B_{2,20} + 5.68068\cdot10^{11}B_{3,20} + 4.97822 \\ &\quad \cdot10^{11}B_{4,20} + 4.27617\cdot10^{11}B_{5,20} + 3.57404\cdot10^{11}B_{6,20} + 2.87302\cdot10^{11}B_{7,20} + 2.17075\cdot10^{11}B_{8,20} \\ &\quad + 1.4711\cdot10^{11}B_{9,20} + 7.68685\cdot10^{10}B_{10,20} + 6.98833\cdot10^{9}B_{11,20} - 6.31634\cdot10^{10}B_{12,20} \\ &\quad - 1.33061\cdot10^{11}B_{13,20} - 2.03075\cdot10^{11}B_{14,20} - 2.72984\cdot10^{11}B_{15,20} - 3.42898\cdot10^{11}B_{16,20} \\ &\quad - 4.12773\cdot10^{11}B_{17,20} - 4.82622\cdot10^{11}B_{18,20} - 5.52443\cdot10^{11}B_{19,20} - 6.22236\cdot10^{11}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 8.90566 \cdot 10^7$.
Bounding polynomials M and m:

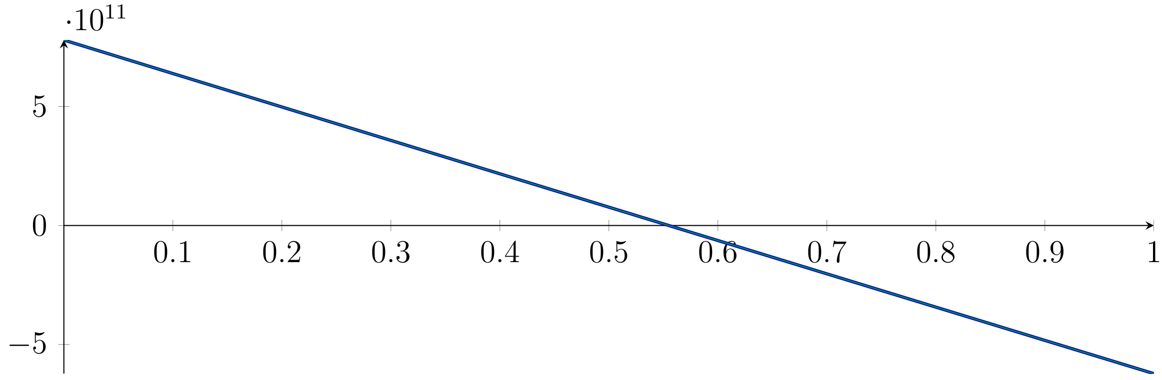
$$M = -2.50501 \cdot 10^6 X^3 + 5.35276 \cdot 10^9 X^2 - 1.40655 \cdot 10^{12} X + 7.79055 \cdot 10^{11}$$

$$m = -2.50501 \cdot 10^6 X^3 + 5.35276 \cdot 10^9 X^2 - 1.40655 \cdot 10^{12} X + 7.78877 \cdot 10^{11}$$

Root of M and m:

$$N(M) = \{0.555048, 306.163, 1830.1\} \qquad N(m) = \{0.554921, 306.163, 1830.1\}$$

Intersection intervals:



$$[0.554921, 0.555048]$$

Longest intersection interval: 0.000127168
$\Longrightarrow$ Selective recursion: interval 1: $[3, 3]$,

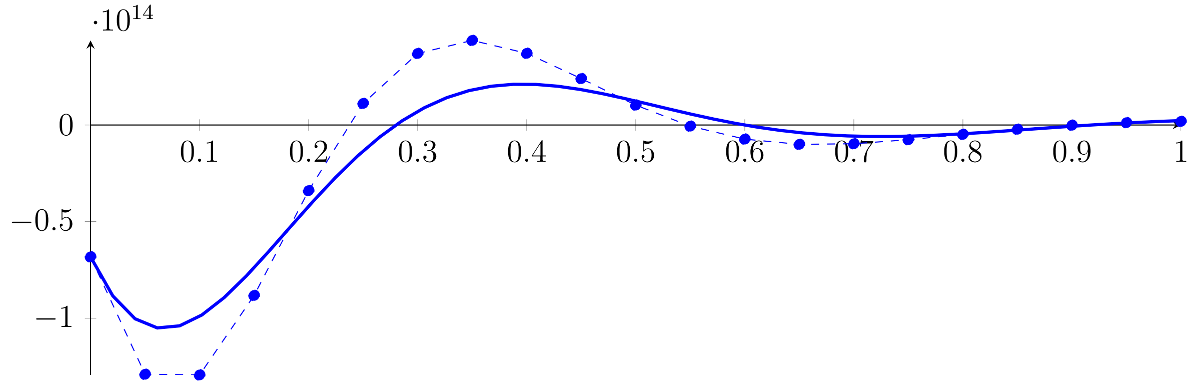
3.15 Recursion Branch 1 1 1 1 2 2 1 1 in Interval 1: $[3, 3]$

Found root in interval $[3, 3]$ at recursion depth 8!

3.16 Recursion Branch 1 1 1 2 on the Second Half $[3.125, 6.25]$

Normalized monomial und Bézier representations and the Bézier polygon:

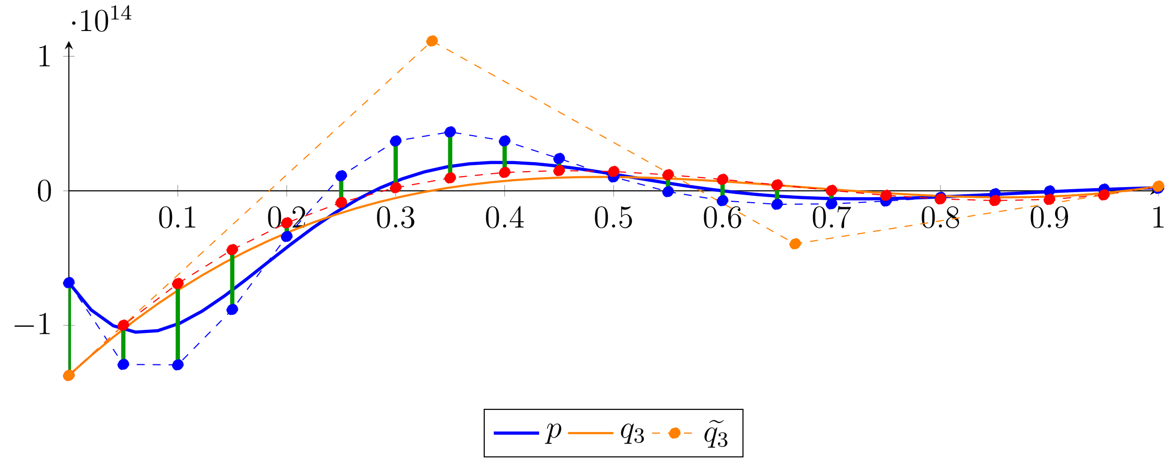
$$\begin{aligned}
p = {} & 7.88859 \cdot 10^9 X^{20} - 3.72342 \cdot 10^{11} X^{19} + 8.07932 \cdot 10^{12} X^{18} - 1.06797 \cdot 10^{14} X^{17} + 9.60483 \cdot 10^{14} X^{16} \\
& - 6.21458 \cdot 10^{15} X^{15} + 2.98115 \cdot 10^{16} X^{14} - 1.07566 \cdot 10^{17} X^{13} + 2.92576 \cdot 10^{17} X^{12} - 5.93362 \cdot 10^{17} X^{11} \\
& + 8.69791 \cdot 10^{17} X^{10} - 8.52613 \cdot 10^{17} X^9 + 4.24784 \cdot 10^{17} X^8 + 1.26126 \cdot 10^{17} X^7 - 3.67434 \cdot 10^{17} X^6 + 2.40127 \\
& \cdot 10^{17} X^5 - 4.54599 \cdot 10^{16} X^4 - 2.16249 \cdot 10^{16} X^3 + 1.14835 \cdot 10^{16} X^2 - 1.2155 \cdot 10^{15} X - 6.82353 \cdot 10^{13} \\
= {} & -6.82353 \cdot 10^{13} B_{0,20}(X) - 1.2901 \cdot 10^{14} B_{1,20}(X) - 1.29346 \cdot 10^{14} B_{2,20}(X) - 8.82108 \\
& \cdot 10^{13} B_{3,20}(X) - 3.39572 \cdot 10^{13} B_{4,20}(X) + 1.11681 \cdot 10^{13} B_{5,20}(X) + 3.70318 \cdot 10^{13} B_{6,20}(X) \\
& + 4.37698 \cdot 10^{13} B_{7,20}(X) + 3.70894 \cdot 10^{13} B_{8,20}(X) + 2.40125 \cdot 10^{13} B_{9,20}(X) + 1.02825 \\
& \cdot 10^{13} B_{10,20}(X) - 6.08666 \cdot 10^{11} B_{11,20}(X) - 7.31328 \cdot 10^{12} B_{12,20}(X) - 1.00112 \cdot 10^{13} B_{13,20}(X) \\
& - 9.69955 \cdot 10^{12} B_{14,20}(X) - 7.61291 \cdot 10^{12} B_{15,20}(X) - 4.85196 \cdot 10^{12} B_{16,20}(X) - 2.20819 \\
& \cdot 10^{12} B_{17,20}(X) - 1.32423 \cdot 10^{11} B_{18,20}(X) + 1.21228 \cdot 10^{12} B_{19,20}(X) + 1.86285 \cdot 10^{12} B_{20,20}(X)
\end{aligned}$$



Degree reduction and raising:

$$q_3 = 5.92859\cdot 10^{14}X^3 - 1.19797\cdot 10^{15}X^2 + 7.45636\cdot 10^{14}X - 1.37175\cdot 10^{14}$$
$$= -1.37175\cdot 10^{14}B_{0,3} + 1.1137\cdot 10^{14}B_{1,3} - 3.94071\cdot 10^{13}B_{2,3} + 3.351\cdot 10^{12}B_{3,3}$$

$$\widetilde{q}_3 = 1.91322\cdot 10^{16}X^{20} - 1.92167\cdot 10^{17}X^{19} + 8.93359\cdot 10^{17}X^{18} - 2.55035\cdot 10^{18}X^{17} + 5.00205\cdot 10^{18}X^{16} - 7.14484\cdot 10^{18}X^{15} + 7.68758\cdot 10^{18}X^{14} - 6.35722\cdot 10^{18}X^{13} + 4.08648\cdot 10^{18}X^{12} - 2.05207\cdot 10^{18}X^{11} + 8.04795\cdot 10^{17}X^{10} - 2.45243\cdot 10^{17}X^9 + 5.74704\cdot 10^{16}X^8 - 1.01865\cdot 10^{16}X^7 + 1.33202\cdot 10^{15}X^6 - 1.23862\cdot 10^{14}X^5 + 7.76045\cdot 10^{12}X^4 + 5.92555\cdot 10^{14}X^3 - 1.19796\cdot 10^{15}X^2 + 7.45636\cdot 10^{14}X - 1.37175\cdot 10^{14}$$
$$= -1.37175\cdot 10^{14}B_{0,20} - 9.98932\cdot 10^{13}B_{1,20} - 6.89164\cdot 10^{13}B_{2,20} - 4.3725\cdot 10^{13}B_{3,20} - 2.37974\cdot 10^{13}B_{4,20} - 8.61871\cdot 10^{12}B_{5,20} + 2.34606\cdot 10^{12}B_{6,20} + 9.58125\cdot 10^{12}B_{7,20} + 1.36776\cdot 10^{13}B_{8,20} + 1.50382\cdot 10^{13}B_{9,20} + 1.43455\cdot 10^{13}B_{10,20} + 1.193\cdot 10^{13}B_{11,20} + 8.49821\cdot 10^{12}B_{12,20} + 4.41537\cdot 10^{12}B_{13,20} + 3.10093\cdot 10^{11}B_{14,20} - 3.36181\cdot 10^{12}B_{15,20} - 6.04854\cdot 10^{12}B_{16,20} - 7.24287\cdot 10^{12}B_{17,20} - 6.42059\cdot 10^{12}B_{18,20} - 3.06272\cdot 10^{12}B_{19,20} + 3.351\cdot 10^{12}B_{20,20}$$



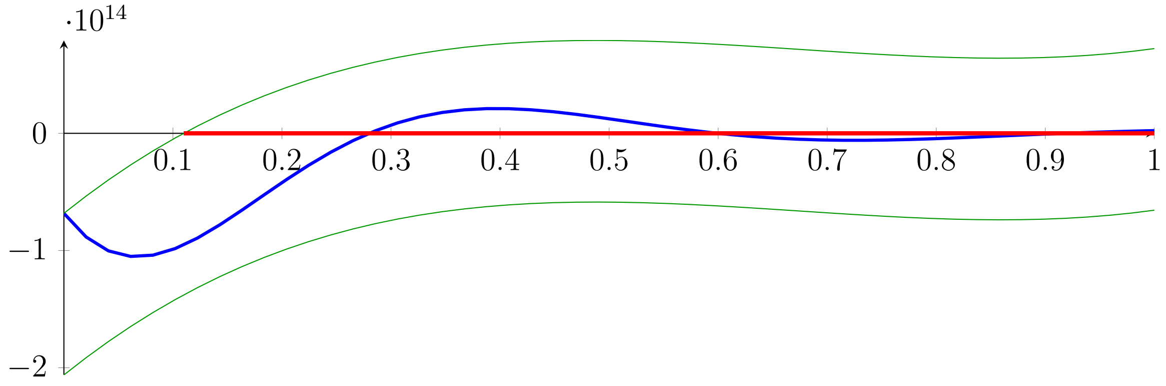
The maximum difference of the Bézier coefficients is $\delta = 6.89397\cdot 10^{13}$.
Bounding polynomials M and m:

$$M = 5.92859\cdot 10^{14}X^3 - 1.19797\cdot 10^{15}X^2 + 7.45636\cdot 10^{14}X - 6.82353\cdot 10^{13}$$
$$m = 5.92859\cdot 10^{14}X^3 - 1.19797\cdot 10^{15}X^2 + 7.45636\cdot 10^{14}X - 2.06115\cdot 10^{14}$$

Root of M and m:

$$N(M) = \{0.109844\} \qquad N(m) = \{1.22621\}$$

Intersection intervals:



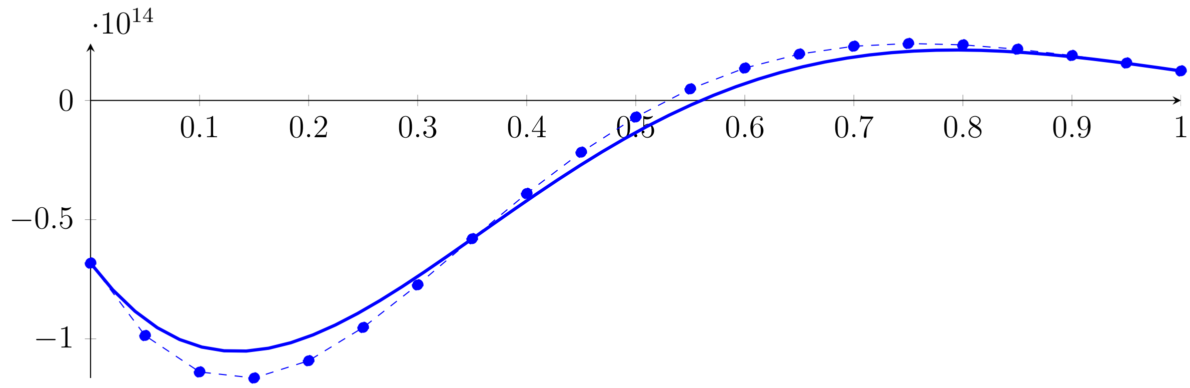
[0.109844, 1]

Longest intersection interval: 0.890156
$\Longrightarrow$ Bisection: first half [3.125, 4.6875] und second half [4.6875, 6.25]

3.17 Recursion Branch 1 1 1 2 1 on the First Half [3.125, 4.6875]

Normalized monomial und Bézier representations and the Bézier polygon:

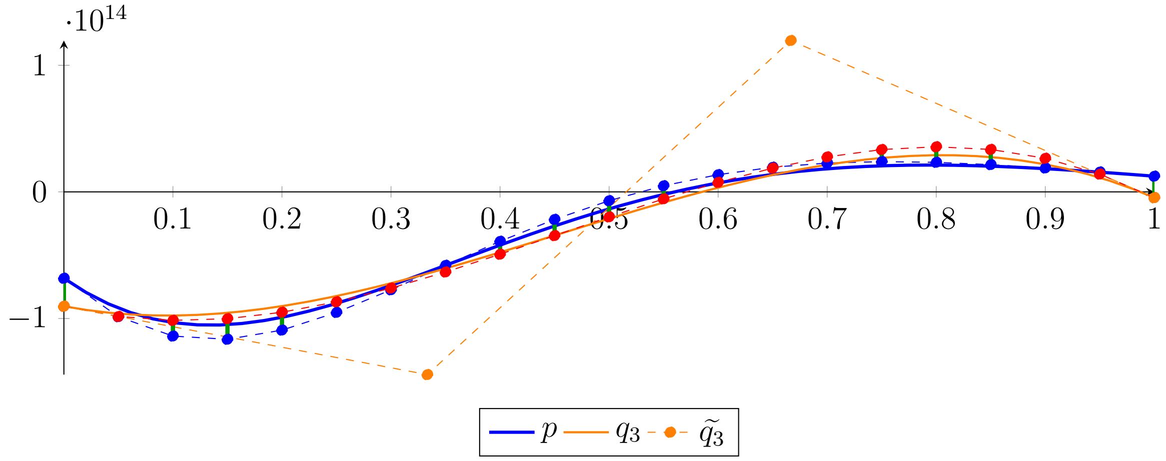
$$\begin{aligned} p = {} & 34696.8X^{20} - 584847X^{19} + 3.25592{\cdot}10^{7}X^{18} - 8.1589{\cdot}10^{8}X^{17} + 1.46784{\cdot}10^{10}X^{16} - 1.89675{\cdot}10^{11}X^{15} \\ & + 1.81956{\cdot}10^{12}X^{14} - 1.31307{\cdot}10^{13}X^{13} + 7.14297{\cdot}10^{13}X^{12} - 2.89728{\cdot}10^{14}X^{11} + 8.49406{\cdot}10^{14}X^{10} \\ & - 1.66526{\cdot}10^{15}X^{9} + 1.65931{\cdot}10^{15}X^{8} + 9.85362{\cdot}10^{14}X^{7} - 5.74116{\cdot}10^{15}X^{6} + 7.50397{\cdot}10^{15}X^{5} \\ & - 2.84125{\cdot}10^{15}X^{4} - 2.70311{\cdot}10^{15}X^{3} + 2.87089{\cdot}10^{15}X^{2} - 6.07752{\cdot}10^{14}X - 6.82353{\cdot}10^{13} \\ = {} & -6.82353{\cdot}10^{13}B_{0,20}(X) - 9.86229{\cdot}10^{13}B_{1,20}(X) - 1.13901{\cdot}10^{14}B_{2,20}(X) - 1.16439 \\ & {\cdot}10^{14}B_{3,20}(X) - 1.09197{\cdot}10^{14}B_{4,20}(X) - 9.52335{\cdot}10^{13}B_{5,20}(X) - 7.73753{\cdot}10^{13}B_{6,20}(X) \\ & - 5.80151{\cdot}10^{13}B_{7,20}(X) - 3.90206{\cdot}10^{13}B_{8,20}(X) - 2.17241{\cdot}10^{13}B_{9,20}(X) - 6.96521 \\ & {\cdot}10^{12}B_{10,20}(X) + 4.83558{\cdot}10^{12}B_{11,20}(X) + 1.35903{\cdot}10^{13}B_{12,20}(X) + 1.94553{\cdot}10^{13}B_{13,20}(X) \\ & + 2.27507{\cdot}10^{13}B_{14,20}(X) + 2.38903{\cdot}10^{13}B_{15,20}(X) + 2.33265{\cdot}10^{13}B_{16,20}(X) + 2.15075 \\ & {\cdot}10^{13}B_{17,20}(X) + 1.88477{\cdot}10^{13}B_{18,20}(X) + 1.57094{\cdot}10^{13}B_{19,20}(X) + 1.23927{\cdot}10^{13}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -7.05808{\cdot}10^{14}X^{3} + 9.53556{\cdot}10^{14}X^{2} - 1.62009{\cdot}10^{14}X - 9.03207{\cdot}10^{13} \\ &= -9.03207{\cdot}10^{13}B_{0,3} - 1.44324{\cdot}10^{14}B_{1,3} + 1.19526{\cdot}10^{14}B_{2,3} - 4.58092{\cdot}10^{12}B_{3,3} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 = &-2.09596\cdot 10^{15}X^{20} + 2.14797\cdot 10^{16}X^{19} - 1.01894\cdot 10^{17}X^{18} + 2.96774\cdot 10^{17}X^{17} - 5.93493\cdot 10^{17}X^{16}\\ &+ 8.63304\cdot 10^{17}X^{15} - 9.44024\cdot 10^{17}X^{14} + 7.9115\cdot 10^{17}X^{13} - 5.13739\cdot 10^{17}X^{12} + 2.59946\cdot 10^{17}X^{11}\\ &- 1.02723\cdot 10^{17}X^{10} + 3.16775\cdot 10^{16}X^{9} - 7.56897\cdot 10^{15}X^{8} + 1.3705\cdot 10^{15}X^{7} - 1.79298\cdot 10^{14}X^{6} + 1.57888\\ &\cdot 10^{13}X^{5} - 8.69681\cdot 10^{11}X^{4} - 7.05782\cdot 10^{14}X^{3} + 9.53556\cdot 10^{14}X^{2} - 1.62009\cdot 10^{14}X - 9.03207\cdot 10^{13}\\ = &-9.03207\cdot 10^{13}B_{0,20} - 9.84212\cdot 10^{13}B_{1,20} - 1.01503\cdot 10^{14}B_{2,20} - 1.00185\cdot 10^{14}B_{3,20} - 9.50868\\ &\cdot 10^{13}B_{4,20} - 8.68267\cdot 10^{13}B_{5,20} - 7.60259\cdot 10^{13}B_{6,20} - 6.32991\cdot 10^{13}B_{7,20} - 4.92738\cdot 10^{13}B_{8,20}\\ &- 3.45544\cdot 10^{13}B_{9,20} - 1.97805\cdot 10^{13}B_{10,20} - 5.54979\cdot 10^{12}B_{11,20} + 7.49939\cdot 10^{12}B_{12,20}\\ &+ 1.87633\cdot 10^{13}B_{13,20} + 2.76129\cdot 10^{13}B_{14,20} + 3.34342\cdot 10^{13}B_{15,20} + 3.56058\cdot 10^{13}B_{16,20}\\ &+ 3.35093\cdot 10^{13}B_{17,20} + 2.65254\cdot 10^{13}B_{18,20} + 1.40351\cdot 10^{13}B_{19,20} - 4.58092\cdot 10^{12}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.20854\cdot 10^{13}$.

Bounding polynomials M and m:

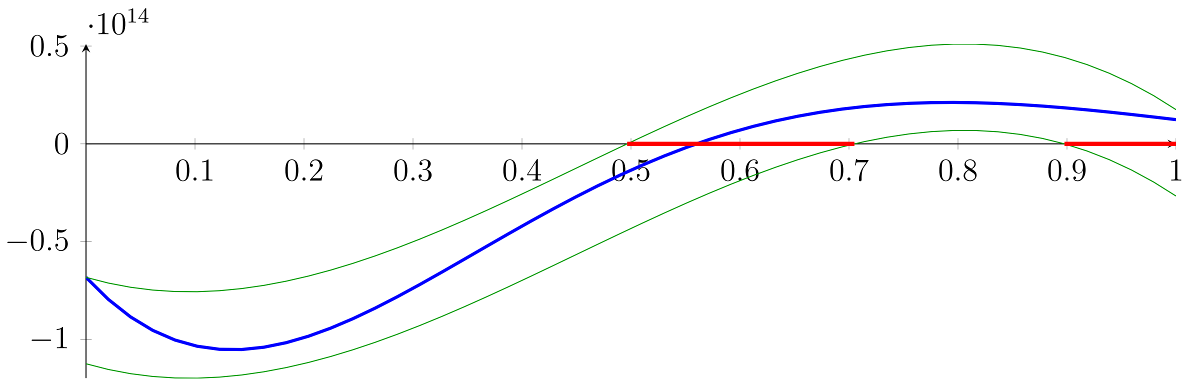
$$M = -7.05808\cdot 10^{14}X^3 + 9.53556\cdot 10^{14}X^2 - 1.62009\cdot 10^{14}X - 6.82353\cdot 10^{13}$$

$$m = -7.05808\cdot 10^{14}X^3 + 9.53556\cdot 10^{14}X^2 - 1.62009\cdot 10^{14}X - 1.12406\cdot 10^{14}$$

Root of M and m:

$$N(M) = \{-0.186965, 0.496483, 1.0415\} \qquad N(m) = \{-0.251653, 0.704983, 0.897683\}$$

Intersection intervals:



$$[0.496483, 0.704983],\ [0.897683, 1]$$

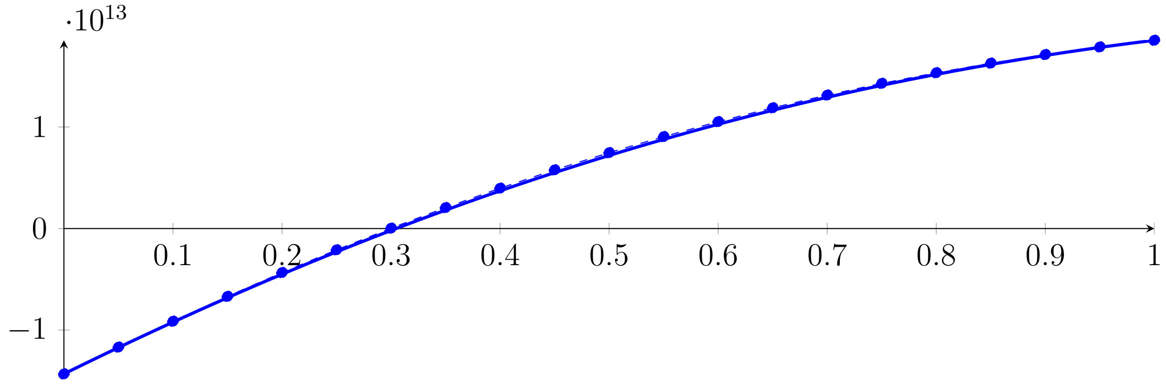
Longest intersection interval: 0.208501

$\Longrightarrow$ Selective recursion: interval 1: $[3.90075, 4.22654]$, interval 2: $[4.52763, 4.6875]$,

3.18 Recursion Branch 1 1 1 2 1 1 in Interval 1: $[3.90075, 4.22654]$

Normalized monomial und Bézier representations and the Bézier polygon:

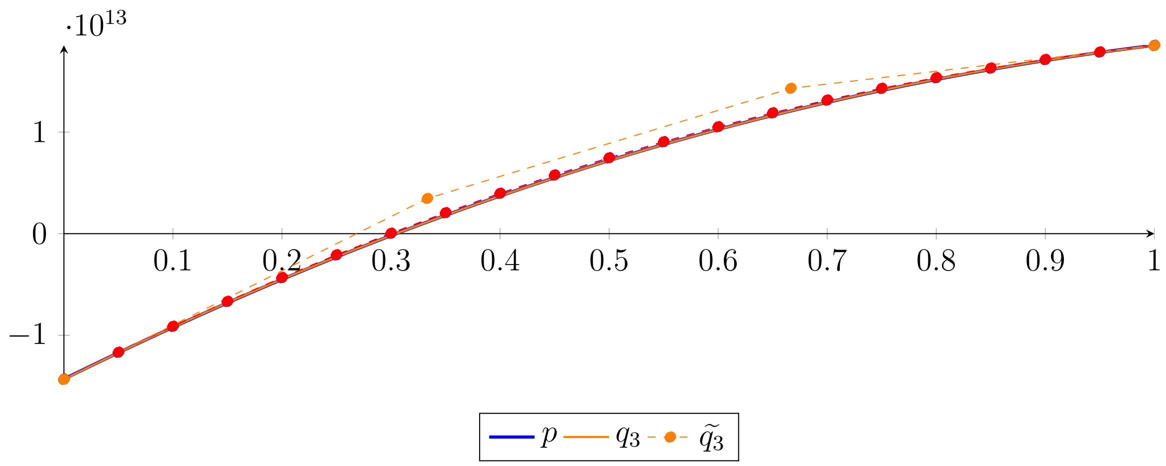
$$p = -5273.86X^{20} + 59854.1X^{19} - 67714.6X^{18} + 882344X^{17} - 2.79853\cdot10^{6}X^{16} + 1.83217\cdot10^{6}X^{15} - 49131.3X^{14} + 304175X^{13} + 104073X^{12} - 918203X^{11} + 1.03633\cdot10^{7}X^{10} + 5.49442\cdot10^{7}X^{9} - 1.77951\cdot10^{9}X^{8} + 1.49862\cdot10^{10}X^{7} - 2.82181\cdot10^{10}X^{6} - 4.16862\cdot10^{11}X^{5} + 2.99182\cdot10^{12}X^{4} - 4.44302\cdot10^{12}X^{3} - 1.81017\cdot10^{13}X^{2} + 5.28436\cdot10^{13}X - 1.43173\cdot10^{13}$$
$$= -1.43173\cdot10^{13}B_{0,20}(X) - 1.16751\cdot10^{13}B_{1,20}(X) - 9.12821\cdot10^{12}B_{2,20}(X) - 6.68048\cdot10^{12}B_{3,20}(X) - 4.33519\cdot10^{12}B_{4,20}(X) - 2.09504\cdot10^{12}B_{5,20}(X) + 3.78457\cdot10^{10}B_{6,20}(X) + 2.06187\cdot10^{12}B_{7,20}(X) + 3.97596\cdot10^{12}B_{8,20}(X) + 5.7795\cdot10^{12}B_{9,20}(X) + 7.47233\cdot10^{12}B_{10,20}(X) + 9.05471\cdot10^{12}B_{11,20}(X) + 1.05273\cdot10^{13}B_{12,20}(X) + 1.18911\cdot10^{13}B_{13,20}(X) + 1.31475\cdot10^{13}B_{14,20}(X) + 1.42982\cdot10^{13}B_{15,20}(X) + 1.5345\cdot10^{13}B_{16,20}(X) + 1.62902\cdot10^{13}B_{17,20}(X) + 1.71362\cdot10^{13}B_{18,20}(X) + 1.78857\cdot10^{13}B_{19,20}(X) + 1.85415\cdot10^{13}B_{20,20}(X)$$



Degree reduction and raising:

$$q_3 = 3.37444\cdot10^{11}X^{3} - 2.09151\cdot10^{13}X^{2} + 5.34399\cdot10^{13}X - 1.43463\cdot10^{13}$$
$$= -1.43463\cdot10^{13}B_{0,3} + 3.46705\cdot10^{12}B_{1,3} + 1.43087\cdot10^{13}B_{2,3} + 1.8516\cdot10^{13}B_{3,3}$$

$$\widetilde{q}_3 = 4.95697\cdot10^{14}X^{20} - 4.96396\cdot10^{15}X^{19} + 2.29664\cdot10^{16}X^{18} - 6.51563\cdot10^{16}X^{17} + 1.26892\cdot10^{17}X^{16} - 1.79995\cdot10^{17}X^{15} + 1.92603\cdot10^{17}X^{14} - 1.58879\cdot10^{17}X^{13} + 1.02361\cdot10^{17}X^{12} - 5.18367\cdot10^{16}X^{11} + 2.06406\cdot10^{16}X^{10} - 6.42263\cdot10^{15}X^{9} + 1.53965\cdot10^{15}X^{8} - 2.77093\cdot10^{14}X^{7} + 3.58264\cdot10^{13}X^{6} - 3.07628\cdot10^{12}X^{5} + 1.44594\cdot10^{11}X^{4} + 3.35758\cdot10^{11}X^{3} - 2.09152\cdot10^{13}X^{2} + 5.34399\cdot10^{13}X - 1.43463\cdot10^{13}$$
$$= -1.43463\cdot10^{13}B_{0,20} - 1.16743\cdot10^{13}B_{1,20} - 9.11234\cdot10^{12}B_{2,20} - 6.66021\cdot10^{12}B_{3,20} - 4.31754\cdot10^{12}B_{4,20} - 2.08418\cdot10^{12}B_{5,20} + 4.05957\cdot10^{10}B_{6,20} + 2.05612\cdot10^{12}B_{7,20} + 3.96446\cdot10^{12}B_{8,20} + 5.76306\cdot10^{12}B_{9,20} + 7.45622\cdot10^{12}B_{10,20} + 9.03936\cdot10^{12}B_{11,20} + 1.05179\cdot10^{13}B_{12,20} + 1.18879\cdot10^{13}B_{13,20} + 1.31523\cdot10^{13}B_{14,20} + 1.431\cdot10^{13}B_{15,20} + 1.53619\cdot10^{13}B_{16,20} + 1.63081\cdot10^{13}B_{17,20} + 1.7149\cdot10^{13}B_{18,20} + 1.78849\cdot10^{13}B_{19,20} + 1.8516\cdot10^{13}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 2.89502 \cdot 10^{10}$.
Bounding polynomials M and m:

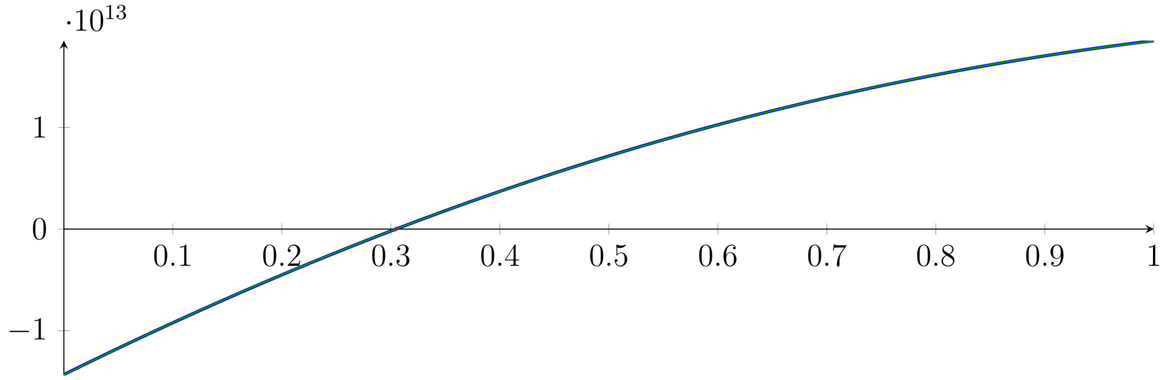
$$M = 3.37444 \cdot 10^{11} X^3 - 2.09151 \cdot 10^{13} X^2 + 5.34399 \cdot 10^{13} X - 1.43173 \cdot 10^{13}$$

$$m = 3.37444 \cdot 10^{11} X^3 - 2.09151 \cdot 10^{13} X^2 + 5.34399 \cdot 10^{13} X - 1.43752 \cdot 10^{13}$$

Root of M and m:

$$N(M) = \{0.303877, 2.35361, 59.3235\} \qquad N(m) = \{0.305296, 2.35214, 59.3235\}$$

Intersection intervals:



$$[0.303877, 0.305296]$$

Longest intersection interval: 0.00141938
$\Longrightarrow$ Selective recursion: interval 1: $[3.99975, 4.00021]$,

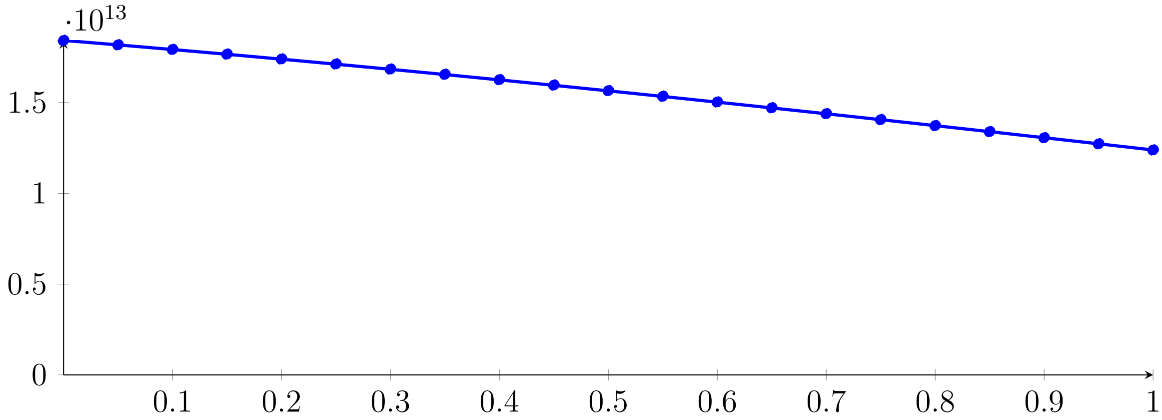
3.19 Recursion Branch 1 1 1 2 1 1 1 in Interval 1: $[3.99975, 4.00021]$

Found root in interval $[3.99975, 4.00021]$ at recursion depth 7!

3.20 Recursion Branch 1 1 1 2 1 2 in Interval 2: $[4.52763, 4.6875]$

Normalized monomial und Bézier representations and the Bézier polygon:

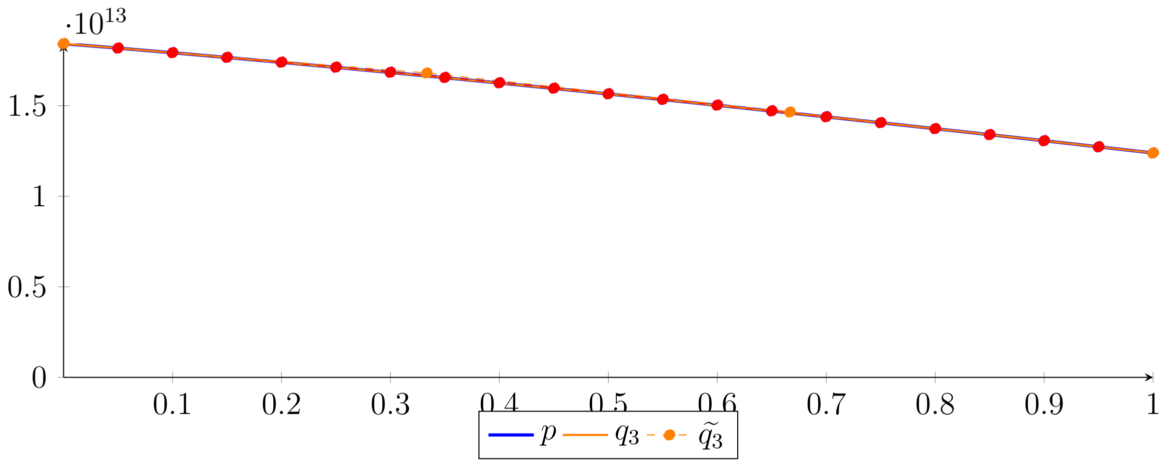
$$\begin{aligned} p = & -15686.9 X^{20} + 80487.3 X^{19} - 499543 X^{18} + 1.92203 \cdot 10^{6} X^{17} - 1.03752 \cdot 10^{7} X^{16} + 8.09048 \\ & \cdot 10^{6} X^{15} - 3.22632 \cdot 10^{6} X^{14} - 1.57553 \cdot 10^{6} X^{13} - 8.26531 \cdot 10^{6} X^{12} - 1.26692 \cdot 10^{6} X^{11} - 2.41698 \\ & \cdot 10^{6} X^{10} - 180426 X^{9} - 1.34089 \cdot 10^{6} X^{8} - 1.13285 \cdot 10^{7} X^{7} + 5.05689 \cdot 10^{8} X^{6} - 4.4203 \cdot 10^{9} X^{5} \\ & - 1.79317 \cdot 10^{10} X^{4} + 4.72836 \cdot 10^{11} X^{3} - 1.62878 \cdot 10^{12} X^{2} - 4.85707 \cdot 10^{12} X + 1.84276 \cdot 10^{13} \\ = & \, 1.84276 \cdot 10^{13} B_{0,20}(X) + 1.81848 \cdot 10^{13} B_{1,20}(X) + 1.79333 \cdot 10^{13} B_{2,20}(X) + 1.76737 \\ & \cdot 10^{13} B_{3,20}(X) + 1.74064 \cdot 10^{13} B_{4,20}(X) + 1.71317 \cdot 10^{13} B_{5,20}(X) + 1.68501 \cdot 10^{13} B_{6,20}(X) \\ & + 1.6562 \cdot 10^{13} B_{7,20}(X) + 1.62677 \cdot 10^{13} B_{8,20}(X) + 1.59677 \cdot 10^{13} B_{9,20}(X) + 1.56622 \\ & \cdot 10^{13} B_{10,20}(X) + 1.53518 \cdot 10^{13} B_{11,20}(X) + 1.50368 \cdot 10^{13} B_{12,20}(X) + 1.47175 \cdot 10^{13} B_{13,20}(X) \\ & + 1.43943 \cdot 10^{13} B_{14,20}(X) + 1.40676 \cdot 10^{13} B_{15,20}(X) + 1.37376 \cdot 10^{13} B_{16,20}(X) + 1.34049 \\ & \cdot 10^{13} B_{17,20}(X) + 1.30696 \cdot 10^{13} B_{18,20}(X) + 1.27321 \cdot 10^{13} B_{19,20}(X) + 1.23927 \cdot 10^{13} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_3 = 4.26333\cdot 10^{11} X^3 - 1.59677\cdot 10^{12} X^2 - 4.8644\cdot 10^{12} X + 1.8428\cdot 10^{13}$$
$$= 1.8428\cdot 10^{13} B_{0,3} + 1.68065\cdot 10^{13} B_{1,3} + 1.46528\cdot 10^{13} B_{2,3} + 1.23931\cdot 10^{13} B_{3,3}$$

$$\widetilde{q}_3 = -2.70734\cdot 10^{15} X^{20} + 2.71464\cdot 10^{16} X^{19} - 1.26044\cdot 10^{17} X^{18} + 3.59528\cdot 10^{17} X^{17} - 7.04772\cdot 10^{17} X^{16} + 1.00628\cdot 10^{18} X^{15} - 1.0822\cdot 10^{18} X^{14} + 8.94204\cdot 10^{17} X^{13} - 5.73969\cdot 10^{17} X^{12} + 2.87501\cdot 10^{17} X^{11} - 1.12289\cdot 10^{17} X^{10} + 3.40037\cdot 10^{16} X^9 - 7.90476\cdot 10^{15} X^8 + 1.39249\cdot 10^{15} X^7 - 1.83212\cdot 10^{14} X^6 + 1.76863\cdot 10^{13} X^5 - 1.2206\cdot 10^{12} X^4 + 4.8318\cdot 10^{11} X^3 - 1.59826\cdot 10^{12} X^2 - 4.86439\cdot 10^{12} X + 1.8428\cdot 10^{13}$$
$$= 1.8428\cdot 10^{13} B_{0,20} + 1.81848\cdot 10^{13} B_{1,20} + 1.79331\cdot 10^{13} B_{2,20} + 1.76735\cdot 10^{13} B_{3,20} + 1.74061\cdot 10^{13} B_{4,20} + 1.71319\cdot 10^{13} B_{5,20} + 1.68493\cdot 10^{13} B_{6,20} + 1.65636\cdot 10^{13} B_{7,20} + 1.62653\cdot 10^{13} B_{8,20} + 1.59711\cdot 10^{13} B_{9,20} + 1.56586\cdot 10^{13} B_{10,20} + 1.53549\cdot 10^{13} B_{11,20} + 1.50343\cdot 10^{13} B_{12,20} + 1.4719\cdot 10^{13} B_{13,20} + 1.43934\cdot 10^{13} B_{14,20} + 1.40677\cdot 10^{13} B_{15,20} + 1.37373\cdot 10^{13} B_{16,20} + 1.34046\cdot 10^{13} B_{17,20} + 1.30694\cdot 10^{13} B_{18,20} + 1.27321\cdot 10^{13} B_{19,20} + 1.23931\cdot 10^{13} B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 3.68594\cdot 10^9$.

Bounding polynomials M and m:

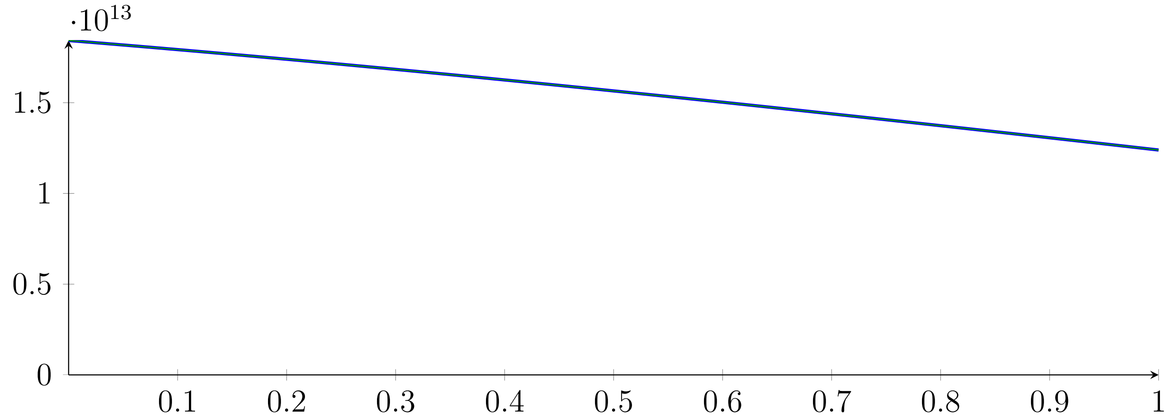
$$M = 4.26333\cdot 10^{11} X^3 - 1.59677\cdot 10^{12} X^2 - 4.8644\cdot 10^{12} X + 1.84317\cdot 10^{13}$$

$$m = 4.26333\cdot 10^{11} X^3 - 1.59677\cdot 10^{12} X^2 - 4.8644\cdot 10^{12} X + 1.84243\cdot 10^{13}$$

Root of M and m:

$$N(M) = \{-3.38819\} \qquad N(m) = \{-3.38783\}$$

Intersection intervals:

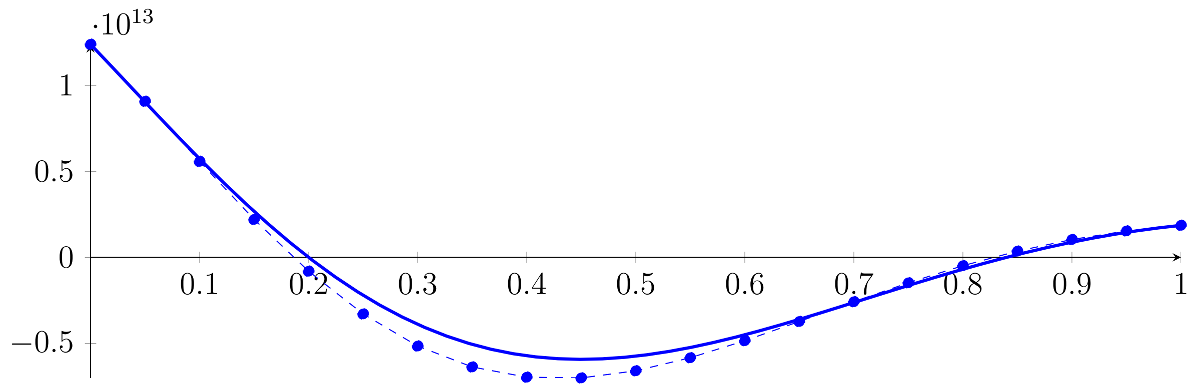


No intersection intervals with the x axis.

3.21 Recursion Branch 1 1 1 2 2 on the Second Half $[4.6875, 6.25]$

Normalized monomial und Bézier representations and the Bézier polygon:

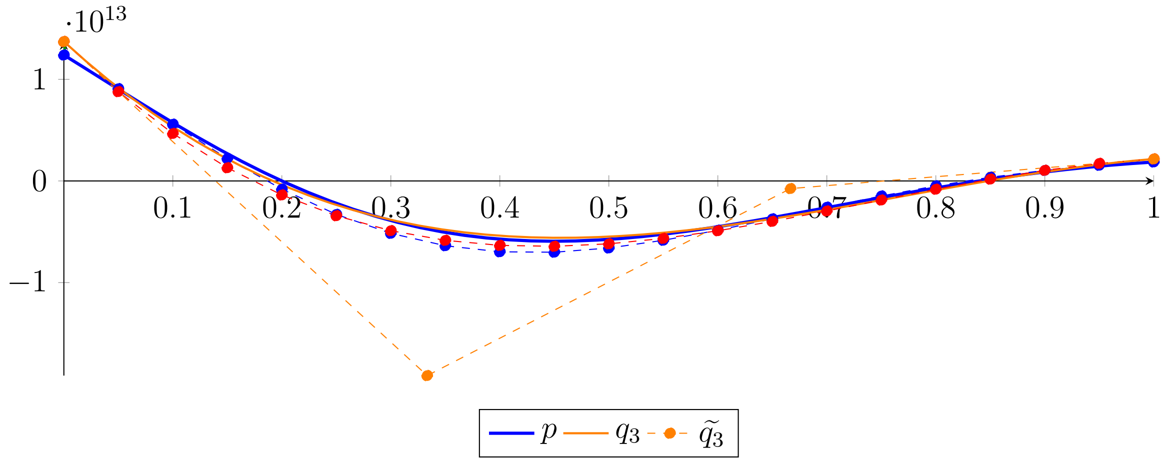
$$
\begin{aligned}
p = {} & 13943.5X^{20} - 587258X^{19} + 1.89008{\cdot}10^{7}X^{18} - 3.73664{\cdot}10^{8}X^{17} + 4.87208{\cdot}10^{9}X^{16} - 4.34631{\cdot}10^{10}X^{15} \\
& + 2.65721{\cdot}10^{11}X^{14} - 1.05722{\cdot}10^{12}X^{13} + 2.18629{\cdot}10^{12}X^{12} + 1.53487{\cdot}10^{12}X^{11} - 2.39754{\cdot}10^{13}X^{10} \\
& + 6.26713{\cdot}10^{13}X^{9} - 3.75532{\cdot}10^{13}X^{8} - 1.53878{\cdot}10^{14}X^{7} + 3.47765{\cdot}10^{14}X^{6} - 1.50066{\cdot}10^{14}X^{5} \\
& - 3.00387{\cdot}10^{14}X^{4} + 3.42221{\cdot}10^{14}X^{3} - 3.38862{\cdot}10^{13}X^{2} - 6.63332{\cdot}10^{13}X + 1.23927{\cdot}10^{13} \\
= {} & 1.23927{\cdot}10^{13}B_{0,20}(X) + 9.07608{\cdot}10^{12}B_{1,20}(X) + 5.58107{\cdot}10^{12}B_{2,20}(X) + 2.20791 \\
& {\cdot}10^{12}B_{3,20}(X) - 8.05212{\cdot}10^{11}B_{4,20}(X) - 3.29178{\cdot}10^{12}B_{5,20}(X) - 5.15766{\cdot}10^{12}B_{6,20}(X) \\
& - 6.37482{\cdot}10^{12}B_{7,20}(X) - 6.97037{\cdot}10^{12}B_{8,20}(X) - 7.01303{\cdot}10^{12}B_{9,20}(X) - 6.5991 \\
& {\cdot}10^{12}B_{10,20}(X) - 5.83916{\cdot}10^{12}B_{11,20}(X) - 4.8467{\cdot}10^{12}B_{12,20}(X) - 3.72904{\cdot}10^{12}B_{13,20}(X) \\
& - 2.58078{\cdot}10^{12}B_{14,20}(X) - 1.47983{\cdot}10^{12}B_{15,20}(X) - 4.85456{\cdot}10^{11}B_{16,20}(X) + 3.6178 \\
& {\cdot}10^{11}B_{17,20}(X) + 1.03875{\cdot}10^{12}B_{18,20}(X) + 1.53757{\cdot}10^{12}B_{19,20}(X) + 1.86285{\cdot}10^{12}B_{20,20}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_3 &= -6.67489{\cdot}10^{13}X^{3} + 1.53795{\cdot}10^{14}X^{2} - 9.85979{\cdot}10^{13}X + 1.37157{\cdot}10^{13} \\
&= 1.37157{\cdot}10^{13}B_{0,3} - 1.91502{\cdot}10^{13}B_{1,3} - 7.51182{\cdot}10^{11}B_{2,3} + 2.1639{\cdot}10^{12}B_{3,3}
\end{aligned}
$$

$$\widetilde{q}_3 = -1.6977\cdot 10^{15}X^{20} + 1.70588\cdot 10^{16}X^{19} - 7.93195\cdot 10^{16}X^{18} + 2.26447\cdot 10^{17}X^{17} - 4.44096\cdot 10^{17}X^{16} + 6.3424\cdot 10^{17}X^{15} - 6.82309\cdot 10^{17}X^{14} + 5.64183\cdot 10^{17}X^{13} - 3.62683\cdot 10^{17}X^{12} + 1.82181\cdot 10^{17}X^{11} - 7.14984\cdot 10^{16}X^{10} + 2.18141\cdot 10^{16}X^{9} - 5.12027\cdot 10^{15}X^{8} + 9.08365\cdot 10^{14}X^{7} - 1.18372\cdot 10^{14}X^{6} + 1.08372\cdot 10^{13}X^{5} - 6.49507\cdot 10^{11}X^{4} - 6.6726\cdot 10^{13}X^{3} + 1.53795\cdot 10^{14}X^{2} - 9.85979\cdot 10^{13}X + 1.37157\cdot 10^{13}$$
$$= 1.37157\cdot 10^{13}B_{0,20} + 8.78585\cdot 10^{12}B_{1,20} + 4.6654\cdot 10^{12}B_{2,20} + 1.29587\cdot 10^{12}B_{3,20} - 1.38142\cdot 10^{12}B_{4,20} - 3.42456\cdot 10^{12}B_{5,20} - 4.89344\cdot 10^{12}B_{6,20} - 5.84344\cdot 10^{12}B_{7,20} - 6.33937\cdot 10^{12}B_{8,20} - 6.42938\cdot 10^{12}B_{9,20} - 6.18648\cdot 10^{12}B_{10,20} - 5.65238\cdot 10^{12}B_{11,20} - 4.90226\cdot 10^{12}B_{12,20} - 3.98087\cdot 10^{12}B_{13,20} - 2.95631\cdot 10^{12}B_{14,20} - 1.88157\cdot 10^{12}B_{15,20} - 8.1789\cdot 10^{11}B_{16,20} + 1.77224\cdot 10^{11}B_{17,20} + 1.04489\cdot 10^{12}B_{18,20} + 1.72664\cdot 10^{12}B_{19,20} + 2.1639\cdot 10^{12}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.323\cdot 10^{12}$.

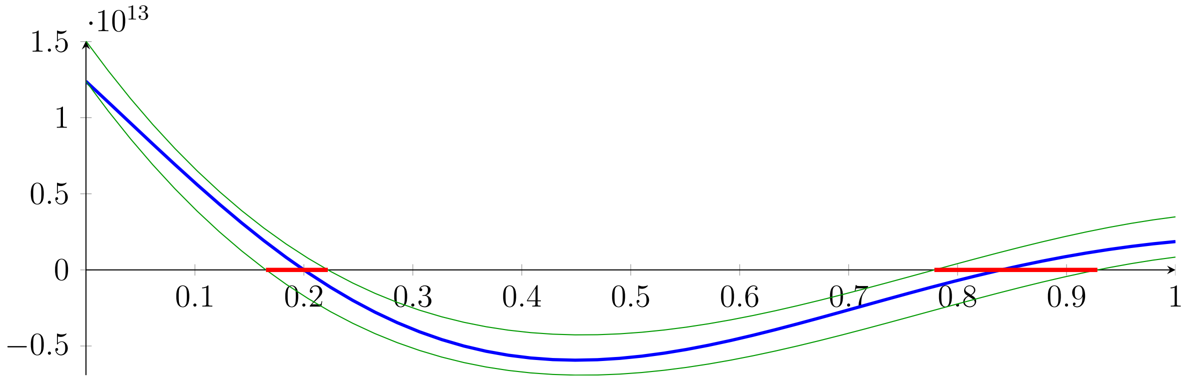
Bounding polynomials M and m:

$$M = -6.67489\cdot 10^{13}X^3 + 1.53795\cdot 10^{14}X^2 - 9.85979\cdot 10^{13}X + 1.50387\cdot 10^{13}$$
$$m = -6.67489\cdot 10^{13}X^3 + 1.53795\cdot 10^{14}X^2 - 9.85979\cdot 10^{13}X + 1.23927\cdot 10^{13}$$

Root of M and m:

$$N(M) = \{0.221984, 0.778696, 1.3034\} \qquad N(m) = \{0.165212, 0.928324, 1.21054\}$$

Intersection intervals:



$$[0.165212, 0.221984],\ [0.778696, 0.928324]$$

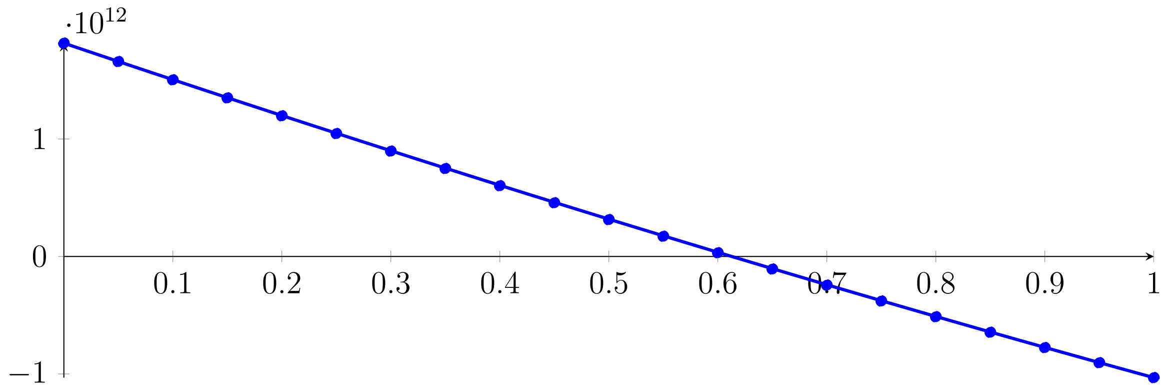
Longest intersection interval: 0.149628

$\Longrightarrow$ Selective recursion: interval 1: $[4.94564, 5.03435]$, interval 2: $[5.90421, 6.13801]$,

3.22 Recursion Branch 1 1 1 2 2 1 in Interval 1: $[4.94564, 5.03435]$

Normalized monomial und Bézier representations and the Bézier polygon:

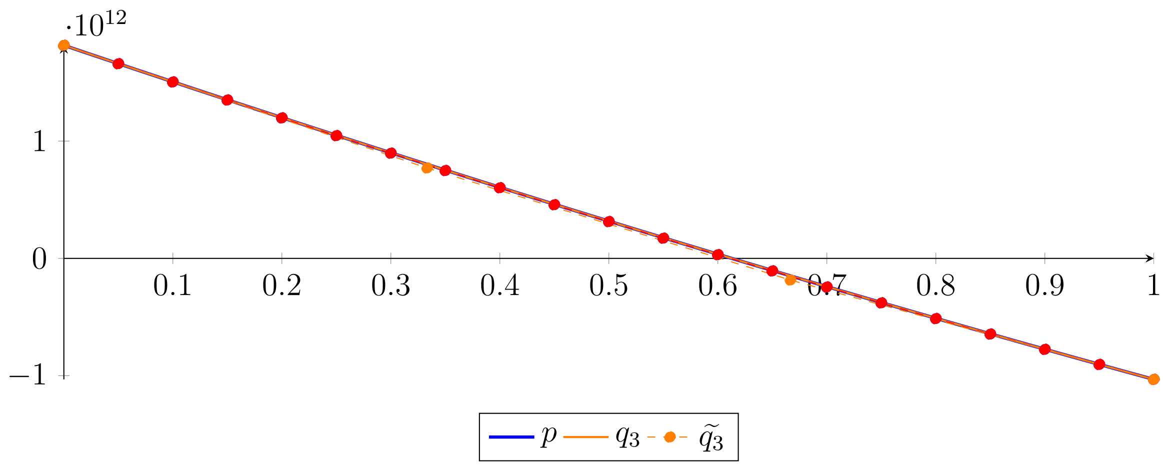
$$\begin{aligned}p &= -466.887X^{20} - 304.531X^{19} - 20726.3X^{18} + 38933.7X^{17} - 361142X^{16} + 301601X^{15} \\ &\quad - 174420X^{14} - 66921.6X^{13} - 427609X^{12} - 109568X^{11} - 154805X^{10} - 27555.9X^{9} \\ &\quad - 246.035X^{8} - 293274X^{7} + 5.39635\cdot 10^{6}X^{6} + 6.0233\cdot 10^{7}X^{5} - 3.19253 \\ &\quad \cdot 10^{9}X^{4} + 2.37758\cdot 10^{10}X^{3} + 2.68294\cdot 10^{11}X^{2} - 3.13663\cdot 10^{12}X + 1.81626\cdot 10^{12} \\ &= 1.81626\cdot 10^{12}B_{0,20}(X) + 1.65943\cdot 10^{12}B_{1,20}(X) + 1.50401\cdot 10^{12}B_{2,20}(X) + 1.35002 \\ &\quad \cdot 10^{12}B_{3,20}(X) + 1.19749\cdot 10^{12}B_{4,20}(X) + 1.04643\cdot 10^{12}B_{5,20}(X) + 8.9686\cdot 10^{11}B_{6,20}(X) \\ &\quad + 7.488\cdot 10^{11}B_{7,20}(X) + 6.02268\cdot 10^{11}B_{8,20}(X) + 4.5728\cdot 10^{11}B_{9,20}(X) + 3.13853 \\ &\quad \cdot 10^{11}B_{10,20}(X) + 1.72003\cdot 10^{11}B_{11,20}(X) + 3.17435\cdot 10^{10}B_{12,20}(X) - 1.0691\cdot 10^{11}B_{13,20}(X) \\ &\quad - 2.43943\cdot 10^{11}B_{14,20}(X) - 3.79344\cdot 10^{11}B_{15,20}(X) - 5.13098\cdot 10^{11}B_{16,20}(X) - 6.45196 \\ &\quad \cdot 10^{11}B_{17,20}(X) - 7.75624\cdot 10^{11}B_{18,20}(X) - 9.04372\cdot 10^{11}B_{19,20}(X) - 1.03143\cdot 10^{12}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= 1.75749\cdot 10^{10}X^{3} + 2.72239\cdot 10^{11}X^{2} - 3.13751\cdot 10^{12}X + 1.8163\cdot 10^{12} \\ &= 1.8163\cdot 10^{12}B_{0,3} + 7.7047\cdot 10^{11}B_{1,3} - 1.84619\cdot 10^{11}B_{2,3} - 1.03139\cdot 10^{12}B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= -1.62407\cdot 10^{14}X^{20} + 1.62805\cdot 10^{15}X^{19} - 7.55387\cdot 10^{15}X^{18} + 2.15237\cdot 10^{16}X^{17} - 4.21381\cdot 10^{16}X^{16} \\ &\quad + 6.00896\cdot 10^{16}X^{15} - 6.45645\cdot 10^{16}X^{14} + 5.33393\cdot 10^{16}X^{13} - 3.42717\cdot 10^{16}X^{12} + 1.72112\cdot 10^{16}X^{11} \\ &\quad - 6.75234\cdot 10^{15}X^{10} + 2.05773\cdot 10^{15}X^{9} - 4.81854\cdot 10^{14}X^{8} + 8.53439\cdot 10^{13}X^{7} - 1.11964\cdot 10^{13}X^{6} + 1.05688 \\ &\quad \cdot 10^{12}X^{5} - 6.86575\cdot 10^{10}X^{4} + 2.04548\cdot 10^{10}X^{3} + 2.7217\cdot 10^{11}X^{2} - 3.1375\cdot 10^{12}X + 1.8163\cdot 10^{12} \\ &= 1.8163\cdot 10^{12}B_{0,20} + 1.65943\cdot 10^{12}B_{1,20} + 1.50399\cdot 10^{12}B_{2,20} + 1.34999\cdot 10^{12}B_{3,20} + 1.19746 \\ &\quad \cdot 10^{12}B_{4,20} + 1.04643\cdot 10^{12}B_{5,20} + 8.96807\cdot 10^{11}B_{6,20} + 7.48903\cdot 10^{11}B_{7,20} + 6.02137\cdot 10^{11}B_{8,20} \\ &\quad + 4.57501\cdot 10^{11}B_{9,20} + 3.13653\cdot 10^{11}B_{10,20} + 1.72216\cdot 10^{11}B_{11,20} + 3.16139\cdot 10^{10}B_{12,20} \\ &\quad - 1.06816\cdot 10^{11}B_{13,20} - 2.43995\cdot 10^{11}B_{14,20} - 3.79344\cdot 10^{11}B_{15,20} - 5.13131\cdot 10^{11}B_{16,20} \\ &\quad - 6.45224\cdot 10^{11}B_{17,20} - 7.75646\cdot 10^{11}B_{18,20} - 9.04371\cdot 10^{11}B_{19,20} - 1.03139\cdot 10^{12}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.21011 \cdot 10^8$.
Bounding polynomials M and m:

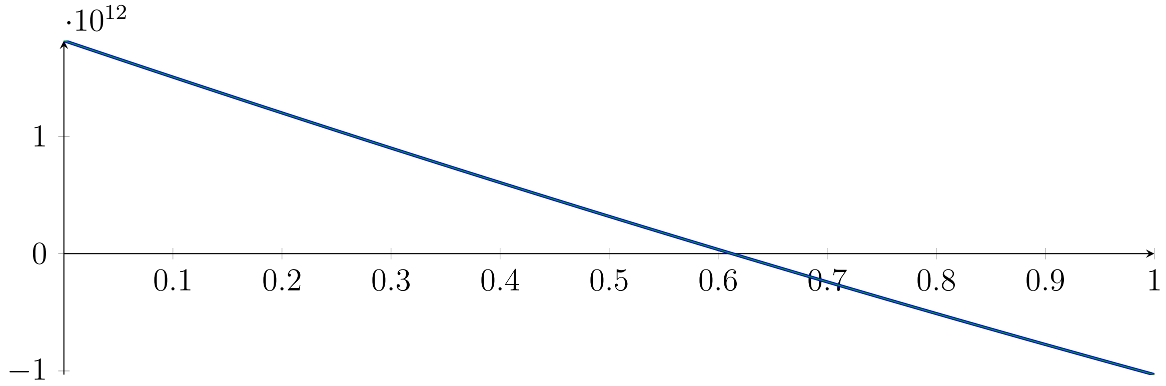
$$M = 1.75749 \cdot 10^{10} X^3 + 2.72239 \cdot 10^{11} X^2 - 3.13751 \cdot 10^{12} X + 1.81653 \cdot 10^{12}$$

$$m = 1.75749 \cdot 10^{10} X^3 + 2.72239 \cdot 10^{11} X^2 - 3.13751 \cdot 10^{12} X + 1.81608 \cdot 10^{12}$$

Root of M and m:

$$N(M) = \{-23.3315, 0.61285, 7.22853\} \qquad N(m) = \{-23.3315, 0.612691, 7.22865\}$$

Intersection intervals:



$$[0.612691, 0.61285]$$

Longest intersection interval: 0.000158768
$\Longrightarrow$ Selective recursion: interval 1: $[4.99999, 5.00001]$,

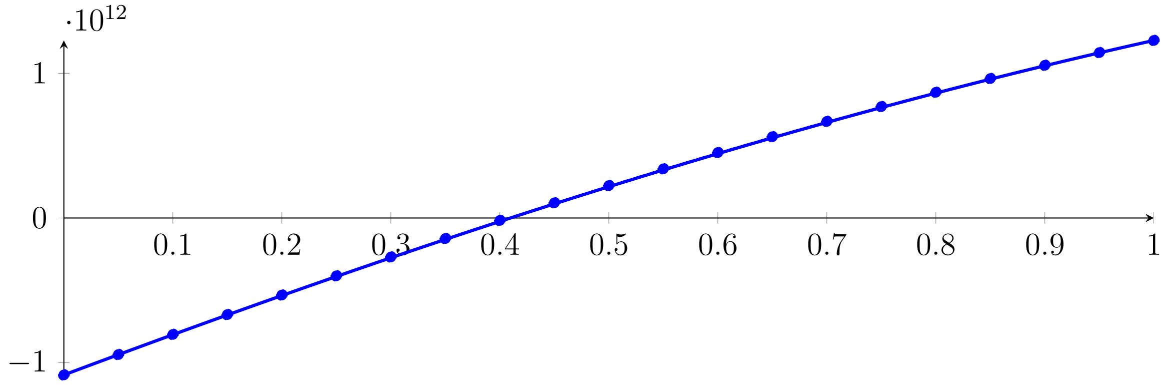
3.23 Recursion Branch 1 1 1 2 2 1 1 in Interval 1: $[4.99999, 5.00001]$

Found root in interval $[4.99999, 5.00001]$ at recursion depth 7!

3.24 Recursion Branch 1 1 1 2 2 2 in Interval 2: $[5.90421, 6.13801]$

Normalized monomial und Bézier representations and the Bézier polygon:

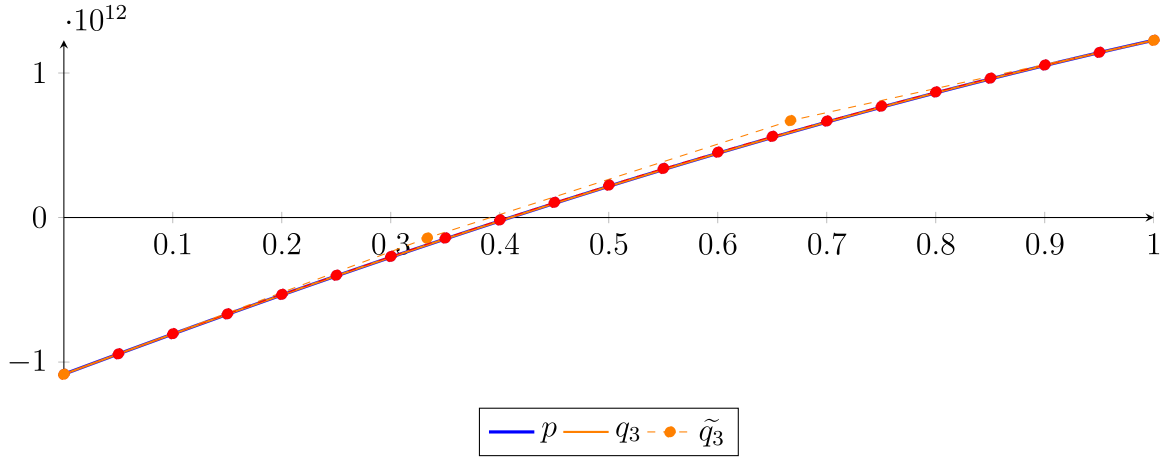
$$\begin{aligned} p = & -88.6415 X^{20} + 2699.94 X^{19} + 3172.32 X^{18} + 26205.4 X^{17} - 14491.8 X^{16} - 21849.8 X^{15} \\ & + 45994.4 X^{14} + 42157.2 X^{13} + 125616 X^{12} + 56895.6 X^{11} + 72373.3 X^{10} - 410079 X^9 \\ & + 3.0944 \cdot 10^6 X^8 + 4.21787 \cdot 10^7 X^7 - 7.60186 \cdot 10^8 X^6 + 1.29326 \cdot 10^9 X^5 + 4.02755 \\ & \cdot 10^{10} X^4 - 2.07686 \cdot 10^{11} X^3 - 3.3951 \cdot 10^{11} X^2 + 2.8174 \cdot 10^{12} X - 1.08416 \cdot 10^{12} \\ = & -1.08416 \cdot 10^{12} B_{0,20}(X) - 9.43291 \cdot 10^{11} B_{1,20}(X) - 8.04208 \cdot 10^{11} B_{2,20}(X) - 6.67094 \\ & \cdot 10^{11} B_{3,20}(X) - 5.32123 \cdot 10^{11} B_{4,20}(X) - 3.9946 \cdot 10^{11} B_{5,20}(X) - 2.69263 \cdot 10^{11} B_{6,20}(X) \\ & - 1.4168 \cdot 10^{11} B_{7,20}(X) - 1.68508 \cdot 10^{10} B_{8,20}(X) + 1.05093 \cdot 10^{11} B_{9,20}(X) + 2.24029 \\ & \cdot 10^{11} B_{10,20}(X) + 3.39842 \cdot 10^{11} B_{11,20}(X) + 4.52426 \cdot 10^{11} B_{12,20}(X) + 5.61684 \cdot 10^{11} B_{13,20}(X) \\ & + 6.67527 \cdot 10^{11} B_{14,20}(X) + 7.69874 \cdot 10^{11} B_{15,20}(X) + 8.68652 \cdot 10^{11} B_{16,20}(X) + 9.63795 \\ & \cdot 10^{11} B_{17,20}(X) + 1.05525 \cdot 10^{12} B_{18,20}(X) + 1.14296 \cdot 10^{12} B_{19,20}(X) + 1.22689 \cdot 10^{12} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_3 = -1.2591\cdot 10^{11}X^3 - 3.92101\cdot 10^{11}X^2 + 2.82907\cdot 10^{12}X - 1.08474\cdot 10^{12}$$
$$= -1.08474\cdot 10^{12}B_{0,3} - 1.4172\cdot 10^{11}B_{1,3} + 6.70603\cdot 10^{11}B_{2,3} + 1.22632\cdot 10^{12}B_{3,3}$$

$$\widetilde{q}_3 = 6.1133\cdot 10^{13}X^{20} - 6.12522\cdot 10^{14}X^{19} + 2.83875\cdot 10^{15}X^{18} - 8.07512\cdot 10^{15}X^{17} + 1.57779\cdot 10^{16}X^{16} - 2.24556\cdot 10^{16}X^{15} + 2.40922\cdot 10^{16}X^{14} - 1.98944\cdot 10^{16}X^{13} + 1.27975\cdot 10^{16}X^{12} - 6.44842\cdot 10^{15}X^{11} + 2.54469\cdot 10^{15}X^{10} - 7.81856\cdot 10^{14}X^9 + 1.84789\cdot 10^{14}X^8 - 3.29454\cdot 10^{13}X^7 + 4.30303\cdot 10^{12}X^6 - 3.93555\cdot 10^{11}X^5 + 2.32605\cdot 10^{10}X^4 - 1.26703\cdot 10^{11}X^3 - 3.92086\cdot 10^{11}X^2 + 2.82907\cdot 10^{12}X - 1.08474\cdot 10^{12}$$
$$= -1.08474\cdot 10^{12}B_{0,20} - 9.4329\cdot 10^{11}B_{1,20} - 8.039\cdot 10^{11}B_{2,20} - 6.66685\cdot 10^{11}B_{3,20} - 5.31751\cdot 10^{11}B_{4,20} - 3.99225\cdot 10^{11}B_{5,20} - 2.69169\cdot 10^{11}B_{6,20} - 1.41808\cdot 10^{11}B_{7,20} - 1.70304\cdot 10^{10}B_{8,20} + 1.04692\cdot 10^{11}B_{9,20} + 2.23753\cdot 10^{11}B_{10,20} + 3.39437\cdot 10^{11}B_{11,20} + 4.52246\cdot 10^{11}B_{12,20} + 5.61563\cdot 10^{11}B_{13,20} + 6.67623\cdot 10^{11}B_{14,20} + 7.70113\cdot 10^{11}B_{15,20} + 8.69022\cdot 10^{11}B_{16,20} + 9.642\cdot 10^{11}B_{17,20} + 1.05555\cdot 10^{12}B_{18,20} + 1.14296\cdot 10^{12}B_{19,20} + 1.22632\cdot 10^{12}B_{20,20}$$



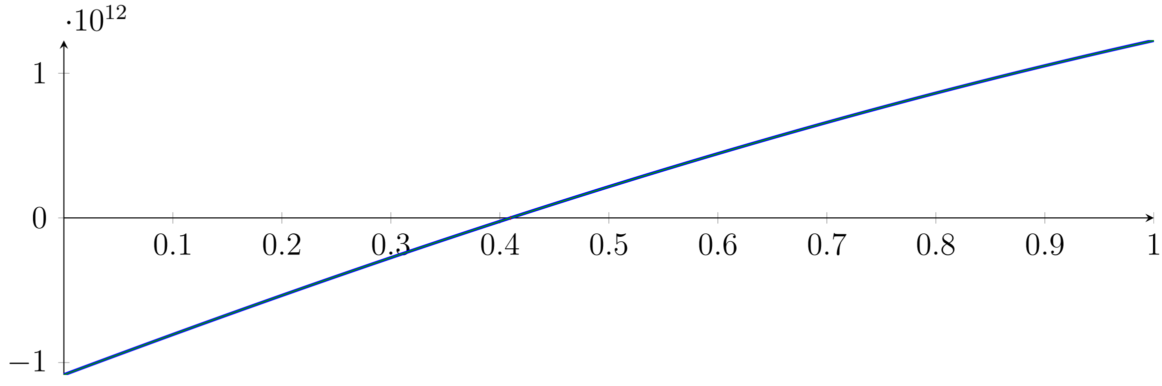
The maximum difference of the Bézier coefficients is $\delta = 5.82963\cdot 10^8$.
Bounding polynomials M and m:

$$M = -1.2591\cdot 10^{11}X^3 - 3.92101\cdot 10^{11}X^2 + 2.82907\cdot 10^{12}X - 1.08416\cdot 10^{12}$$
$$m = -1.2591\cdot 10^{11}X^3 - 3.92101\cdot 10^{11}X^2 + 2.82907\cdot 10^{12}X - 1.08533\cdot 10^{12}$$

Root of M and m:

$$N(M) = \{-6.67407, 0.409522, 3.15041\} \qquad N(m) = \{-6.67421, 0.409999, 3.15006\}$$

Intersection intervals:



$[0.409522, 0.409999]$

Longest intersection interval: 0.000476995
$\Longrightarrow$ Selective recursion: interval 1: $[5.99996, 6.00007]$,

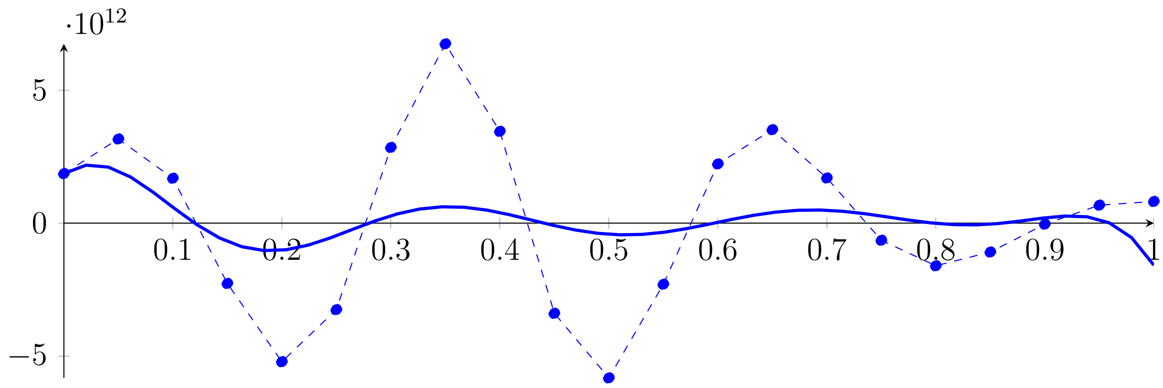
3.25 Recursion Branch 1 1 1 2 2 2 1 in Interval 1: $[5.99996, 6.00007]$

Found root in interval $[5.99996, 6.00007]$ at recursion depth 7!

3.26 Recursion Branch 1 1 2 on the Second Half $[6.25, 12.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

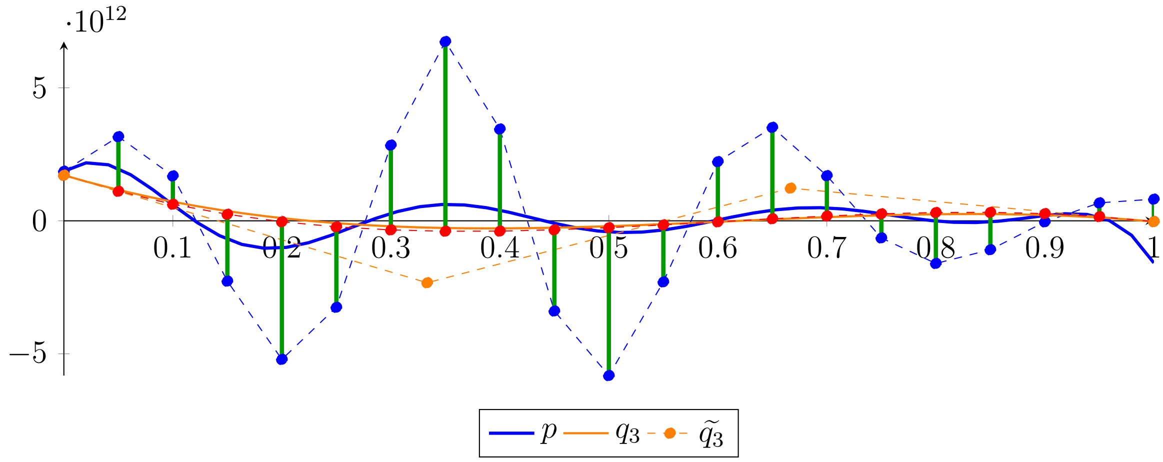
$$p = 8.27181\cdot10^{15}X^{20} - 1.12497\cdot10^{17}X^{19} + 6.56318\cdot10^{17}X^{18} - 2.10324\cdot10^{18}X^{17} + 3.83361\cdot10^{18}X^{16} - 3.25611\cdot10^{18}X^{15} - 1.18134\cdot10^{18}X^{14} + 5.65844\cdot10^{18}X^{13} - 4.66119\cdot10^{18}X^{12} - 3.70393\cdot10^{17}X^{11} + 2.95436\cdot10^{18}X^{10} - 1.48062\cdot10^{18}X^{9} - 3.2208\cdot10^{17}X^{8} + 4.91145\cdot10^{17}X^{7} - 8.64752\cdot10^{16}X^{6} - 4.35417\cdot10^{16}X^{5} + 1.55034\cdot10^{16}X^{4} + 3.36768\cdot10^{14}X^{3} - 5.27545\cdot10^{14}X^{2} + 2.60227\cdot10^{13}X + 1.86285\cdot10^{12}$$
$$= 1.86285\cdot10^{12}B_{0,20}(X) + 3.16399\cdot10^{12}B_{1,20}(X) + 1.68857\cdot10^{12}B_{2,20}(X) - 2.268\cdot10^{12}B_{3,20}(X) - 5.21041\cdot10^{12}B_{4,20}(X) - 3.25192\cdot10^{12}B_{5,20}(X) + 2.84625\cdot10^{12}B_{6,20}(X) + 6.74009\cdot10^{12}B_{7,20}(X) + 3.45161\cdot10^{12}B_{8,20}(X) - 3.39194\cdot10^{12}B_{9,20}(X) - 5.81848\cdot10^{12}B_{10,20}(X) - 2.29738\cdot10^{12}B_{11,20}(X) + 2.22447\cdot10^{12}B_{12,20}(X) + 3.51385\cdot10^{12}B_{13,20}(X) + 1.69765\cdot10^{12}B_{14,20}(X) - 6.43381\cdot10^{11}B_{15,20}(X) - 1.60376\cdot10^{12}B_{16,20}(X) - 1.08654\cdot10^{12}B_{17,20}(X) - 4.06339\cdot10^{10}B_{18,20}(X) + 6.75764\cdot10^{11}B_{19,20}(X) + 8.1419\cdot10^{11}B_{20,20}(X)$$



Degree reduction and raising:

$$q_3 = -1.24152\cdot10^{13}X^3 + 2.28015\cdot10^{13}X^2 - 1.21315\cdot10^{13}X + 1.71511\cdot10^{12}$$
$$= 1.71511\cdot10^{12}B_{0,3} - 2.32872\cdot10^{12}B_{1,3} + 1.22793\cdot10^{12}B_{2,3} - 3.01564\cdot10^{10}B_{3,3}$$

$$\begin{aligned}\widetilde{q}_3 = &-3.10581{\cdot}10^{14}X^{20} + 3.12143{\cdot}10^{15}X^{19} - 1.45204{\cdot}10^{16}X^{18} + 4.14799{\cdot}10^{16}X^{17} - 8.14092{\cdot}10^{16}X^{16} \\ &+ 1.16357{\cdot}10^{17}X^{15} - 1.25267{\cdot}10^{17}X^{14} + 1.03637{\cdot}10^{17}X^{13} - 6.66411{\cdot}10^{16}X^{12} + 3.34715{\cdot}10^{16}X^{11} \\ &- 1.31292{\cdot}10^{16}X^{10} + 4.00187{\cdot}10^{15}X^{9} - 9.38276{\cdot}10^{14}X^{8} + 1.66416{\cdot}10^{14}X^{7} - 2.17656{\cdot}10^{13}X^{6} + 2.022 \\ &{\cdot}10^{12}X^{5} - 1.26511{\cdot}10^{11}X^{4} - 1.24103{\cdot}10^{13}X^{3} + 2.28014{\cdot}10^{13}X^{2} - 1.21315{\cdot}10^{13}X + 1.71511{\cdot}10^{12} \\ = &\ 1.71511{\cdot}10^{12}B_{0,20} + 1.10854{\cdot}10^{12}B_{1,20} + 6.21969{\cdot}10^{11}B_{2,20} + 2.44522{\cdot}10^{11}B_{3,20} - 3.4717 \\ &{\cdot}10^{10}B_{4,20} - 2.26555{\cdot}10^{11}B_{5,20} - 3.42128{\cdot}10^{11}B_{6,20} - 3.91742{\cdot}10^{11}B_{7,20} - 3.87441{\cdot}10^{11}B_{8,20} \\ &- 3.38205{\cdot}10^{11}B_{9,20} - 2.57576{\cdot}10^{11}B_{10,20} - 1.53362{\cdot}10^{11}B_{11,20} - 3.94753{\cdot}10^{10}B_{12,20} \\ &+ 7.56994{\cdot}10^{10}B_{13,20} + 1.79517{\cdot}10^{11}B_{14,20} + 2.62123{\cdot}10^{11}B_{15,20} + 3.12114{\cdot}10^{11}B_{16,20} \\ &+ 3.18808{\cdot}10^{11}B_{17,20} + 2.71246{\cdot}10^{11}B_{18,20} + 1.58556{\cdot}10^{11}B_{19,20} - 3.01564{\cdot}10^{10}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 7.13183{\cdot}10^{12}$.

Bounding polynomials M and m:

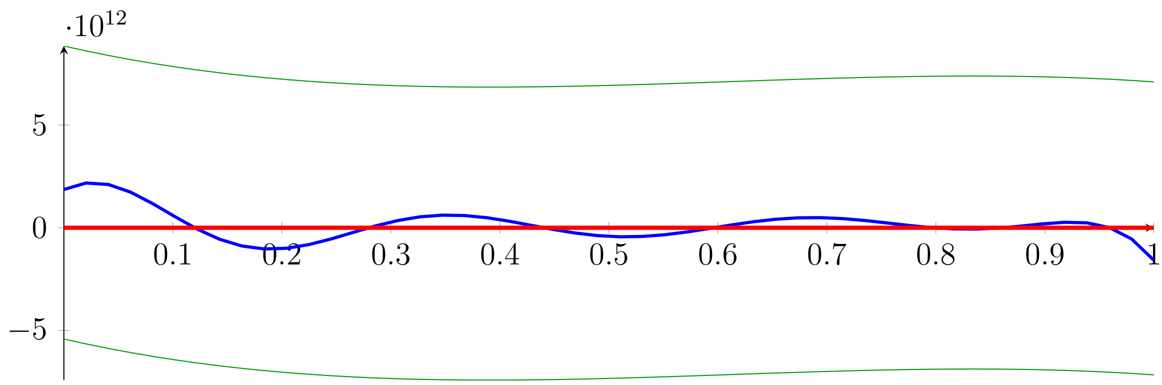
$$M = -1.24152{\cdot}10^{13}X^3 + 2.28015{\cdot}10^{13}X^2 - 1.21315{\cdot}10^{13}X + 8.84695{\cdot}10^{12}$$

$$m = -1.24152{\cdot}10^{13}X^3 + 2.28015{\cdot}10^{13}X^2 - 1.21315{\cdot}10^{13}X - 5.41672{\cdot}10^{12}$$

Root of M and m:

$$N(M) = \{1.50187\} \qquad N(m) = \{-0.27855\}$$

Intersection intervals:



$[0, 1]$

Longest intersection interval: 1

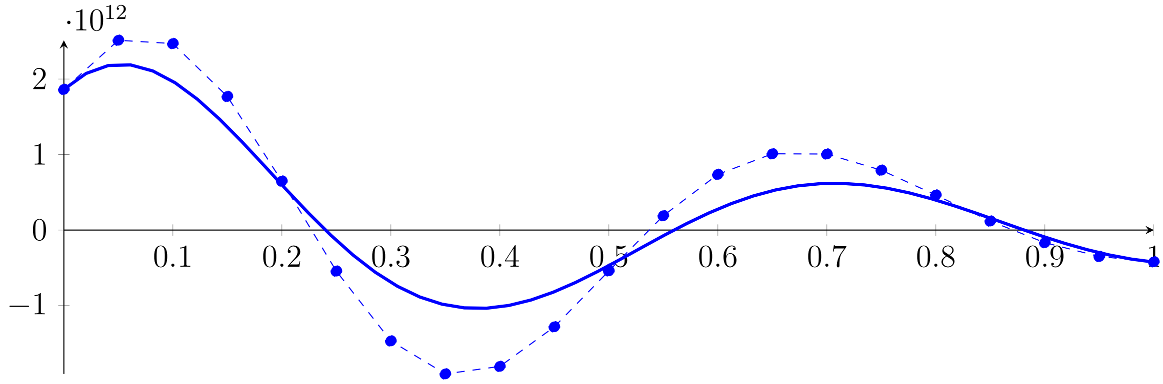
$\Longrightarrow$ Bisection: first half $[6.25, 9.375]$ und second half $[9.375, 12.5]$

Bisection point is very near to a root?!?

3.27 Recursion Branch 1 1 2 1 on the First Half $[6.25, 9.375]$

Normalized monomial und Bézier representations and the Bézier polygon:

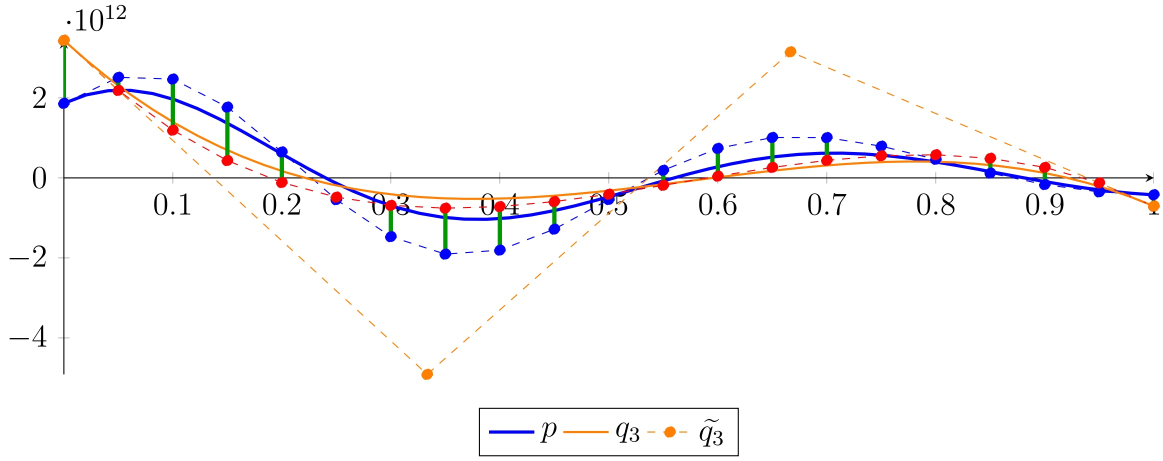
$$p = 7.88861\cdot10^{9}X^{20} - 2.1457\cdot10^{11}X^{19} + 2.50366\cdot10^{12}X^{18} - 1.60464\cdot10^{13}X^{17} + 5.84963\cdot10^{13}X^{16} - 9.93687\cdot10^{13}X^{15} - 7.21032\cdot10^{13}X^{14} + 6.90728\cdot10^{14}X^{13} - 1.13799\cdot10^{15}X^{12} - 1.80856\cdot10^{14}X^{11} + 2.88511\cdot10^{15}X^{10} - 2.89183\cdot10^{15}X^{9} - 1.25813\cdot10^{15}X^{8} + 3.83707\cdot10^{15}X^{7} - 1.35117\cdot10^{15}X^{6} - 1.36068\cdot10^{15}X^{5} + 9.68965\cdot10^{14}X^{4} + 4.2096\cdot10^{13}X^{3} - 1.31886\cdot10^{14}X^{2} + 1.30114\cdot10^{13}X + 1.86285\cdot10^{12}$$
$$= 1.86285\cdot10^{12}B_{0,20}(X) + 2.51342\cdot10^{12}B_{1,20}(X) + 2.46985\cdot10^{12}B_{2,20}(X) + 1.76906\cdot10^{12}B_{3,20}(X) + 6.47986\cdot10^{11}B_{4,20}(X) - 5.44235\cdot10^{11}B_{5,20}(X) - 1.46885\cdot10^{12}B_{6,20}(X) - 1.90547\cdot10^{12}B_{7,20}(X) - 1.80595\cdot10^{12}B_{8,20}(X) - 1.28171\cdot10^{12}B_{9,20}(X) - 5.41242\cdot10^{11}B_{10,20}(X) + 1.90115\cdot10^{11}B_{11,20}(X) + 7.36986\cdot10^{11}B_{12,20}(X) + 1.00973\cdot10^{12}B_{13,20}(X) + 1.00677\cdot10^{12}B_{14,20}(X) + 7.92436\cdot10^{11}B_{15,20}(X) + 4.63782\cdot10^{11}B_{16,20}(X) + 1.1866\cdot10^{11}B_{17,20}(X) - 1.67068\cdot10^{11}B_{18,20}(X) - 3.50344\cdot10^{11}B_{19,20}(X) - 4.20945\cdot10^{11}B_{20,20}(X)$$



Degree reduction and raising:

$$q_3 = -2.83276\cdot10^{13}X^{3} + 4.92306\cdot10^{13}X^{2} - 2.50468\cdot10^{13}X + 3.43777\cdot10^{12}$$
$$= 3.43777\cdot10^{12}B_{0,3} - 4.91117\cdot10^{12}B_{1,3} + 3.1501\cdot10^{12}B_{2,3} - 7.06063\cdot10^{11}B_{3,3}$$

$$\widetilde{q}_3 = -6.79074\cdot10^{14}X^{20} + 6.82581\cdot10^{15}X^{19} - 3.17571\cdot10^{16}X^{18} + 9.07339\cdot10^{16}X^{17} - 1.78106\cdot10^{17}X^{16} + 2.54608\cdot10^{17}X^{15} - 2.74152\cdot10^{17}X^{14} + 2.26853\cdot10^{17}X^{13} - 1.45896\cdot10^{17}X^{12} + 7.32925\cdot10^{16}X^{11} - 2.87553\cdot10^{16}X^{10} + 8.76732\cdot10^{15}X^{9} - 2.05631\cdot10^{15}X^{8} + 3.64867\cdot10^{14}X^{7} - 4.77448\cdot10^{13}X^{6} + 4.43952\cdot10^{12}X^{5} - 2.78525\cdot10^{11}X^{4} - 2.83167\cdot10^{13}X^{3} + 4.92304\cdot10^{13}X^{2} - 2.50468\cdot10^{13}X + 3.43777\cdot10^{12}$$
$$= 3.43777\cdot10^{12}B_{0,20} + 2.18543\cdot10^{12}B_{1,20} + 1.19219\cdot10^{12}B_{2,20} + 4.33229\cdot10^{11}B_{3,20} - 1.16365\cdot10^{11}B_{4,20} - 4.81255\cdot10^{11}B_{5,20} - 6.86826\cdot10^{11}B_{6,20} - 7.56649\cdot10^{11}B_{7,20} - 7.18097\cdot10^{11}B_{8,20} - 5.91836\cdot10^{11}B_{9,20} - 4.0852\cdot10^{11}B_{10,20} - 1.86253\cdot10^{11}B_{11,20} + 4.35134\cdot10^{10}B_{12,20} + 2.61402\cdot10^{11}B_{13,20} + 4.38732\cdot10^{11}B_{14,20} + 5.52916\cdot10^{11}B_{15,20} + 5.77989\cdot10^{11}B_{16,20} + 4.89554\cdot10^{11}B_{17,20} + 2.62615\cdot10^{11}B_{18,20} - 1.27639\cdot10^{11}B_{19,20} - 7.06063\cdot10^{11}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.57492{\cdot}10^{12}$.

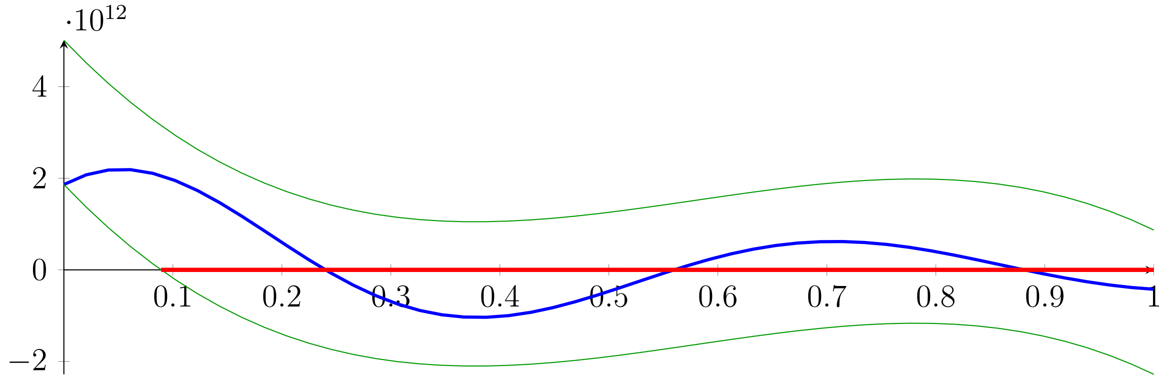
Bounding polynomials M and m:

$$M = -2.83276{\cdot}10^{13}X^3 + 4.92306{\cdot}10^{13}X^2 - 2.50468{\cdot}10^{13}X + 5.01268{\cdot}10^{12}$$
$$m = -2.83276{\cdot}10^{13}X^3 + 4.92306{\cdot}10^{13}X^2 - 2.50468{\cdot}10^{13}X + 1.86285{\cdot}10^{12}$$

Root of M and m:

$$N(M) = \{1.06245\} \qquad N(m) = \{0.0892166\}$$

Intersection intervals:



$$[0.0892166, 1]$$

Longest intersection interval: 0.910783

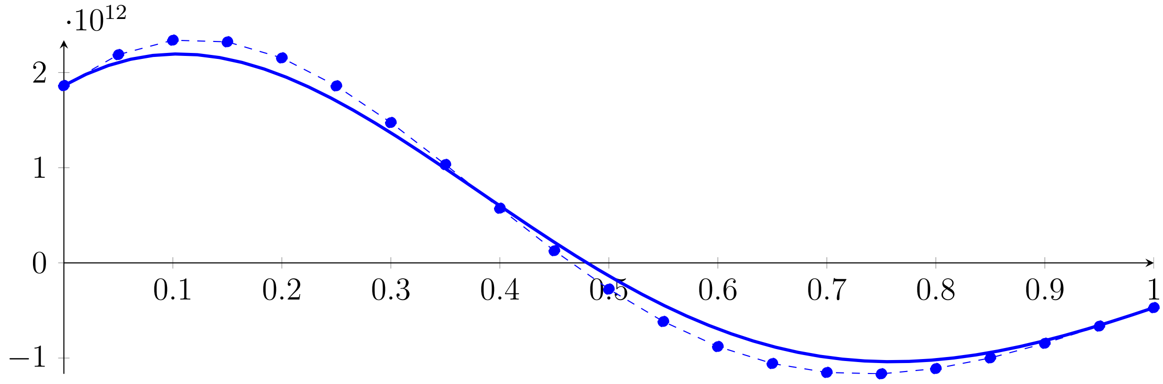
$\Longrightarrow$ Bisection: first half $[6.25, 7.8125]$ und second half $[7.8125, 9.375]$

Bisection point is very near to a root?!?

3.28 Recursion Branch 1 1 2 1 1 on the First Half $[6.25, 7.8125]$

Normalized monomial und Bézier representations and the Bézier polygon:

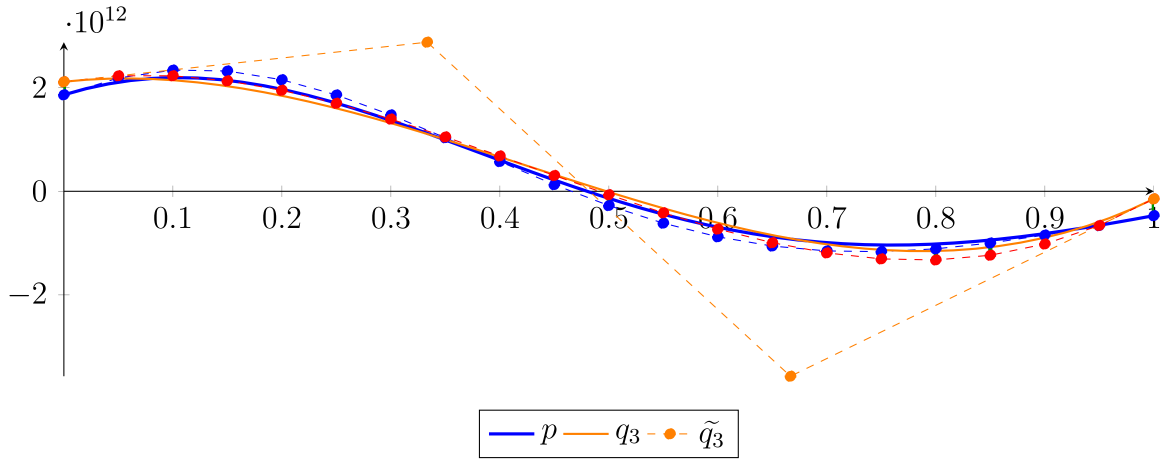
$$\begin{aligned} p = {} & 7302.07X^{20} - 415766X^{19} + 9.52338{\cdot}10^{6}X^{18} - 1.22457{\cdot}10^{8}X^{17} + 8.92307{\cdot}10^{8}X^{16} - 3.03216 \\ & {\cdot}10^{9}X^{15} - 4.40106{\cdot}10^{9}X^{14} + 8.43171{\cdot}10^{10}X^{13} - 2.77829{\cdot}10^{11}X^{12} - 8.83088{\cdot}10^{10}X^{11} + 2.81749 \\ & {\cdot}10^{12}X^{10} - 5.64811{\cdot}10^{12}X^{9} - 4.91456{\cdot}10^{12}X^{8} + 2.99771{\cdot}10^{13}X^{7} - 2.11121{\cdot}10^{13}X^{6} - 4.25212 \\ & {\cdot}10^{13}X^{5} + 6.05603{\cdot}10^{13}X^{4} + 5.262{\cdot}10^{12}X^{3} - 3.29716{\cdot}10^{13}X^{2} + 6.50568{\cdot}10^{12}X + 1.86285{\cdot}10^{12} \\ = {} & 1.86285{\cdot}10^{12}B_{0,20}(X) + 2.18813{\cdot}10^{12}B_{1,20}(X) + 2.33988{\cdot}10^{12}B_{2,20}(X) + 2.32271 \\ & {\cdot}10^{12}B_{3,20}(X) + 2.15374{\cdot}10^{12}B_{4,20}(X) + 1.85984{\cdot}10^{12}B_{5,20}(X) + 1.47434{\cdot}10^{12}B_{6,20}(X) \\ & + 1.03362{\cdot}10^{12}B_{7,20}(X) + 5.7382{\cdot}10^{11}B_{8,20}(X) + 1.28041{\cdot}10^{11}B_{9,20}(X) - 2.75764 \\ & {\cdot}10^{11}B_{10,20}(X) - 6.16213{\cdot}10^{11}B_{11,20}(X) - 8.79156{\cdot}10^{11}B_{12,20}(X) - 1.05766{\cdot}10^{12}B_{13,20}(X) \\ & - 1.15145{\cdot}10^{12}B_{14,20}(X) - 1.1659{\cdot}10^{12}B_{15,20}(X) - 1.11081{\cdot}10^{12}B_{16,20}(X) - 9.99056 \\ & {\cdot}10^{11}B_{17,20}(X) - 8.45188{\cdot}10^{11}B_{18,20}(X) - 6.64233{\cdot}10^{11}B_{19,20}(X) - 4.70618{\cdot}10^{11}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_3 = 1.70887{\cdot}10^{13}X^3 - 2.16418{\cdot}10^{13}X^2 + 2.29105{\cdot}10^{12}X + 2.11534{\cdot}10^{12}$$
$$= 2.11534{\cdot}10^{12}B_{0,3} + 2.87902{\cdot}10^{12}B_{1,3} - 3.57123{\cdot}10^{12}B_{2,3} - 1.46762{\cdot}10^{11}B_{3,3}$$

$$\widetilde{q}_3 = 9.8011{\cdot}10^{13}X^{20} - 9.94152{\cdot}10^{14}X^{19} + 4.67019{\cdot}10^{15}X^{18} - 1.34777{\cdot}10^{16}X^{17} + 2.67231{\cdot}10^{16}X^{16} - 3.85689{\cdot}10^{16}X^{15} + 4.18829{\cdot}10^{16}X^{14} - 3.48907{\cdot}10^{16}X^{13} + 2.25403{\cdot}10^{16}X^{12} - 1.13492{\cdot}10^{16}X^{11} + 4.45735{\cdot}10^{15}X^{10} - 1.36157{\cdot}10^{15}X^9 + 3.20926{\cdot}10^{14}X^8 - 5.73841{\cdot}10^{13}X^7 + 7.54781{\cdot}10^{12}X^6 - 7.01074{\cdot}10^{11}X^5 + 4.46629{\cdot}10^{10}X^4 + 1.70868{\cdot}10^{13}X^3 - 2.16418{\cdot}10^{13}X^2 + 2.29105{\cdot}10^{12}X + 2.11534{\cdot}10^{12}$$
$$= 2.11534{\cdot}10^{12}B_{0,20} + 2.22989{\cdot}10^{12}B_{1,20} + 2.23054{\cdot}10^{12}B_{2,20} + 2.13227{\cdot}10^{12}B_{3,20} + 1.95009{\cdot}10^{12}B_{4,20} + 1.69895{\cdot}10^{12}B_{5,20} + 1.39392{\cdot}10^{12}B_{6,20} + 1.04981{\cdot}10^{12}B_{7,20} + 6.8199{\cdot}10^{11}B_{8,20} + 3.0479{\cdot}10^{11}B_{9,20} - 6.58962{\cdot}10^{10}B_{10,20} - 4.16065{\cdot}10^{11}B_{11,20} - 7.29814{\cdot}10^{11}B_{12,20} - 9.92899{\cdot}10^{11}B_{13,20} - 1.18981{\cdot}10^{12}B_{14,20} - 1.30586{\cdot}10^{12}B_{15,20} - 1.3259{\cdot}10^{12}B_{16,20} - 1.23501{\cdot}10^{12}B_{17,20} - 1.01819{\cdot}10^{12}B_{18,20} - 6.60432{\cdot}10^{11}B_{19,20} - 1.46762{\cdot}10^{11}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 3.23856{\cdot}10^{11}$.

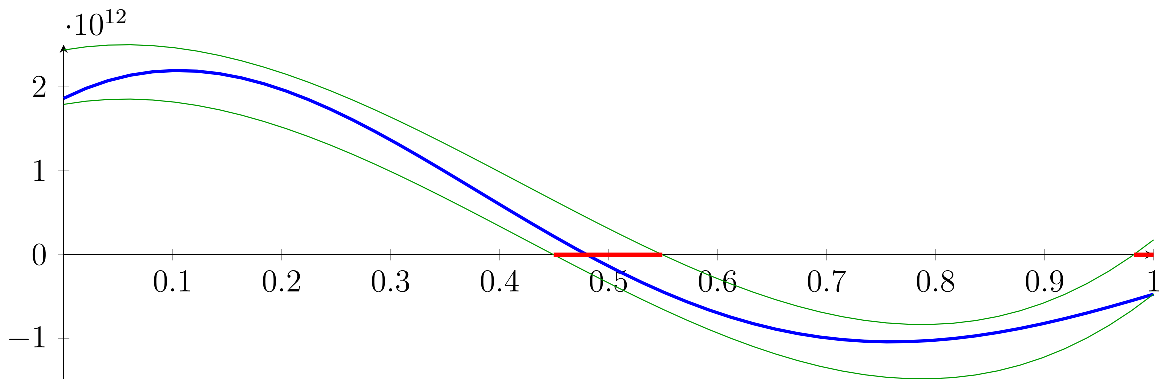
Bounding polynomials M and m:

$$M = 1.70887{\cdot}10^{13}X^3 - 2.16418{\cdot}10^{13}X^2 + 2.29105{\cdot}10^{12}X + 2.43919{\cdot}10^{12}$$
$$m = 1.70887{\cdot}10^{13}X^3 - 2.16418{\cdot}10^{13}X^2 + 2.29105{\cdot}10^{12}X + 1.79148{\cdot}10^{12}$$

Root of M and m:

$$N(M) = \{-0.264669, 0.549293, 0.981819\} \qquad N(m) = \{-0.224022, 0.449588, 1.04088\}$$

Intersection intervals:



$$[0.449588, 0.549293],\ [0.981819, 1]$$

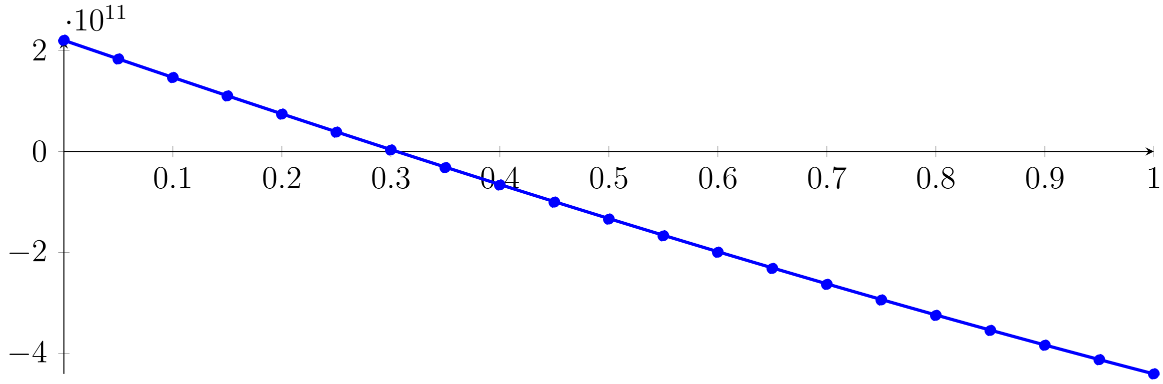
Longest intersection interval: 0.0997056

$\Longrightarrow$ Selective recursion: interval 1: $[6.95248, 7.10827]$, interval 2: $[7.78409, 7.8125]$,

3.29 Recursion Branch 1 1 2 1 1 1 in Interval 1: $[6.95248, 7.10827]$

Normalized monomial und Bézier representations and the Bézier polygon:

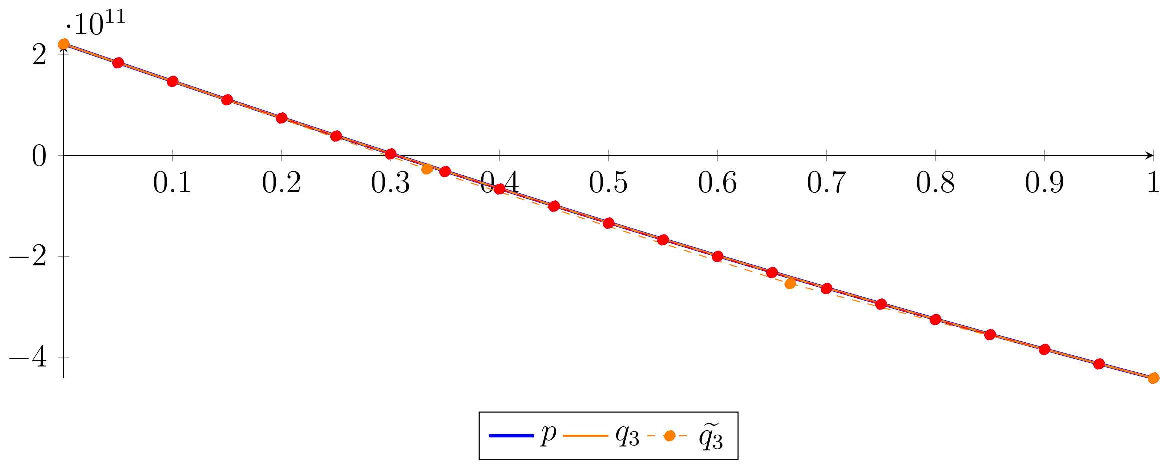
$$p = 95.8899X^{20} - 1115X^{19} + 1531.65X^{18} - 15924.1X^{17} + 50992.1X^{16} - 30671.6X^{15} + 3890.43X^{14} - 6266.8X^{13} - 1656.89X^{12} - 11773.8X^{11} - 5858.2X^{10} - 251.161X^{9} - 70854.3X^{8} - 214848X^{7} + 2.36872{\cdot}10^{7}X^{6} - 1.4045{\cdot}10^{8}X^{5} - 2.59179{\cdot}10^{9}X^{4} + 2.4718{\cdot}10^{10}X^{3} + 5.83238{\cdot}10^{10}X^{2} - 7.40587{\cdot}10^{11}X + 2.20031{\cdot}10^{11}$$
$$= 2.20031{\cdot}10^{11}B_{0,20}(X) + 1.83002{\cdot}10^{11}B_{1,20}(X) + 1.46279{\cdot}10^{11}B_{2,20}(X) + 1.09886{\cdot}10^{11}B_{3,20}(X) + 7.38418{\cdot}10^{10}B_{4,20}(X) + 3.81683{\cdot}10^{10}B_{5,20}(X) + 2.88522{\cdot}10^{9}B_{6,20}(X) - 3.19879{\cdot}10^{10}B_{7,20}(X) - 6.64322{\cdot}10^{10}B_{8,20}(X) - 1.00429{\cdot}10^{11}B_{9,20}(X) - 1.33961{\cdot}10^{11}B_{10,20}(X) - 1.67011{\cdot}10^{11}B_{11,20}(X) - 1.99562{\cdot}10^{11}B_{12,20}(X) - 2.31599{\cdot}10^{11}B_{13,20}(X) - 2.63105{\cdot}10^{11}B_{14,20}(X) - 2.94066{\cdot}10^{11}B_{15,20}(X) - 3.24468{\cdot}10^{11}B_{16,20}(X) - 3.54298{\cdot}10^{11}B_{17,20}(X) - 3.83541{\cdot}10^{11}B_{18,20}(X) - 4.12187{\cdot}10^{11}B_{19,20}(X) - 4.40222{\cdot}10^{11}B_{20,20}(X)$$



Degree reduction and raising:

$$q_3 = 1.92222{\cdot}10^{10}X^{3} + 6.19155{\cdot}10^{10}X^{2} - 7.41391{\cdot}10^{11}X + 2.20071{\cdot}10^{11}$$
$$= 2.20071{\cdot}10^{11}B_{0,3} - 2.70588{\cdot}10^{10}B_{1,3} - 2.5355{\cdot}10^{11}B_{2,3} - 4.40182{\cdot}10^{11}B_{3,3}$$

$$\widetilde{q}_3 = -4.7582{\cdot}10^{12}X^{20} + 4.75914{\cdot}10^{13}X^{19} - 2.19526{\cdot}10^{14}X^{18} + 6.20003{\cdot}10^{14}X^{17} - 1.20099{\cdot}10^{15}X^{16} + 1.69469{\cdot}10^{15}X^{15} - 1.80683{\cdot}10^{15}X^{14} + 1.49029{\cdot}10^{15}X^{13} - 9.65373{\cdot}10^{14}X^{12} + 4.95085{\cdot}10^{14}X^{11} - 2.01179{\cdot}10^{14}X^{10} + 6.42788{\cdot}10^{13}X^{9} - 1.58469{\cdot}10^{13}X^{8} + 2.91051{\cdot}10^{12}X^{7} - 3.74589{\cdot}10^{11}X^{6} + 2.99812{\cdot}10^{10}X^{5} - 9.63668{\cdot}10^{8}X^{4} + 1.9185{\cdot}10^{10}X^{3} + 6.19182{\cdot}10^{10}X^{2} - 7.41391{\cdot}10^{11}X + 2.20071{\cdot}10^{11}$$
$$= 2.20071{\cdot}10^{11}B_{0,20} + 1.83002{\cdot}10^{11}B_{1,20} + 1.46258{\cdot}10^{11}B_{2,20} + 1.09857{\cdot}10^{11}B_{3,20} + 7.38158{\cdot}10^{10}B_{4,20} + 3.81519{\cdot}10^{10}B_{5,20} + 2.87807{\cdot}10^{9}B_{6,20} - 3.19791{\cdot}10^{10}B_{7,20} - 6.64193{\cdot}10^{10}B_{8,20} - 1.004{\cdot}10^{11}B_{9,20} - 1.33941{\cdot}10^{11}B_{10,20} - 1.66978{\cdot}10^{11}B_{11,20} - 1.99548{\cdot}10^{11}B_{12,20} - 2.31589{\cdot}10^{11}B_{13,20} - 2.63111{\cdot}10^{11}B_{14,20} - 2.94083{\cdot}10^{11}B_{15,20} - 3.24494{\cdot}10^{11}B_{16,20} - 3.54326{\cdot}10^{11}B_{17,20} - 3.83563{\cdot}10^{11}B_{18,20} - 4.12187{\cdot}10^{11}B_{19,20} - 4.40182{\cdot}10^{11}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 4.09394 \cdot 10^{7}$.
Bounding polynomials M and m:

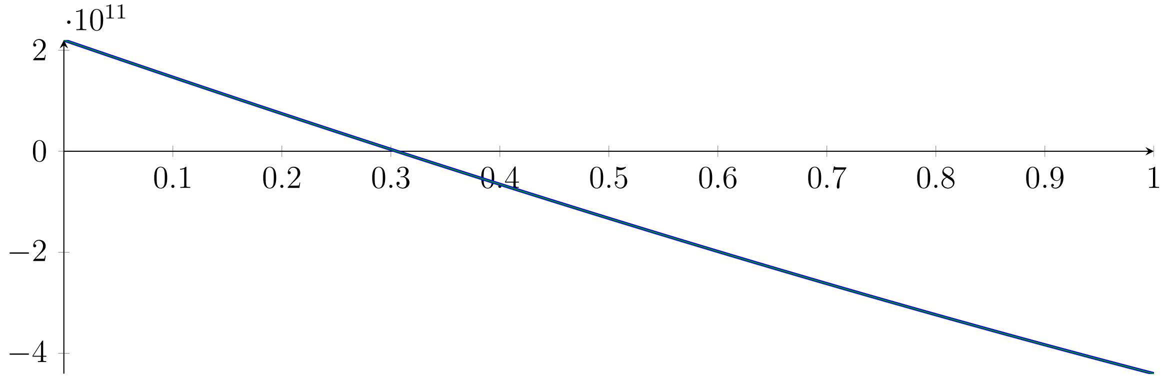
$$M = 1.92222 \cdot 10^{10} X^3 + 6.19155 \cdot 10^{10} X^2 - 7.41391 \cdot 10^{11} X + 2.20112 \cdot 10^{11}$$

$$m = 1.92222 \cdot 10^{10} X^3 + 6.19155 \cdot 10^{10} X^2 - 7.41391 \cdot 10^{11} X + 2.20031 \cdot 10^{11}$$

Root of M and m:

$$N(M) = \{-8.13516, 0.30542, 4.60869\} \qquad N(m) = \{-8.13512, 0.305303, 4.60877\}$$

Intersection intervals:



$[0.305303, 0.30542]$

Longest intersection interval: 0.000117271
$\Longrightarrow$ Selective recursion: interval 1: $[7.00004, 7.00006]$,

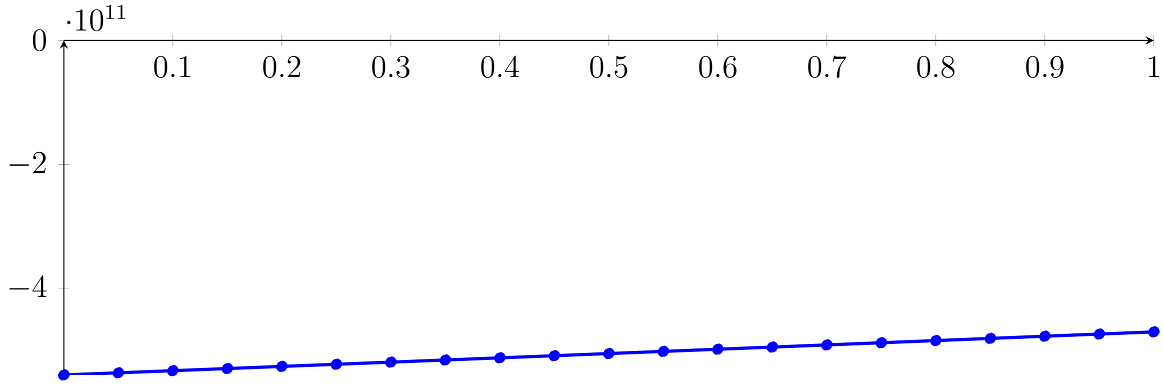
3.30 Recursion Branch 1 1 2 1 1 1 1 in Interval 1: $[7.00004, 7.00006]$

Found root in interval $[7.00004, 7.00006]$ at recursion depth 7!

3.31 Recursion Branch 1 1 2 1 1 2 in Interval 2: $[7.78409, 7.8125]$

Normalized monomial und Bézier representations and the Bézier polygon:

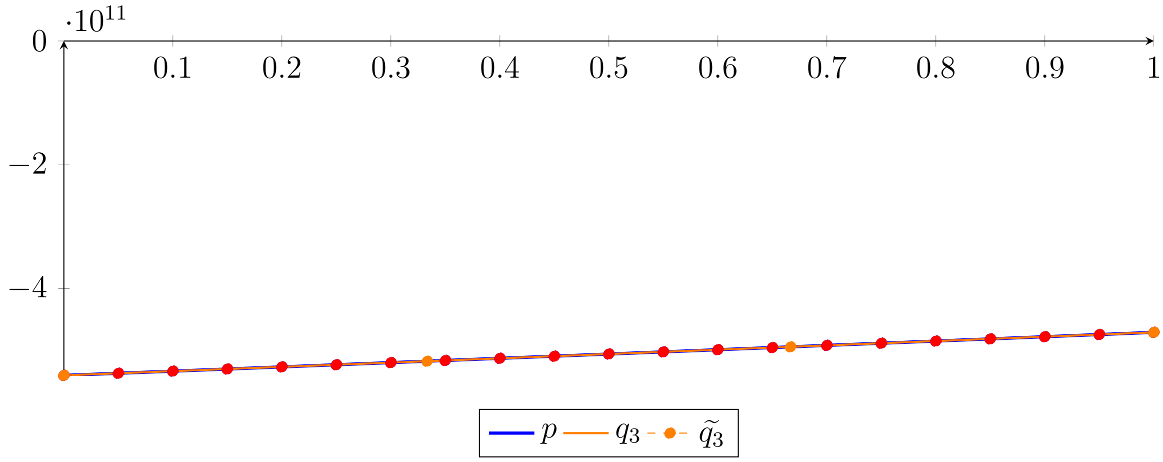
$$\begin{aligned}
p = {} & 520.078 X^{20} - 2689.22 X^{19} + 15692.8 X^{18} - 58269.2 X^{17} + 328259 X^{16} \\
& - 252380 X^{15} + 107122 X^{14} + 30125.1 X^{13} + 250102 X^{12} + 28693.9 X^{11} \\
& + 74166.3 X^{10} + 8847.01 X^{9} + 1791.44 X^{8} - 269.692 X^{7} + 172.698 X^{6} + 34095.7 X^{5} \\
& + 149943 X^{4} - 9.97756 \cdot 10^{7} X^{3} + 1.09323 \cdot 10^{9} X^{2} + 6.85156 \cdot 10^{10} X - 5.40127 \cdot 10^{11} \\
= {} & -5.40127 \cdot 10^{11} B_{0,20}(X) - 5.36701 \cdot 10^{11} B_{1,20}(X) - 5.3327 \cdot 10^{11} B_{2,20}(X) - 5.29832 \\
& \cdot 10^{11} B_{3,20}(X) - 5.2639 \cdot 10^{11} B_{4,20}(X) - 5.22941 \cdot 10^{11} B_{5,20}(X) - 5.19488 \cdot 10^{11} B_{6,20}(X) \\
& - 5.16029 \cdot 10^{11} B_{7,20}(X) - 5.12565 \cdot 10^{11} B_{8,20}(X) - 5.09095 \cdot 10^{11} B_{9,20}(X) - 5.05621 \\
& \cdot 10^{11} B_{10,20}(X) - 5.02141 \cdot 10^{11} B_{11,20}(X) - 4.98657 \cdot 10^{11} B_{12,20}(X) - 4.95168 \cdot 10^{11} B_{13,20}(X) \\
& - 4.91674 \cdot 10^{11} B_{14,20}(X) - 4.88176 \cdot 10^{11} B_{15,20}(X) - 4.84673 \cdot 10^{11} B_{16,20}(X) - 4.81166 \\
& \cdot 10^{11} B_{17,20}(X) - 4.77654 \cdot 10^{11} B_{18,20}(X) - 4.74138 \cdot 10^{11} B_{19,20}(X) - 4.70618 \cdot 10^{11} B_{20,20}(X)
\end{aligned}$$



Degree reduction and raising:

$$
\begin{aligned}
q_3 &= -9.93818{\cdot}10^7 X^3 + 1.09295{\cdot}10^9 X^2 + 6.85157{\cdot}10^{10} X - 5.40127{\cdot}10^{11} \\
&= -5.40127{\cdot}10^{11} B_{0,3} - 5.17288{\cdot}10^{11} B_{1,3} - 4.94086{\cdot}10^{11} B_{2,3} - 4.70618{\cdot}10^{11} B_{3,3}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_3 &= 8.36579{\cdot}10^{13} X^{20} - 8.3886{\cdot}10^{14} X^{19} + 3.89516{\cdot}10^{15} X^{18} - 1.11116{\cdot}10^{16} X^{17} + 2.1784{\cdot}10^{16} X^{16} \\
&\quad - 3.11066{\cdot}10^{16} X^{15} + 3.34559{\cdot}10^{16} X^{14} - 2.76442{\cdot}10^{16} X^{13} + 1.77425{\cdot}10^{16} X^{12} - 8.88516{\cdot}10^{15} X^{11} \\
&\quad + 3.46892{\cdot}10^{15} X^{10} - 1.0499{\cdot}10^{15} X^9 + 2.43914{\cdot}10^{14} X^8 - 4.29468{\cdot}10^{13} X^7 + 5.65141{\cdot}10^{12} X^6 - 5.4642 \\
&\quad {\cdot}10^{11} X^5 + 3.78672{\cdot}10^{10} X^4 - 1.87454{\cdot}10^9 X^3 + 1.13953{\cdot}10^9 X^2 + 6.85152{\cdot}10^{10} X - 5.40127{\cdot}10^{11} \\
&= -5.40127{\cdot}10^{11} B_{0,20} - 5.36701{\cdot}10^{11} B_{1,20} - 5.33269{\cdot}10^{11} B_{2,20} - 5.29833{\cdot}10^{11} B_{3,20} - 5.26387 \\
&\quad {\cdot}10^{11} B_{4,20} - 5.22951{\cdot}10^{11} B_{5,20} - 5.19464{\cdot}10^{11} B_{6,20} - 5.16078{\cdot}10^{11} B_{7,20} - 5.12485{\cdot}10^{11} B_{8,20} \\
&\quad - 5.09194{\cdot}10^{11} B_{9,20} - 5.05499{\cdot}10^{11} B_{10,20} - 5.02229{\cdot}10^{11} B_{11,20} - 4.98575{\cdot}10^{11} B_{12,20} \\
&\quad - 4.95213{\cdot}10^{11} B_{13,20} - 4.91649{\cdot}10^{11} B_{14,20} - 4.88186{\cdot}10^{11} B_{15,20} - 4.8467{\cdot}10^{11} B_{16,20} \\
&\quad - 4.81166{\cdot}10^{11} B_{17,20} - 4.77654{\cdot}10^{11} B_{18,20} - 4.74138{\cdot}10^{11} B_{19,20} - 4.70618{\cdot}10^{11} B_{20,20}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 1.2152{\cdot}10^8$.

Bounding polynomials M and m:

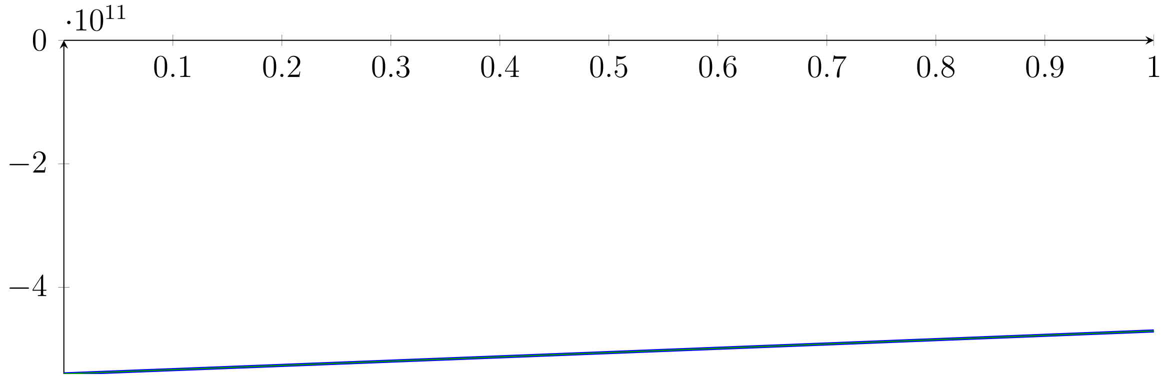
$$M = -9.93818{\cdot}10^7 X^3 + 1.09295{\cdot}10^9 X^2 + 6.85157{\cdot}10^{10} X - 5.40005{\cdot}10^{11}$$

$$m = -9.93818{\cdot}10^7 X^3 + 1.09295{\cdot}10^9 X^2 + 6.85157{\cdot}10^{10} X - 5.40249{\cdot}10^{11}$$

Root of M and m:

$$N(M) = \{-25.0979, 7.59681, 28.4986\} \qquad N(m) = \{-25.0993, 7.60039, 28.4964\}$$

Intersection intervals:

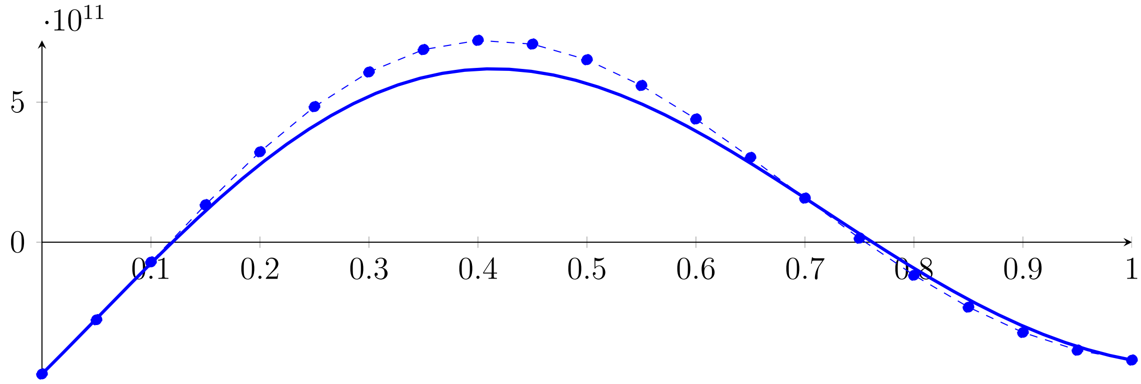


No intersection intervals with the x axis.

3.32 Recursion Branch 1 1 2 1 2 on the Second Half $[7.8125, 9.375]$

Normalized monomial und Bézier representations and the Bézier polygon:

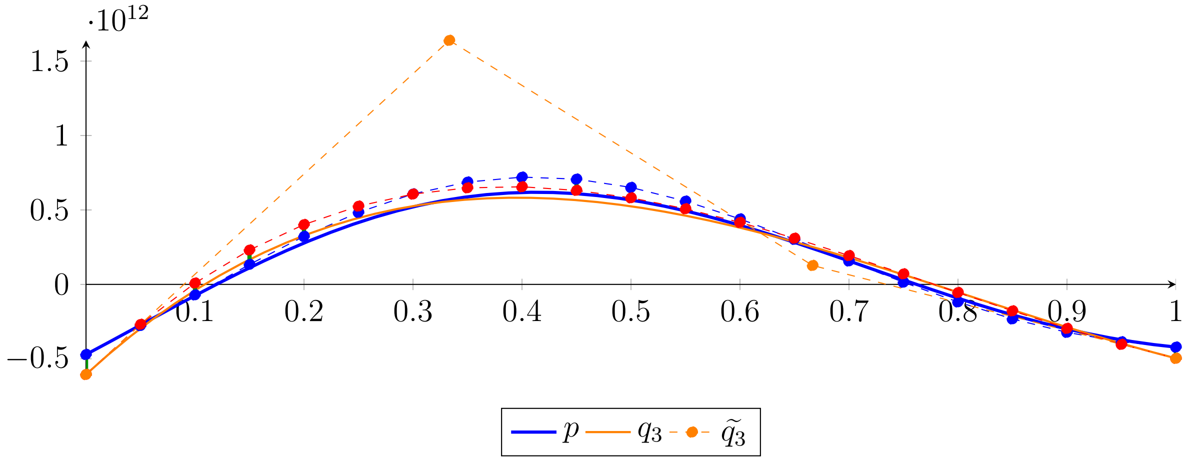
$$p = 6877.98X^{20} - 256526X^{19} + 3.18676\cdot 10^{6}X^{18} - 1.18426\cdot 10^{7}X^{17} - 8.79174\cdot 10^{7}X^{16} + 9.23149\cdot 10^{8}X^{15} - 1.26914\cdot 10^{9}X^{14} - 1.59203\cdot 10^{10}X^{13} + 6.26004\cdot 10^{10}X^{12} + 7.11942\cdot 10^{10}X^{11} - 7.3925\cdot 10^{11}X^{10} + 5.09162\cdot 10^{11}X^{9} + 3.6295\cdot 10^{12}X^{8} - 5.56929\cdot 10^{12}X^{7} - 7.06545\cdot 10^{12}X^{6} + 1.64355\cdot 10^{13}X^{5} + 2.9001\cdot 10^{12}X^{4} - 1.64458\cdot 10^{13}X^{3} + 2.40542\cdot 10^{12}X^{2} + 3.8723\cdot 10^{12}X - 4.70618\cdot 10^{11}$$
$$= -4.70618\cdot 10^{11}B_{0,20}(X) - 2.77003\cdot 10^{11}B_{1,20}(X) - 7.07277\cdot 10^{10}B_{2,20}(X) + 1.33781\cdot 10^{11}B_{3,20}(X) + 3.22697\cdot 10^{11}B_{4,20}(X) + 4.8385\cdot 10^{11}B_{5,20}(X) + 6.07608\cdot 10^{11}B_{6,20}(X) + 6.87499\cdot 10^{11}B_{7,20}(X) + 7.20537\cdot 10^{11}B_{8,20}(X) + 7.07242\cdot 10^{11}B_{9,20}(X) + 6.51366\cdot 10^{11}B_{10,20}(X) + 5.59383\cdot 10^{11}B_{11,20}(X) + 4.398\cdot 10^{11}B_{12,20}(X) + 3.02359\cdot 10^{11}B_{13,20}(X) + 1.57223\cdot 10^{11}B_{14,20}(X) + 1.42063\cdot 10^{10}B_{15,20}(X) - 1.17894\cdot 10^{11}B_{16,20}(X) - 2.31815\cdot 10^{11}B_{17,20}(X) - 3.22175\cdot 10^{11}B_{18,20}(X) - 3.85644\cdot 10^{11}B_{19,20}(X) - 4.20945\cdot 10^{11}B_{20,20}(X)$$



Degree reduction and raising:

$$q_3 = 4.64373\cdot 10^{12}X^{3} - 1.12665\cdot 10^{13}X^{2} + 6.73102\cdot 10^{12}X - 6.04724\cdot 10^{11}$$
$$= -6.04724\cdot 10^{11}B_{0,3} + 1.63895\cdot 10^{12}B_{1,3} + 1.27129\cdot 10^{11}B_{2,3} - 4.96452\cdot 10^{11}B_{3,3}$$

$$\widetilde{q_3} = 8.11791\cdot 10^{13}X^{20} - 8.16523\cdot 10^{14}X^{19} + 3.79985\cdot 10^{15}X^{18} - 1.08556\cdot 10^{16}X^{17} + 2.13012\cdot 10^{16}X^{16} - 3.04345\cdot 10^{16}X^{15} + 3.27511\cdot 10^{16}X^{14} - 2.70858\cdot 10^{16}X^{13} + 1.74128\cdot 10^{16}X^{12} - 8.74648\cdot 10^{15}X^{11} + 3.43292\cdot 10^{15}X^{10} - 1.04793\cdot 10^{15}X^{9} + 2.46238\cdot 10^{14}X^{8} - 4.37053\cdot 10^{13}X^{7} + 5.67024\cdot 10^{12}X^{6} - 5.09275\cdot 10^{11}X^{5} + 2.88466\cdot 10^{10}X^{4} + 4.64285\cdot 10^{12}X^{3} - 1.12665\cdot 10^{13}X^{2} + 6.73102\cdot 10^{12}X - 6.04724\cdot 10^{11}$$
$$= -6.04724\cdot 10^{11}B_{0,20} - 2.68174\cdot 10^{11}B_{1,20} + 9.08015\cdot 10^{9}B_{2,20} + 2.31109\cdot 10^{11}B_{3,20} + 4.01993\cdot 10^{11}B_{4,20} + 5.25782\cdot 10^{11}B_{5,20} + 6.06615\cdot 10^{11}B_{6,20} + 6.48414\cdot 10^{11}B_{7,20} + 6.5555\cdot 10^{11}B_{8,20} + 6.31597\cdot 10^{11}B_{9,20} + 5.81324\cdot 10^{11}B_{10,20} + 5.07999\cdot 10^{11}B_{11,20} + 4.16491\cdot 10^{11}B_{12,20} + 3.10215\cdot 10^{11}B_{13,20} + 1.93693\cdot 10^{11}B_{14,20} + 7.07405\cdot 10^{10}B_{15,20} - 5.44463\cdot 10^{10}B_{16,20} - 1.7784\cdot 10^{11}B_{17,20} - 2.95353\cdot 10^{11}B_{18,20} - 4.02915\cdot 10^{11}B_{19,20} - 4.96452\cdot 10^{11}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.34107 \cdot 10^{11}$.

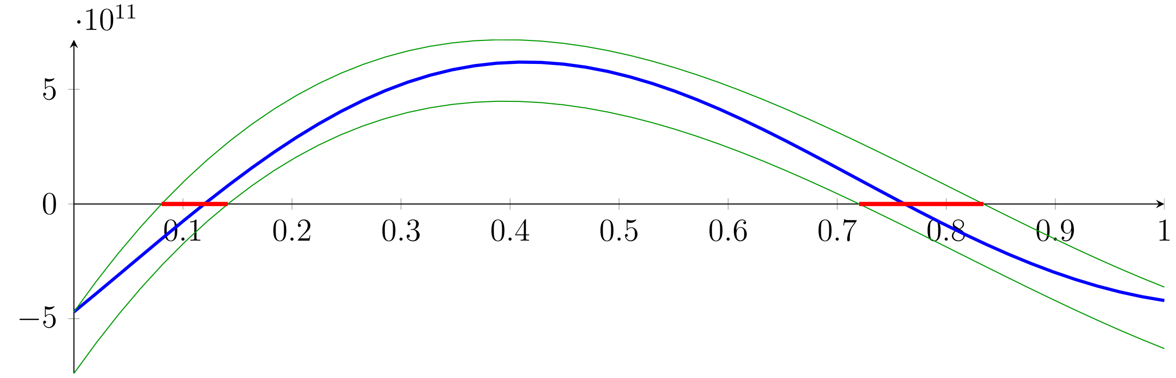
Bounding polynomials M and m:

$$M = 4.64373 \cdot 10^{12} X^3 - 1.12665 \cdot 10^{13} X^2 + 6.73102 \cdot 10^{12} X - 4.70618 \cdot 10^{11}$$
$$m = 4.64373 \cdot 10^{12} X^3 - 1.12665 \cdot 10^{13} X^2 + 6.73102 \cdot 10^{12} X - 7.38831 \cdot 10^{11}$$

Root of M and m:

$$N(M) = \{0.0803718, 0.834162, 1.51164\} \qquad N(m) = \{0.14119, 0.720099, 1.56488\}$$

Intersection intervals:



$$[0.0803718, 0.14119], \ [0.720099, 0.834162]$$

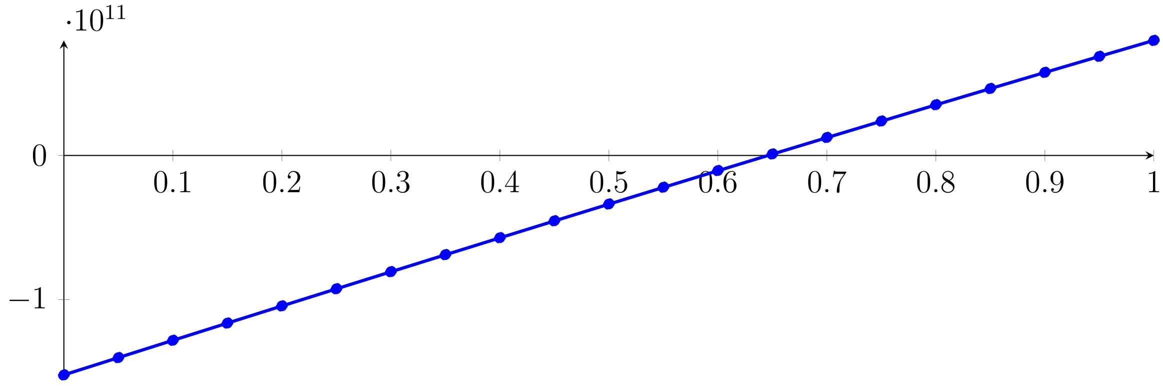
Longest intersection interval: 0.114064

$\Longrightarrow$ Selective recursion: interval 1: $[7.93808, 8.03311]$, interval 2: $[8.93765, 9.11588]$,

3.33 Recursion Branch 1 1 2 1 2 1 in Interval 1: $[7.93808, 8.03311]$

Normalized monomial und Bézier representations and the Bézier polygon:

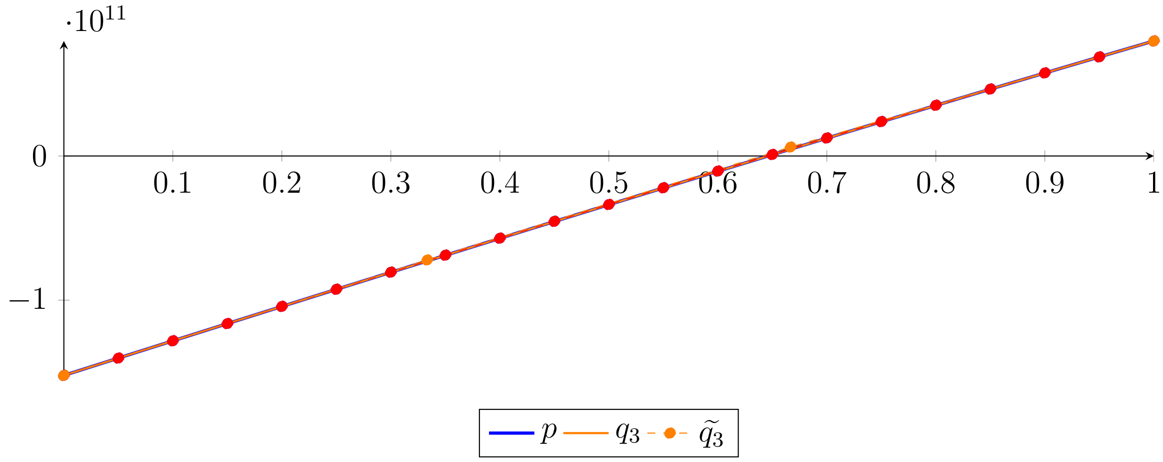
$$\begin{aligned} p = {} & 45.416 X^{20} - 25.7904 X^{19} + 1922.54 X^{18} - 4097.95 X^{17} + 33642.8 X^{16} - 29932.5 X^{15} \\ & + 16161.4 X^{14} + 6247.87 X^{13} + 39300.3 X^{12} + 7837.24 X^{11} + 13481.2 X^{10} \\ & + 1768.38 X^9 + 888.033 X^8 - 9744.41 X^7 - 482022 X^6 + 1.03019 \cdot 10^7 X^5 + 1.1944 \\ & \cdot 10^8 X^4 - 3.26927 \cdot 10^9 X^3 - 5.05627 \cdot 10^9 X^2 + 2.40206 \cdot 10^{11} X - 1.5222 \cdot 10^{11} \\ = {} & -1.5222 \cdot 10^{11} B_{0,20}(X) - 1.4021 \cdot 10^{11} B_{1,20}(X) - 1.28226 \cdot 10^{11} B_{2,20}(X) - 1.16272 \\ & \cdot 10^{11} B_{3,20}(X) - 1.0435 \cdot 10^{11} B_{4,20}(X) - 9.24631 \cdot 10^{10} B_{5,20}(X) - 8.06143 \cdot 10^{10} B_{6,20}(X) \\ & - 6.88061 \cdot 10^{10} B_{7,20}(X) - 5.70414 \cdot 10^{10} B_{8,20}(X) - 4.53229 \cdot 10^{10} B_{9,20}(X) - 3.36532 \\ & \cdot 10^{10} B_{10,20}(X) - 2.20349 \cdot 10^{10} B_{11,20}(X) - 1.04708 \cdot 10^{10} B_{12,20}(X) + 1.03662 \cdot 10^{9} B_{13,20}(X) \\ & + 1.24848 \cdot 10^{10} B_{14,20}(X) + 2.38712 \cdot 10^{10} B_{15,20}(X) + 3.51933 \cdot 10^{10} B_{16,20}(X) + 4.64486 \\ & \cdot 10^{10} B_{17,20}(X) + 5.76348 \cdot 10^{10} B_{18,20}(X) + 6.87493 \cdot 10^{10} B_{19,20}(X) + 7.97899 \cdot 10^{10} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -3.00341\cdot 10^9 X^3 - 5.23278\cdot 10^9 X^2 + 2.40246\cdot 10^{11} X - 1.52222\cdot 10^{11} \\ &= -1.52222\cdot 10^{11} B_{0,3} - 7.214\cdot 10^{10} B_{1,3} + 6.1978\cdot 10^9 B_{2,3} + 7.97879\cdot 10^{10} B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 = {} & 1.41684\cdot 10^{13} X^{20} - 1.42034\cdot 10^{14} X^{19} + 6.59057\cdot 10^{14} X^{18} - 1.87808\cdot 10^{15} X^{17} + 3.6773\cdot 10^{15} X^{16} - 5.24455 \\ & \cdot 10^{15} X^{15} + 5.63568\cdot 10^{15} X^{14} - 4.65604\cdot 10^{15} X^{13} + 2.99142\cdot 10^{15} X^{12} - 1.50199\cdot 10^{15} X^{11} + 5.89051 \\ & \cdot 10^{14} X^{10} - 1.79416\cdot 10^{14} X^9 + 4.19884\cdot 10^{13} X^8 - 7.43373\cdot 10^{12} X^7 + 9.75649\cdot 10^{11} X^6 - 9.23221 \\ & \cdot 10^{10} X^5 + 6.03832\cdot 10^9 X^4 - 3.25993\cdot 10^9 X^3 - 5.22659\cdot 10^9 X^2 + 2.40246\cdot 10^{11} X - 1.52222\cdot 10^{11} \\ = {} & -1.52222\cdot 10^{11} B_{0,20} - 1.4021\cdot 10^{11} B_{1,20} - 1.28225\cdot 10^{11} B_{2,20} - 1.1627\cdot 10^{11} B_{3,20} - 1.04348 \\ & \cdot 10^{11} B_{4,20} - 9.24639\cdot 10^{10} B_{5,20} - 8.06098\cdot 10^{10} B_{6,20} - 6.88148\cdot 10^{10} B_{7,20} - 5.70292\cdot 10^{10} B_{8,20} \\ & - 4.53412\cdot 10^{10} B_{9,20} - 3.36346\cdot 10^{10} B_{10,20} - 2.20525\cdot 10^{10} B_{11,20} - 1.04588\cdot 10^{10} B_{12,20} \\ & + 1.02864\cdot 10^9 B_{13,20} + 1.24891\cdot 10^{10} B_{14,20} + 2.38705\cdot 10^{10} B_{15,20} + 3.51951\cdot 10^{10} B_{16,20} \\ & + 4.645\cdot 10^{10} B_{17,20} + 5.76359\cdot 10^{10} B_{18,20} + 6.87494\cdot 10^{10} B_{19,20} + 7.97879\cdot 10^{10} B_{20,20} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.85928\cdot 10^7$.
Bounding polynomials M and m:

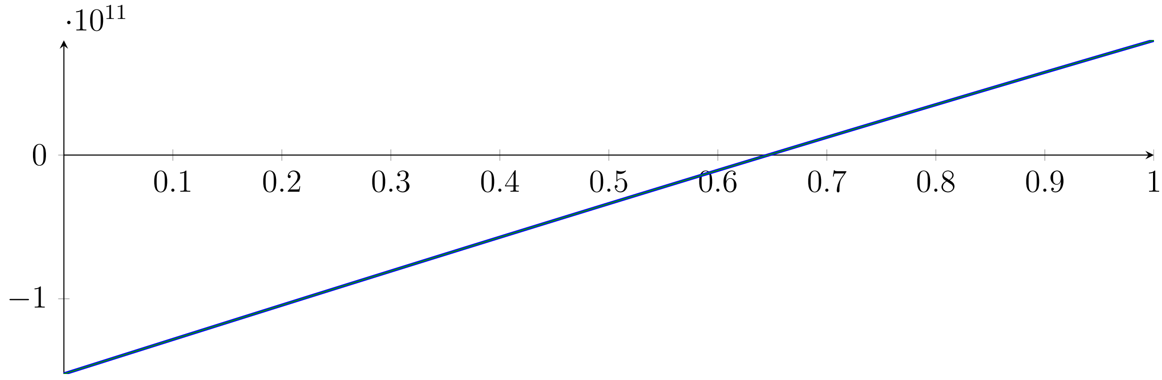
$$M = -3.00341\cdot 10^9 X^3 - 5.23278\cdot 10^9 X^2 + 2.40246\cdot 10^{11} X - 1.52203\cdot 10^{11}$$

$$m = -3.00341\cdot 10^9 X^3 - 5.23278\cdot 10^9 X^2 + 2.40246\cdot 10^{11} X - 1.52241\cdot 10^{11}$$

Root of M and m:

$$N(M) = \{-10.1314, 0.645991, 7.7431\} \qquad N(m) = \{-10.1314, 0.646152, 7.743\}$$

Intersection intervals:



$[0.645991, 0.646152]$

Longest intersection interval: 0.000161871
$\Longrightarrow$ Selective recursion: interval 1: $[7.99947, 7.99948]$,

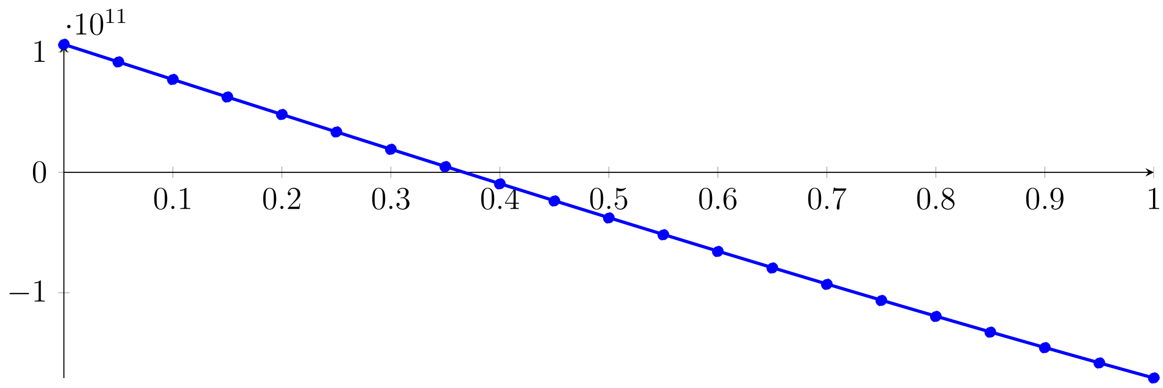
3.34 Recursion Branch 1 1 2 1 2 1 1 in Interval 1: $[7.99947, 7.99948]$

Found root in interval $[7.99947, 7.99948]$ at recursion depth 7!

3.35 Recursion Branch 1 1 2 1 2 2 in Interval 2: $[8.93765, 9.11588]$

Normalized monomial und Bézier representations and the Bézier polygon:

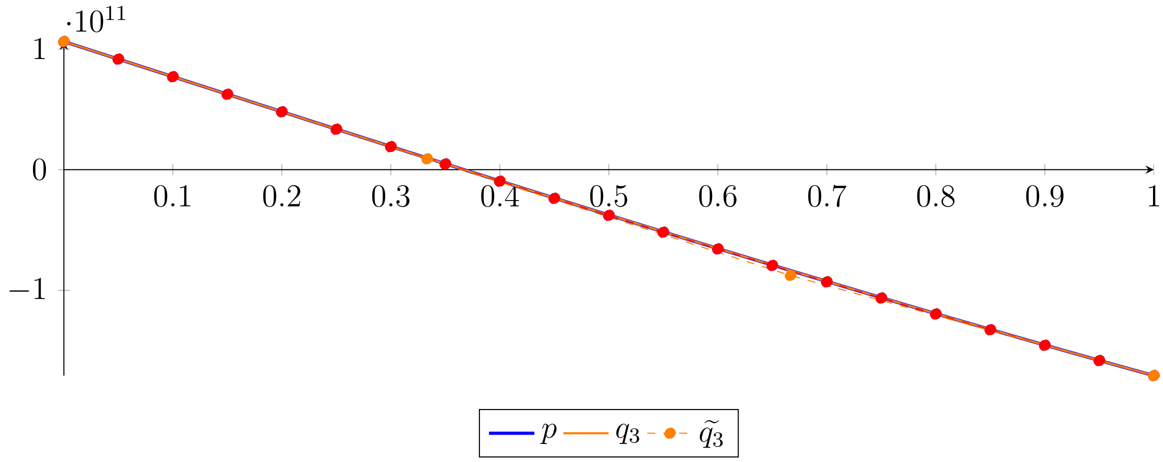
$$\begin{aligned} p = {} & 23.4782X^{20} - 390.589X^{19} + 102.729X^{18} - 4694.05X^{17} + 9496.75X^{16} - 4279.59X^{15} \\ & - 2113.77X^{14} - 3967.32X^{13} - 8468.99X^{12} - 6053.49X^{11} - 4617.77X^{10} \\ & - 3859.68X^{9} - 45681.8X^{8} + 944934X^{7} + 7.51682\cdot 10^{6}X^{6} - 1.85749\cdot 10^{8}X^{5} - 4.37481 \\ & \cdot 10^{8}X^{4} + 1.44796\cdot 10^{10}X^{3} + 7.51879\cdot 10^{8}X^{2} - 2.91498\cdot 10^{11}X + 1.06187\cdot 10^{11} \\ = {} & 1.06187\cdot 10^{11}B_{0,20}(X) + 9.16117\cdot 10^{10}B_{1,20}(X) + 7.70407\cdot 10^{10}B_{2,20}(X) + 6.24865 \\ & \cdot 10^{10}B_{3,20}(X) + 4.79614\cdot 10^{10}B_{4,20}(X) + 3.34782\cdot 10^{10}B_{5,20}(X) + 1.90492\cdot 10^{10}B_{6,20}(X) \\ & + 4.68654\cdot 10^{9}B_{7,20}(X) - 9.5975\cdot 10^{9}B_{8,20}(X) - 2.3791\cdot 10^{10}B_{9,20}(X) - 3.78821 \\ & \cdot 10^{10}B_{10,20}(X) - 5.18591\cdot 10^{10}B_{11,20}(X) - 6.57107\cdot 10^{10}B_{12,20}(X) - 7.94254\cdot 10^{10}B_{13,20}(X) \\ & - 9.29923\cdot 10^{10}B_{14,20}(X) - 1.064\cdot 10^{11}B_{15,20}(X) - 1.19639\cdot 10^{11}B_{16,20}(X) - 1.32698 \\ & \cdot 10^{11}B_{17,20}(X) - 1.45567\cdot 10^{11}B_{18,20}(X) - 1.58236\cdot 10^{11}B_{19,20}(X) - 1.70695\cdot 10^{11}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 1.3117\cdot 10^{10}X^{3} + 1.72905\cdot 10^{9}X^{2} - 2.91726\cdot 10^{11}X + 1.06198\cdot 10^{11} \\ &= 1.06198\cdot 10^{11}B_{0,3} + 8.95627\cdot 10^{9}B_{1,3} - 8.77094\cdot 10^{10}B_{2,3} - 1.70682\cdot 10^{11}B_{3,3} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 = &-4.4688\cdot10^{12}X^{20} + 4.4758\cdot10^{13}X^{19} - 2.07222\cdot10^{14}X^{18} + 5.8857\cdot10^{14}X^{17} - 1.14792\cdot10^{15}X^{16} \\ &+ 1.63088\cdot10^{15}X^{15} - 1.7476\cdot10^{15}X^{14} + 1.44299\cdot10^{15}X^{13} - 9.29846\cdot10^{14}X^{12} + 4.70475\cdot10^{14}X^{11} \\ &- 1.8694\cdot10^{14}X^{10} + 5.7976\cdot10^{13}X^{9} - 1.38451\cdot10^{13}X^{8} + 2.48747\cdot10^{12}X^{7} - 3.24074\cdot10^{11}X^{6} + 2.88345 \\ &\cdot10^{10}X^{5} - 1.55083\cdot10^{9}X^{4} + 1.31567\cdot10^{10}X^{3} + 1.72869\cdot10^{9}X^{2} - 2.91726\cdot10^{11}X + 1.06198\cdot10^{11} \\ = &\,1.06198\cdot10^{11}B_{0,20} + 9.1612\cdot10^{10}B_{1,20} + 7.70348\cdot10^{10}B_{2,20} + 6.24782\cdot10^{10}B_{3,20} + 4.79535 \\ &\cdot10^{10}B_{4,20} + 3.34734\cdot10^{10}B_{5,20} + 1.90458\cdot10^{10}B_{6,20} + 4.69059\cdot10^{9}B_{7,20} - 9.59665\cdot10^{9}B_{8,20} \\ &- 2.37786\cdot10^{10}B_{9,20} - 3.78801\cdot10^{10}B_{10,20} - 5.18454\cdot10^{10}B_{11,20} - 6.57086\cdot10^{10}B_{12,20} \\ &- 7.94206\cdot10^{10}B_{13,20} - 9.29946\cdot10^{10}B_{14,20} - 1.06405\cdot10^{11}B_{15,20} - 1.19647\cdot10^{11}B_{16,20} \\ &- 1.32707\cdot10^{11}B_{17,20} - 1.45574\cdot10^{11}B_{18,20} - 1.58236\cdot10^{11}B_{19,20} - 1.70682\cdot10^{11}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.37656\cdot10^{7}$.

Bounding polynomials M and m:

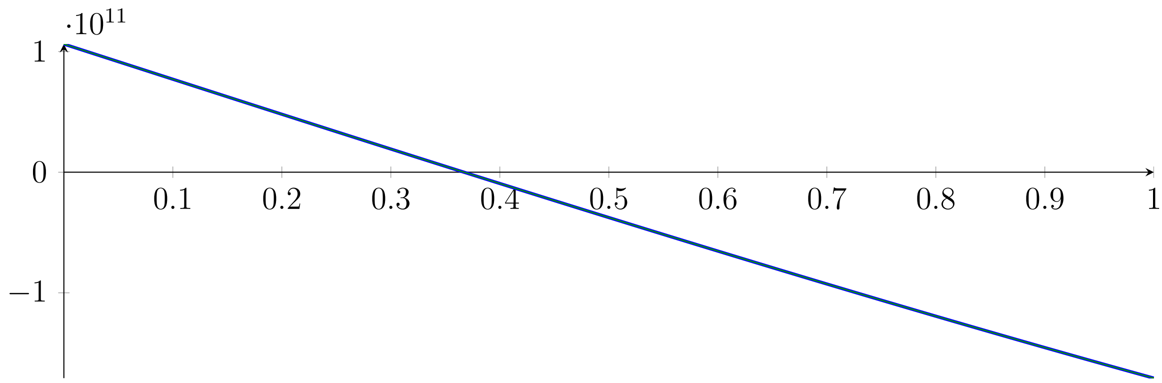
$$M = 1.3117\cdot10^{10}X^3 + 1.72905\cdot10^{9}X^2 - 2.91726\cdot10^{11}X + 1.06212\cdot10^{11}$$

$$m = 1.3117\cdot10^{10}X^3 + 1.72905\cdot10^{9}X^2 - 2.91726\cdot10^{11}X + 1.06185\cdot10^{11}$$

Root of M and m:

$$N(M) = \{-4.95258, 0.367105, 4.45366\} \qquad N(m) = \{-4.95254, 0.367008, 4.45371\}$$

Intersection intervals:



$$[0.367008, 0.367105]$$

Longest intersection interval: $9.65481\cdot10^{-05}$

$\Longrightarrow$ Selective recursion: interval 1: $[9.00306, 9.00308]$,

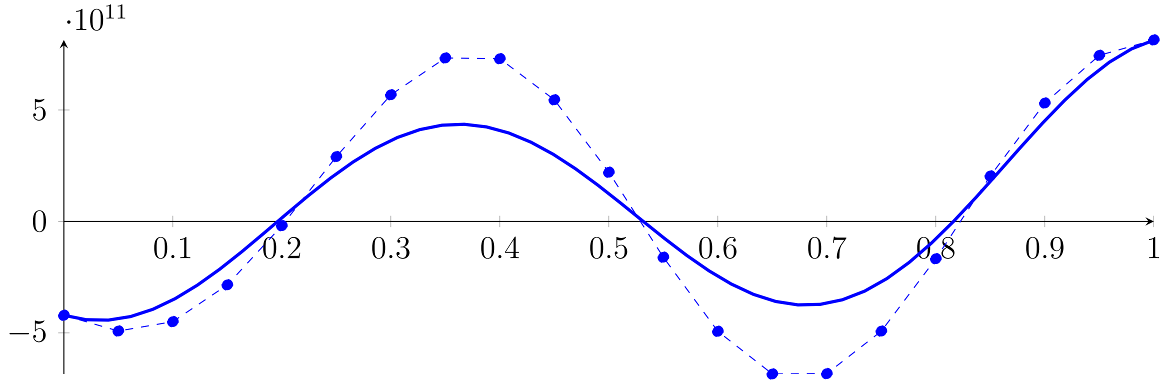
3.36 Recursion Branch 1 1 2 1 2 2 1 in Interval 1: $[9.00306, 9.00308]$

Found root in interval $[9.00306, 9.00308]$ at recursion depth 7!

3.37 Recursion Branch 1 1 2 2 on the Second Half $[9.375, 12.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

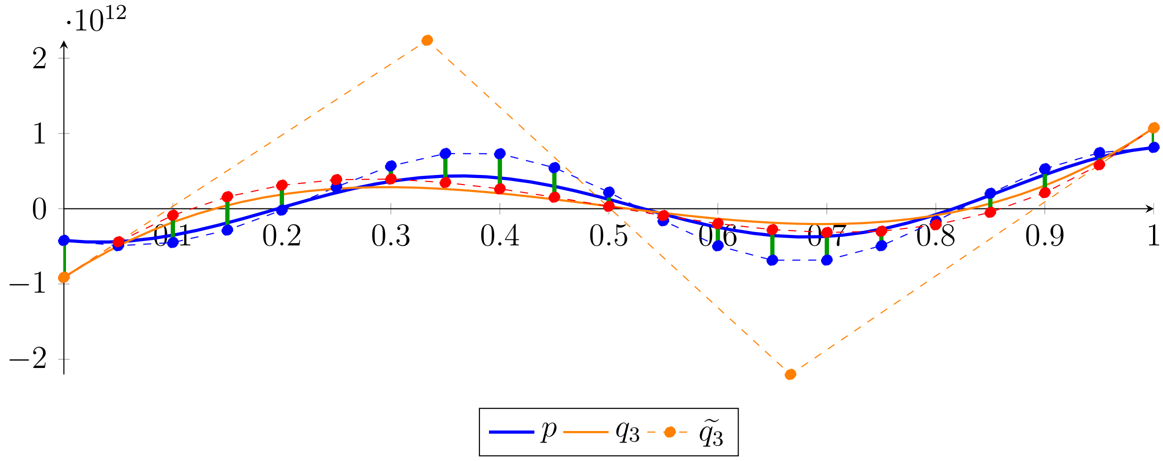
$$\begin{aligned} p = & 7.88861\cdot 10^{9} X^{20} - 5.6798\cdot 10^{10} X^{19} - 7.43423\cdot 10^{10} X^{18} + 1.32089\cdot 10^{12} X^{17} - 9.3169\cdot 10^{11} X^{16} - 1.21266 \\ & \cdot 10^{13} X^{15} + 1.72866\cdot 10^{13} X^{14} + 5.61608\cdot 10^{13} X^{13} - 1.04782\cdot 10^{14} X^{12} - 1.38659\cdot 10^{14} X^{11} + 3.15838 \\ & \cdot 10^{14} X^{10} + 1.75102\cdot 10^{14} X^{9} - 5.05882\cdot 10^{14} X^{8} - 9.20246\cdot 10^{13} X^{7} + 4.17973\cdot 10^{14} X^{6} - 4.84112 \\ & \cdot 10^{11} X^{5} - 1.59085\cdot 10^{14} X^{4} + 1.16549\cdot 10^{13} X^{3} + 2.14084\cdot 10^{13} X^{2} - 1.41201\cdot 10^{12} X - 4.20945\cdot 10^{11} \\ = & -4.20945\cdot 10^{11} B_{0,20}(X) - 4.91545\cdot 10^{11} B_{1,20}(X) - 4.4947\cdot 10^{11} B_{2,20}(X) - 2.84495 \\ & \cdot 10^{11} B_{3,20}(X) - 1.92322\cdot 10^{10} B_{4,20}(X) + 2.90841\cdot 10^{11} B_{5,20}(X) + 5.68134\cdot 10^{11} B_{6,20}(X) \\ & + 7.3329\cdot 10^{11} B_{7,20}(X) + 7.29931\cdot 10^{11} B_{8,20}(X) + 5.45616\cdot 10^{11} B_{9,20}(X) + 2.20619 \\ & \cdot 10^{11} B_{10,20}(X) - 1.60453\cdot 10^{11} B_{11,20}(X) - 4.92917\cdot 10^{11} B_{12,20}(X) - 6.84241\cdot 10^{11} B_{13,20}(X) \\ & - 6.82665\cdot 10^{11} B_{14,20}(X) - 4.91903\cdot 10^{11} B_{15,20}(X) - 1.67279\cdot 10^{11} B_{16,20}(X) + 2.0413 \\ & \cdot 10^{11} B_{17,20}(X) + 5.31271\cdot 10^{11} B_{18,20}(X) + 7.44977\cdot 10^{11} B_{19,20}(X) + 8.1419\cdot 10^{11} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = & 1.53024\cdot 10^{13} X^{3} - 2.27622\cdot 10^{13} X^{2} + 9.44558\cdot 10^{12} X - 9.11555\cdot 10^{11} \\ = & -9.11555\cdot 10^{11} B_{0,3} + 2.23697\cdot 10^{12} B_{1,3} - 2.20191\cdot 10^{12} B_{2,3} + 1.07418\cdot 10^{12} B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 = & 2.99683\cdot 10^{14} X^{20} - 3.0143\cdot 10^{15} X^{19} + 1.40341\cdot 10^{16} X^{18} - 4.01271\cdot 10^{16} X^{17} + 7.88281\cdot 10^{16} X^{16} - 1.12773 \\ & \cdot 10^{17} X^{15} + 1.21519\cdot 10^{17} X^{14} - 1.00622\cdot 10^{17} X^{13} + 6.47541\cdot 10^{16} X^{12} - 3.25502\cdot 10^{16} X^{11} + 1.27798 \\ & \cdot 10^{16} X^{10} - 3.90018\cdot 10^{15} X^{9} + 9.15924\cdot 10^{14} X^{8} - 1.62759\cdot 10^{14} X^{7} + 2.13265\cdot 10^{13} X^{6} - 1.98611 \\ & \cdot 10^{12} X^{5} + 1.2527\cdot 10^{11} X^{4} + 1.52974\cdot 10^{13} X^{3} - 2.27621\cdot 10^{13} X^{2} + 9.44558\cdot 10^{12} X - 9.11555\cdot 10^{11} \\ = & -9.11555\cdot 10^{11} B_{0,20} - 4.39277\cdot 10^{11} B_{1,20} - 8.67984\cdot 10^{10} B_{2,20} + 1.59298\cdot 10^{11} B_{3,20} + 3.12457 \\ & \cdot 10^{11} B_{4,20} + 3.86021\cdot 10^{11} B_{5,20} + 3.93653\cdot 10^{11} B_{6,20} + 3.48207\cdot 10^{11} B_{7,20} + 2.64226\cdot 10^{11} B_{8,20} \\ & + 1.53277\cdot 10^{11} B_{9,20} + 3.13583\cdot 10^{10} B_{10,20} - 9.10884\cdot 10^{10} B_{11,20} - 1.97733\cdot 10^{11} B_{12,20} \\ & - 2.77558\cdot 10^{11} B_{13,20} - 3.15455\cdot 10^{11} B_{14,20} - 2.98996\cdot 10^{11} B_{15,20} - 2.14264\cdot 10^{11} B_{16,20} \\ & - 4.8038\cdot 10^{10} B_{17,20} + 2.13171\cdot 10^{11} B_{18,20} + 5.8277\cdot 10^{11} B_{19,20} + 1.07418\cdot 10^{12} B_{20,20} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 4.90611 \cdot 10^{11}$.

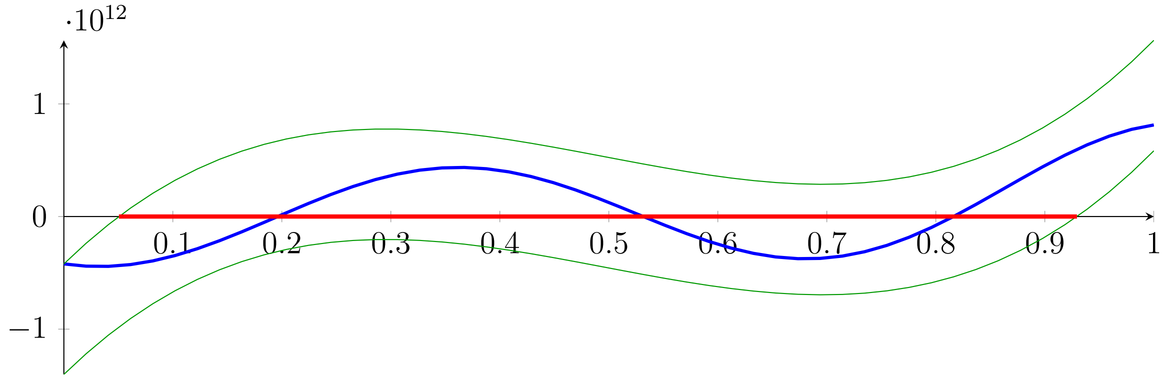
Bounding polynomials M and m:

$$M = 1.53024 \cdot 10^{13} X^3 - 2.27622 \cdot 10^{13} X^2 + 9.44558 \cdot 10^{12} X - 4.20945 \cdot 10^{11}$$
$$m = 1.53024 \cdot 10^{13} X^3 - 2.27622 \cdot 10^{13} X^2 + 9.44558 \cdot 10^{12} X - 1.40217 \cdot 10^{12}$$

Root of M and m:

$$N(M) = \{0.050503\} \qquad N(m) = \{0.929448\}$$

Intersection intervals:



$$[0.050503, 0.929448]$$

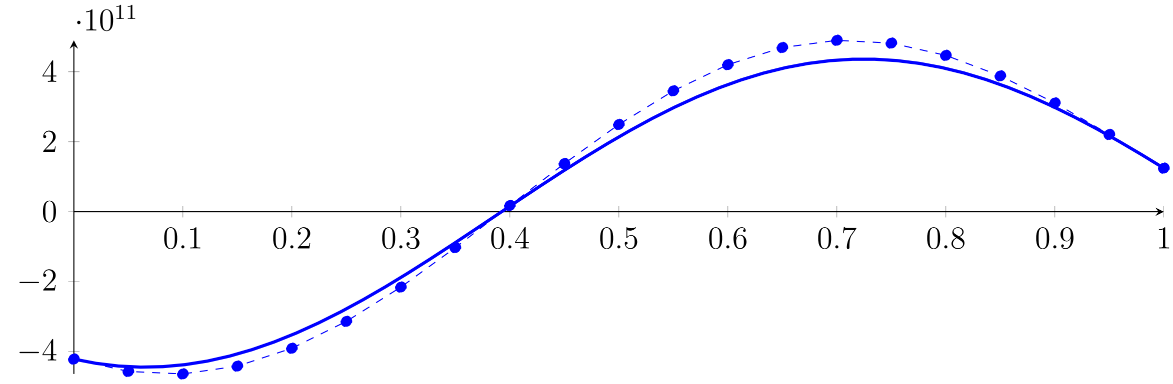
Longest intersection interval: 0.878945

$\Longrightarrow$ Bisection: first half $[9.375, 10.9375]$ und second half $[10.9375, 12.5]$

3.38 Recursion Branch 1 1 2 2 1 on the First Half $[9.375, 10.9375]$

Normalized monomial und Bézier representations and the Bézier polygon:

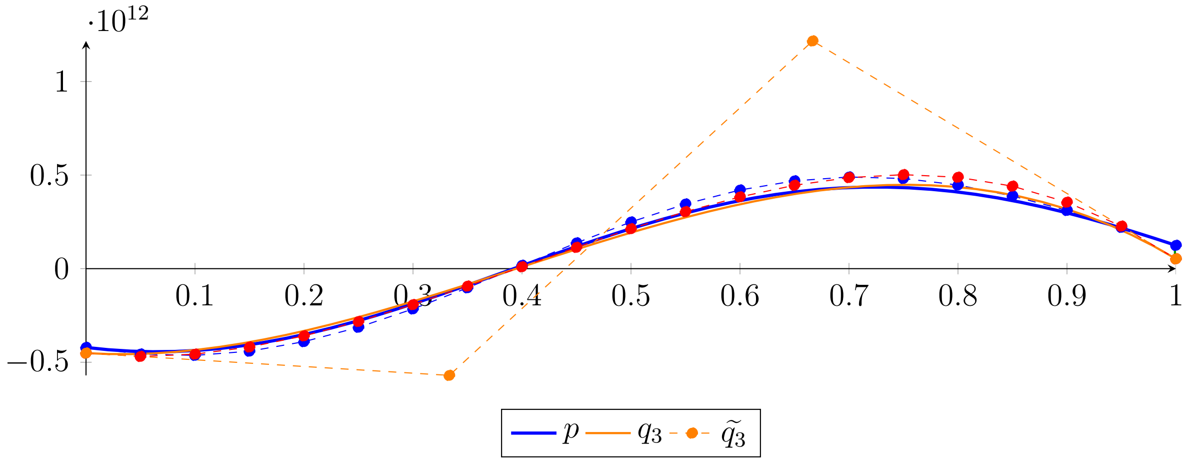
$$\begin{aligned} p = {} & 7412.69 X^{20} - 105681 X^{19} - 281108 X^{18} + 1.01062 \cdot 10^7 X^{17} - 1.42567 \cdot 10^7 X^{16} - 3.70075 \cdot 10^8 X^{15} \\ & + 1.05512 \cdot 10^9 X^{14} + 6.8556 \cdot 10^9 X^{13} - 2.55814 \cdot 10^{10} X^{12} - 6.77046 \cdot 10^{10} X^{11} + 3.08436 \cdot 10^{11} X^{10} \\ & + 3.41997 \cdot 10^{11} X^9 - 1.9761 \cdot 10^{12} X^8 - 7.18943 \cdot 10^{11} X^7 + 6.53083 \cdot 10^{12} X^6 - 1.51285 \cdot 10^{10} X^5 \\ & - 9.94282 \cdot 10^{12} X^4 + 1.45686 \cdot 10^{12} X^3 + 5.3521 \cdot 10^{12} X^2 - 7.06004 \cdot 10^{11} X - 4.20945 \cdot 10^{11} \\ = {} & -4.20945 \cdot 10^{11} B_{0,20}(X) - 4.56245 \cdot 10^{11} B_{1,20}(X) - 4.63376 \cdot 10^{11} B_{2,20}(X) - 4.4106 \\ & \cdot 10^{11} B_{3,20}(X) - 3.90072 \cdot 10^{11} B_{4,20}(X) - 3.13239 \cdot 10^{11} B_{5,20}(X) - 2.15273 \cdot 10^{11} B_{6,20}(X) \\ & - 1.02447 \cdot 10^{11} B_{7,20}(X) + 1.78698 \cdot 10^{10} B_{8,20}(X) + 1.37766 \cdot 10^{11} B_{9,20}(X) + 2.49392 \\ & \cdot 10^{11} B_{10,20}(X) + 3.45561 \cdot 10^{11} B_{11,20}(X) + 4.2028 \cdot 10^{11} B_{12,20}(X) + 4.69189 \cdot 10^{11} B_{13,20}(X) \\ & + 4.89846 \cdot 10^{11} B_{14,20}(X) + 4.81857 \cdot 10^{11} B_{15,20}(X) + 4.46838 \cdot 10^{11} B_{16,20}(X) + 3.88213 \\ & \cdot 10^{11} B_{17,20}(X) + 3.10886 \cdot 10^{11} B_{18,20}(X) + 2.20816 \cdot 10^{11} B_{19,20}(X) + 1.24532 \cdot 10^{11} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -4.85733\cdot 10^{12} X^3 + 5.7182\cdot 10^{12} X^2 - 3.57195\cdot 10^{11} X - 4.51214\cdot 10^{11} \\ &= -4.51214\cdot 10^{11} B_{0,3} - 5.70279\cdot 10^{11} B_{1,3} + 1.21672\cdot 10^{12} B_{2,3} + 5.24625\cdot 10^{10} B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= -5.42396\cdot 10^{13} X^{20} + 5.47126\cdot 10^{14} X^{19} - 2.55601\cdot 10^{15} X^{18} + 7.33618\cdot 10^{15} X^{17} - 1.44693\cdot 10^{16} X^{16} \\ &\quad + 2.07805\cdot 10^{16} X^{15} - 2.24676\cdot 10^{16} X^{14} + 1.86491\cdot 10^{16} X^{13} - 1.20144\cdot 10^{16} X^{12} + 6.03637\cdot 10^{15} X^{11} \\ &\quad - 2.36541\cdot 10^{15} X^{10} + 7.19912\cdot 10^{14} X^9 - 1.68676\cdot 10^{14} X^8 + 2.99718\cdot 10^{13} X^7 - 3.94757\cdot 10^{12} X^6 + 3.74484 \\ &\quad \cdot 10^{11} X^5 - 2.49938\cdot 10^{10} X^4 - 4.85622\cdot 10^{12} X^3 + 5.71817\cdot 10^{12} X^2 - 3.57195\cdot 10^{11} X - 4.51214\cdot 10^{11} \\ &= -4.51214\cdot 10^{11} B_{0,20} - 4.69074\cdot 10^{11} B_{1,20} - 4.56838\cdot 10^{11} B_{2,20} - 4.18766\cdot 10^{11} B_{3,20} - 3.59123 \\ &\quad \cdot 10^{11} B_{4,20} - 2.82156\cdot 10^{11} B_{5,20} - 1.92169\cdot 10^{11} B_{6,20} - 9.33173\cdot 10^{10} B_{7,20} + 9.92932\cdot 10^{9} B_{8,20} \\ &\quad + 1.13658\cdot 10^{11} B_{9,20} + 2.13127\cdot 10^{11} B_{10,20} + 3.04621\cdot 10^{11} B_{11,20} + 3.83362\cdot 10^{11} B_{12,20} \\ &\quad + 4.45515\cdot 10^{11} B_{13,20} + 4.86515\cdot 10^{11} B_{14,20} + 5.02283\cdot 10^{11} B_{15,20} + 4.88467\cdot 10^{11} B_{16,20} \\ &\quad + 4.40844\cdot 10^{11} B_{17,20} + 3.55142\cdot 10^{11} B_{18,20} + 2.27102\cdot 10^{11} B_{19,20} + 5.24625\cdot 10^{10} B_{20,20} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 7.2069\cdot 10^{10}$.
Bounding polynomials M and m:

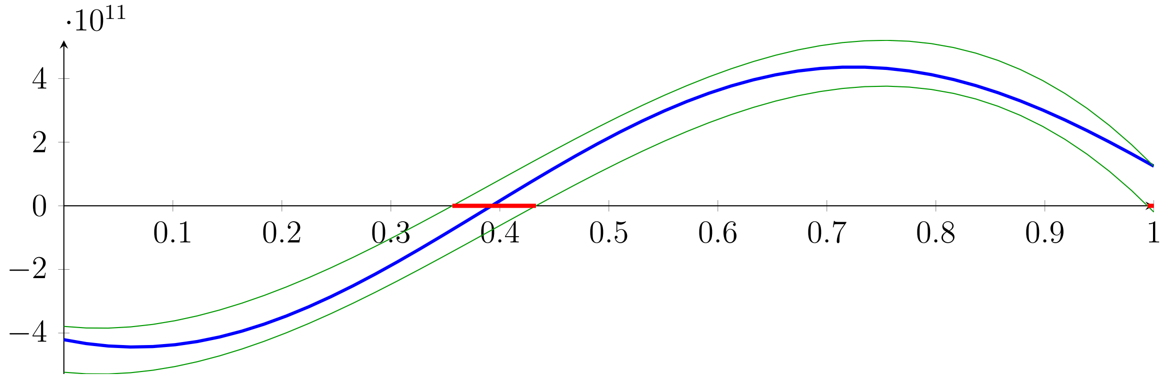
$$M = -4.85733\cdot 10^{12} X^3 + 5.7182\cdot 10^{12} X^2 - 3.57195\cdot 10^{11} X - 3.79145\cdot 10^{11}$$

$$m = -4.85733\cdot 10^{12} X^3 + 5.7182\cdot 10^{12} X^2 - 3.57195\cdot 10^{11} X - 5.23283\cdot 10^{11}$$

Root of M and m:

$$N(M) = \{-0.212041, 0.356406, 1.03287\} \qquad N(m) = \{-0.25017, 0.433097, 0.994305\}$$

Intersection intervals:



$[0.356406, 0.433097]$, $[0.994305, 1]$

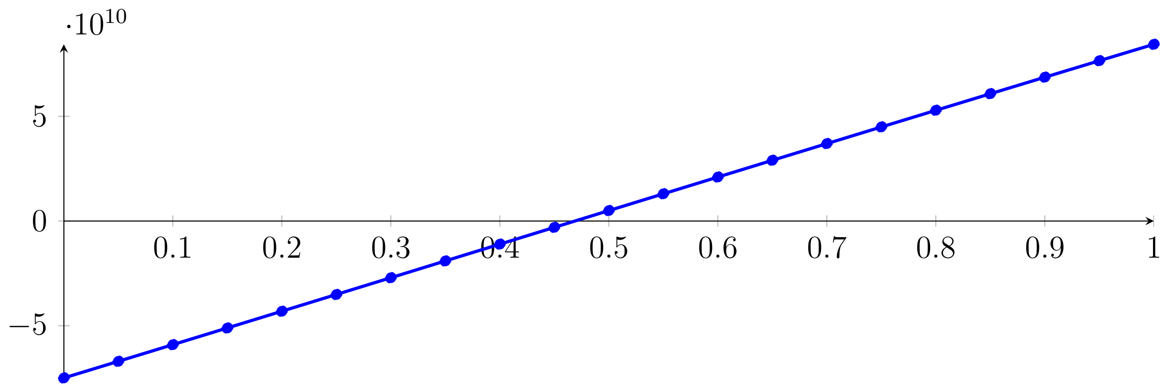
Longest intersection interval: 0.076691

$\Longrightarrow$ Selective recursion: interval 1: $[9.93188, 10.0517]$, interval 2: $[10.9286, 10.9375]$,

3.39 Recursion Branch 1 1 2 2 1 1 in Interval 1: $[9.93188, 10.0517]$

Normalized monomial und Bézier representations and the Bézier polygon:

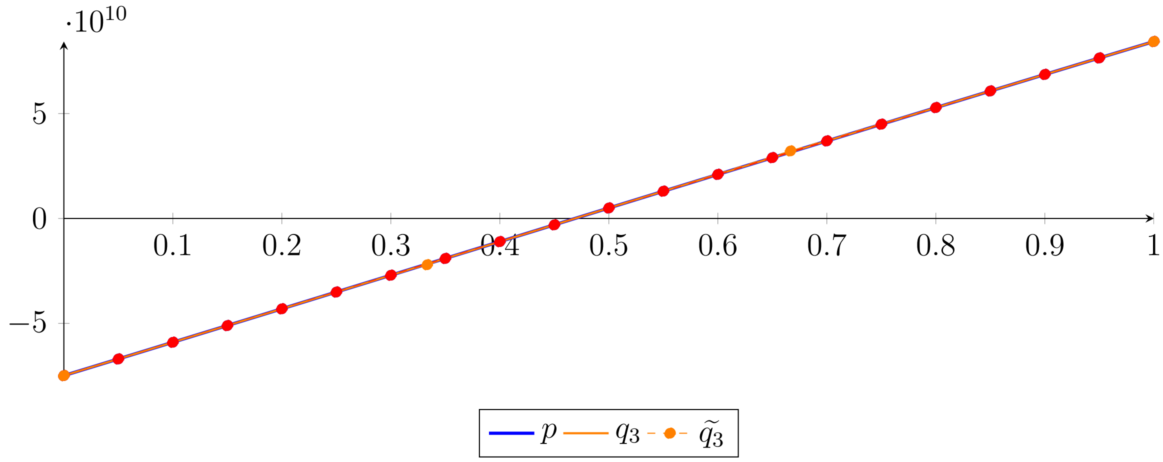
$$\begin{aligned}
p &= 3.93141X^{20} + 132.929X^{19} + 465.401X^{18} + 603.938X^{17} + 5182.78X^{16} - 6073.52X^{15} \\
&\quad + 5053.78X^{14} + 3377.07X^{13} + 13364.7X^{12} + 4282.55X^{11} + 5054.74X^{10} \\
&\quad + 1186.61X^{9} + 294.089X^{8} - 55609.9X^{7} - 33448.4X^{6} + 2.14586\cdot 10^{7}X^{5} - 1.85352 \\
&\quad \cdot 10^{7}X^{4} - 3.51085\cdot 10^{9}X^{3} + 4.22101\cdot 10^{9}X^{2} + 1.58519\cdot 10^{11}X - 7.48937\cdot 10^{10} \\
&= -7.48937\cdot 10^{10}B_{0,20}(X) - 6.69678\cdot 10^{10}B_{1,20}(X) - 5.90196\cdot 10^{10}B_{2,20}(X) - 5.10523 \\
&\quad \cdot 10^{10}B_{3,20}(X) - 4.3069\cdot 10^{10}B_{4,20}(X) - 3.50727\cdot 10^{10}B_{5,20}(X) - 2.70665\cdot 10^{10}B_{6,20}(X) \\
&\quad - 1.90535\cdot 10^{10}B_{7,20}(X) - 1.10368\cdot 10^{10}B_{8,20}(X) - 3.01944\cdot 10^{9}B_{9,20}(X) + 4.99543 \\
&\quad \cdot 10^{9}B_{10,20}(X) + 1.30048\cdot 10^{10}B_{11,20}(X) + 2.10055\cdot 10^{10}B_{12,20}(X) + 2.89947\cdot 10^{10}B_{13,20}(X) \\
&\quad + 3.69691\cdot 10^{10}B_{14,20}(X) + 4.49258\cdot 10^{10}B_{15,20}(X) + 5.28618\cdot 10^{10}B_{16,20}(X) + 6.0774 \\
&\quad \cdot 10^{10}B_{17,20}(X) + 6.86594\cdot 10^{10}B_{18,20}(X) + 7.65151\cdot 10^{10}B_{19,20}(X) + 8.43382\cdot 10^{10}B_{20,20}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 &= -3.48863\cdot 10^{9}X^{3} + 4.19407\cdot 10^{9}X^{2} + 1.58526\cdot 10^{11}X - 7.48941\cdot 10^{10} \\
&= -7.48941\cdot 10^{10}B_{0,3} - 2.2052\cdot 10^{10}B_{1,3} + 3.21881\cdot 10^{10}B_{2,3} + 8.43376\cdot 10^{10}B_{3,3}
\end{aligned}$$

$$\widetilde{q}_3 = 5.06581{\cdot}10^{12}X^{20} - 5.07702{\cdot}10^{13}X^{19} + 2.35422{\cdot}10^{14}X^{18} - 6.70195{\cdot}10^{14}X^{17} + 1.31067{\cdot}10^{15}X^{16} - 1.86709{\cdot}10^{15}X^{15} + 2.00466{\cdot}10^{15}X^{14} - 1.65601{\cdot}10^{15}X^{13} + 1.06504{\cdot}10^{15}X^{12} - 5.3612{\cdot}10^{14}X^{11} + 2.11167{\cdot}10^{14}X^{10} - 6.47062{\cdot}10^{13}X^{9} + 1.52467{\cdot}10^{13}X^{8} - 2.7131{\cdot}10^{12}X^{7} + 3.55302{\cdot}10^{11}X^{6} - 3.29706{\cdot}10^{10}X^{5} + 2.03728{\cdot}10^{9}X^{4} - 3.56586{\cdot}10^{9}X^{3} + 4.19572{\cdot}10^{9}X^{2} + 1.58526{\cdot}10^{11}X - 7.48941{\cdot}10^{10}$$
$$= -7.48941{\cdot}10^{10}B_{0,20} - 6.69678{\cdot}10^{10}B_{1,20} - 5.90194{\cdot}10^{10}B_{2,20} - 5.10521{\cdot}10^{10}B_{3,20} - 4.30685{\cdot}10^{10}B_{4,20} - 3.5073{\cdot}10^{10}B_{5,20} - 2.70649{\cdot}10^{10}B_{6,20} - 1.90564{\cdot}10^{10}B_{7,20} - 1.10324{\cdot}10^{10}B_{8,20} - 3.02593{\cdot}10^{9}B_{9,20} + 5.00198{\cdot}10^{9}B_{10,20} + 1.29981{\cdot}10^{10}B_{11,20} + 2.10096{\cdot}10^{10}B_{12,20} + 2.89918{\cdot}10^{10}B_{13,20} + 3.69704{\cdot}10^{10}B_{14,20} + 4.49255{\cdot}10^{10}B_{15,20} + 5.28622{\cdot}10^{10}B_{16,20} + 6.07743{\cdot}10^{10}B_{17,20} + 6.86598{\cdot}10^{10}B_{18,20} + 7.65152{\cdot}10^{10}B_{19,20} + 8.43376{\cdot}10^{10}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 6.68824{\cdot}10^{6}$.

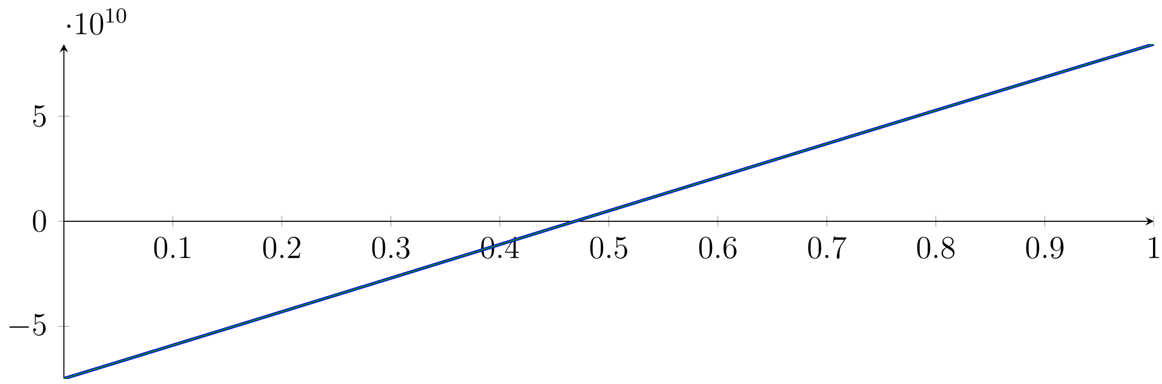
Bounding polynomials M and m:

$$M = -3.48863{\cdot}10^{9}X^{3} + 4.19407{\cdot}10^{9}X^{2} + 1.58526{\cdot}10^{11}X - 7.48874{\cdot}10^{10}$$
$$m = -3.48863{\cdot}10^{9}X^{3} + 4.19407{\cdot}10^{9}X^{2} + 1.58526{\cdot}10^{11}X - 7.49008{\cdot}10^{10}$$

Root of M and m:

$$N(M) = \{-6.40968, 0.46885, 7.14305\} \qquad N(m) = \{-6.40973, 0.468933, 7.143\}$$

Intersection intervals:



$$[0.46885, 0.468933]$$

Longest intersection interval: $8.35203{\cdot}10^{-05}$

$\Longrightarrow$ Selective recursion: interval 1: $[9.98807, 9.98808]$,

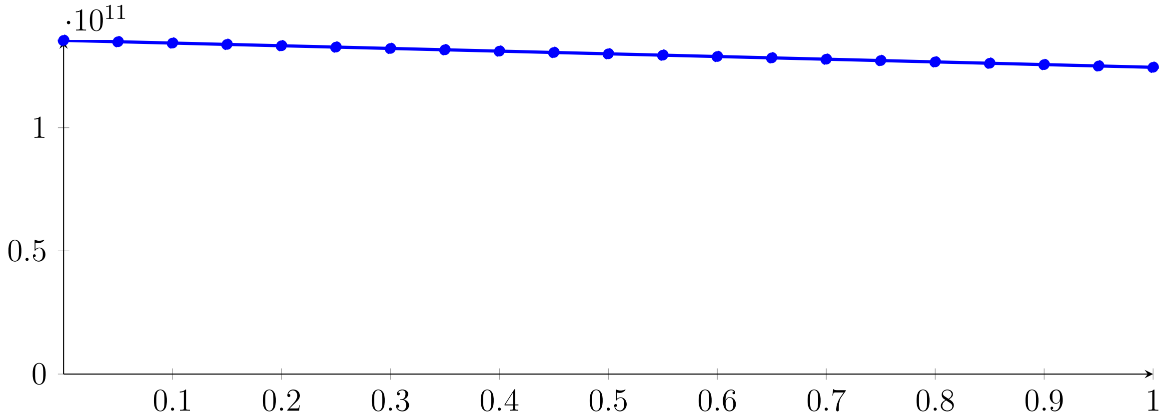
3.40 Recursion Branch 1 1 2 2 1 1 1 in Interval 1: $[9.98807, 9.98808]$

Found root in interval $[9.98807, 9.98808]$ at recursion depth 7!

3.41 Recursion Branch 1 1 2 2 1 2 in Interval 2: $[10.9286, 10.9375]$

Normalized monomial und Bézier representations and the Bézier polygon:

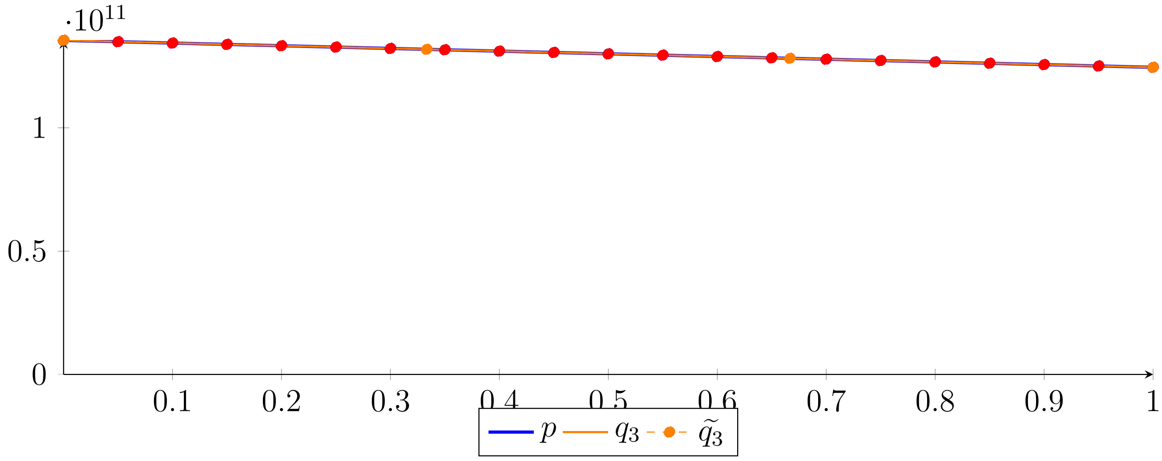
$$
\begin{aligned}
p = & -128.452X^{20} + 696.229X^{19} - 3920.87X^{18} + 15843.6X^{17} - 83609X^{16} \\
& + 64410.8X^{15} - 26824.3X^{14} - 9393.1X^{13} - 67957.6X^{12} - 7598.9X^{11} \\
& - 19065.9X^{10} - 2309.14X^{9} - 282.556X^{8} - 5.91431X^{7} - 101.135X^{6} - 46.6047X^{5} \\
& + 3127.85X^{4} + 1.3628\cdot 10^{6}X^{3} - 4.24103\cdot 10^{7}X^{2} - 1.08869\cdot 10^{10}X + 1.35459\cdot 10^{11} \\
= & \ 1.35459\cdot 10^{11}B_{0,20}(X) + 1.34915\cdot 10^{11}B_{1,20}(X) + 1.34371\cdot 10^{11}B_{2,20}(X) + 1.33826 \\
& \cdot 10^{11}B_{3,20}(X) + 1.33281\cdot 10^{11}B_{4,20}(X) + 1.32736\cdot 10^{11}B_{5,20}(X) + 1.3219\cdot 10^{11}B_{6,20}(X) \\
& + 1.31644\cdot 10^{11}B_{7,20}(X) + 1.31099\cdot 10^{11}B_{8,20}(X) + 1.30552\cdot 10^{11}B_{9,20}(X) + 1.30006 \\
& \cdot 10^{11}B_{10,20}(X) + 1.2946\cdot 10^{11}B_{11,20}(X) + 1.28913\cdot 10^{11}B_{12,20}(X) + 1.28366\cdot 10^{11}B_{13,20}(X) \\
& + 1.27819\cdot 10^{11}B_{14,20}(X) + 1.27271\cdot 10^{11}B_{15,20}(X) + 1.26724\cdot 10^{11}B_{16,20}(X) + 1.26176 \\
& \cdot 10^{11}B_{17,20}(X) + 1.25628\cdot 10^{11}B_{18,20}(X) + 1.2508\cdot 10^{11}B_{19,20}(X) + 1.24532\cdot 10^{11}B_{20,20}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_3 = & \ 1.36894\cdot 10^{6}X^{3} - 4.24142\cdot 10^{7}X^{2} - 1.08869\cdot 10^{10}X + 1.35459\cdot 10^{11} \\
= & \ 1.35459\cdot 10^{11}B_{0,3} + 1.31831\cdot 10^{11}B_{1,3} + 1.28187\cdot 10^{11}B_{2,3} + 1.24532\cdot 10^{11}B_{3,3}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q_3} = & -2.12581\cdot 10^{13}X^{20} + 2.13162\cdot 10^{14}X^{19} - 9.89808\cdot 10^{14}X^{18} + 2.82365\cdot 10^{15}X^{17} - 5.53585\cdot 10^{15}X^{16} \\
& + 7.90511\cdot 10^{15}X^{15} - 8.50229\cdot 10^{15}X^{14} + 7.02535\cdot 10^{15}X^{13} - 4.50889\cdot 10^{15}X^{12} + 2.25786\cdot 10^{15}X^{11} \\
& - 8.81426\cdot 10^{14}X^{10} + 2.66736\cdot 10^{14}X^{9} - 6.19594\cdot 10^{13}X^{8} + 1.09082\cdot 10^{13}X^{7} - 1.43548\cdot 10^{12}X^{6} + 1.38853 \\
& \cdot 10^{11}X^{5} - 9.63322\cdot 10^{9}X^{4} + 4.53745\cdot 10^{8}X^{3} - 5.43005\cdot 10^{7}X^{2} - 1.08867\cdot 10^{10}X + 1.35459\cdot 10^{11} \\
= & \ 1.35459\cdot 10^{11}B_{0,20} + 1.34915\cdot 10^{11}B_{1,20} + 1.34371\cdot 10^{11}B_{2,20} + 1.33826\cdot 10^{11}B_{3,20} + 1.3328 \\
& \cdot 10^{11}B_{4,20} + 1.32738\cdot 10^{11}B_{5,20} + 1.32184\cdot 10^{11}B_{6,20} + 1.31657\cdot 10^{11}B_{7,20} + 1.31078\cdot 10^{11}B_{8,20} \\
& + 1.30578\cdot 10^{11}B_{9,20} + 1.29975\cdot 10^{11}B_{10,20} + 1.29482\cdot 10^{11}B_{11,20} + 1.28892\cdot 10^{11}B_{12,20} \\
& + 1.28377\cdot 10^{11}B_{13,20} + 1.27812\cdot 10^{11}B_{14,20} + 1.27274\cdot 10^{11}B_{15,20} + 1.26723\cdot 10^{11}B_{16,20} \\
& + 1.26176\cdot 10^{11}B_{17,20} + 1.25628\cdot 10^{11}B_{18,20} + 1.2508\cdot 10^{11}B_{19,20} + 1.24532\cdot 10^{11}B_{20,20}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 3.09014 \cdot 10^{7}$.

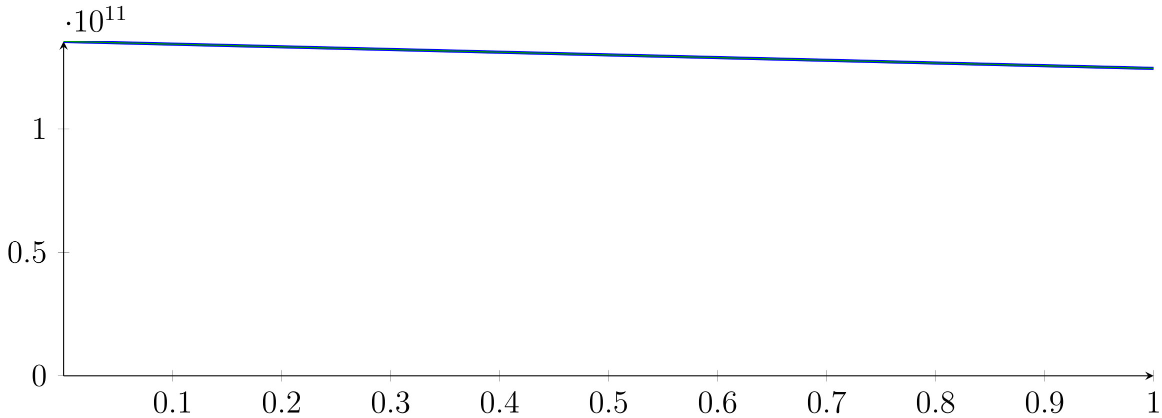
Bounding polynomials M and m:

$$M = 1.36894 \cdot 10^{6} X^{3} - 4.24142 \cdot 10^{7} X^{2} - 1.08869 \cdot 10^{10} X + 1.3549 \cdot 10^{11}$$
$$m = 1.36894 \cdot 10^{6} X^{3} - 4.24142 \cdot 10^{7} X^{2} - 1.08869 \cdot 10^{10} X + 1.35429 \cdot 10^{11}$$

Root of M and m:

$$N(M) = \{-81.4991, 12.0978, 100.385\} \qquad N(m) = \{-81.4964, 12.0923, 100.387\}$$

Intersection intervals:

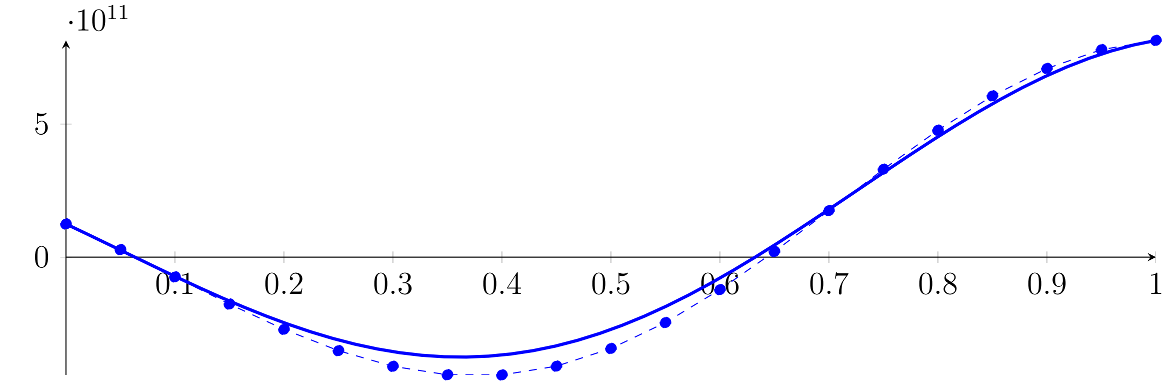


No intersection intervals with the x axis.

3.42 Recursion Branch 1 1 2 2 2 on the Second Half $[10.9375, 12.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

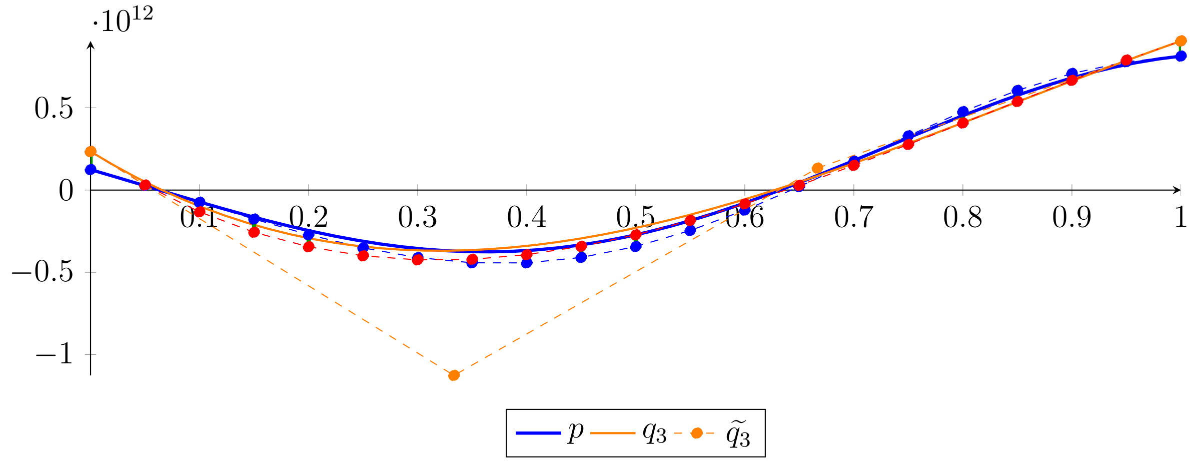
$$\begin{aligned}
p &= 7874.96 X^{20} + 41504.1 X^{19} - 902121 X^{18} - 5.01703 \cdot 10^{6} X^{17} + 4.54393 \cdot 10^{7} X^{16} + 2.38133 \cdot 10^{8} X^{15} \\
&\quad - 1.18503 \cdot 10^{9} X^{14} - 5.9933 \cdot 10^{9} X^{13} + 1.78815 \cdot 10^{10} X^{12} + 8.56274 \cdot 10^{10} X^{11} - 1.58071 \cdot 10^{11} X^{10} \\
&\quad - 7.03711 \cdot 10^{11} X^{9} + 7.99866 \cdot 10^{11} X^{8} + 3.21659 \cdot 10^{12} X^{7} - 2.16687 \cdot 10^{12} X^{6} - 7.4915 \cdot 10^{12} X^{5} \\
&\quad + 2.76126 \cdot 10^{12} X^{4} + 7.44201 \cdot 10^{12} X^{3} - 1.18084 \cdot 10^{12} X^{2} - 1.92569 \cdot 10^{12} X + 1.24532 \cdot 10^{11} \\
&= 1.24532 \cdot 10^{11} B_{0,20}(X) + 2.82469 \cdot 10^{10} B_{1,20}(X) - 7.42527 \cdot 10^{10} B_{2,20}(X) - 1.76439 \\
&\quad \cdot 10^{11} B_{3,20}(X) - 2.71214 \cdot 10^{11} B_{4,20}(X) - 3.51394 \cdot 10^{11} B_{5,20}(X) - 4.10245 \cdot 10^{11} B_{6,20}(X) \\
&\quad - 4.42042 \cdot 10^{11} B_{7,20}(X) - 4.42583 \cdot 10^{11} B_{8,20}(X) - 4.09631 \cdot 10^{11} B_{9,20}(X) - 3.43218 \\
&\quad \cdot 10^{11} B_{10,20}(X) - 2.45778 \cdot 10^{11} B_{11,20}(X) - 1.22096 \cdot 10^{11} B_{12,20}(X) + 2.09582 \cdot 10^{10} B_{13,20}(X) \\
&\quad + 1.74882 \cdot 10^{11} B_{14,20}(X) + 3.3015 \cdot 10^{11} B_{15,20}(X) + 4.76935 \cdot 10^{11} B_{16,20}(X) + 6.05883 \\
&\quad \cdot 10^{11} B_{17,20}(X) + 7.08854 \cdot 10^{11} B_{18,20}(X) + 7.79584 \cdot 10^{11} B_{19,20}(X) + 8.1419 \cdot 10^{11} B_{20,20}(X)
\end{aligned}$$



Degree reduction and raising:

$$q_3 = -3.10845\cdot 10^{12}X^3 + 7.86325\cdot 10^{12}X^2 - 4.08303\cdot 10^{12}X + 2.34464\cdot 10^{11}$$
$$= 2.34464\cdot 10^{11}B_{0,3} - 1.12655\cdot 10^{12}B_{1,3} + 1.33529\cdot 10^{11}B_{2,3} + 9.06233\cdot 10^{11}B_{3,3}$$

$$\widetilde{q}_3 = -5.29697\cdot 10^{13}X^{20} + 5.32913\cdot 10^{14}X^{19} - 2.48137\cdot 10^{15}X^{18} + 7.09443\cdot 10^{15}X^{17} - 1.39336\cdot 10^{16}X^{16} + 1.99248\cdot 10^{16}X^{15} - 2.14533\cdot 10^{16}X^{14} + 1.77413\cdot 10^{16}X^{13} - 1.13939\cdot 10^{16}X^{12} + 5.71009\cdot 10^{15}X^{11} - 2.23268\cdot 10^{15}X^{10} + 6.78001\cdot 10^{14}X^9 - 1.58378\cdot 10^{14}X^8 + 2.79809\cdot 10^{13}X^7 - 3.63174\cdot 10^{12}X^6 + 3.30233\cdot 10^{11}X^5 - 1.95071\cdot 10^{10}X^4 - 3.10779\cdot 10^{12}X^3 + 7.86324\cdot 10^{12}X^2 - 4.08303\cdot 10^{12}X + 2.34464\cdot 10^{11}$$
$$= 2.34464\cdot 10^{11}B_{0,20} + 3.03124\cdot 10^{10}B_{1,20} - 1.32454\cdot 10^{11}B_{2,20} - 2.5656\cdot 10^{11}B_{3,20} - 3.44738\cdot 10^{11}B_{4,20} - 3.99699\cdot 10^{11}B_{5,20} - 4.24212\cdot 10^{11}B_{6,20} - 4.20904\cdot 10^{11}B_{7,20} - 3.927\cdot 10^{11}B_{8,20} - 3.41996\cdot 10^{11}B_{9,20} - 2.71975\cdot 10^{11}B_{10,20} - 1.84839\cdot 10^{11}B_{11,20} - 8.38294\cdot 10^{10}B_{12,20} + 2.8755\cdot 10^{10}B_{13,20} + 1.49891\cdot 10^{11}B_{14,20} + 2.77024\cdot 10^{11}B_{15,20} + 4.07344\cdot 10^{11}B_{16,20} + 5.38157\cdot 10^{11}B_{17,20} + 6.66727\cdot 10^{11}B_{18,20} + 7.90328\cdot 10^{11}B_{19,20} + 9.06233\cdot 10^{11}B_{20,20}$$



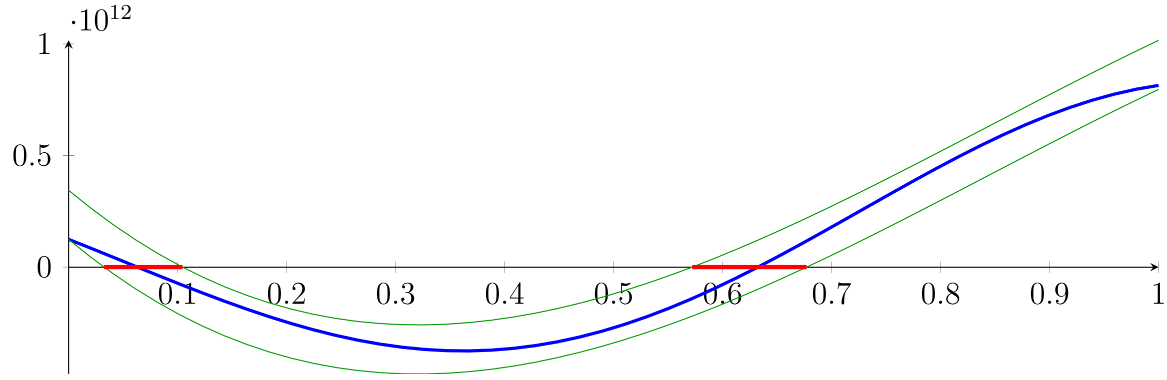
The maximum difference of the Bézier coefficients is $\delta = 1.09932\cdot 10^{11}$.
Bounding polynomials M and m:

$$M = -3.10845\cdot 10^{12}X^3 + 7.86325\cdot 10^{12}X^2 - 4.08303\cdot 10^{12}X + 3.44396\cdot 10^{11}$$
$$m = -3.10845\cdot 10^{12}X^3 + 7.86325\cdot 10^{12}X^2 - 4.08303\cdot 10^{12}X + 1.24532\cdot 10^{11}$$

Root of M and m:

$$N(M) = \{0.104516, 0.57206, 1.85306\} \qquad N(m) = \{0.0325089, 0.677105, 1.82002\}$$

Intersection intervals:



$[0.0325089, 0.104516]$, $[0.57206, 0.677105]$

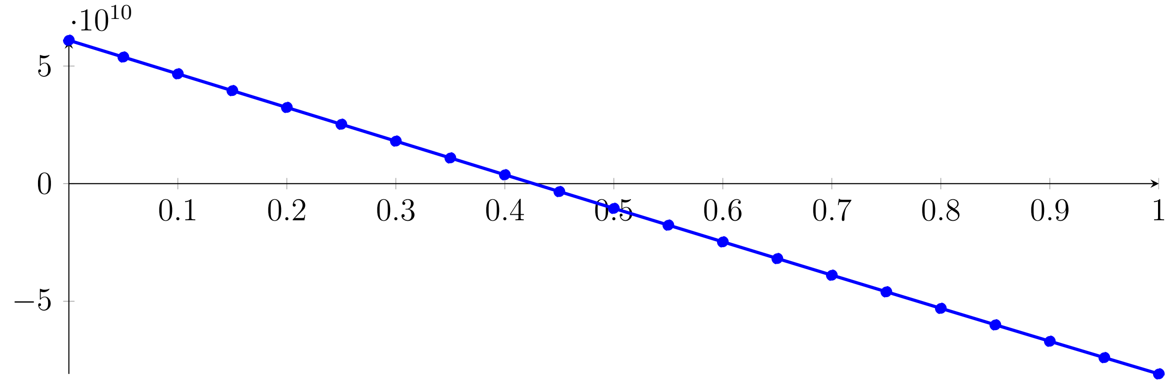
Longest intersection interval: 0.105046

$\Longrightarrow$ Selective recursion: interval 1: $[10.9883, 11.1008]$, interval 2: $[11.8313, 11.9955]$,

3.43 Recursion Branch 1 1 2 2 2 1 in Interval 1: $[10.9883, 11.1008]$

Normalized monomial und Bézier representations and the Bézier polygon:

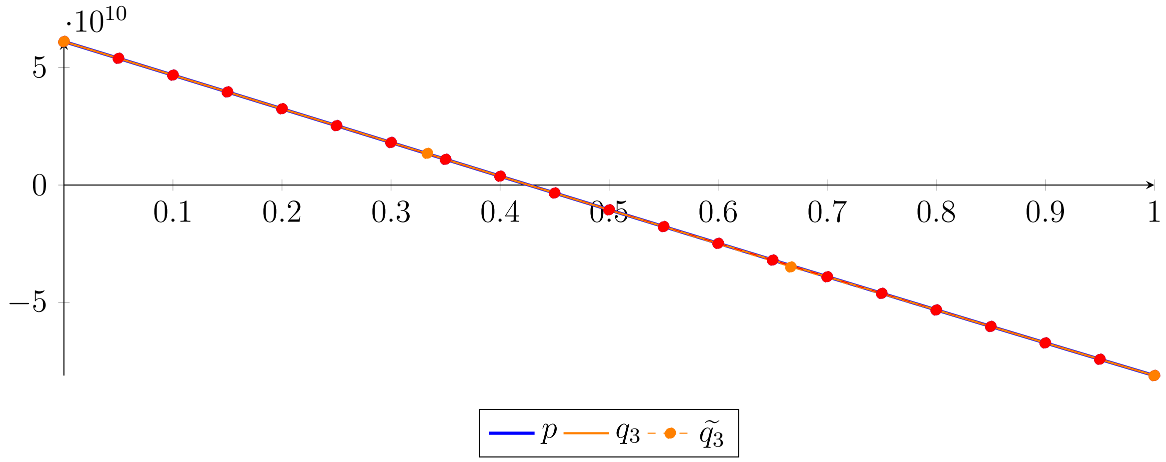
$$\begin{aligned}p &= 3.02665X^{20} - 152.696X^{19} - 209.614X^{18} - 1218.11X^{17} - 895.241X^{16} + 2304.57X^{15}\\ &\quad - 3050.3X^{14} - 2785.05X^{13} - 9113.87X^{12} - 3225.37X^{11} - 3646.57X^{10}\\ &\quad - 939.29X^{9} + 435.367X^{8} + 34110.8X^{7} - 197040X^{6} - 1.518\cdot 10^{7}X^{5} + 4.06802\\ &\quad \cdot 10^{7}X^{4} + 2.88255\cdot 10^{9}X^{3} - 2.28216\cdot 10^{9}X^{2} - 1.42469\cdot 10^{11}X + 6.09399\cdot 10^{10}\\ &= 6.09399\cdot 10^{10}B_{0,20}(X) + 5.38165\cdot 10^{10}B_{1,20}(X) + 4.6681\cdot 10^{10}B_{2,20}(X) + 3.9536\\ &\quad \cdot 10^{10}B_{3,20}(X) + 3.23842\cdot 10^{10}B_{4,20}(X) + 2.52279\cdot 10^{10}B_{5,20}(X) + 1.80697\cdot 10^{10}B_{6,20}(X)\\ &\quad + 1.09123\cdot 10^{10}B_{7,20}(X) + 3.75812\cdot 10^{9}B_{8,20}(X) - 3.39021\cdot 10^{9}B_{9,20}(X) - 1.05302\\ &\quad \cdot 10^{10}B_{10,20}(X) - 1.76591\cdot 10^{10}B_{11,20}(X) - 2.47746\cdot 10^{10}B_{12,20}(X) - 3.18739\cdot 10^{10}B_{13,20}(X)\\ &\quad - 3.89546\cdot 10^{10}B_{14,20}(X) - 4.6014\cdot 10^{10}B_{15,20}(X) - 5.30497\cdot 10^{10}B_{16,20}(X) - 6.0059\\ &\quad \cdot 10^{10}B_{17,20}(X) - 6.70394\cdot 10^{10}B_{18,20}(X) - 7.39884\cdot 10^{10}B_{19,20}(X) - 8.09033\cdot 10^{10}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= 2.92121\cdot 10^{9}X^{3} - 2.29781\cdot 10^{9}X^{2} - 1.42467\cdot 10^{11}X + 6.09398\cdot 10^{10}\\ &= 6.09398\cdot 10^{10}B_{0,3} + 1.3451\cdot 10^{10}B_{1,3} - 3.48038\cdot 10^{10}B_{2,3} - 8.09033\cdot 10^{10}B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 = &-3.56674\cdot 10^{12}X^{20} + 3.57414\cdot 10^{13}X^{19} - 1.65671\cdot 10^{14}X^{18} + 4.71359\cdot 10^{14}X^{17} - 9.21186\cdot 10^{14}X^{16} \\ &+ 1.31138\cdot 10^{15}X^{15} - 1.40733\cdot 10^{15}X^{14} + 1.16247\cdot 10^{15}X^{13} - 7.4805\cdot 10^{14}X^{12} + 3.7709\cdot 10^{14}X^{11} \\ &- 1.48887\cdot 10^{14}X^{10} + 4.57745\cdot 10^{13}X^{9} - 1.08264\cdot 10^{13}X^{8} + 1.93184\cdot 10^{12}X^{7} - 2.52658\cdot 10^{11}X^{6} + 2.3184 \\ &\cdot 10^{10}X^{5} - 1.38381\cdot 10^{9}X^{4} + 2.96964\cdot 10^{9}X^{3} - 2.29875\cdot 10^{9}X^{2} - 1.42467\cdot 10^{11}X + 6.09398\cdot 10^{10} \\ = &\, 6.09398\cdot 10^{10}B_{0,20} + 5.38165\cdot 10^{10}B_{1,20} + 4.66811\cdot 10^{10}B_{2,20} + 3.95361\cdot 10^{10}B_{3,20} + 3.2384 \\ &\cdot 10^{10}B_{4,20} + 2.52283\cdot 10^{10}B_{5,20} + 1.80686\cdot 10^{10}B_{6,20} + 1.09143\cdot 10^{10}B_{7,20} + 3.75497\cdot 10^{9}B_{8,20} \\ &- 3.38581\cdot 10^{9}B_{9,20} - 1.05349\cdot 10^{10}B_{10,20} - 1.76544\cdot 10^{10}B_{11,20} - 2.47774\cdot 10^{10}B_{12,20} \\ &- 3.18719\cdot 10^{10}B_{13,20} - 3.89555\cdot 10^{10}B_{14,20} - 4.60137\cdot 10^{10}B_{15,20} - 5.30498\cdot 10^{10}B_{16,20} \\ &- 6.0059\cdot 10^{10}B_{17,20} - 6.70394\cdot 10^{10}B_{18,20} - 7.39884\cdot 10^{10}B_{19,20} - 8.09033\cdot 10^{10}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 4.74596\cdot 10^{6}$.

Bounding polynomials M and m:

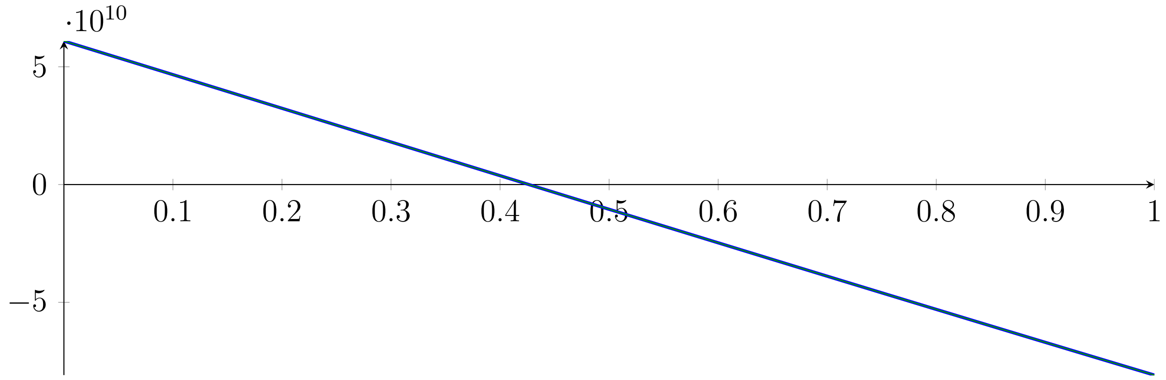
$$M = 2.92121\cdot 10^{9}X^3 - 2.29781\cdot 10^{9}X^2 - 1.42467\cdot 10^{11}X + 6.09446\cdot 10^{10}$$

$$m = 2.92121\cdot 10^{9}X^3 - 2.29781\cdot 10^{9}X^2 - 1.42467\cdot 10^{11}X + 6.09351\cdot 10^{10}$$

Root of M and m:

$$N(M) = \{-6.81675, 0.426439, 7.17691\} \qquad N(m) = \{-6.81672, 0.426372, 7.17694\}$$

Intersection intervals:



$$[0.426372, 0.426439]$$

Longest intersection interval: $6.64548\cdot 10^{-05}$

$\Longrightarrow$ Selective recursion: interval 1: $[11.0363, 11.0363]$,

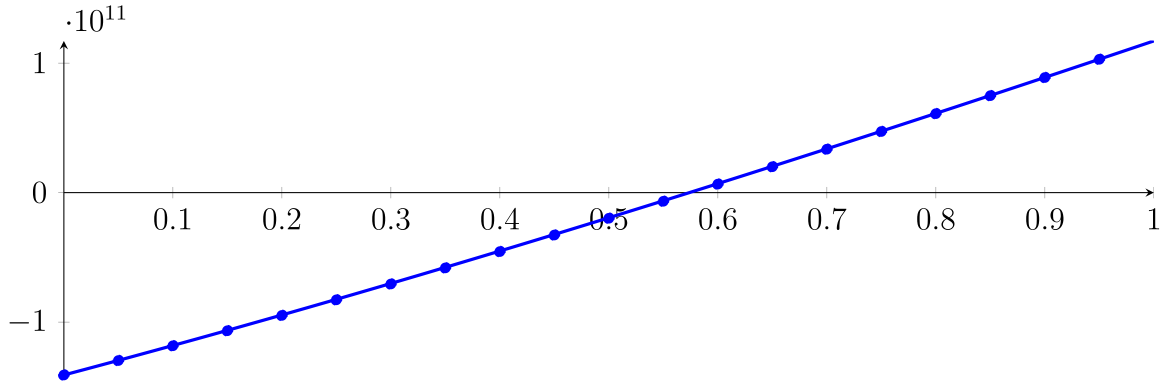
3.44 Recursion Branch 1 1 2 2 2 1 1 in Interval 1: $[11.0363, 11.0363]$

Found root in interval $[11.0363, 11.0363]$ at recursion depth 7!

3.45 Recursion Branch 1 1 2 2 2 2 in Interval 2: $[11.8313, 11.9955]$

Normalized monomial und Bézier representations and the Bézier polygon:

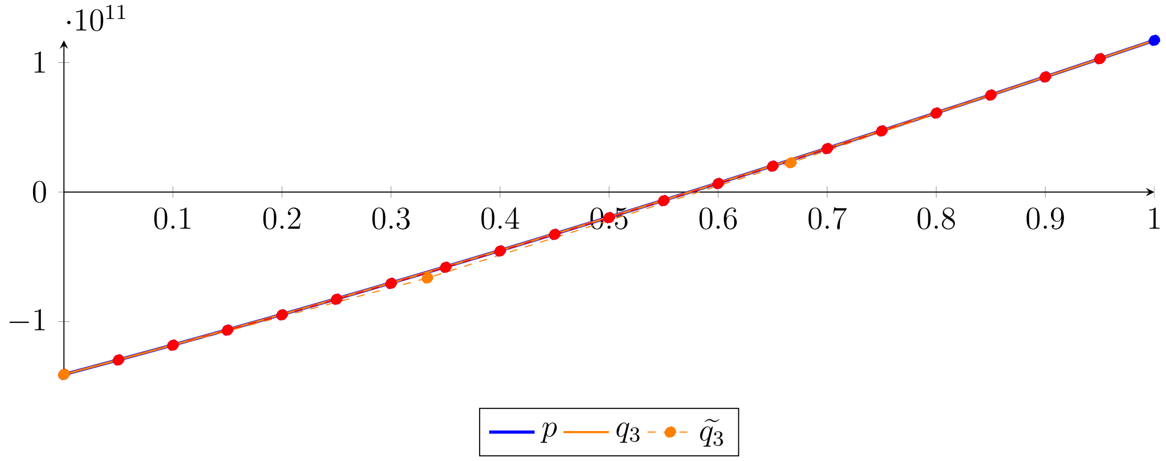
$$
\begin{aligned}
p = {} & 32.495X^{20} + 67.1509X^{19} + 1553.56X^{18} - 2283.76X^{17} + 26644.5X^{16} - 23700.8X^{15} \\
& + 12287.6X^{14} + 8715.32X^{13} + 34614.1X^{12} + 9287.83X^{11} + 11705.1X^{10} + 2521.86X^{9} \\
& - 31338.7X^{8} - 210521X^{7} + 8.31212 \cdot 10^{6}X^{6} + 6.64489 \cdot 10^{7}X^{5} - 9.97275 \\
& \cdot 10^{8}X^{4} - 7.34796 \cdot 10^{9}X^{3} + 4.27606 \cdot 10^{10}X^{2} + 2.23491 \cdot 10^{11}X - 1.40843 \cdot 10^{11} \\
= {} & -1.40843 \cdot 10^{11}B_{0,20}(X) - 1.29669 \cdot 10^{11}B_{1,20}(X) - 1.18269 \cdot 10^{11}B_{2,20}(X) - 1.06651 \\
& \cdot 10^{11}B_{3,20}(X) - 9.48206 \cdot 10^{10}B_{4,20}(X) - 8.27853 \cdot 10^{10}B_{5,20}(X) - 7.0552 \cdot 10^{10}B_{6,20}(X) \\
& - 5.81278 \cdot 10^{10}B_{7,20}(X) - 4.55203 \cdot 10^{10}B_{8,20}(X) - 3.2737 \cdot 10^{10}B_{9,20}(X) - 1.97858 \\
& \cdot 10^{10}B_{10,20}(X) - 6.67448 \cdot 10^{9}B_{11,20}(X) + 6.58871 \cdot 10^{9}B_{12,20}(X) + 1.99955 \cdot 10^{10}B_{13,20}(X) \\
& + 3.35375 \cdot 10^{10}B_{14,20}(X) + 4.7206 \cdot 10^{10}B_{15,20}(X) + 6.09924 \cdot 10^{10}B_{16,20}(X) + 7.48878 \\
& \cdot 10^{10}B_{17,20}(X) + 8.88832 \cdot 10^{10}B_{18,20}(X) + 1.0297 \cdot 10^{11}B_{19,20}(X) + 1.17138 \cdot 10^{11}B_{20,20}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_3 = {} & -9.13113 \cdot 10^{9}X^{3} + 4.38588 \cdot 10^{10}X^{2} + 2.23252 \cdot 10^{11}X - 1.40831 \cdot 10^{11} \\
= {} & -1.40831 \cdot 10^{11}B_{0,3} - 6.64139 \cdot 10^{10}B_{1,3} + 2.26231 \cdot 10^{10}B_{2,3} + 1.17149 \cdot 10^{11}B_{3,3}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_3 = {} & 1.18922 \cdot 10^{13}X^{20} - 1.19205 \cdot 10^{14}X^{19} + 5.53018 \cdot 10^{14}X^{18} - 1.57545 \cdot 10^{15}X^{17} + 3.08371 \cdot 10^{15}X^{16} \\
& - 4.39659 \cdot 10^{15}X^{15} + 4.72354 \cdot 10^{15}X^{14} - 3.90258 \cdot 10^{15}X^{13} + 2.50833 \cdot 10^{15}X^{12} - 1.26055 \cdot 10^{15}X^{11} \\
& + 4.95083 \cdot 10^{14}X^{10} - 1.51096 \cdot 10^{14}X^{9} + 3.54402 \cdot 10^{13}X^{8} - 6.2856 \cdot 10^{12}X^{7} + 8.24824 \cdot 10^{11}X^{6} - 7.76996 \\
& \cdot 10^{10}X^{5} + 5.01595 \cdot 10^{9}X^{4} - 9.33911 \cdot 10^{9}X^{3} + 4.38637 \cdot 10^{10}X^{2} + 2.23252 \cdot 10^{11}X - 1.40831 \cdot 10^{11} \\
= {} & -1.40831 \cdot 10^{11}B_{0,20} - 1.29669 \cdot 10^{11}B_{1,20} - 1.18275 \cdot 10^{11}B_{2,20} - 1.06659 \cdot 10^{11}B_{3,20} - 9.48274 \\
& \cdot 10^{10}B_{4,20} - 8.27914 \cdot 10^{10}B_{5,20} - 7.05498 \cdot 10^{10}B_{6,20} - 5.81328 \cdot 10^{10}B_{7,20} - 4.55047 \\
& \cdot 10^{10}B_{8,20} - 3.2745 \cdot 10^{10}B_{9,20} - 1.97622 \cdot 10^{10}B_{10,20} - 6.68231 \cdot 10^{9}B_{11,20} + 6.60364 \cdot 10^{9}B_{12,20} \\
& + 1.99906 \cdot 10^{10}B_{13,20} + 3.3539 \cdot 10^{10}B_{14,20} + 4.71999 \cdot 10^{10}B_{15,20} + 6.09857 \cdot 10^{10}B_{16,20} \\
& + 7.488 \cdot 10^{10}B_{17,20} + 8.88776 \cdot 10^{10}B_{18,20} + 1.0297 \cdot 10^{11}B_{19,20} + 1.17149 \cdot 10^{11}B_{20,20}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 2.35304 \cdot 10^{7}$.
Bounding polynomials M and m:

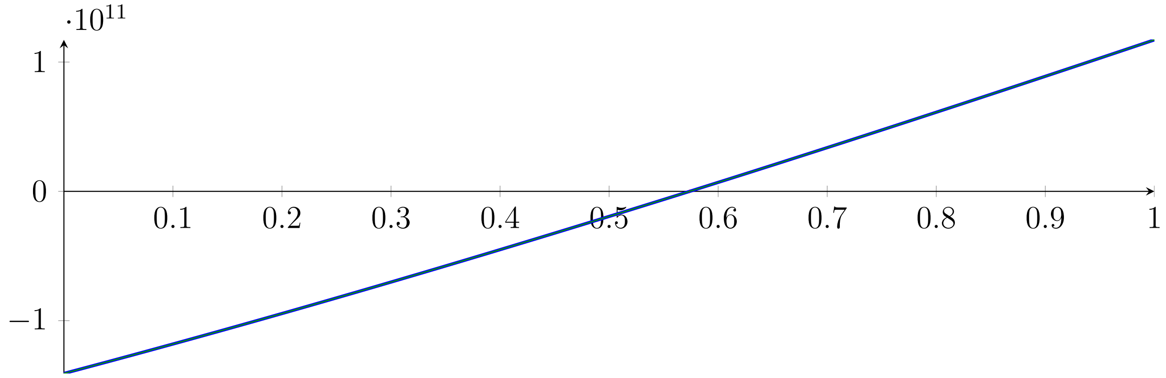
$$M = -9.13113 \cdot 10^{9} X^3 + 4.38588 \cdot 10^{10} X^2 + 2.23252 \cdot 10^{11} X - 1.40808 \cdot 10^{11}$$

$$m = -9.13113 \cdot 10^{9} X^3 + 4.38588 \cdot 10^{10} X^2 + 2.23252 \cdot 10^{11} X - 1.40855 \cdot 10^{11}$$

Root of M and m:

$$N(M) = \{-3.48423, 0.573764, 7.71369\} \qquad N(m) = \{-3.48434, 0.573942, 7.71362\}$$

Intersection intervals:



$$[0.573764, 0.573942]$$

Longest intersection interval: 0.000177878
$\Longrightarrow$ Selective recursion: interval 1: $[11.9255, 11.9255]$,

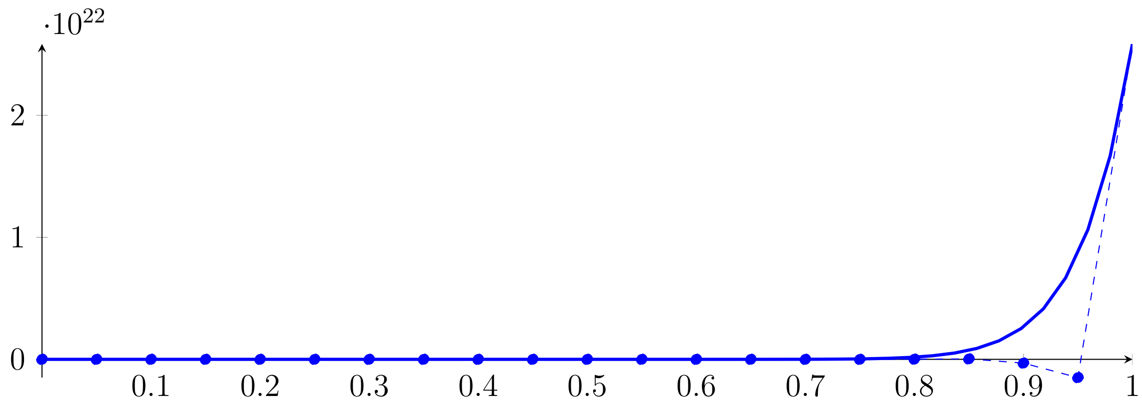
3.46 Recursion Branch 1 1 2 2 2 2 1 in Interval 1: $[11.9255, 11.9255]$

Found root in interval $[11.9255, 11.9255]$ at recursion depth 7!

3.47 Recursion Branch 1 2 on the Second Half $[12.5, 25]$

Normalized monomial und Bézier representations and the Bézier polygon:

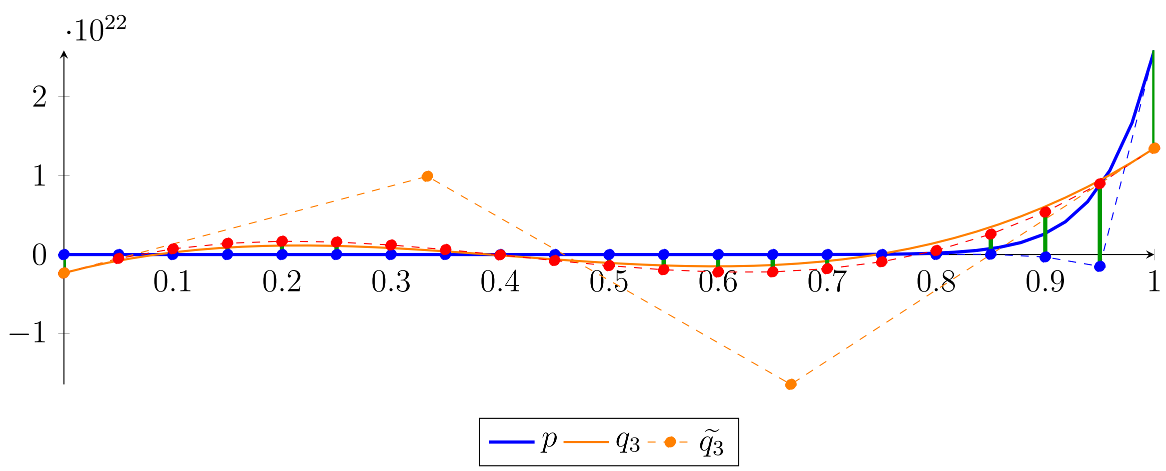
$$p = 8.67362\cdot10^{21}X^{20} + 2.77556\cdot10^{22}X^{19} + 2.3731\cdot10^{22}X^{18} - 1.26565\cdot10^{22}X^{17} - 2.8638\cdot10^{22}X^{16} - 6.33435\cdot10^{21}X^{15} + 1.06357\cdot10^{22}X^{14} + 5.39429\cdot10^{21}X^{13} - 1.50133\cdot10^{21}X^{12} - 1.39249\cdot10^{21}X^{11} + 1.05296\cdot10^{19}X^{10} + 1.67885\cdot10^{20}X^{9} + 1.71006\cdot10^{19}X^{8} - 9.83957\cdot10^{18}X^{7} - 1.53217\cdot10^{18}X^{6} + 2.57478\cdot10^{17}X^{5} + 4.72654\cdot10^{16}X^{4} - 2.266\cdot10^{15}X^{3} - 4.39258\cdot10^{14}X^{2} + 5.53708\cdot10^{12}X + 8.1419\cdot10^{11}$$
$$= 8.1419\cdot10^{11}B_{0,20}(X) + 1.09104\cdot10^{12}B_{1,20}(X) - 9.43984\cdot10^{11}B_{2,20}(X) - 7.27862\cdot10^{12}B_{3,20}(X) - 1.01451\cdot10^{13}B_{4,20}(X) + 2.45871\cdot10^{13}B_{5,20}(X) + 1.34488\cdot10^{14}B_{6,20}(X) + 1.71188\cdot10^{14}B_{7,20}(X) - 5.46645\cdot10^{14}B_{8,20}(X) - 2.59384\cdot10^{15}B_{9,20}(X) - 1.47677\cdot10^{15}B_{10,20}(X) + 2.00018\cdot10^{16}B_{11,20}(X) + 5.97972\cdot10^{16}B_{12,20}(X) - 8.43638\cdot10^{16}B_{13,20}(X) - 9.00155\cdot10^{17}B_{14,20}(X) - 6.30584\cdot10^{17}B_{15,20}(X) + 1.35026\cdot10^{19}B_{16,20}(X) + 3.45757\cdot10^{19}B_{17,20}(X) - 3.09468\cdot10^{20}B_{18,20}(X) - 1.49659\cdot10^{21}B_{19,20}(X) + 2.5852\cdot10^{22}B_{20,20}(X)$$



Degree reduction and raising:

$$q_3 = 9.48062\cdot10^{22}X^{3} - 1.15734\cdot10^{23}X^{2} + 3.67172\cdot10^{22}X - 2.33492\cdot10^{21}$$
$$= -2.33492\cdot10^{21}B_{0,3} + 9.90415\cdot10^{21}B_{1,3} - 1.64348\cdot10^{22}B_{2,3} + 1.34546\cdot10^{22}B_{3,3}$$

$$\widetilde{q}_3 = 1.66059\cdot10^{24}X^{20} - 1.67105\cdot10^{25}X^{19} + 7.78411\cdot10^{25}X^{18} - 2.22692\cdot10^{26}X^{17} + 4.37726\cdot10^{26}X^{16} - 6.26604\cdot10^{26}X^{15} + 6.7562\cdot10^{26}X^{14} - 5.59806\cdot10^{26}X^{13} + 3.60517\cdot10^{26}X^{12} - 1.81377\cdot10^{26}X^{11} + 7.12872\cdot10^{25}X^{10} - 2.17849\cdot10^{25}X^{9} + 5.1243\cdot10^{24}X^{8} - 9.12231\cdot10^{23}X^{7} + 1.19776\cdot10^{23}X^{6} - 1.11957\cdot10^{22}X^{5} + 7.13535\cdot10^{20}X^{4} + 9.47771\cdot10^{22}X^{3} - 1.15733\cdot10^{23}X^{2} + 3.67172\cdot10^{22}X - 2.33492\cdot10^{21}$$
$$= -2.33492\cdot10^{21}B_{0,20} - 4.99057\cdot10^{20}B_{1,20} + 7.2768\cdot10^{20}B_{2,20} + 1.42843\cdot10^{21}B_{3,20} + 1.68649\cdot10^{21}B_{4,20} + 1.58455\cdot10^{21}B_{5,20} + 1.20713\cdot10^{21}B_{6,20} + 6.34209\cdot10^{20}B_{7,20} - 4.48131\cdot10^{19}B_{8,20} - 7.5709\cdot10^{20}B_{9,20} - 1.40515\cdot10^{21}B_{10,20} - 1.92237\cdot10^{21}B_{11,20} - 2.20951\cdot10^{21}B_{12,20} - 2.19669\cdot10^{21}B_{13,20} - 1.79144\cdot10^{21}B_{14,20} - 9.16084\cdot10^{20}B_{15,20} + 5.1526\cdot10^{20}B_{16,20} + 2.58464\cdot10^{21}B_{17,20} + 5.37559\cdot10^{21}B_{18,20} + 8.97117\cdot10^{21}B_{19,20} + 1.34546\cdot10^{22}B_{20,20}$$



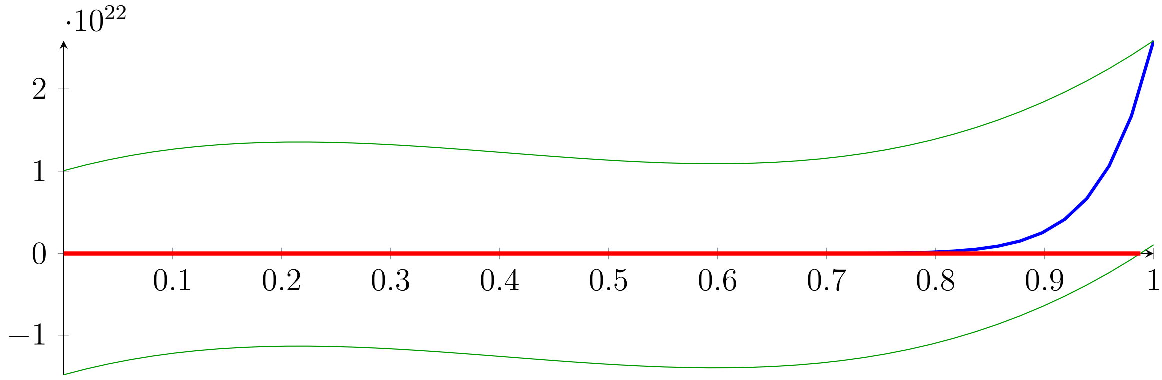
The maximum difference of the Bézier coefficients is $\delta = 1.23974 \cdot 10^{22}$.
Bounding polynomials M and m:

$$M = 9.48062 \cdot 10^{22} X^3 - 1.15734 \cdot 10^{23} X^2 + 3.67172 \cdot 10^{22} X + 1.00625 \cdot 10^{22}$$
$$m = 9.48062 \cdot 10^{22} X^3 - 1.15734 \cdot 10^{23} X^2 + 3.67172 \cdot 10^{22} X - 1.47324 \cdot 10^{22}$$

Root of M and m:

$$N(M) = \{-0.17012\} \qquad N(m) = \{0.987939\}$$

Intersection intervals:



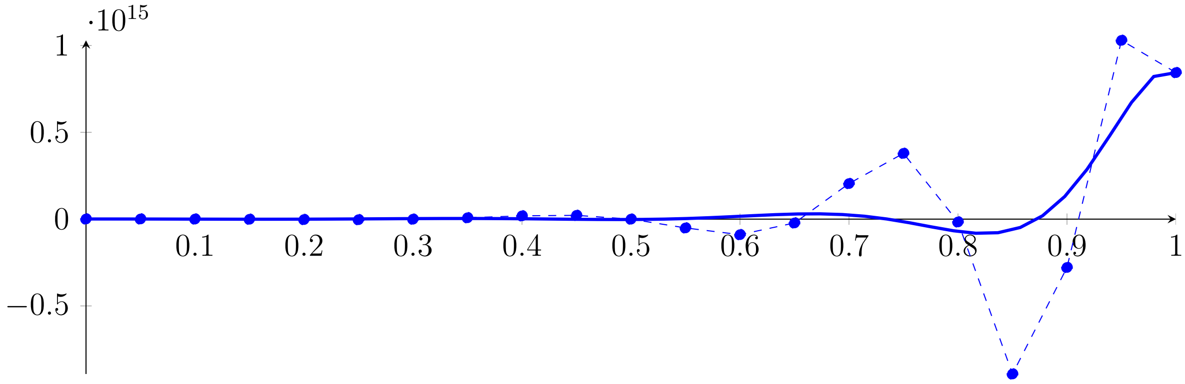
$[0, 0.987939]$

Longest intersection interval: 0.987939
$\Longrightarrow$ Bisection: first half $[12.5, 18.75]$ und second half $[18.75, 25]$

3.48 Recursion Branch 1 2 1 on the First Half $[12.5, 18.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

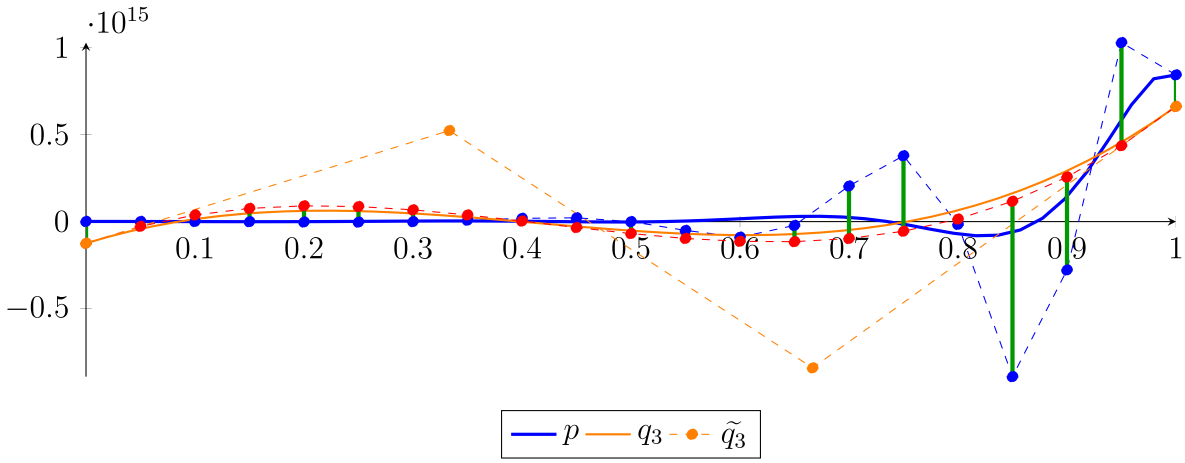
$$\begin{aligned} p = {} & 8.27181 \cdot 10^{15} X^{20} + 5.29396 \cdot 10^{16} X^{19} + 9.05266 \cdot 10^{16} X^{18} - 9.65618 \cdot 10^{16} X^{17} - 4.36981 \cdot 10^{17} X^{16} \\ & - 1.93309 \cdot 10^{17} X^{15} + 6.49154 \cdot 10^{17} X^{14} + 6.58483 \cdot 10^{17} X^{13} - 3.66535 \cdot 10^{17} X^{12} - 6.79925 \cdot 10^{17} X^{11} \\ & + 1.02828 \cdot 10^{16} X^{10} + 3.279 \cdot 10^{17} X^9 + 6.67991 \cdot 10^{16} X^8 - 7.68717 \cdot 10^{16} X^7 - 2.39402 \cdot 10^{16} X^6 + 8.04618 \\ & \cdot 10^{15} X^5 + 2.95408 \cdot 10^{15} X^4 - 2.8325 \cdot 10^{14} X^3 - 1.09814 \cdot 10^{14} X^2 + 2.76854 \cdot 10^{12} X + 8.1419 \cdot 10^{11} \\ = {} & 8.1419 \cdot 10^{11} B_{0,20}(X) + 9.52617 \cdot 10^{11} B_{1,20}(X) + 5.13074 \cdot 10^{11} B_{2,20}(X) - 7.52905 \\ & \cdot 10^{11} B_{3,20}(X) - 2.48407 \cdot 10^{12} B_{4,20}(X) - 3.19047 \cdot 10^{12} B_{5,20}(X) - 3.5214 \cdot 10^{11} B_{6,20}(X) \\ & + 7.87292 \cdot 10^{12} B_{7,20}(X) + 1.88702 \cdot 10^{13} B_{8,20}(X) + 2.17404 \cdot 10^{13} B_{9,20}(X) - 6.61543 \\ & \cdot 10^{10} B_{10,20}(X) - 5.06363 \cdot 10^{13} B_{11,20}(X) - 8.94122 \cdot 10^{13} B_{12,20}(X) - 2.20403 \cdot 10^{13} B_{13,20}(X) \\ & + 2.04834 \cdot 10^{14} B_{14,20}(X) + 3.789 \cdot 10^{14} B_{15,20}(X) - 1.62511 \cdot 10^{13} B_{16,20}(X) - 8.91971 \\ & \cdot 10^{14} B_{17,20}(X) - 2.7844 \cdot 10^{14} B_{18,20}(X) + 1.02974 \cdot 10^{15} B_{19,20}(X) + 8.43944 \cdot 10^{14} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_3 = 4.8828{\cdot}10^{15}X^3 - 6.04152{\cdot}10^{15}X^2 + 1.94544{\cdot}10^{15}X - 1.24697{\cdot}10^{14}$$
$$= -1.24697{\cdot}10^{14}B_{0,3} + 5.23784{\cdot}10^{14}B_{1,3} - 8.41574{\cdot}10^{14}B_{2,3} + 6.62027{\cdot}10^{14}B_{3,3}$$

$$\widetilde{q}_3 = 8.58275{\cdot}10^{16}X^{20} - 8.63665{\cdot}10^{17}X^{19} + 4.02308{\cdot}10^{18}X^{18} - 1.15093{\cdot}10^{19}X^{17} + 2.26224{\cdot}10^{19}X^{16} - 3.23832{\cdot}10^{19}X^{15} + 3.49155{\cdot}10^{19}X^{14} - 2.89294{\cdot}10^{19}X^{13} + 1.86297{\cdot}10^{19}X^{12} - 9.37196{\cdot}10^{18}X^{11} + 3.68314{\cdot}10^{18}X^{10} - 1.1254{\cdot}10^{18}X^9 + 2.64682{\cdot}10^{17}X^8 - 4.71121{\cdot}10^{16}X^7 + 6.18509{\cdot}10^{15}X^6 - 5.78046{\cdot}10^{14}X^5 + 3.6827{\cdot}10^{13}X^4 + 4.8813{\cdot}10^{15}X^3 - 6.04148{\cdot}10^{15}X^2 + 1.94544{\cdot}10^{15}X - 1.24697{\cdot}10^{14}$$
$$= -1.24697{\cdot}10^{14}B_{0,20} - 2.7425{\cdot}10^{13}B_{1,20} + 3.80498{\cdot}10^{13}B_{2,20} + 7.60091{\cdot}10^{13}B_{3,20} + 9.07425{\cdot}10^{13}B_{4,20} + 8.65097{\cdot}10^{13}B_{5,20} + 6.7663{\cdot}10^{13}B_{6,20} + 3.83216{\cdot}10^{13}B_{7,20} + 3.09055{\cdot}10^{12}B_{8,20} - 3.428{\cdot}10^{13}B_{9,20} - 6.87674{\cdot}10^{13}B_{10,20} - 9.69433{\cdot}10^{13}B_{11,20} - 1.13694{\cdot}10^{14}B_{12,20} - 1.15422{\cdot}10^{14}B_{13,20} - 9.73641{\cdot}10^{13}B_{14,20} - 5.55206{\cdot}10^{13}B_{15,20} + 1.45326{\cdot}10^{13}B_{16,20} + 1.17021{\cdot}10^{14}B_{17,20} + 2.56246{\cdot}10^{14}B_{18,20} + 4.36487{\cdot}10^{14}B_{19,20} + 6.62027{\cdot}10^{14}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.00899{\cdot}10^{15}$.

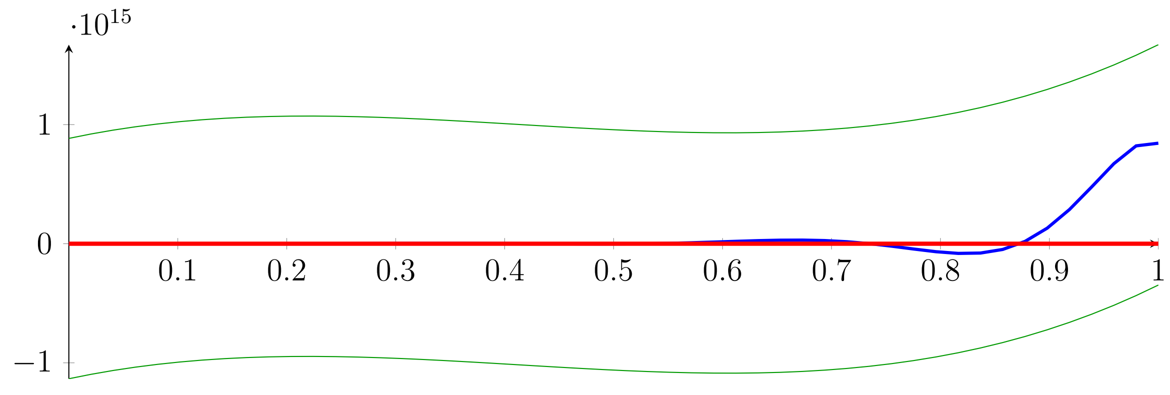
Bounding polynomials M and m:

$$M = 4.8828{\cdot}10^{15}X^3 - 6.04152{\cdot}10^{15}X^2 + 1.94544{\cdot}10^{15}X + 8.84295{\cdot}10^{14}$$
$$m = 4.8828{\cdot}10^{15}X^3 - 6.04152{\cdot}10^{15}X^2 + 1.94544{\cdot}10^{15}X - 1.13369{\cdot}10^{15}$$

Root of M and m:

$N(M) = \{-0.240333\}$ $\qquad$ $N(m) = \{1.06781\}$

Intersection intervals:



$[0, 1]$

Longest intersection interval: 1

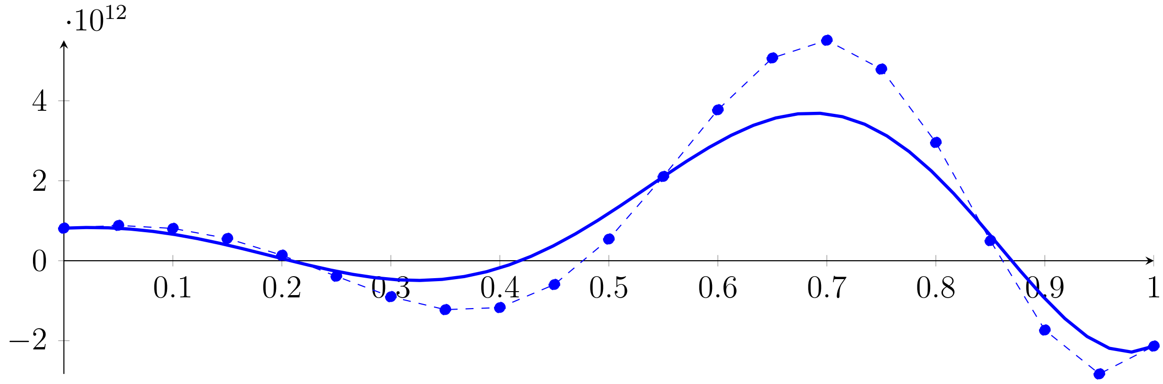
$\Longrightarrow$ Bisection: first half $[12.5, 15.625]$ und second half $[15.625, 18.75]$

Bisection point is very near to a root?!?

3.49 Recursion Branch 1 2 1 1 on the First Half [12.5, 15.625]

Normalized monomial und Bézier representations and the Bézier polygon:

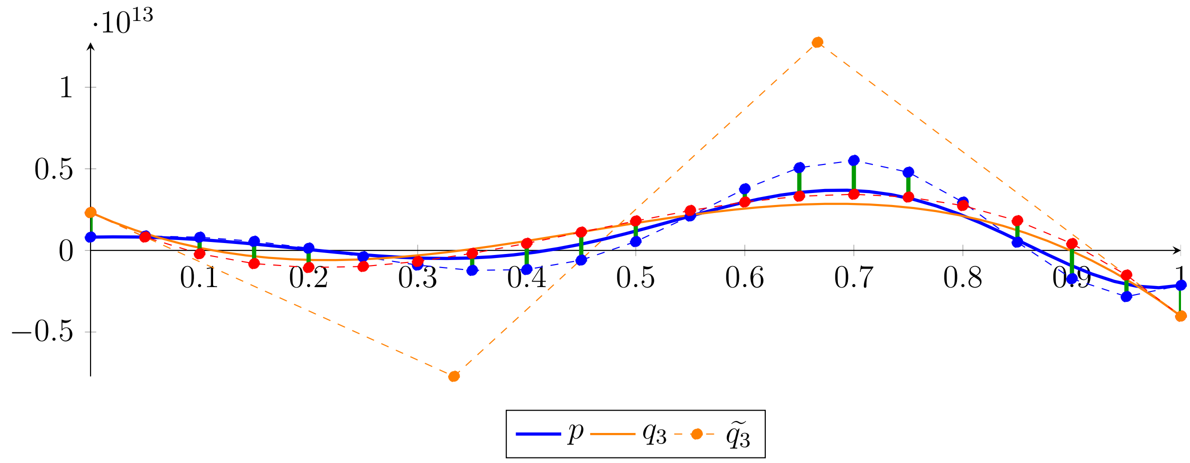
$$p = 7.88861\cdot 10^{9}X^{20} + 1.00974\cdot 10^{11}X^{19} + 3.45332\cdot 10^{11}X^{18} - 7.36708\cdot 10^{11}X^{17} - 6.66779\cdot 10^{12}X^{16} - 5.89932\cdot 10^{12}X^{15} + 3.96212\cdot 10^{13}X^{14} + 8.03812\cdot 10^{13}X^{13} - 8.94862\cdot 10^{13}X^{12} - 3.31995\cdot 10^{14}X^{11} + 1.00418\cdot 10^{13}X^{10} + 6.4043\cdot 10^{14}X^{9} + 2.60934\cdot 10^{14}X^{8} - 6.0056\cdot 10^{14}X^{7} - 3.74065\cdot 10^{14}X^{6} + 2.51443\cdot 10^{14}X^{5} + 1.8463\cdot 10^{14}X^{4} - 3.54063\cdot 10^{13}X^{3} - 2.74536\cdot 10^{13}X^{2} + 1.38427\cdot 10^{12}X + 8.1419\cdot 10^{11}$$
$$= 8.1419\cdot 10^{11}B_{0,20}(X) + 8.83404\cdot 10^{11}B_{1,20}(X) + 8.08125\cdot 10^{11}B_{2,20}(X) + 5.57295\cdot 10^{11}B_{3,20}(X) + 1.37963\cdot 10^{11}B_{4,20}(X) - 3.88495\cdot 10^{11}B_{5,20}(X) - 8.99813\cdot 10^{11}B_{6,20}(X) - 1.22366\cdot 10^{12}B_{7,20}(X) - 1.17156\cdot 10^{12}B_{8,20}(X) - 5.95624\cdot 10^{11}B_{9,20}(X) + 5.41725\cdot 10^{11}B_{10,20}(X) + 2.10687\cdot 10^{12}B_{11,20}(X) + 3.77349\cdot 10^{12}B_{12,20}(X) + 5.07064\cdot 10^{12}B_{13,20}(X) + 5.51323\cdot 10^{12}B_{14,20}(X) + 4.79225\cdot 10^{12}B_{15,20}(X) + 2.95806\cdot 10^{12}B_{16,20}(X) + 5.02527\cdot 10^{11}B_{17,20}(X) - 1.7341\cdot 10^{12}B_{18,20}(X) - 2.83115\cdot 10^{12}B_{19,20}(X) - 2.1354\cdot 10^{12}B_{20,20}(X)$$



Degree reduction and raising:

$$q_3 = -6.77572\cdot 10^{13}X^{3} + 9.15356\cdot 10^{13}X^{2} - 3.01307\cdot 10^{13}X + 2.32514\cdot 10^{12}$$
$$= 2.32514\cdot 10^{12}B_{0,3} - 7.71842\cdot 10^{12}B_{1,3} + 1.27499\cdot 10^{13}B_{2,3} - 4.0271\cdot 10^{12}B_{3,3}$$

$$\widetilde{q}_3 = -1.33558\cdot 10^{15}X^{20} + 1.34346\cdot 10^{16}X^{19} - 6.25614\cdot 10^{16}X^{18} + 1.78933\cdot 10^{17}X^{17} - 3.51636\cdot 10^{17}X^{16} + 5.0325\cdot 10^{17}X^{15} - 5.42452\cdot 10^{17}X^{14} + 4.49257\cdot 10^{17}X^{13} - 2.89109\cdot 10^{17}X^{12} + 1.45283\cdot 10^{17}X^{11} - 5.70039\cdot 10^{16}X^{10} + 1.73795\cdot 10^{16}X^{9} - 4.07673\cdot 10^{15}X^{8} + 7.2397\cdot 10^{14}X^{7} - 9.50158\cdot 10^{13}X^{6} + 8.91585\cdot 10^{12}X^{5} - 5.74411\cdot 10^{11}X^{4} - 6.77333\cdot 10^{13}X^{3} + 9.1535\cdot 10^{13}X^{2} - 3.01307\cdot 10^{13}X + 2.32514\cdot 10^{12}$$
$$= 2.32514\cdot 10^{12}B_{0,20} + 8.18609\cdot 10^{11}B_{1,20} - 2.06161\cdot 10^{11}B_{2,20} - 8.08584\cdot 10^{11}B_{3,20} - 1.04819\cdot 10^{12}B_{4,20} - 9.84063\cdot 10^{11}B_{5,20} - 6.76693\cdot 10^{11}B_{6,20} - 1.82989\cdot 10^{11}B_{7,20} + 4.32612\cdot 10^{11}B_{8,20} + 1.11897\cdot 10^{12}B_{9,20} + 1.80514\cdot 10^{12}B_{10,20} + 2.44499\cdot 10^{12}B_{11,20} + 2.96617\cdot 10^{12}B_{12,20} + 3.31993\cdot 10^{12}B_{13,20} + 3.4393\cdot 10^{12}B_{14,20} + 3.26931\cdot 10^{12}B_{15,20} + 2.74829\cdot 10^{12}B_{16,20} + 1.81774\cdot 10^{12}B_{17,20} + 4.17913\cdot 10^{11}B_{18,20} - 1.51055\cdot 10^{12}B_{19,20} - 4.0271\cdot 10^{12}B_{20,20}$$



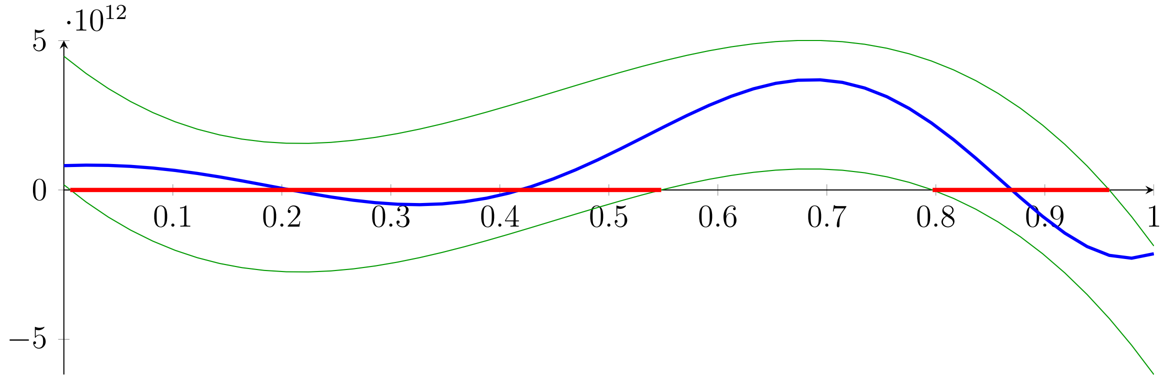
The maximum difference of the Bézier coefficients is $\delta = 2.15202 \cdot 10^{12}$.
Bounding polynomials M and m:

$$M = -6.77572 \cdot 10^{13} X^3 + 9.15356 \cdot 10^{13} X^2 - 3.01307 \cdot 10^{13} X + 4.47716 \cdot 10^{12}$$
$$m = -6.77572 \cdot 10^{13} X^3 + 9.15356 \cdot 10^{13} X^2 - 3.01307 \cdot 10^{13} X + 1.73127 \cdot 10^{11}$$

Root of M and m:

$$N(M) = \{0.959128\} \qquad N(m) = \{0.00584936, 0.54806, 0.797027\}$$

Intersection intervals:



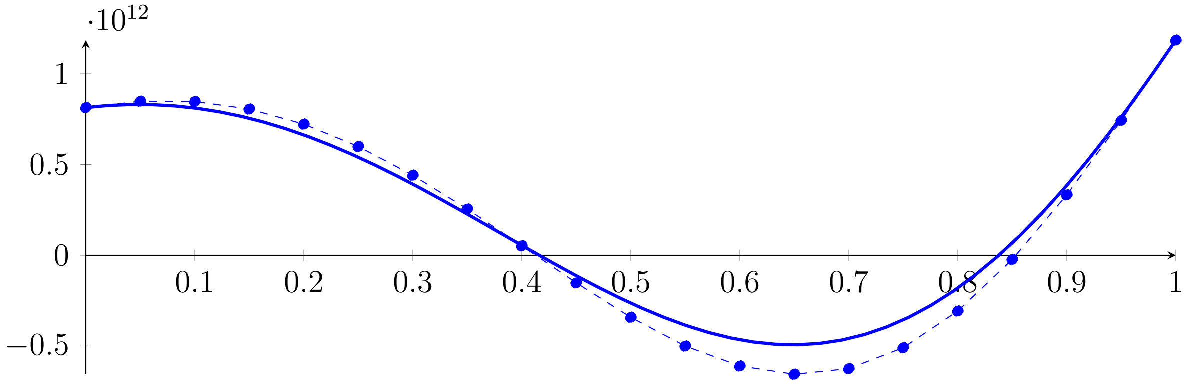
$$[0.00584936, 0.54806], \; [0.797027, 0.959128]$$

Longest intersection interval: 0.54221
$\Longrightarrow$ Bisection: first half $[12.5, 14.0625]$ und second half $[14.0625, 15.625]$

3.50 Recursion Branch 1 2 1 1 1 on the First Half $[12.5, 14.0625]$

Normalized monomial und Bézier representations and the Bézier polygon:

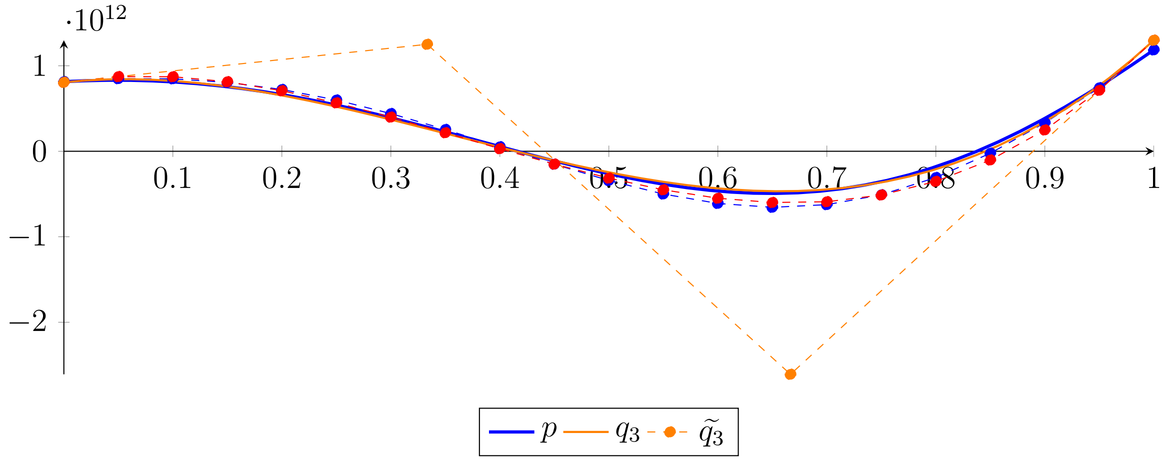
$$\begin{aligned} p = {} & 7635.4X^{20} + 188595X^{19} + 1.31037 \cdot 10^6 X^{18} - 5.65847 \cdot 10^6 X^{17} - 1.0173 \cdot 10^8 X^{16} - 1.7999 \\ & \cdot 10^8 X^{15} + 2.41822 \cdot 10^9 X^{14} + 9.8121 \cdot 10^9 X^{13} - 2.18474 \cdot 10^{10} X^{12} - 1.62107 \cdot 10^{11} X^{11} + 9.80637 \\ & \cdot 10^9 X^{10} + 1.25084 \cdot 10^{12} X^9 + 1.01927 \cdot 10^{12} X^8 - 4.69187 \cdot 10^{12} X^7 - 5.84477 \cdot 10^{12} X^6 + 7.8576 \\ & \cdot 10^{12} X^5 + 1.15394 \cdot 10^{13} X^4 - 4.42579 \cdot 10^{12} X^3 - 6.8634 \cdot 10^{12} X^2 + 6.92135 \cdot 10^{11} X + 8.1419 \cdot 10^{11} \\ = {} & 8.1419 \cdot 10^{11} B_{0,20}(X) + 8.48797 \cdot 10^{11} B_{1,20}(X) + 8.47281 \cdot 10^{11} B_{2,20}(X) + 8.05759 \\ & \cdot 10^{11} B_{3,20}(X) + 7.22731 \cdot 10^{11} B_{4,20}(X) + 5.99585 \cdot 10^{11} B_{5,20}(X) + 4.40954 \cdot 10^{11} B_{6,20}(X) \\ & + 2.54859 \cdot 10^{11} B_{7,20}(X) + 5.25918 \cdot 10^{10} B_{8,20}(X) - 1.51705 \cdot 10^{11} B_{9,20}(X) - 3.41772 \\ & \cdot 10^{11} B_{10,20}(X) - 5.00267 \cdot 10^{11} B_{11,20}(X) - 6.10048 \cdot 10^{11} B_{12,20}(X) - 6.55614 \cdot 10^{11} B_{13,20}(X) \\ & - 6.24598 \cdot 10^{11} B_{14,20}(X) - 5.09162 \cdot 10^{11} B_{15,20}(X) - 3.07139 \cdot 10^{11} B_{16,20}(X) - 2.27736 \\ & \cdot 10^{10} B_{17,20}(X) + 3.33058 \cdot 10^{11} B_{18,20}(X) + 7.43235 \cdot 10^{11} B_{19,20}(X) + 1.1854 \cdot 10^{12} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_3 = 1.20726{\cdot}10^{13}X^3 - 1.29137{\cdot}10^{13}X^2 + 1.33303{\cdot}10^{12}X + 8.06838{\cdot}10^{11}$$
$$= 8.06838{\cdot}10^{11}B_{0,3} + 1.25118{\cdot}10^{12}B_{1,3} - 2.60905{\cdot}10^{12}B_{2,3} + 1.29872{\cdot}10^{12}B_{3,3}$$

$$\widetilde{q}_3 = 1.34941{\cdot}10^{14}X^{20} - 1.36102{\cdot}10^{15}X^{19} + 6.35591{\cdot}10^{15}X^{18} - 1.82321{\cdot}10^{16}X^{17} + 3.59355{\cdot}10^{16}X^{16} - 5.15785{\cdot}10^{16}X^{15} + 5.57468{\cdot}10^{16}X^{14} - 4.62811{\cdot}10^{16}X^{13} + 2.98461{\cdot}10^{16}X^{12} - 1.50275{\cdot}10^{16}X^{11} + 5.90895{\cdot}10^{15}X^{10} - 1.80684{\cdot}10^{15}X^9 + 4.25593{\cdot}10^{14}X^8 - 7.59406{\cdot}10^{13}X^7 + 1.00001{\cdot}10^{13}X^6 - 9.39394{\cdot}10^{11}X^5 + 6.09539{\cdot}10^{10}X^4 + 1.207{\cdot}10^{13}X^3 - 1.29137{\cdot}10^{13}X^2 + 1.33302{\cdot}10^{12}X + 8.06838{\cdot}10^{11}$$
$$= 8.06838{\cdot}10^{11}B_{0,20} + 8.73489{\cdot}10^{11}B_{1,20} + 8.72174{\cdot}10^{11}B_{2,20} + 8.13479{\cdot}10^{11}B_{3,20} + 7.08007{\cdot}10^{11}B_{4,20} + 5.66307{\cdot}10^{11}B_{5,20} + 3.99084{\cdot}10^{11}B_{6,20} + 2.16662{\cdot}10^{11}B_{7,20} + 3.0148{\cdot}10^{10}B_{8,20} - 1.50725{\cdot}10^{11}B_{9,20} - 3.14183{\cdot}10^{11}B_{10,20} - 4.50984{\cdot}10^{11}B_{11,20} - 5.49248{\cdot}10^{11}B_{12,20} - 5.99448{\cdot}10^{11}B_{13,20} - 5.90247{\cdot}10^{11}B_{14,20} - 5.11494{\cdot}10^{11}B_{15,20} - 3.52381{\cdot}10^{11}B_{16,20} - 1.02409{\cdot}10^{11}B_{17,20} + 2.49042{\cdot}10^{11}B_{18,20} + 7.12555{\cdot}10^{11}B_{19,20} + 1.29872{\cdot}10^{12}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.13321{\cdot}10^{11}$.

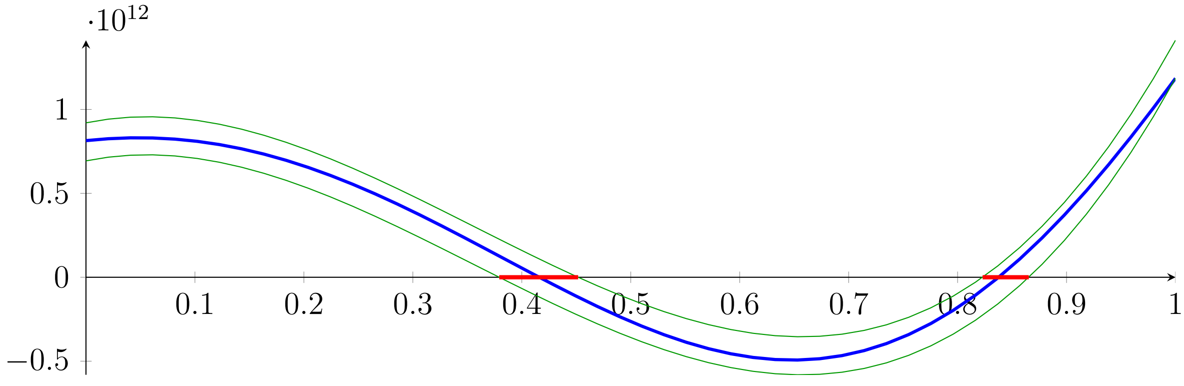
Bounding polynomials M and m:

$$M = 1.20726{\cdot}10^{13}X^3 - 1.29137{\cdot}10^{13}X^2 + 1.33303{\cdot}10^{12}X + 9.20159{\cdot}10^{11}$$
$$m = 1.20726{\cdot}10^{13}X^3 - 1.29137{\cdot}10^{13}X^2 + 1.33303{\cdot}10^{12}X + 6.93517{\cdot}10^{11}$$

Root of M and m:

$$N(M) = \{-0.205022, 0.451731, 0.822965\} \qquad N(m) = \{-0.175008, 0.379315, 0.865366\}$$

Intersection intervals:



$$[0.379315, 0.451731],\ [0.822965, 0.865366]$$

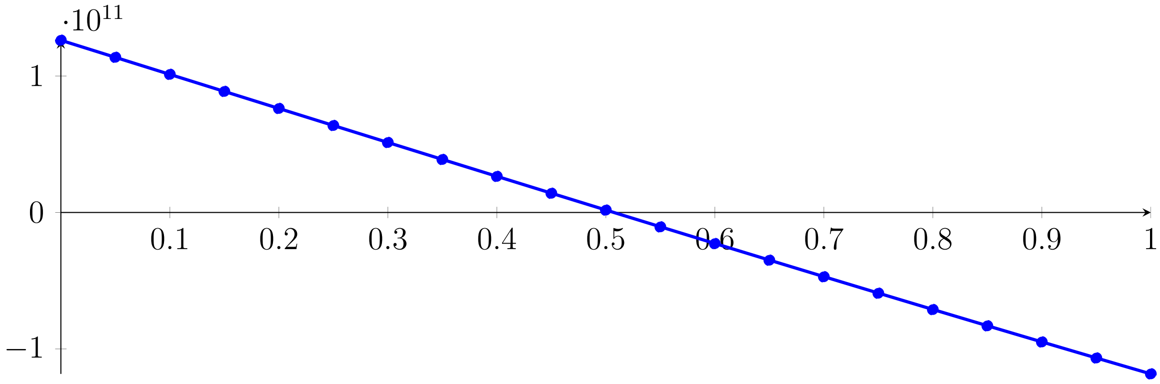
Longest intersection interval: 0.0724162

$\Longrightarrow$ Selective recursion: interval 1: $[13.0927, 13.2058]$, interval 2: $[13.7859, 13.8521]$,

3.51 Recursion Branch 1 2 1 1 1 1 in Interval 1: $[13.0927, 13.2058]$

Normalized monomial und Bézier representations and the Bézier polygon:

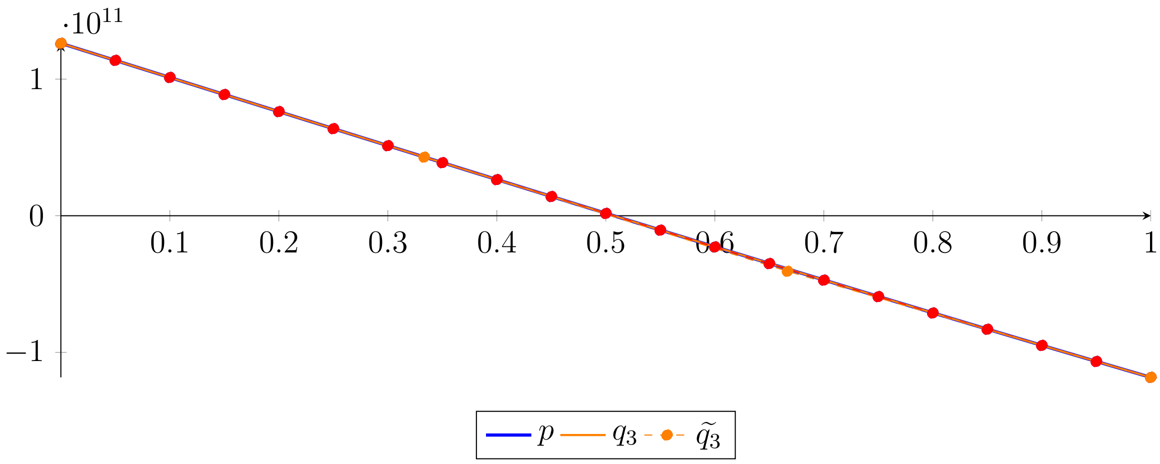
$$\begin{aligned} p = & -14.591X^{20} - 160.02X^{19} - 1014X^{18} + 144.953X^{17} - 14404.7X^{16} + 14367.7X^{15} \\ & - 8694.62X^{14} - 5567.73X^{13} - 24905.3X^{12} - 6773.66X^{11} - 9012.83X^{10} \\ & - 2037.48X^{9} + 2773.66X^{8} + 39239.1X^{7} - 1.30461 \cdot 10^{6}X^{6} - 2.68057 \cdot 10^{7}X^{5} \\ & + 2.06874 \cdot 10^{8}X^{4} + 5.83282 \cdot 10^{9}X^{3} - 5.43785 \cdot 10^{8}X^{2} - 2.4997 \cdot 10^{11}X + 1.26191 \cdot 10^{11} \\ = & \ 1.26191 \cdot 10^{11}B_{0,20}(X) + 1.13693 \cdot 10^{11}B_{1,20}(X) + 1.01191 \cdot 10^{11}B_{2,20}(X) + 8.86923 \\ & \cdot 10^{10}B_{3,20}(X) + 7.62006 \cdot 10^{10}B_{4,20}(X) + 6.37216 \cdot 10^{10}B_{5,20}(X) + 5.12604 \cdot 10^{10}B_{6,20}(X) \\ & + 3.88223 \cdot 10^{10}B_{7,20}(X) + 2.64127 \cdot 10^{10}B_{8,20}(X) + 1.40368 \cdot 10^{10}B_{9,20}(X) + 1.70014 \\ & \cdot 10^{9}B_{10,20}(X) - 1.0592 \cdot 10^{10}B_{11,20}(X) - 2.28341 \cdot 10^{10}B_{12,20}(X) - 3.50207 \cdot 10^{10}B_{13,20}(X) \\ & - 4.71463 \cdot 10^{10}B_{14,20}(X) - 5.92055 \cdot 10^{10}B_{15,20}(X) - 7.11928 \cdot 10^{10}B_{16,20}(X) - 8.31024 \\ & \cdot 10^{10}B_{17,20}(X) - 9.4929 \cdot 10^{10}B_{18,20}(X) - 1.06667 \cdot 10^{11}B_{19,20}(X) - 1.18311 \cdot 10^{11}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = & \ 6.16791 \cdot 10^{9}X^{3} - 7.41911 \cdot 10^{8}X^{2} - 2.49928 \cdot 10^{11}X + 1.26189 \cdot 10^{11} \\ = & \ 1.26189 \cdot 10^{11}B_{0,3} + 4.288 \cdot 10^{10}B_{1,3} - 4.06765 \cdot 10^{10}B_{2,3} - 1.18312 \cdot 10^{11}B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 = & -9.42805 \cdot 10^{12}X^{20} + 9.44968 \cdot 10^{13}X^{19} - 4.38283 \cdot 10^{14}X^{18} + 1.24813 \cdot 10^{15}X^{17} - 2.44192 \cdot 10^{15}X^{16} \\ & + 3.47998 \cdot 10^{15}X^{15} - 3.73745 \cdot 10^{15}X^{14} + 3.08748 \cdot 10^{15}X^{13} - 1.9849 \cdot 10^{15}X^{12} + 9.98221 \cdot 10^{14}X^{11} \\ & - 3.92558 \cdot 10^{14}X^{10} + 1.20026 \cdot 10^{14}X^{9} - 2.82117 \cdot 10^{13}X^{8} + 5.01085 \cdot 10^{12}X^{7} - 6.5667 \cdot 10^{11}X^{6} + 6.1352 \\ & \cdot 10^{10}X^{5} - 3.86873 \cdot 10^{9}X^{4} + 6.321 \cdot 10^{9}X^{3} - 7.45344 \cdot 10^{8}X^{2} - 2.49927 \cdot 10^{11}X + 1.26189 \cdot 10^{11} \\ = & \ 1.26189 \cdot 10^{11}B_{0,20} + 1.13693 \cdot 10^{11}B_{1,20} + 1.01193 \cdot 10^{11}B_{2,20} + 8.86938 \cdot 10^{10}B_{3,20} + 7.62015 \\ & \cdot 10^{10}B_{4,20} + 6.37235 \cdot 10^{10}B_{5,20} + 5.12578 \cdot 10^{10}B_{6,20} + 3.88274 \cdot 10^{10}B_{7,20} + 2.64033 \cdot 10^{10}B_{8,20} \\ & + 1.40472 \cdot 10^{10}B_{9,20} + 1.68604 \cdot 10^{9}B_{10,20} - 1.05815 \cdot 10^{10}B_{11,20} - 2.28429 \cdot 10^{10}B_{12,20} \\ & - 3.50158 \cdot 10^{10}B_{13,20} - 4.71486 \cdot 10^{10}B_{14,20} - 5.92037 \cdot 10^{10}B_{15,20} - 7.11919 \cdot 10^{10}B_{16,20} \\ & - 8.31011 \cdot 10^{10}B_{17,20} - 9.49281 \cdot 10^{10}B_{18,20} - 1.06667 \cdot 10^{11}B_{19,20} - 1.18312 \cdot 10^{11}B_{20,20} \end{aligned}$$



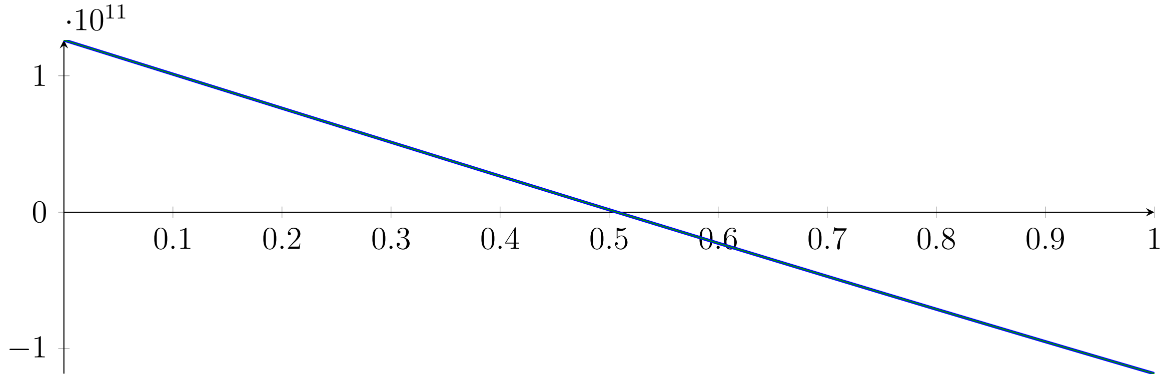
The maximum difference of the Bézier coefficients is $\delta = 1.40968 \cdot 10^7$.
Bounding polynomials M and m:

$$M = 6.16791 \cdot 10^9 X^3 - 7.41911 \cdot 10^8 X^2 - 2.49928 \cdot 10^{11} X + 1.26203 \cdot 10^{11}$$
$$m = 6.16791 \cdot 10^9 X^3 - 7.41911 \cdot 10^8 X^2 - 2.49928 \cdot 10^{11} X + 1.26175 \cdot 10^{11}$$

Root of M and m:

$$N(M) = \{-6.54665, 0.507419, 6.15951\} \qquad N(m) = \{-6.5466, 0.507305, 6.15958\}$$

Intersection intervals:



$$[0.507305, 0.507419]$$

Longest intersection interval: 0.000114647
$\Longrightarrow$ Selective recursion: interval 1: $[13.1501, 13.1501]$,

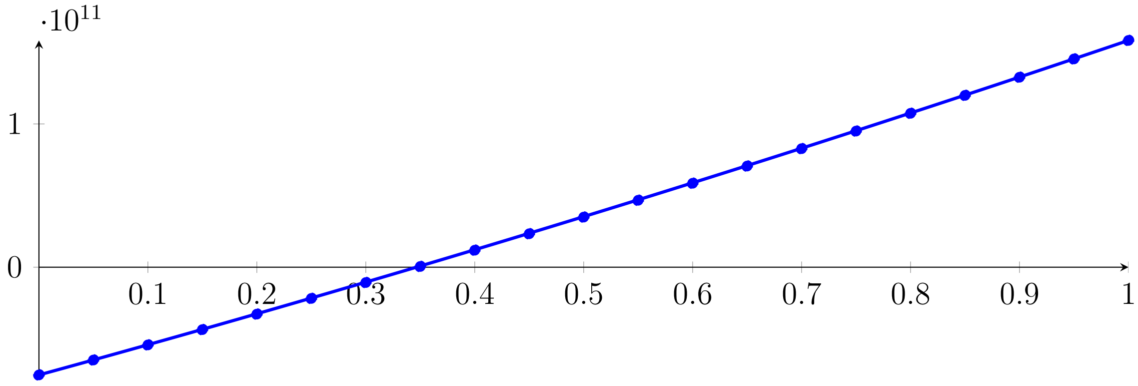
3.52 Recursion Branch 1 2 1 1 1 1 1 in Interval 1: $[13.1501, 13.1501]$

Found root in interval $[13.1501, 13.1501]$ at recursion depth 7!

3.53 Recursion Branch 1 2 1 1 1 2 in Interval 2: $[13.7859, 13.8521]$

Normalized monomial und Bézier representations and the Bézier polygon:

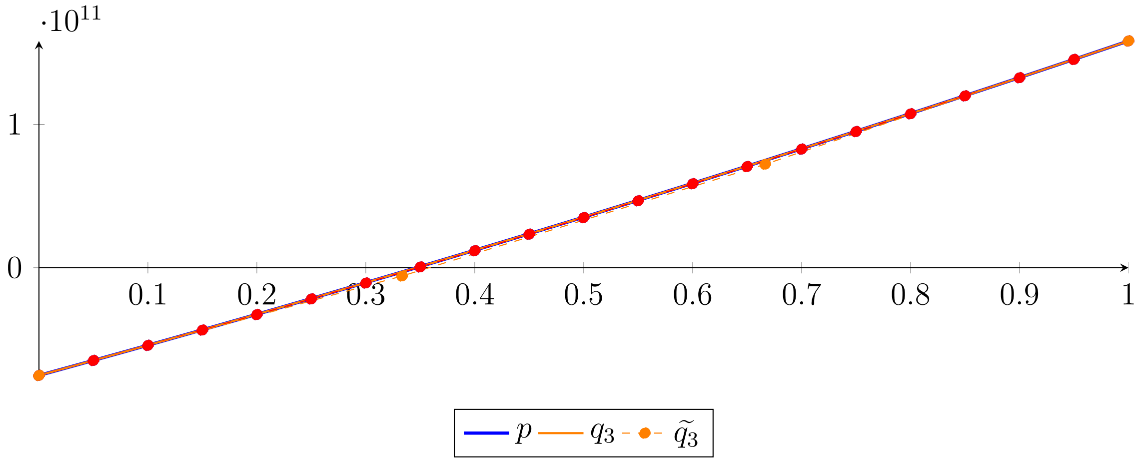
$$\begin{aligned} p = & -21.9015 X^{20} + 348.348 X^{19} - 194.946 X^{18} + 4216.17 X^{17} - 10616.3 X^{16} \\ & + 5532.48 X^{15} + 1001.88 X^{14} + 3002.1 X^{13} + 4732.33 X^{12} + 4454.26 X^{11} + 3219.47 X^{10} \\ & + 968.763 X^9 + 11.5329 X^8 + 2226.15 X^7 + 98682.6 X^6 - 962112 X^5 - 9.24854 \\ & \cdot 10^7 X^4 - 3.95032 \cdot 10^8 X^3 + 2.57457 \cdot 10^{10} X^2 + 2.08202 \cdot 10^{11} X - 7.5184 \cdot 10^{10} \\ = & -7.5184 \cdot 10^{10} B_{0,20}(X) - 6.47739 \cdot 10^{10} B_{1,20}(X) - 5.42284 \cdot 10^{10} B_{2,20}(X) - 4.35476 \\ & \cdot 10^{10} B_{3,20}(X) - 3.27321 \cdot 10^{10} B_{4,20}(X) - 2.17821 \cdot 10^{10} B_{5,20}(X) - 1.06982 \cdot 10^{10} B_{6,20}(X) \\ & + 5.1933 \cdot 10^8 B_{7,20}(X) + 1.187 \cdot 10^{10} B_{8,20}(X) + 2.33533 \cdot 10^{10} B_{9,20}(X) + 3.49689 \\ & \cdot 10^{10} B_{10,20}(X) + 4.67161 \cdot 10^{10} B_{11,20}(X) + 5.85945 \cdot 10^{10} B_{12,20}(X) + 7.06035 \cdot 10^{10} B_{13,20}(X) \\ & + 8.27426 \cdot 10^{10} B_{14,20}(X) + 9.50112 \cdot 10^{10} B_{15,20}(X) + 1.07409 \cdot 10^{11} B_{16,20}(X) + 1.19934 \\ & \cdot 10^{11} B_{17,20}(X) + 1.32588 \cdot 10^{11} B_{18,20}(X) + 1.45368 \cdot 10^{11} B_{19,20}(X) + 1.58275 \cdot 10^{11} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_3 = -5.82339\cdot 10^{8}X^{3} + 2.58666\cdot 10^{10}X^{2} + 2.08175\cdot 10^{11}X - 7.51827\cdot 10^{10}$$
$$= -7.51827\cdot 10^{10}B_{0,3} - 5.7911\cdot 10^{9}B_{1,3} + 7.22227\cdot 10^{10}B_{2,3} + 1.58276\cdot 10^{11}B_{3,3}$$

$$\widetilde{q}_3 = 2.45235\cdot 10^{12}X^{20} - 2.45568\cdot 10^{13}X^{19} + 1.1358\cdot 10^{14}X^{18} - 3.22069\cdot 10^{14}X^{17} + 6.26883\cdot 10^{14}X^{16} - 8.88908\cdot 10^{14}X^{15} + 9.51368\cdot 10^{14}X^{14} - 7.85792\cdot 10^{14}X^{13} + 5.07747\cdot 10^{14}X^{12} - 2.58439\cdot 10^{14}X^{11} + 1.03672\cdot 10^{14}X^{10} - 3.25607\cdot 10^{13}X^{9} + 7.88347\cdot 10^{12}X^{8} - 1.43112\cdot 10^{12}X^{7} + 1.86095\cdot 10^{11}X^{6} - 1.60344\cdot 10^{10}X^{5} + 7.59839\cdot 10^{8}X^{4} - 5.92077\cdot 10^{8}X^{3} + 2.58663\cdot 10^{10}X^{2} + 2.08175\cdot 10^{11}X - 7.51827\cdot 10^{10}$$
$$= -7.51827\cdot 10^{10}B_{0,20} - 6.47739\cdot 10^{10}B_{1,20} - 5.42291\cdot 10^{10}B_{2,20} - 4.35486\cdot 10^{10}B_{3,20} - 3.27328\cdot 10^{10}B_{4,20} - 2.17831\cdot 10^{10}B_{5,20} - 1.06976\cdot 10^{10}B_{6,20} + 5.18126\cdot 10^{8}B_{7,20} + 1.18724\cdot 10^{10}B_{8,20} + 2.33508\cdot 10^{10}B_{9,20} + 3.49725\cdot 10^{10}B_{10,20} + 4.67127\cdot 10^{10}B_{11,20} + 5.85965\cdot 10^{10}B_{12,20} + 7.06023\cdot 10^{10}B_{13,20} + 8.27429\cdot 10^{10}B_{14,20} + 9.50105\cdot 10^{10}B_{15,20} + 1.07408\cdot 10^{11}B_{16,20} + 1.19933\cdot 10^{11}B_{17,20} + 1.32587\cdot 10^{11}B_{18,20} + 1.45368\cdot 10^{11}B_{19,20} + 1.58276\cdot 10^{11}B_{20,20}$$



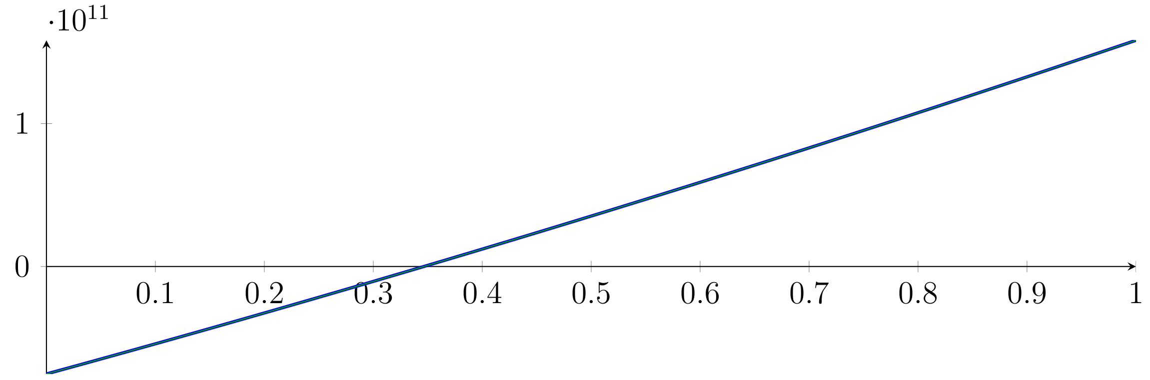
The maximum difference of the Bézier coefficients is $\delta = 3.67522\cdot 10^{6}$.
Bounding polynomials M and m:

$$M = -5.82339\cdot 10^{8}X^{3} + 2.58666\cdot 10^{10}X^{2} + 2.08175\cdot 10^{11}X - 7.5179\cdot 10^{10}$$
$$m = -5.82339\cdot 10^{8}X^{3} + 2.58666\cdot 10^{10}X^{2} + 2.08175\cdot 10^{11}X - 7.51864\cdot 10^{10}$$

Root of M and m:

$$N(M) = \{-7.26126, 0.346345, 51.3333\} \qquad N(m) = \{-7.26129, 0.346378, 51.3333\}$$

Intersection intervals:



$[0.346345, 0.346378]$

Longest intersection interval: $3.25409 \cdot 10^{-05}$
$\Longrightarrow$ Selective recursion: interval 1: $[13.8088, 13.8088]$,

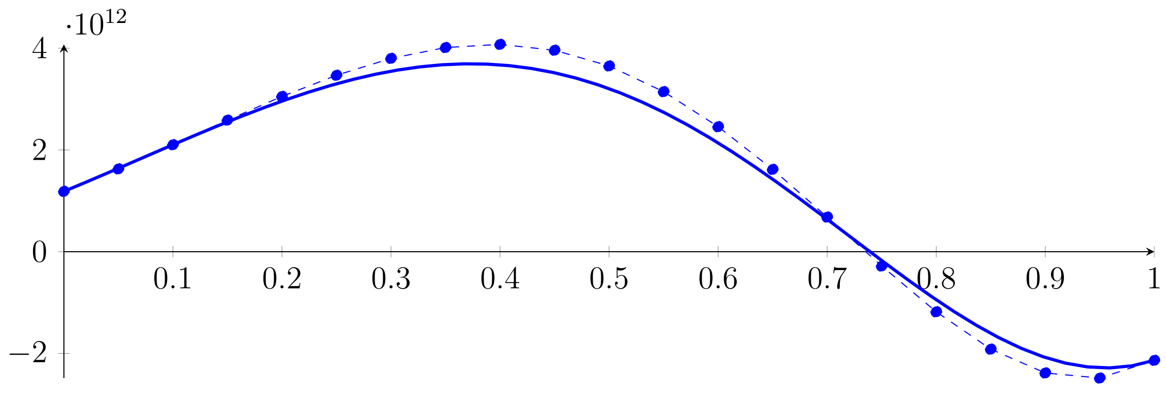
3.54 Recursion Branch 1 2 1 1 1 2 1 in Interval 1: $[13.8088, 13.8088]$

Found root in interval $[13.8088, 13.8088]$ at recursion depth 7!

3.55 Recursion Branch 1 2 1 1 2 on the Second Half $[14.0625, 15.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

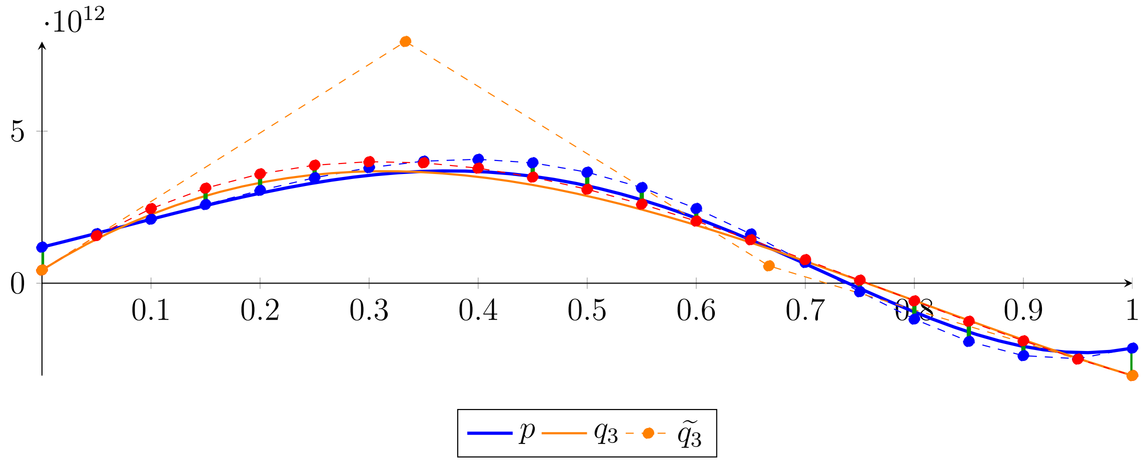
$$\begin{aligned} p &= 3986.15X^{20} + 355982X^{19} + 6.295\cdot 10^{6}X^{18} + 6.00178\cdot 10^{7}X^{17} + 2.24902\cdot 10^{8}X^{16} - 6.32265 \\ &\quad \cdot 10^{8}X^{15} - 9.75189\cdot 10^{9}X^{14} - 2.84929\cdot 10^{10}X^{13} + 5.90133\cdot 10^{10}X^{12} + 5.19357\cdot 10^{11}X^{11} + 6.2382 \\ &\quad \cdot 10^{11}X^{10} - 2.63478\cdot 10^{12}X^{9} - 7.48493\cdot 10^{12}X^{8} + 1.62878\cdot 10^{12}X^{7} + 2.42459\cdot 10^{13}X^{6} + 1.56831 \\ &\quad \cdot 10^{13}X^{5} - 2.53581\cdot 10^{13}X^{4} - 2.54855\cdot 10^{13}X^{3} + 6.07786\cdot 10^{12}X^{2} + 8.8433\cdot 10^{12}X + 1.1854\cdot 10^{12} \\ &= 1.1854\cdot 10^{12}B_{0,20}(X) + 1.62756\cdot 10^{12}B_{1,20}(X) + 2.10172\cdot 10^{12}B_{2,20}(X) + 2.58551 \\ &\quad \cdot 10^{12}B_{3,20}(X) + 3.05134\cdot 10^{12}B_{4,20}(X) + 3.4674\cdot 10^{12}B_{5,20}(X) + 3.79929\cdot 10^{12}B_{6,20}(X) \\ &\quad + 4.01233\cdot 10^{12}B_{7,20}(X) + 4.07439\cdot 10^{12}B_{8,20}(X) + 3.95934\cdot 10^{12}B_{9,20}(X) + 3.65071 \\ &\quad \cdot 10^{12}B_{10,20}(X) + 3.14537\cdot 10^{12}B_{11,20}(X) + 2.4568\cdot 10^{12}B_{12,20}(X) + 1.61759\cdot 10^{12}B_{13,20}(X) \\ &\quad + 6.80535\cdot 10^{11}B_{14,20}(X) - 2.82012\cdot 10^{11}B_{15,20}(X) - 1.18103\cdot 10^{12}B_{16,20}(X) - 1.91608 \\ &\quad \cdot 10^{12}B_{17,20}(X) - 2.38295\cdot 10^{12}B_{18,20}(X) - 2.48328\cdot 10^{12}B_{19,20}(X) - 2.1354\cdot 10^{12}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 1.86596\cdot 10^{13}X^{3} - 4.46732\cdot 10^{13}X^{2} + 2.25434\cdot 10^{13}X + 4.32321\cdot 10^{11} \\ &= 4.32321\cdot 10^{11}B_{0,3} + 7.94677\cdot 10^{12}B_{1,3} + 5.70165\cdot 10^{11}B_{2,3} - 3.03792\cdot 10^{12}B_{3,3} \end{aligned}$$

$$\begin{aligned}
\widetilde{q}_3 &= 4.71521\cdot 10^{13}X^{20} - 4.83541\cdot 10^{14}X^{19} + 2.28704\cdot 10^{15}X^{18} - 6.62238\cdot 10^{15}X^{17} + 1.31403\cdot 10^{16}X^{16} \\
&\quad - 1.89466\cdot 10^{16}X^{15} + 2.05384\cdot 10^{16}X^{14} - 1.70811\cdot 10^{16}X^{13} + 1.10278\cdot 10^{16}X^{12} - 5.56299\cdot 10^{15}X^{11} \\
&\quad + 2.20025\cdot 10^{15}X^{10} - 6.82605\cdot 10^{14}X^{9} + 1.64586\cdot 10^{14}X^{8} - 2.97015\cdot 10^{13}X^{7} + 3.63193\cdot 10^{12}X^{6} - 2.3036 \\
&\quad \cdot 10^{11}X^{5} - 3.56116\cdot 10^{9}X^{4} + 1.86612\cdot 10^{13}X^{3} - 4.46732\cdot 10^{13}X^{2} + 2.25434\cdot 10^{13}X + 4.32321\cdot 10^{11} \\
&= 4.32321\cdot 10^{11}B_{0,20} + 1.55949\cdot 10^{12}B_{1,20} + 2.45153\cdot 10^{12}B_{2,20} + 3.12483\cdot 10^{12}B_{3,20} + 3.59573 \\
&\quad \cdot 10^{12}B_{4,20} + 3.88061\cdot 10^{12}B_{5,20} + 3.99587\cdot 10^{12}B_{6,20} + 3.95779\cdot 10^{12}B_{7,20} + 3.78291\cdot 10^{12}B_{8,20} \\
&\quad + 3.48727\cdot 10^{12}B_{9,20} + 3.08769\cdot 10^{12}B_{10,20} + 2.60007\cdot 10^{12}B_{11,20} + 2.04126\cdot 10^{12}B_{12,20} \\
&\quad + 1.42723\cdot 10^{12}B_{13,20} + 7.74536\cdot 10^{11}B_{14,20} + 9.94961\cdot 10^{10}B_{15,20} - 5.81526\cdot 10^{11}B_{16,20} \\
&\quad - 1.25214\cdot 10^{12}B_{17,20} - 1.89599\cdot 10^{12}B_{18,20} - 2.49671\cdot 10^{12}B_{19,20} - 3.03792\cdot 10^{12}B_{20,20}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 9.02519\cdot 10^{11}$.

Bounding polynomials M and m:

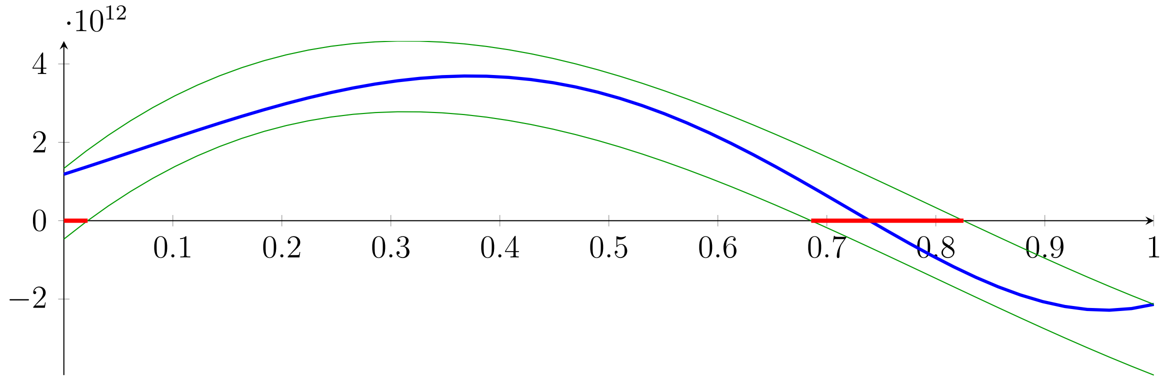
$$M = 1.86596\cdot 10^{13}X^3 - 4.46732\cdot 10^{13}X^2 + 2.25434\cdot 10^{13}X + 1.33484\cdot 10^{12}$$

$$m = 1.86596\cdot 10^{13}X^3 - 4.46732\cdot 10^{13}X^2 + 2.25434\cdot 10^{13}X - 4.70198\cdot 10^{11}$$

Root of M and m:

$$N(M) = \{-0.0534289, 0.825388, 1.62216\} \qquad N(m) = \{0.0217898, 0.685627, 1.6867\}$$

Intersection intervals:



$$[0, 0.0217898],\ [0.685627, 0.825388]$$

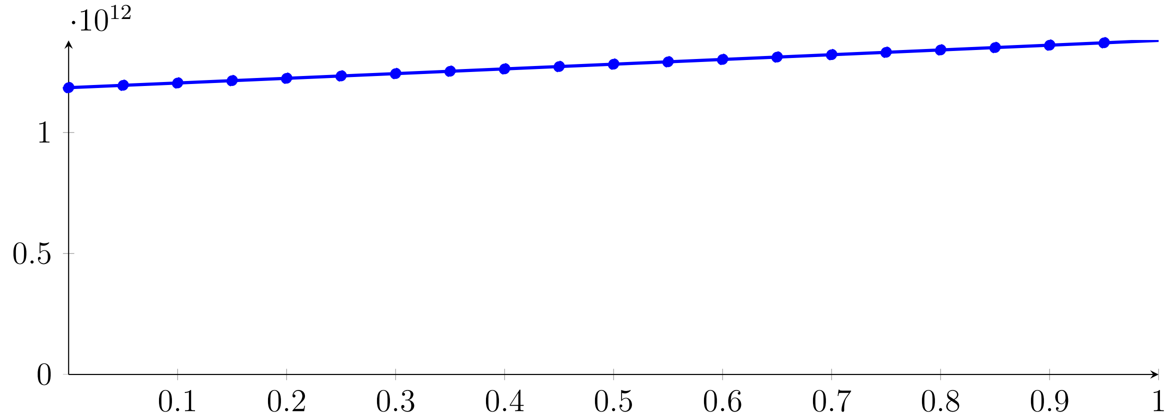
Longest intersection interval: 0.139761

$\Longrightarrow$ Selective recursion: interval 1: $[14.0625, 14.0965]$, interval 2: $[15.1338, 15.3522]$,

3.56 Recursion Branch 1 2 1 1 2 1 in Interval 1: $[14.0625, 14.0965]$

Normalized monomial und Bézier representations and the Bézier polygon:

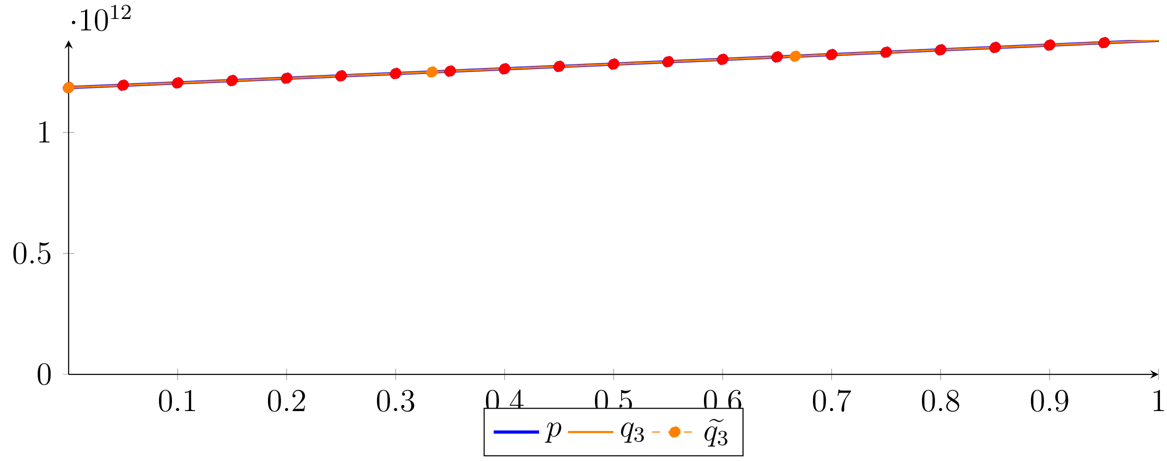
$$p = -1227.06X^{20} + 6812.28X^{19} - 36770.3X^{18} + 153843X^{17} - 820397X^{16} + 644574X^{15} - 263996X^{14} - 67943.6X^{13} - 644428X^{12} - 64215.2X^{11} - 193507X^{10} - 12055.7X^{9} - 4490.14X^{8} + 492.07X^{7} + 1646.54X^{6} + 77073.4X^{5} - 5.71655\cdot 10^{6}X^{4} - 2.63666\cdot 10^{8}X^{3} + 2.88575\cdot 10^{9}X^{2} + 1.92694\cdot 10^{11}X + 1.1854\cdot 10^{12}$$
$$= 1.1854\cdot 10^{12}B_{0,20}(X) + 1.19503\cdot 10^{12}B_{1,20}(X) + 1.20468\cdot 10^{12}B_{2,20}(X) + 1.21435\cdot 10^{12}B_{3,20}(X) + 1.22403\cdot 10^{12}B_{4,20}(X) + 1.23372\cdot 10^{12}B_{5,20}(X) + 1.24343\cdot 10^{12}B_{6,20}(X) + 1.25315\cdot 10^{12}B_{7,20}(X) + 1.26289\cdot 10^{12}B_{8,20}(X) + 1.27264\cdot 10^{12}B_{9,20}(X) + 1.2824\cdot 10^{12}B_{10,20}(X) + 1.29218\cdot 10^{12}B_{11,20}(X) + 1.30197\cdot 10^{12}B_{12,20}(X) + 1.31177\cdot 10^{12}B_{13,20}(X) + 1.32158\cdot 10^{12}B_{14,20}(X) + 1.33141\cdot 10^{12}B_{15,20}(X) + 1.34125\cdot 10^{12}B_{16,20}(X) + 1.3511\cdot 10^{12}B_{17,20}(X) + 1.36096\cdot 10^{12}B_{18,20}(X) + 1.37083\cdot 10^{12}B_{19,20}(X) + 1.38071\cdot 10^{12}B_{20,20}(X)$$



Degree reduction and raising:

$$q_3 = -2.74877\cdot 10^{8}X^{3} + 2.89291\cdot 10^{9}X^{2} + 1.92692\cdot 10^{11}X + 1.1854\cdot 10^{12}$$
$$= 1.1854\cdot 10^{12}B_{0,3} + 1.24963\cdot 10^{12}B_{1,3} + 1.31483\cdot 10^{12}B_{2,3} + 1.38071\cdot 10^{12}B_{3,3}$$

$$\widetilde{q}_3 = -1.98539\cdot 10^{14}X^{20} + 1.99086\cdot 10^{15}X^{19} - 9.24508\cdot 10^{15}X^{18} + 2.63763\cdot 10^{16}X^{17} - 5.17175\cdot 10^{16}X^{16} + 7.38603\cdot 10^{16}X^{15} - 7.94464\cdot 10^{16}X^{14} + 6.56464\cdot 10^{16}X^{13} - 4.21277\cdot 10^{16}X^{12} + 2.10904\cdot 10^{16}X^{11} - 8.22973\cdot 10^{15}X^{10} + 2.48894\cdot 10^{15}X^{9} - 5.77735\cdot 10^{14}X^{8} + 1.01657\cdot 10^{14}X^{7} - 1.3381\cdot 10^{13}X^{6} + 1.29699\cdot 10^{12}X^{5} - 9.04565\cdot 10^{10}X^{4} + 4.00796\cdot 10^{9}X^{3} + 2.77965\cdot 10^{9}X^{2} + 1.92694\cdot 10^{11}X + 1.1854\cdot 10^{12}$$
$$= 1.1854\cdot 10^{12}B_{0,20} + 1.19503\cdot 10^{12}B_{1,20} + 1.20468\cdot 10^{12}B_{2,20} + 1.21435\cdot 10^{12}B_{3,20} + 1.22402\cdot 10^{12}B_{4,20} + 1.23374\cdot 10^{12}B_{5,20} + 1.24337\cdot 10^{12}B_{6,20} + 1.25327\cdot 10^{12}B_{7,20} + 1.2627\cdot 10^{12}B_{8,20} + 1.27287\cdot 10^{12}B_{9,20} + 1.28211\cdot 10^{12}B_{10,20} + 1.29238\cdot 10^{12}B_{11,20} + 1.30177\cdot 10^{12}B_{12,20} + 1.31187\cdot 10^{12}B_{13,20} + 1.32152\cdot 10^{12}B_{14,20} + 1.33143\cdot 10^{12}B_{15,20} + 1.34124\cdot 10^{12}B_{16,20} + 1.3511\cdot 10^{12}B_{17,20} + 1.36096\cdot 10^{12}B_{18,20} + 1.37083\cdot 10^{12}B_{19,20} + 1.38071\cdot 10^{12}B_{20,20}$$



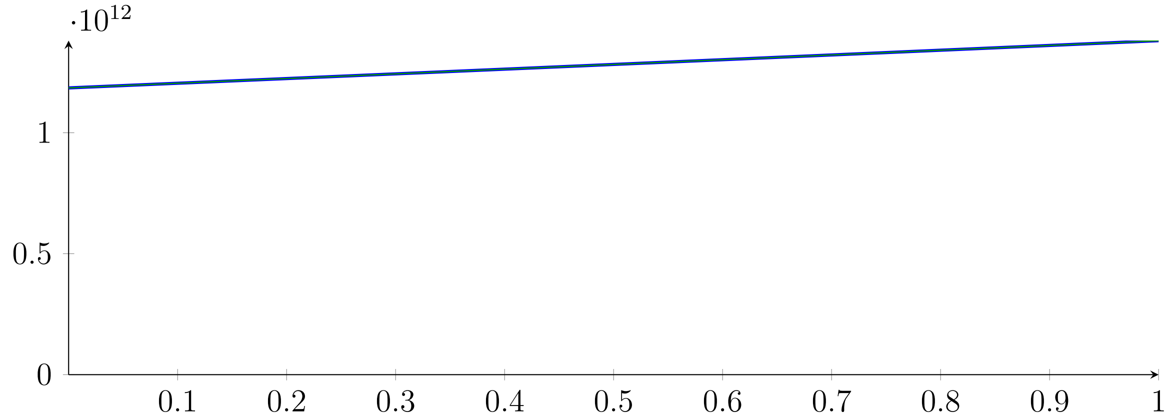
The maximum difference of the Bézier coefficients is $\delta = 2.89524{\cdot}10^{8}$.
Bounding polynomials M and m:

$$M = -2.74877{\cdot}10^{8}X^3 + 2.89291{\cdot}10^{9}X^2 + 1.92692{\cdot}10^{11}X + 1.18569{\cdot}10^{12}$$
$$m = -2.74877{\cdot}10^{8}X^3 + 2.89291{\cdot}10^{9}X^2 + 1.92692{\cdot}10^{11}X + 1.18511{\cdot}10^{12}$$

Root of M and m:

$$N(M) = \{-16.268, -7.68961, 34.482\} \qquad N(m) = \{-16.2729, -7.68379, 34.481\}$$

Intersection intervals:

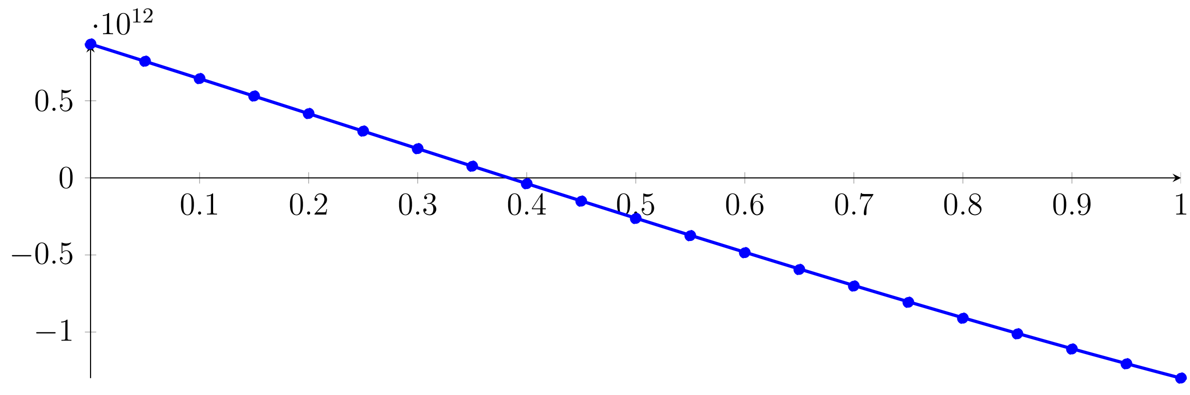


No intersection intervals with the x axis.

3.57 Recursion Branch 1 2 1 1 2 2 in Interval 2: $[15.1338, 15.3522]$

Normalized monomial und Bézier representations and the Bézier polygon:

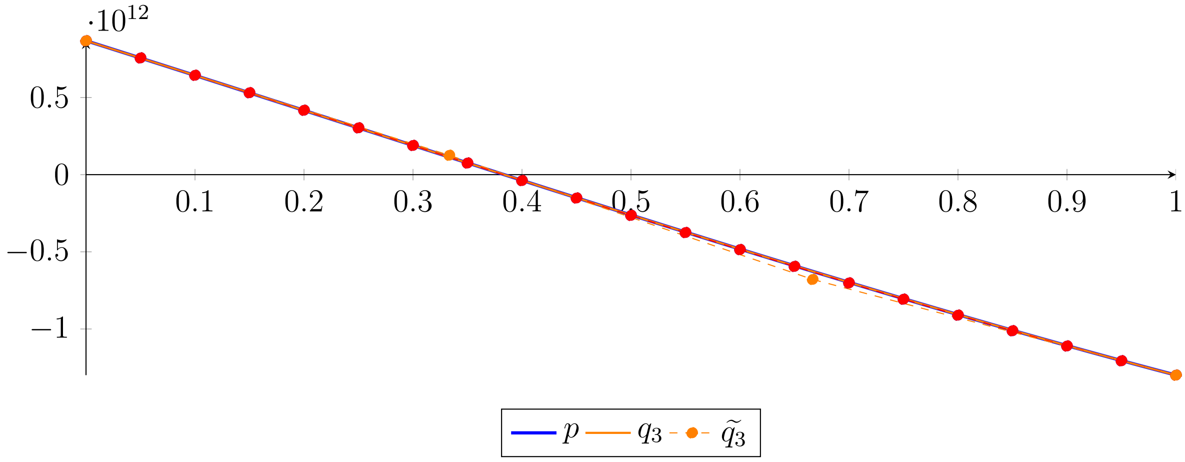
$$\begin{aligned}p &= 127.706X^{20} - 2813.3X^{19} - 404.701X^{18} - 30337.9X^{17} + 48880X^{16} - 8522.28X^{15}\\&\quad - 31383.7X^{14} - 33527X^{13} - 91171.4X^{12} - 47382.3X^{11} - 42783.5X^{10} + 227193X^{9}\\&\quad + 2.16017{\cdot}10^{6}X^{8} - 2.34126{\cdot}10^{7}X^{7} - 5.35002{\cdot}10^{8}X^{6} - 1.45838{\cdot}10^{9}X^{5} + 2.97571\\&\quad {\cdot}10^{10}X^{4} + 1.93256{\cdot}10^{11}X^{3} - 1.54765{\cdot}10^{11}X^{2} - 2.23317{\cdot}10^{12}X + 8.67978{\cdot}10^{11}\\&= 8.67978{\cdot}10^{11}B_{0,20}(X) + 7.56319{\cdot}10^{11}B_{1,20}(X) + 6.43846{\cdot}10^{11}B_{2,20}(X) + 5.30728\\&\quad {\cdot}10^{11}B_{3,20}(X) + 4.1714{\cdot}10^{11}B_{4,20}(X) + 3.03265{\cdot}10^{11}B_{5,20}(X) + 1.89289{\cdot}10^{11}B_{6,20}(X)\\&\quad + 7.54073{\cdot}10^{10}B_{7,20}(X) - 3.81819{\cdot}10^{10}B_{8,20}(X) - 1.51274{\cdot}10^{11}B_{9,20}(X) - 2.63658\\&\quad {\cdot}10^{11}B_{10,20}(X) - 3.7512{\cdot}10^{11}B_{11,20}(X) - 4.8544{\cdot}10^{11}B_{12,20}(X) - 5.94391{\cdot}10^{11}B_{13,20}(X)\\&\quad - 7.01745{\cdot}10^{11}B_{14,20}(X) - 8.07268{\cdot}10^{11}B_{15,20}(X) - 9.10722{\cdot}10^{11}B_{16,20}(X) - 1.01186\\&\quad {\cdot}10^{12}B_{17,20}(X) - 1.11045{\cdot}10^{12}B_{18,20}(X) - 1.20624{\cdot}10^{12}B_{19,20}(X) - 1.29896{\cdot}10^{12}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$q_3 = 2.46859\cdot 10^{11}X^3 - 1.87753\cdot 10^{11}X^2 - 2.22602\cdot 10^{12}X + 8.67626\cdot 10^{11}$$
$$= 8.67626\cdot 10^{11}B_{0,3} + 1.25619\cdot 10^{11}B_{1,3} - 6.78973\cdot 10^{11}B_{2,3} - 1.29929\cdot 10^{12}B_{3,3}$$

$$\widetilde{q}_3 = -3.83204\cdot 10^{13}X^{20} + 3.83753\cdot 10^{14}X^{19} - 1.77677\cdot 10^{15}X^{18} + 5.04741\cdot 10^{15}X^{17} - 9.8468\cdot 10^{15}X^{16} + 1.39933\cdot 10^{16}X^{15} - 1.4997\cdot 10^{16}X^{14} + 1.23816\cdot 10^{16}X^{13} - 7.97429\cdot 10^{15}X^{12} + 4.03022\cdot 10^{15}X^{11} - 1.59846\cdot 10^{15}X^{10} + 4.94494\cdot 10^{14}X^9 - 1.17752\cdot 10^{14}X^8 + 2.11106\cdot 10^{13}X^7 - 2.75302\cdot 10^{12}X^6 + 2.47153\cdot 10^{11}X^5 - 1.37104\cdot 10^{10}X^4 + 2.47249\cdot 10^{11}X^3 - 1.87758\cdot 10^{11}X^2 - 2.22602\cdot 10^{12}X + 8.67626\cdot 10^{11}$$
$$= 8.67626\cdot 10^{11}B_{0,20} + 7.56325\cdot 10^{11}B_{1,20} + 6.44035\cdot 10^{11}B_{2,20} + 5.30975\cdot 10^{11}B_{3,20} + 4.17357\cdot 10^{11}B_{4,20} + 3.03409\cdot 10^{11}B_{5,20} + 1.89316\cdot 10^{11}B_{6,20} + 7.5368\cdot 10^{10}B_{7,20} - 3.8357\cdot 10^{10}B_{8,20} - 1.5142\cdot 10^{11}B_{9,20} - 2.63916\cdot 10^{11}B_{10,20} - 3.75251\cdot 10^{11}B_{11,20} - 4.85595\cdot 10^{11}B_{12,20} - 5.94413\cdot 10^{11}B_{13,20} - 7.017\cdot 10^{11}B_{14,20} - 8.07119\cdot 10^{11}B_{15,20} - 9.10509\cdot 10^{11}B_{16,20} - 1.01163\cdot 10^{12}B_{17,20} - 1.11029\cdot 10^{12}B_{18,20} - 1.20624\cdot 10^{12}B_{19,20} - 1.29929\cdot 10^{12}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 3.52081\cdot 10^8$.

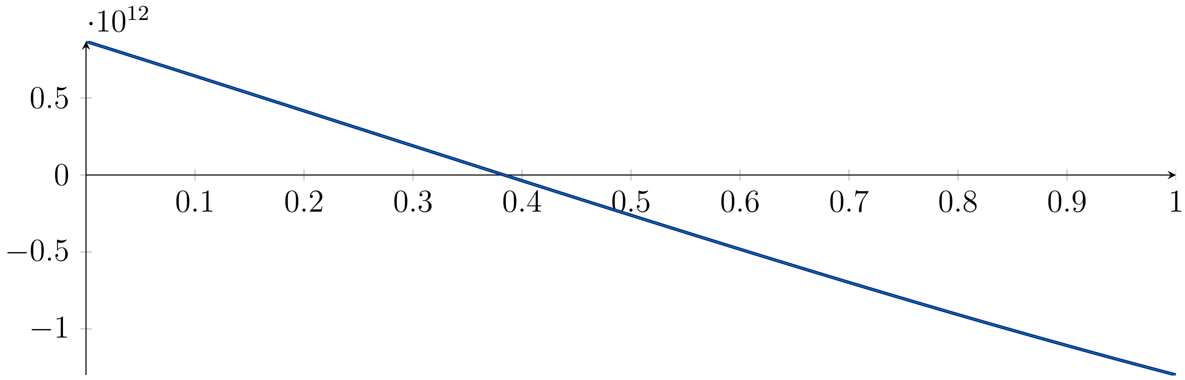
Bounding polynomials M and m:

$$M = 2.46859\cdot 10^{11}X^3 - 1.87753\cdot 10^{11}X^2 - 2.22602\cdot 10^{12}X + 8.67978\cdot 10^{11}$$
$$m = 2.46859\cdot 10^{11}X^3 - 1.87753\cdot 10^{11}X^2 - 2.22602\cdot 10^{12}X + 8.67274\cdot 10^{11}$$

Root of M and m:

$$N(M) = \{-2.84434, 0.383769, 3.22114\} \qquad N(m) = \{-2.84419, 0.383458, 3.2213\}$$

Intersection intervals:



$[0.383458, 0.383769]$

Longest intersection interval: 0.000311426

$\Longrightarrow$ Selective recursion: interval 1: $[15.2175, 15.2176]$,

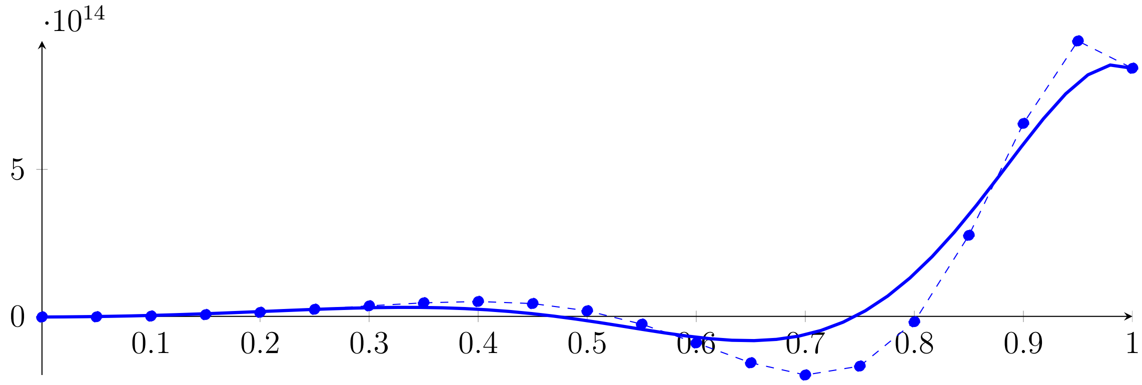
3.58 Recursion Branch 1 2 1 1 2 2 1 in Interval 1: $[15.2175, 15.2176]$

Found root in interval $[15.2175, 15.2176]$ at recursion depth 7!

3.59 Recursion Branch 1 2 1 2 on the Second Half $[15.625, 18.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

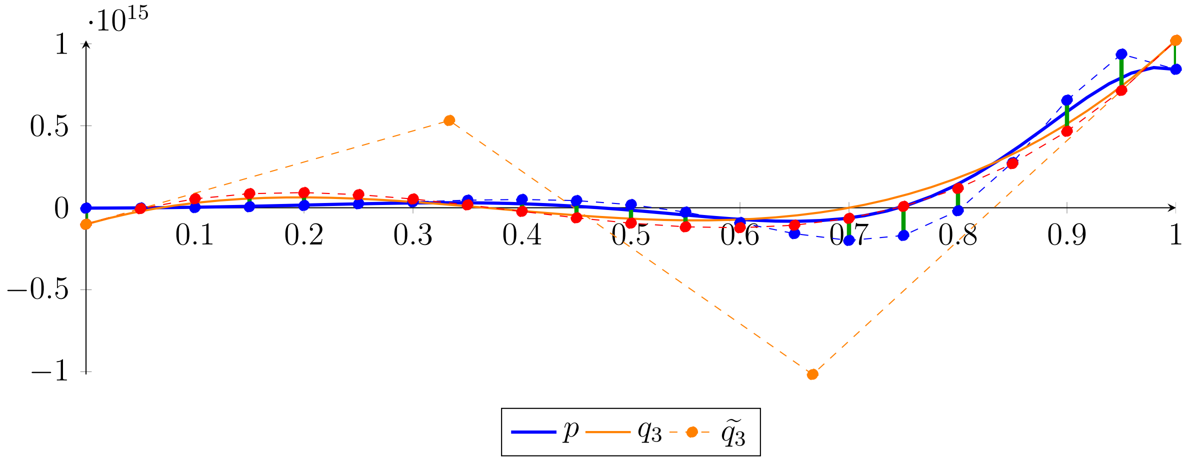
$$\begin{aligned}p &= 7.88858\cdot 10^{9}X^{20} + 2.58746\cdot 10^{11}X^{19} + 3.76268\cdot 10^{12}X^{18} + 3.17389\cdot 10^{13}X^{17} + 1.69708\cdot 10^{14}X^{16} \\ &\quad + 5.82695\cdot 10^{14}X^{15} + 1.18664\cdot 10^{15}X^{14} + 8.38279\cdot 10^{14}X^{13} - 2.32497\cdot 10^{15}X^{12} - 7.06233\cdot 10^{15}X^{11} \\ &\quad - 6.4407\cdot 10^{15}X^{10} + 3.31615\cdot 10^{15}X^{9} + 1.18856\cdot 10^{16}X^{8} + 7.3503\cdot 10^{15}X^{7} - 3.10022\cdot 10^{15}X^{6} - 5.3941 \\ &\quad \cdot 10^{15}X^{5} - 1.29591\cdot 10^{15}X^{4} + 7.44661\cdot 10^{14}X^{3} + 3.40631\cdot 10^{14}X^{2} + 1.3915\cdot 10^{13}X - 2.1354\cdot 10^{12} \\ &= -2.1354\cdot 10^{12}B_{0,20}(X) - 1.43965\cdot 10^{12}B_{1,20}(X) + 1.04889\cdot 10^{12}B_{2,20}(X) + 5.98344 \\ &\quad \cdot 10^{12}B_{3,20}(X) + 1.37497\cdot 10^{13}B_{4,20}(X) + 2.41181\cdot 10^{13}B_{5,20}(X) + 3.58157\cdot 10^{13}B_{6,20}(X) \\ &\quad + 4.61131\cdot 10^{13}B_{7,20}(X) + 5.06156\cdot 10^{13}B_{8,20}(X) + 4.35612\cdot 10^{13}B_{9,20}(X) + 1.90286 \\ &\quad \cdot 10^{13}B_{10,20}(X) - 2.65368\cdot 10^{13}B_{11,20}(X) - 9.02907\cdot 10^{13}B_{12,20}(X) - 1.57924\cdot 10^{14}B_{13,20}(X) \\ &\quad - 1.99362\cdot 10^{14}B_{14,20}(X) - 1.69182\cdot 10^{14}B_{15,20}(X) - 1.82426\cdot 10^{13}B_{16,20}(X) + 2.75733 \\ &\quad \cdot 10^{14}B_{17,20}(X) + 6.56245\cdot 10^{14}B_{18,20}(X) + 9.36841\cdot 10^{14}B_{19,20}(X) + 8.43944\cdot 10^{14}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= 5.7673\cdot 10^{15}X^{3} - 6.54393\cdot 10^{15}X^{2} + 1.89809\cdot 10^{15}X - 1.0062\cdot 10^{14} \\ &= -1.0062\cdot 10^{14}B_{0,3} + 5.32075\cdot 10^{14}B_{1,3} - 1.01654\cdot 10^{15}B_{2,3} + 1.02084\cdot 10^{15}B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= 9.69629\cdot 10^{16}X^{20} - 9.75896\cdot 10^{17}X^{19} + 4.54666\cdot 10^{18}X^{18} - 1.30093\cdot 10^{19}X^{17} + 2.55752\cdot 10^{19}X^{16} \\ &\quad - 3.66167\cdot 10^{19}X^{15} + 3.94886\cdot 10^{19}X^{14} - 3.27275\cdot 10^{19}X^{13} + 2.10839\cdot 10^{19}X^{12} - 1.06124\cdot 10^{19}X^{11} \\ &\quad + 4.17388\cdot 10^{18}X^{10} - 1.27665\cdot 10^{18}X^{9} + 3.00612\cdot 10^{17}X^{8} - 5.35686\cdot 10^{16}X^{7} + 7.0379\cdot 10^{15}X^{6} - 6.57883 \\ &\quad \cdot 10^{14}X^{5} + 4.19316\cdot 10^{13}X^{4} + 5.76559\cdot 10^{15}X^{3} - 6.54389\cdot 10^{15}X^{2} + 1.89808\cdot 10^{15}X - 1.0062\cdot 10^{14} \\ &= -1.0062\cdot 10^{14}B_{0,20} - 5.71624\cdot 10^{12}B_{1,20} + 5.47465\cdot 10^{13}B_{2,20} + 8.58252\cdot 10^{13}B_{3,20} + 9.25861 \\ &\quad \cdot 10^{13}B_{4,20} + 8.00616\cdot 10^{13}B_{5,20} + 5.33896\cdot 10^{13}B_{6,20} + 1.74427\cdot 10^{13}B_{7,20} - 2.23555\cdot 10^{13}B_{8,20} \\ &\quad - 6.15485\cdot 10^{13}B_{9,20} - 9.42415\cdot 10^{13}B_{10,20} - 1.16342\cdot 10^{14}B_{11,20} - 1.21851\cdot 10^{14}B_{12,20} \\ &\quad - 1.06485\cdot 10^{14}B_{13,20} - 6.46441\cdot 10^{13}B_{14,20} + 8.41256\cdot 10^{12}B_{15,20} + 1.17902\cdot 10^{14}B_{16,20} \\ &\quad + 2.68819\cdot 10^{14}B_{17,20} + 4.66244\cdot 10^{14}B_{18,20} + 7.15229\cdot 10^{14}B_{19,20} + 1.02084\cdot 10^{15}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.21612 \cdot 10^{14}$.

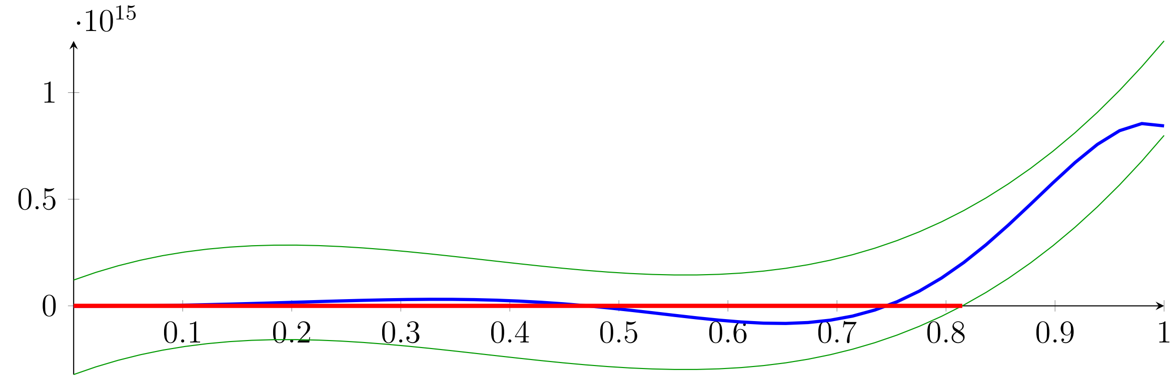
Bounding polynomials M and m:

$$M = 5.7673 \cdot 10^{15} X^3 - 6.54393 \cdot 10^{15} X^2 + 1.89809 \cdot 10^{15} X + 1.20991 \cdot 10^{14}$$
$$m = 5.7673 \cdot 10^{15} X^3 - 6.54393 \cdot 10^{15} X^2 + 1.89809 \cdot 10^{15} X - 3.22232 \cdot 10^{14}$$

Root of M and m:

$$N(M) = \{-0.0534358\} \qquad N(m) = \{0.814943\}$$

Intersection intervals:



$$[0, 0.814943]$$

Longest intersection interval: 0.814943

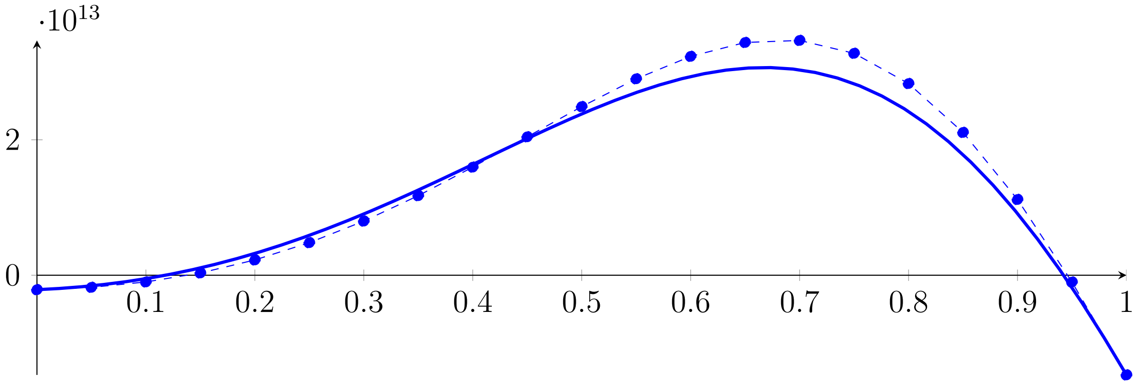
$\Longrightarrow$ Bisection: first half $[15.625, 17.1875]$ und second half $[17.1875, 18.75]$

Bisection point is very near to a root?!?

3.60 Recursion Branch 1 2 1 2 1 on the First Half $[15.625, 17.1875]$

Normalized monomial und Bézier representations and the Bézier polygon:

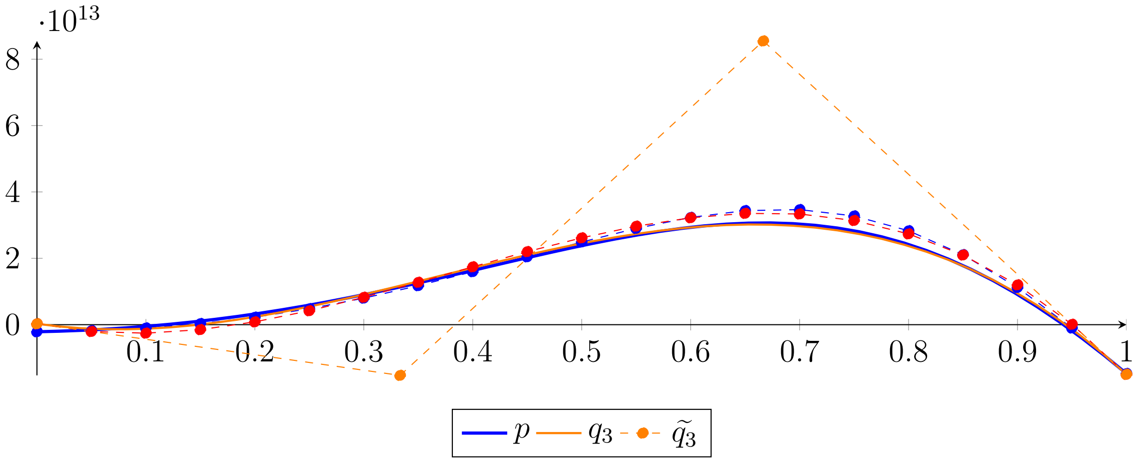
$$\begin{aligned} p = & -12255.1 X^{20} + 678571 X^{19} + 1.39217 \cdot 10^{7} X^{18} + 2.45056 \cdot 10^{8} X^{17} + 2.57809 \cdot 10^{9} X^{16} + 1.77899 \\ & \cdot 10^{10} X^{15} + 7.24246 \cdot 10^{10} X^{14} + 1.02329 \cdot 10^{11} X^{13} - 5.67624 \cdot 10^{11} X^{12} - 3.4484 \cdot 10^{12} X^{11} - 6.28975 \\ & \cdot 10^{12} X^{10} + 6.47685 \cdot 10^{12} X^{9} + 4.6428 \cdot 10^{13} X^{8} + 5.74242 \cdot 10^{13} X^{7} - 4.8441 \cdot 10^{13} X^{6} - 1.68566 \cdot 10^{14} X^{5} \\ & - 8.09942 \cdot 10^{13} X^{4} + 9.30826 \cdot 10^{13} X^{3} + 8.51578 \cdot 10^{13} X^{2} + 6.95749 \cdot 10^{12} X - 2.1354 \cdot 10^{12} \\ = & -2.1354 \cdot 10^{12} B_{0,20}(X) - 1.78753 \cdot 10^{12} B_{1,20}(X) - 9.91453 \cdot 10^{11} B_{2,20}(X) + 3.34471 \\ & \cdot 10^{11} B_{3,20}(X) + 2.25518 \cdot 10^{12} B_{4,20}(X) + 4.80802 \cdot 10^{12} B_{5,20}(X) + 7.99062 \cdot 10^{12} B_{6,20}(X) \\ & + 1.17483 \cdot 10^{13} B_{7,20}(X) + 1.59619 \cdot 10^{13} B_{8,20}(X) + 2.04381 \cdot 10^{13} B_{9,20}(X) + 2.49034 \\ & \cdot 10^{13} B_{10,20}(X) + 2.90046 \cdot 10^{13} B_{11,20}(X) + 3.2318 \cdot 10^{13} B_{12,20}(X) + 3.43704 \cdot 10^{13} B_{13,20}(X) \\ & + 3.46731 \cdot 10^{13} B_{14,20}(X) + 3.27707 \cdot 10^{13} B_{15,20}(X) + 2.83048 \cdot 10^{13} B_{16,20}(X) + 2.10885 \\ & \cdot 10^{13} B_{17,20}(X) + 1.11867 \cdot 10^{13} B_{18,20}(X) - 1.00816 \cdot 10^{12} B_{19,20}(X) - 1.47196 \cdot 10^{13} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_3 = -3.17545\cdot10^{14}X^3 + 3.48953\cdot10^{14}X^2 - 4.66469\cdot10^{13}X + 2.73604\cdot10^{11}$$
$$= 2.73604\cdot10^{11}B_{0,3} - 1.52754\cdot10^{13}B_{1,3} + 8.54933\cdot10^{13}B_{2,3} - 1.4965\cdot10^{13}B_{3,3}$$

$$\widetilde{q}_3 = -6.71986\cdot10^{15}X^{20} + 6.75896\cdot10^{16}X^{19} - 3.14788\cdot10^{17}X^{18} + 9.00606\cdot10^{17}X^{17} - 1.77059\cdot10^{18}X^{16} + 2.53515\cdot10^{18}X^{15} - 2.73367\cdot10^{18}X^{14} + 2.26444\cdot10^{18}X^{13} - 1.45706\cdot10^{18}X^{12} + 7.31791\cdot10^{17}X^{11} - 2.86808\cdot10^{17}X^{10} + 8.72921\cdot10^{16}X^9 - 2.04336\cdot10^{16}X^8 + 3.62416\cdot10^{15}X^7 - 4.76969\cdot10^{14}X^6 + 4.53534\cdot10^{13}X^5 - 3.02706\cdot10^{12}X^4 - 3.17411\cdot10^{14}X^3 + 3.4895\cdot10^{14}X^2 - 4.66469\cdot10^{13}X + 2.73604\cdot10^{11}$$
$$= 2.73604\cdot10^{11}B_{0,20} - 2.05874\cdot10^{12}B_{1,20} - 2.55451\cdot10^{12}B_{2,20} - 1.49213\cdot10^{12}B_{3,20} + 8.49343\cdot10^{11}B_{4,20} + 4.19315\cdot10^{12}B_{5,20} + 8.25546\cdot10^{12}B_{6,20} + 1.27704\cdot10^{13}B_{7,20} + 1.74342\cdot10^{13}B_{8,20} + 2.20102\cdot10^{13}B_{9,20} + 2.61618\cdot10^{13}B_{10,20} + 2.96774\cdot10^{13}B_{11,20} + 3.22139\cdot10^{13}B_{12,20} + 3.35462\cdot10^{13}B_{13,20} + 3.33575\cdot10^{13}B_{14,20} + 3.13923\cdot10^{13}B_{15,20} + 2.73604\cdot10^{13}B_{16,20} + 2.09881\cdot10^{13}B_{17,20} + 1.19952\cdot10^{13}B_{18,20} + 1.03778\cdot10^{11}B_{19,20} - 1.4965\cdot10^{13}B_{20,20}$$



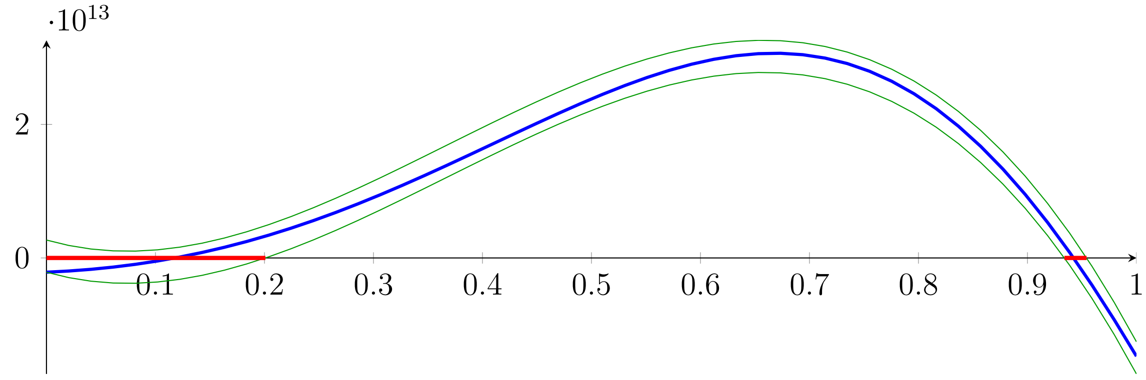
The maximum difference of the Bézier coefficients is $\delta = 2.409\cdot10^{12}$.
Bounding polynomials M and m:

$$M = -3.17545\cdot10^{14}X^3 + 3.48953\cdot10^{14}X^2 - 4.66469\cdot10^{13}X + 2.68261\cdot10^{12}$$
$$m = -3.17545\cdot10^{14}X^3 + 3.48953\cdot10^{14}X^2 - 4.66469\cdot10^{13}X - 2.1354\cdot10^{12}$$

Root of M and m:

$$N(M) = \{0.954245\} \qquad N(m) = \{-0.0358499, 0.200856, 0.933904\}$$

Intersection intervals:



$[0, 0.200856]$, $[0.933904, 0.954245]$

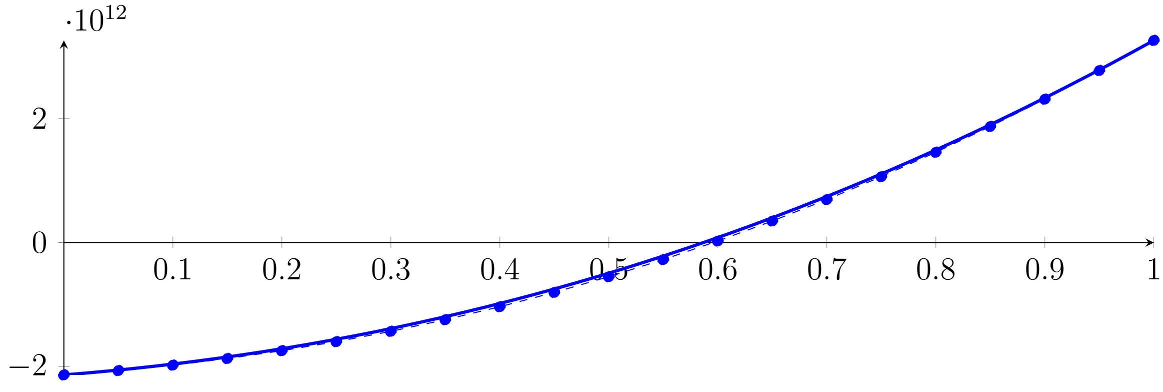
Longest intersection interval: 0.200856

$\Longrightarrow$ Selective recursion: interval 1: $[15.625, 15.9388]$, interval 2: $[17.0842, 17.116]$,

3.61 Recursion Branch 1 2 1 2 1 1 in Interval 1: $[15.625, 15.9388]$

Normalized monomial und Bézier representations and the Bézier polygon:

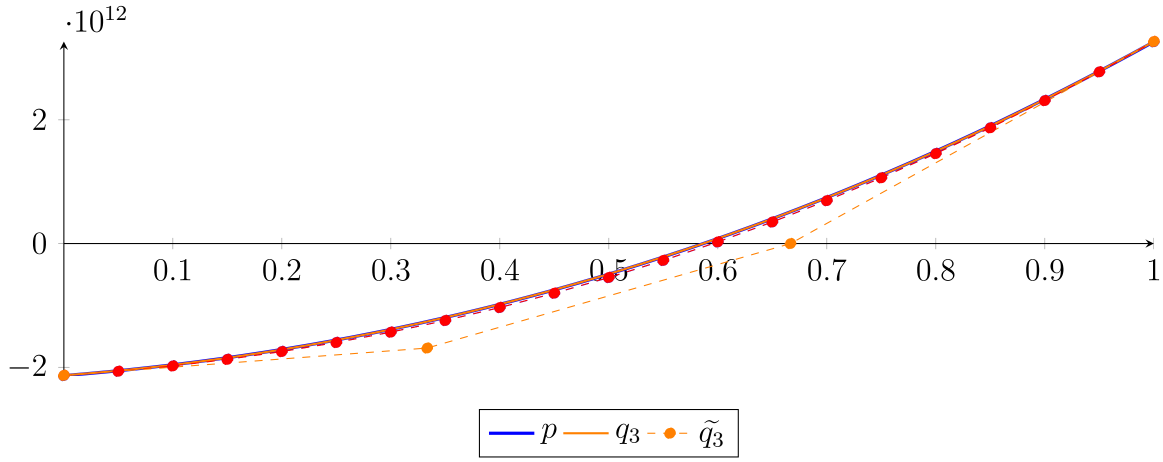
$$p = 752.312X^{20} + 1222.67X^{19} + 31217.8X^{18} - 61599.5X^{17} + 595403X^{16} - 521738X^{15} + 272210X^{14} + 137515X^{13} + 672107X^{12} + 77501.1X^{11} - 448990X^{10} + 3.48263 \cdot 10^{6}X^{9} + 1.22986 \cdot 10^{8}X^{8} + 7.57324 \cdot 10^{8}X^{7} - 3.18065 \cdot 10^{9}X^{6} - 5.51046 \cdot 10^{10}X^{5} - 1.31822 \cdot 10^{11}X^{4} + 7.54258 \cdot 10^{11}X^{3} + 3.43552 \cdot 10^{12}X^{2} + 1.39745 \cdot 10^{12}X - 2.1354 \cdot 10^{12}$$
$$= -2.1354 \cdot 10^{12}B_{0,20}(X) - 2.06553 \cdot 10^{12}B_{1,20}(X) - 1.97757 \cdot 10^{12}B_{2,20}(X) - 1.87088 \cdot 10^{12}B_{3,20}(X) - 1.7448 \cdot 10^{12}B_{4,20}(X) - 1.59874 \cdot 10^{12}B_{5,20}(X) - 1.43214 \cdot 10^{12}B_{6,20}(X) - 1.24445 \cdot 10^{12}B_{7,20}(X) - 1.03519 \cdot 10^{12}B_{8,20}(X) - 8.03914 \cdot 10^{11}B_{9,20}(X) - 5.50231 \cdot 10^{11}B_{10,20}(X) - 2.73798 \cdot 10^{11}B_{11,20}(X) + 2.56669 \cdot 10^{10}B_{12,20}(X) + 3.48387 \cdot 10^{11}B_{13,20}(X) + 6.94518 \cdot 10^{11}B_{14,20}(X) + 1.06415 \cdot 10^{12}B_{15,20}(X) + 1.4573 \cdot 10^{12}B_{16,20}(X) + 1.8739 \cdot 10^{12}B_{17,20}(X) + 2.31381 \cdot 10^{12}B_{18,20}(X) + 2.77681 \cdot 10^{12}B_{19,20}(X) + 3.2626 \cdot 10^{12}B_{20,20}(X)$$



Degree reduction and raising:

$$q_3 = 3.30252 \cdot 10^{11}X^{3} + 3.743 \cdot 10^{12}X^{2} + 1.32522 \cdot 10^{12}X - 2.13167 \cdot 10^{12}$$
$$= -2.13167 \cdot 10^{12}B_{0,3} - 1.68993 \cdot 10^{12}B_{1,3} - 5.27615 \cdot 10^{8}B_{2,3} + 3.2668 \cdot 10^{12}B_{3,3}$$

$$\begin{aligned}\widetilde{q}_3 = {}& 1.95382{\cdot}10^{14}X^{20} - 1.95892{\cdot}10^{15}X^{19} + 9.09019{\cdot}10^{15}X^{18} - 2.59037{\cdot}10^{16}X^{17} + 5.07188{\cdot}10^{16}X^{16} \\ & - 7.23392{\cdot}10^{16}X^{15} + 7.77547{\cdot}10^{16}X^{14} - 6.42807{\cdot}10^{16}X^{13} + 4.13512{\cdot}10^{16}X^{12} - 2.0806{\cdot}10^{16}X^{11} \\ & + 8.185{\cdot}10^{15}X^{10} - 2.50318{\cdot}10^{15}X^{9} + 5.88525{\cdot}10^{14}X^{8} - 1.04634{\cdot}10^{14}X^{7} + 1.37663{\cdot}10^{13}X^{6} - 1.30243 \\ & \cdot 10^{12}X^{5} + 8.50171{\cdot}10^{10}X^{4} + 3.26646{\cdot}10^{11}X^{3} + 3.74309{\cdot}10^{12}X^{2} + 1.32521{\cdot}10^{12}X - 2.13167{\cdot}10^{12} \\ = {}& -2.13167{\cdot}10^{12}B_{0,20} - 2.06541{\cdot}10^{12}B_{1,20} - 1.97945{\cdot}10^{12}B_{2,20} - 1.8735{\cdot}10^{12}B_{3,20} - 1.74726 \\ & \cdot 10^{12}B_{4,20} - 1.60049{\cdot}10^{12}B_{5,20} - 1.43276{\cdot}10^{12}B_{6,20} - 1.24412{\cdot}10^{12}B_{7,20} - 1.03358{\cdot}10^{12}B_{8,20} \\ & - 8.02027{\cdot}10^{11}B_{9,20} - 5.47527{\cdot}10^{11}B_{10,20} - 2.71732{\cdot}10^{11}B_{11,20} + 2.75663{\cdot}10^{10}B_{12,20} \\ & + 3.49067{\cdot}10^{11}B_{13,20} + 6.94184{\cdot}10^{11}B_{14,20} + 1.06253{\cdot}10^{12}B_{15,20} + 1.45474{\cdot}10^{12}B_{16,20} \\ & + 1.87095{\cdot}10^{12}B_{17,20} + 2.31151{\cdot}10^{12}B_{18,20} + 2.7767{\cdot}10^{12}B_{19,20} + 3.2668{\cdot}10^{12}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 4.1989{\cdot}10^{9}$.

Bounding polynomials M and m:

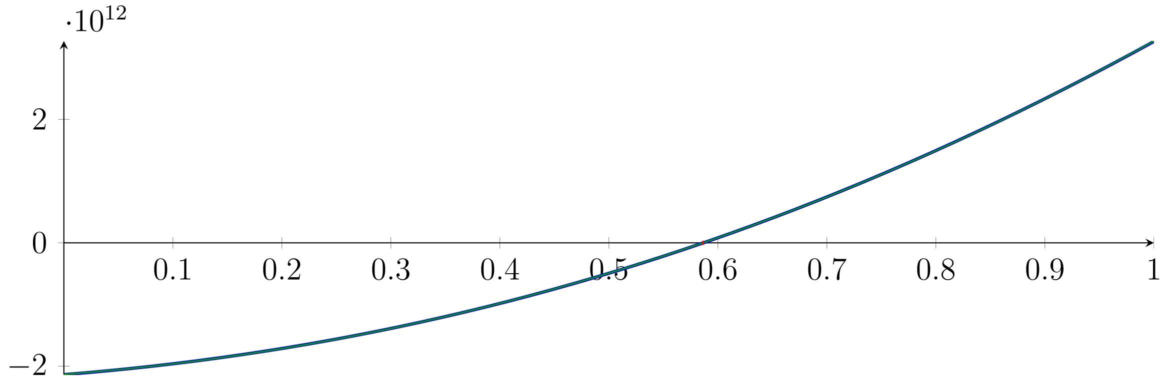
$$M = 3.30252{\cdot}10^{11}X^3 + 3.743{\cdot}10^{12}X^2 + 1.32522{\cdot}10^{12}X - 2.12747{\cdot}10^{12}$$

$$m = 3.30252{\cdot}10^{11}X^3 + 3.743{\cdot}10^{12}X^2 + 1.32522{\cdot}10^{12}X - 2.13587{\cdot}10^{12}$$

Root of M and m:

$$N(M) = \{-10.9119, -1.00769, 0.585853\} \qquad N(m) = \{-10.9117, -1.0093, 0.587239\}$$

Intersection intervals:



$$[0.585853, 0.587239]$$

Longest intersection interval: 0.00138647

$\Longrightarrow$ Selective recursion: interval 1: [15.8089, 15.8093],

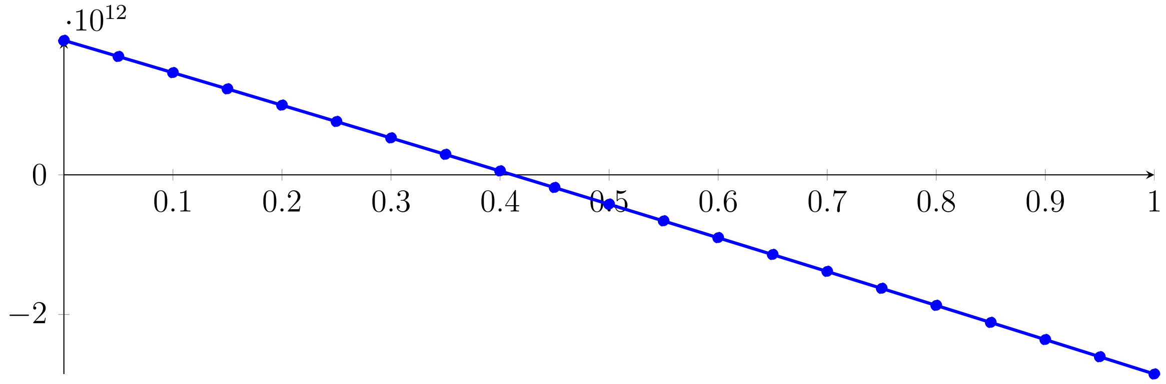
3.62 Recursion Branch 1 2 1 2 1 1 1 in Interval 1: $[15.8089, 15.8093]$

Found root in interval $[15.8089, 15.8093]$ at recursion depth 7!

3.63 Recursion Branch 1 2 1 2 1 2 in Interval 2: $[17.0842, 17.116]$

Normalized monomial und Bézier representations and the Bézier polygon:

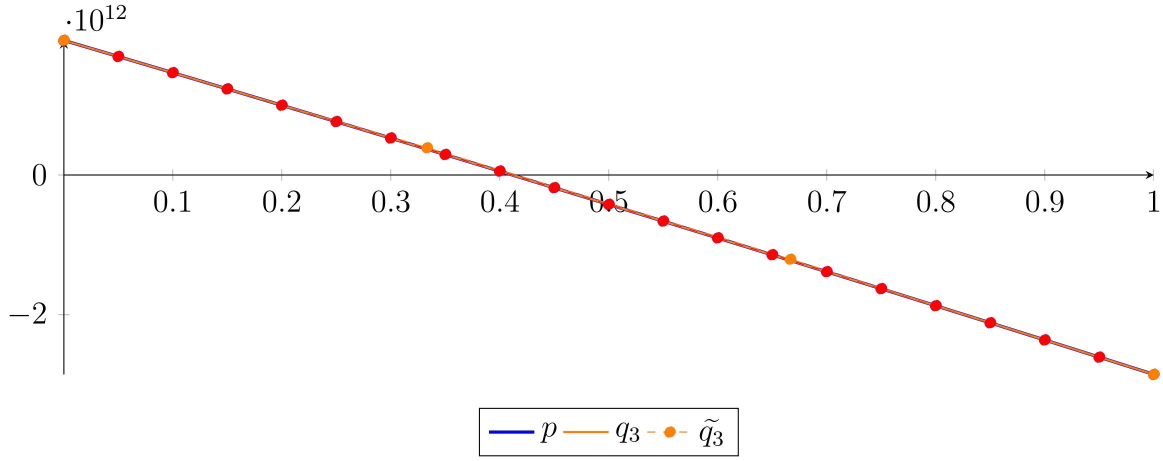
$$\begin{aligned}p &= 165.437X^{20} - 5461.07X^{19} - 5041.87X^{18} - 50470.6X^{17} + 18220.8X^{16} + 46345.5X^{15} \\ &\quad - 84371.1X^{14} - 87739.9X^{13} - 260428X^{12} - 97593.9X^{11} - 112044X^{10} \\ &\quad - 25095.6X^{9} - 861.123X^{8} - 1286.95X^{7} - 21159X^{6} + 3.74771\cdot 10^{6}X^{5} + 2.62443 \\ &\quad \cdot 10^{8}X^{4} + 3.66189\cdot 10^{9}X^{3} - 1.72786\cdot 10^{11}X^{2} - 4.60916\cdot 10^{12}X + 1.92564\cdot 10^{12} \\ &= 1.92564\cdot 10^{12}B_{0,20}(X) + 1.69518\cdot 10^{12}B_{1,20}(X) + 1.46381\cdot 10^{12}B_{2,20}(X) + 1.23154 \\ &\quad \cdot 10^{12}B_{3,20}(X) + 9.98363\cdot 10^{11}B_{4,20}(X) + 7.64287\cdot 10^{11}B_{5,20}(X) + 5.29315\cdot 10^{11}B_{6,20}(X) \\ &\quad + 2.9345\cdot 10^{11}B_{7,20}(X) + 5.6696\cdot 10^{10}B_{8,20}(X) - 1.80944\cdot 10^{11}B_{9,20}(X) - 4.19466 \\ &\quad \cdot 10^{11}B_{10,20}(X) - 6.58867\cdot 10^{11}B_{11,20}(X) - 8.99143\cdot 10^{11}B_{12,20}(X) - 1.14029\cdot 10^{12}B_{13,20}(X) \\ &\quad - 1.3823\cdot 10^{12}B_{14,20}(X) - 1.62518\cdot 10^{12}B_{15,20}(X) - 1.86892\cdot 10^{12}B_{16,20}(X) - 2.11351 \\ &\quad \cdot 10^{12}B_{17,20}(X) - 2.35895\cdot 10^{12}B_{18,20}(X) - 2.60524\cdot 10^{12}B_{19,20}(X) - 2.85238\cdot 10^{12}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= 4.19711\cdot 10^{9}X^{3} - 1.73132\cdot 10^{11}X^{2} - 4.60908\cdot 10^{12}X + 1.92563\cdot 10^{12} \\ &= 1.92563\cdot 10^{12}B_{0,3} + 3.89275\cdot 10^{11}B_{1,3} - 1.2048\cdot 10^{12}B_{2,3} - 2.85238\cdot 10^{12}B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= -1.05756\cdot 10^{14}X^{20} + 1.05974\cdot 10^{15}X^{19} - 4.91144\cdot 10^{15}X^{18} + 1.39702\cdot 10^{16}X^{17} - 2.72935\cdot 10^{16}X^{16} \\ &\quad + 3.88426\cdot 10^{16}X^{15} - 4.16768\cdot 10^{16}X^{14} + 3.44275\cdot 10^{16}X^{13} - 2.21641\cdot 10^{16}X^{12} + 1.11838\cdot 10^{16}X^{11} \\ &\quad - 4.42274\cdot 10^{15}X^{10} + 1.3627\cdot 10^{15}X^{9} - 3.2309\cdot 10^{14}X^{8} + 5.77597\cdot 10^{13}X^{7} - 7.55088\cdot 10^{12}X^{6} + 6.88765 \\ &\quad \cdot 10^{11}X^{5} - 4.03349\cdot 10^{10}X^{4} + 5.54252\cdot 10^{9}X^{3} - 1.73156\cdot 10^{11}X^{2} - 4.60908\cdot 10^{12}X + 1.92563\cdot 10^{12} \\ &= 1.92563\cdot 10^{12}B_{0,20} + 1.69518\cdot 10^{12}B_{1,20} + 1.46382\cdot 10^{12}B_{2,20} + 1.23154\cdot 10^{12}B_{3,20} + 9.98362 \\ &\quad \cdot 10^{11}B_{4,20} + 7.64302\cdot 10^{11}B_{5,20} + 5.29284\cdot 10^{11}B_{6,20} + 2.93511\cdot 10^{11}B_{7,20} + 5.66034\cdot 10^{10}B_{8,20} \\ &\quad - 1.80814\cdot 10^{11}B_{9,20} - 4.19607\cdot 10^{11}B_{10,20} - 6.58726\cdot 10^{11}B_{11,20} - 8.99228\cdot 10^{11}B_{12,20} \\ &\quad - 1.14023\cdot 10^{12}B_{13,20} - 1.38233\cdot 10^{12}B_{14,20} - 1.62517\cdot 10^{12}B_{15,20} - 1.86892\cdot 10^{12}B_{16,20} \\ &\quad - 2.11351\cdot 10^{12}B_{17,20} - 2.35895\cdot 10^{12}B_{18,20} - 2.60524\cdot 10^{12}B_{19,20} - 2.85238\cdot 10^{12}B_{20,20}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.40913 \cdot 10^{8}$.
Bounding polynomials M and m:

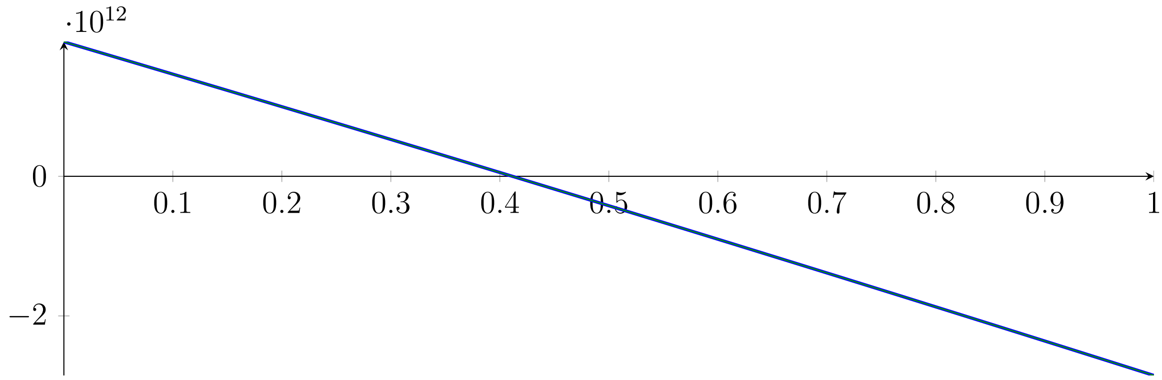
$$M = 4.19711 \cdot 10^{9} X^3 - 1.73132 \cdot 10^{11} X^2 - 4.60908 \cdot 10^{12} X + 1.92578 \cdot 10^{12}$$

$$m = 4.19711 \cdot 10^{9} X^3 - 1.73132 \cdot 10^{11} X^2 - 4.60908 \cdot 10^{12} X + 1.92549 \cdot 10^{12}$$

Root of M and m:

$$N(M) = \{-18.7203, 0.411524, 59.5591\} \qquad N(m) = \{-18.7202, 0.411465, 59.5591\}$$

Intersection intervals:



$$[0.411465, 0.411524]$$

Longest intersection interval: $5.93389 \cdot 10^{-05}$
$\Longrightarrow$ Selective recursion: interval 1: $[17.0973, 17.0973]$,

3.64 Recursion Branch 1 2 1 2 1 2 1 in Interval 1: $[17.0973, 17.0973]$

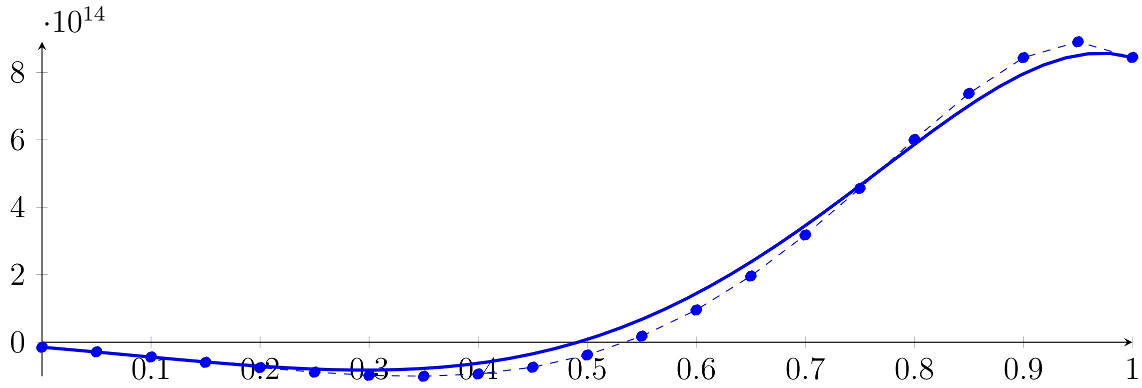
Found root in interval $[17.0973, 17.0973]$ at recursion depth 7!

3.65 Recursion Branch 1 2 1 2 2 on the Second Half [17.1875, 18.75]

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 51309.5X^{20} + 1.13609\cdot10^{6}X^{19} + 2.62513\cdot10^{7}X^{18} + 5.89696\cdot10^{8}X^{17} + 9.46063\cdot10^{9}X^{16} + 1.05848\cdot10^{11}X^{15} + 8.64537\cdot10^{11}X^{14} + 5.14688\cdot10^{12}X^{13} + 2.19482\cdot10^{13}X^{12} + 6.31615\cdot10^{13}X^{11} + 9.96023\cdot10^{13}X^{10} - 2.75387\cdot10^{13}X^{9} - 5.24772\cdot10^{14}X^{8} - 1.10687\cdot10^{15}X^{7} - 7.34813\cdot10^{14}X^{6} + 8.78049\cdot10^{14}X^{5} + 1.86093\cdot10^{15}X^{4} + 8.85216\cdot10^{14}X^{3} - 2.8815\cdot10^{14}X^{2} - 2.74229\cdot10^{14}X - 1.47196\cdot10^{13}$$

$$= -1.47196\cdot10^{13}B_{0,20}(X) - 2.84311\cdot10^{13}B_{1,20}(X) - 4.36591\cdot10^{13}B_{2,20}(X) - 5.96272\cdot10^{13}B_{3,20}(X) - 7.51748\cdot10^{13}B_{4,20}(X) - 8.87006\cdot10^{13}B_{5,20}(X) - 9.81247\cdot10^{13}B_{6,20}(X) - 1.00885\cdot10^{14}B_{7,20}(X) - 9.39824\cdot10^{13}B_{8,20}(X) - 7.41057\cdot10^{13}B_{9,20}(X) - 3.78514\cdot10^{13}B_{10,20}(X) + 1.7921\cdot10^{13}B_{11,20}(X) + 9.55764\cdot10^{13}B_{12,20}(X) + 1.96007\cdot10^{14}B_{13,20}(X) + 3.17738\cdot10^{14}B_{14,20}(X) + 4.5586\cdot10^{14}B_{15,20}(X) + 6.00841\cdot10^{14}B_{16,20}(X) + 7.37367\cdot10^{14}B_{17,20}(X) + 8.43467\cdot10^{14}B_{18,20}(X) + 8.90392\cdot10^{14}B_{19,20}(X) + 8.43944\cdot10^{14}B_{20,20}(X)$$

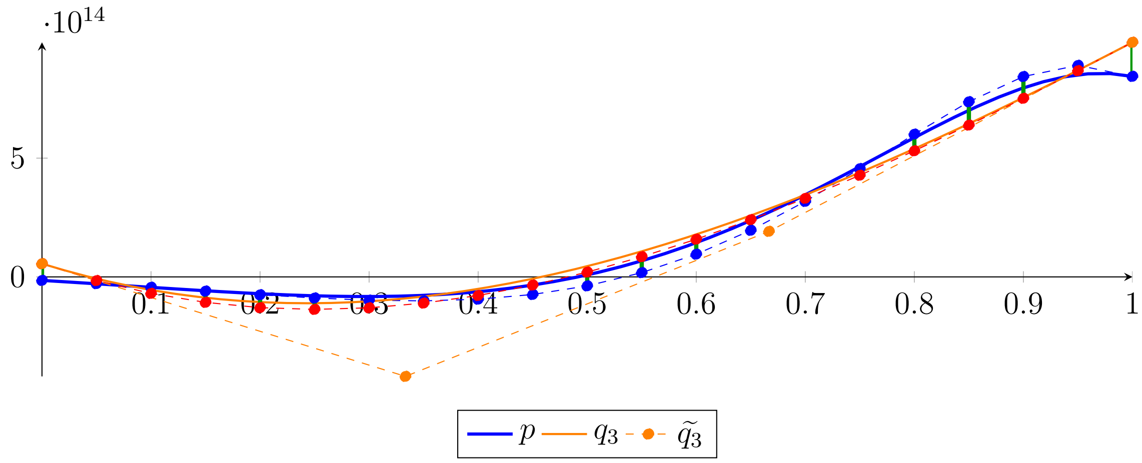


Degree reduction and raising:

$$q_3 = -8.99518\cdot10^{14}X^{3} + 3.25476\cdot10^{15}X^{2} - 1.42347\cdot10^{15}X + 5.56891\cdot10^{13}$$
$$= 5.56891\cdot10^{13}B_{0,3} - 4.18801\cdot10^{14}B_{1,3} + 1.91627\cdot10^{14}B_{2,3} + 9.87456\cdot10^{14}B_{3,3}$$

$$\widetilde{q}_3 = -2.63991\cdot10^{16}X^{20} + 2.65253\cdot10^{17}X^{19} - 1.23427\cdot10^{18}X^{18} + 3.52842\cdot10^{18}X^{17} - 6.9311\cdot10^{18}X^{16} + 9.91318\cdot10^{18}X^{15} - 1.06711\cdot10^{19}X^{14} + 8.81399\cdot10^{18}X^{13} - 5.64455\cdot10^{18}X^{12} + 2.81406\cdot10^{18}X^{11} - 1.09114\cdot10^{18}X^{10} + 3.27416\cdot10^{17}X^{9} - 7.53841\cdot10^{16}X^{8} + 1.3159\cdot10^{16}X^{7} - 1.71214\cdot10^{15}X^{6} + 1.61631\cdot10^{14}X^{5} - 1.06418\cdot10^{13}X^{4} - 8.99062\cdot10^{14}X^{3} + 3.25475\cdot10^{15}X^{2} - 1.42347\cdot10^{15}X + 5.56891\cdot10^{13}$$

$$= 5.56891\cdot10^{13}B_{0,20} - 1.54844\cdot10^{13}B_{1,20} - 6.95277\cdot10^{13}B_{2,20} - 1.07229\cdot10^{14}B_{3,20} - 1.2938\cdot10^{14}B_{4,20} - 1.36763\cdot10^{14}B_{5,20} - 1.30186\cdot10^{14}B_{6,20} - 1.10391\cdot10^{14}B_{7,20} - 7.82641\cdot10^{13}B_{8,20} - 3.44299\cdot10^{13}B_{9,20} + 2.00945\cdot10^{13}B_{10,20} + 8.47815\cdot10^{13}B_{11,20} + 1.5859\cdot10^{14}B_{12,20} + 2.40942\cdot10^{14}B_{13,20} + 3.30895\cdot10^{14}B_{14,20} + 4.27752\cdot10^{14}B_{15,20} + 5.30679\cdot10^{14}B_{16,20} + 6.38905\cdot10^{14}B_{17,20} + 7.51635\cdot10^{14}B_{18,20} + 8.68082\cdot10^{14}B_{19,20} + 9.87456\cdot10^{14}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.43512{\cdot}10^{14}$.

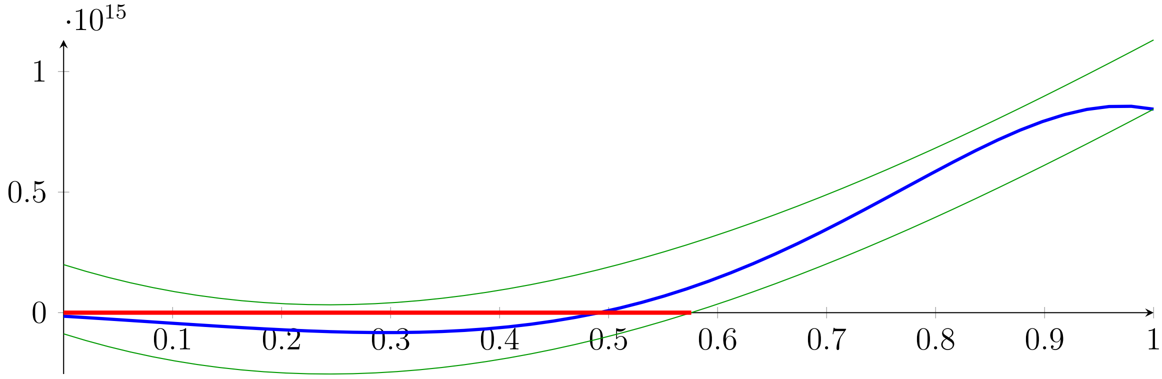
Bounding polynomials M and m:

$$M = -8.99518{\cdot}10^{14}X^3 + 3.25476{\cdot}10^{15}X^2 - 1.42347{\cdot}10^{15}X + 1.99201{\cdot}10^{14}$$
$$m = -8.99518{\cdot}10^{14}X^3 + 3.25476{\cdot}10^{15}X^2 - 1.42347{\cdot}10^{15}X - 8.78233{\cdot}10^{13}$$

Root of M and m:

$$N(M) = \{3.13627\} \qquad N(m) = \{-0.0547412, 0.575855, 3.09722\}$$

Intersection intervals:



$[0, 0.575855]$

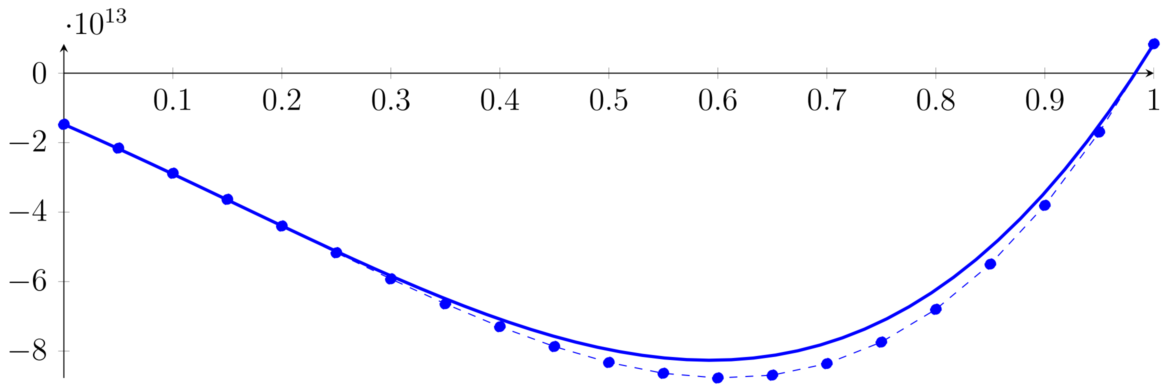
Longest intersection interval: 0.575855

$\Longrightarrow$ Bisection: first half $[17.1875, 17.9688]$ und second half $[17.9688, 18.75]$

3.66 Recursion Branch 1 2 1 2 2 1 on the First Half $[17.1875, 17.9688]$

Normalized monomial und Bézier representations and the Bézier polygon:

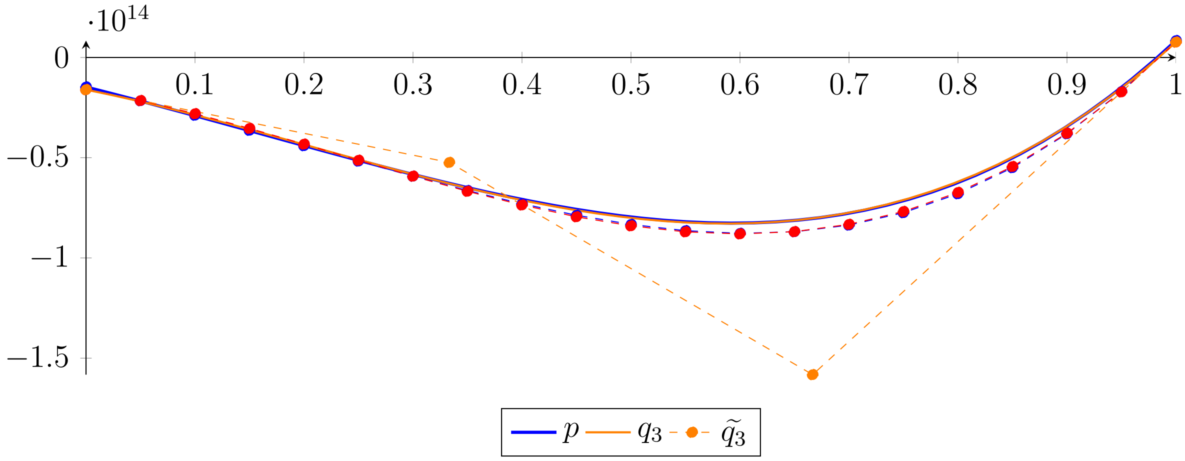
$$\begin{aligned}p &= 77549.2X^{20} - 481986X^{19} + 1.96467{\cdot}10^{6}X^{18} - 9.60214{\cdot}10^{6}X^{17} + 4.74973{\cdot}10^{7}X^{16} - 3.11619\\ &\quad{\cdot}10^{7}X^{15} + 6.44235{\cdot}10^{7}X^{14} + 6.32879{\cdot}10^{8}X^{13} + 5.38653{\cdot}10^{9}X^{12} + 3.08428{\cdot}10^{10}X^{11} + 9.72751\\ &\quad{\cdot}10^{10}X^{10} - 5.37861{\cdot}10^{10}X^{9} - 2.04989{\cdot}10^{12}X^{8} - 8.64744{\cdot}10^{12}X^{7} - 1.14815{\cdot}10^{13}X^{6} + 2.7439\\ &\quad{\cdot}10^{13}X^{5} + 1.16308{\cdot}10^{14}X^{4} + 1.10652{\cdot}10^{14}X^{3} - 7.20374{\cdot}10^{13}X^{2} - 1.37115{\cdot}10^{14}X - 1.47196{\cdot}10^{13}\\ &= -1.47196{\cdot}10^{13}B_{0,20}(X) - 2.15754{\cdot}10^{13}B_{1,20}(X) - 2.88102{\cdot}10^{13}B_{2,20}(X) - 3.63272\\ &\quad{\cdot}10^{13}B_{3,20}(X) - 4.40052{\cdot}10^{13}B_{4,20}(X) - 5.16973{\cdot}10^{13}B_{5,20}(X) - 5.92295{\cdot}10^{13}B_{6,20}(X)\\ &\quad - 6.63994{\cdot}10^{13}B_{7,20}(X) - 7.29757{\cdot}10^{13}B_{8,20}(X) - 7.86984{\cdot}10^{13}B_{9,20}(X) - 8.328\\ &\quad{\cdot}10^{13}B_{10,20}(X) - 8.6407{\cdot}10^{13}B_{11,20}(X) - 8.77436{\cdot}10^{13}B_{12,20}(X) - 8.69362{\cdot}10^{13}B_{13,20}(X)\\ &\quad - 8.36193{\cdot}10^{13}B_{14,20}(X) - 7.74242{\cdot}10^{13}B_{15,20}(X) - 6.79884{\cdot}10^{13}B_{16,20}(X) - 5.49683\\ &\quad{\cdot}10^{13}B_{17,20}(X) - 3.80537{\cdot}10^{13}B_{18,20}(X) - 1.69845{\cdot}10^{13}B_{19,20}(X) + 8.42928{\cdot}10^{12}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$q_3 = 3.41343\cdot10^{14}X^3 - 2.08683\cdot10^{14}X^2 - 1.0893\cdot10^{14}X - 1.60437\cdot10^{13}$$
$$= -1.60437\cdot10^{13}B_{0,3} - 5.23536\cdot10^{13}B_{1,3} - 1.58225\cdot10^{14}B_{2,3} + 7.68597\cdot10^{12}B_{3,3}$$

$$\widetilde{q}_3 = 1.26296\cdot10^{16}X^{20} - 1.26877\cdot10^{17}X^{19} + 5.90301\cdot10^{17}X^{18} - 1.68735\cdot10^{18}X^{17} + 3.31481\cdot10^{18}X^{16} - 4.74295\cdot10^{18}X^{15} + 5.11103\cdot10^{18}X^{14} - 4.23084\cdot10^{18}X^{13} + 2.72009\cdot10^{18}X^{12} - 1.36458\cdot10^{18}X^{11} + 5.33892\cdot10^{17}X^{10} - 1.62063\cdot10^{17}X^9 + 3.78018\cdot10^{16}X^8 - 6.68521\cdot10^{15}X^7 + 8.81987\cdot10^{14}X^6 - 8.52055\cdot10^{13}X^5 + 5.90813\cdot10^{12}X^4 + 3.41064\cdot10^{14}X^3 - 2.08676\cdot10^{14}X^2 - 1.0893\cdot10^{14}X - 1.60437\cdot10^{13}$$
$$= -1.60437\cdot10^{13}B_{0,20} - 2.14902\cdot10^{13}B_{1,20} - 2.8035\cdot10^{13}B_{2,20} - 3.53789\cdot10^{13}B_{3,20} - 4.32215\cdot10^{13}B_{4,20} - 5.12667\cdot10^{13}B_{5,20} - 5.92054\cdot10^{13}B_{6,20} - 6.67617\cdot10^{13}B_{7,20} - 7.35889\cdot10^{13}B_{8,20} - 7.94657\cdot10^{13}B_{9,20} - 8.39846\cdot10^{13}B_{10,20} - 8.69715\cdot10^{13}B_{11,20} - 8.80061\cdot10^{13}B_{12,20} - 8.68894\cdot10^{13}B_{13,20} - 8.32489\cdot10^{13}B_{14,20} - 7.68299\cdot10^{13}B_{15,20} - 6.73098\cdot10^{13}B_{16,20} - 5.43995\cdot10^{13}B_{17,20} - 3.77959\cdot10^{13}B_{18,20} - 1.72006\cdot10^{13}B_{19,20} + 7.68597\cdot10^{12}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.32407\cdot10^{12}$.

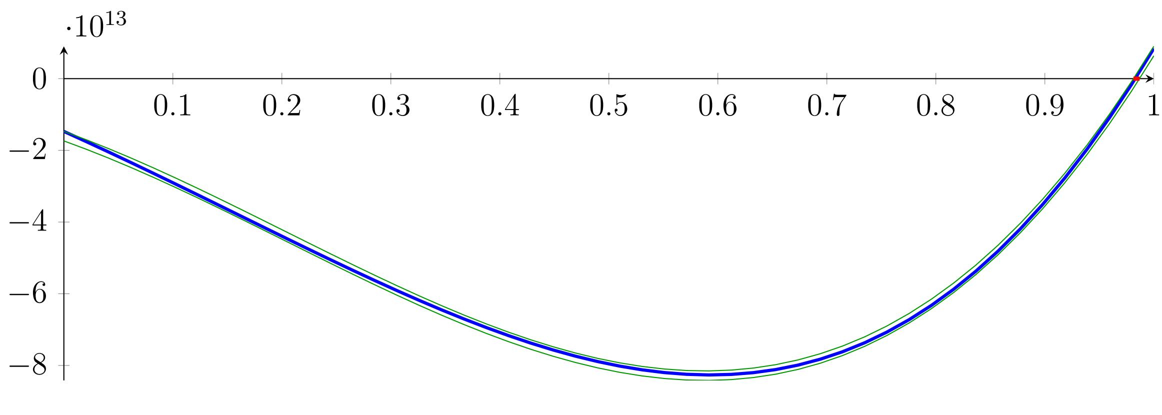
Bounding polynomials M and m:

$$M = 3.41343\cdot10^{14}X^3 - 2.08683\cdot10^{14}X^2 - 1.0893\cdot10^{14}X - 1.47196\cdot10^{13}$$
$$m = 3.41343\cdot10^{14}X^3 - 2.08683\cdot10^{14}X^2 - 1.0893\cdot10^{14}X - 1.73678\cdot10^{13}$$

Root of M and m:

$$N(M) = \{0.981331\} \qquad N(m) = \{0.98694\}$$

Intersection intervals:



$[0.981331, 0.98694]$

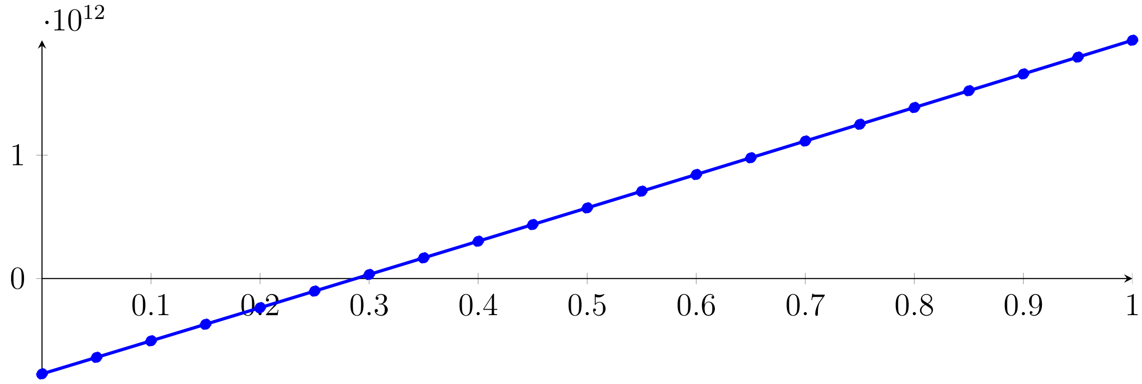
Longest intersection interval: 0.00560903

$\Longrightarrow$ Selective recursion: interval 1: $[17.9542, 17.9585]$,

3.67 Recursion Branch 1 2 1 2 2 1 1 in Interval 1: $[17.9542, 17.9585]$

Normalized monomial und Bézier representations and the Bézier polygon:

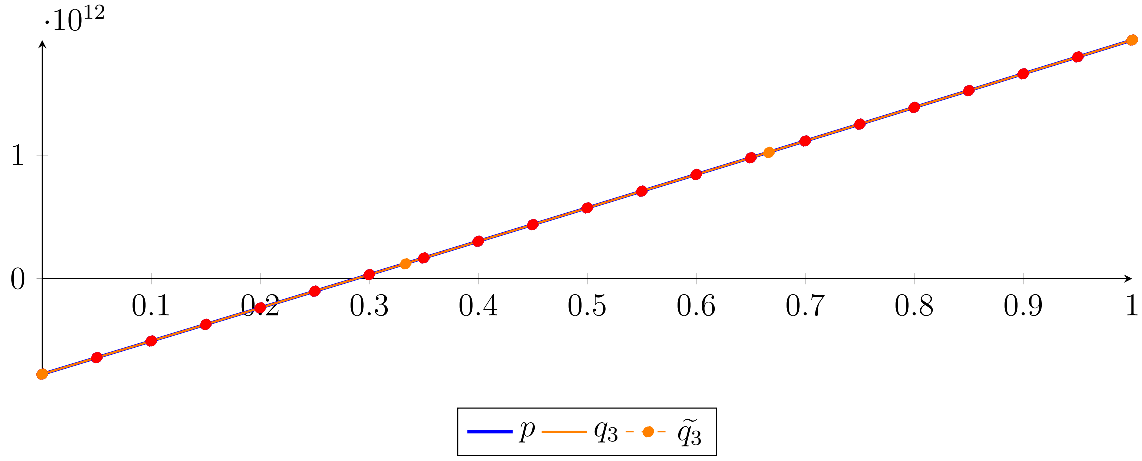
$$p = -412.257X^{20} + 4905.91X^{19} - 6704.32X^{18} + 67214.1X^{17} - 224908X^{16} + 142909X^{15} - 20998.2X^{14} + 20723.7X^{13} - 18667.9X^{12} + 42113X^{11} + 13757.5X^{10} + 10128.4X^{9} - 461.316X^{8} + 56.7773X^{7} + 274.424X^{6} - 1593.55X^{5} - 307138X^{4} + 4.25303\cdot 10^{7}X^{3} + 2.55672\cdot 10^{10}X^{2} + 2.67938\cdot 10^{12}X - 7.73315\cdot 10^{11}$$
$$= -7.73315\cdot 10^{11}B_{0,20}(X) - 6.39346\cdot 10^{11}B_{1,20}(X) - 5.05243\cdot 10^{11}B_{2,20}(X) - 3.71005\cdot 10^{11}B_{3,20}(X) - 2.36632\cdot 10^{11}B_{4,20}(X) - 1.02125\cdot 10^{11}B_{5,20}(X) + 3.25173\cdot 10^{10}B_{6,20}(X) + 1.67294\cdot 10^{11}B_{7,20}(X) + 3.02206\cdot 10^{11}B_{8,20}(X) + 4.37252\cdot 10^{11}B_{9,20}(X) + 5.72433\cdot 10^{11}B_{10,20}(X) + 7.07749\cdot 10^{11}B_{11,20}(X) + 8.432\cdot 10^{11}B_{12,20}(X) + 9.78786\cdot 10^{11}B_{13,20}(X) + 1.11451\cdot 10^{12}B_{14,20}(X) + 1.25036\cdot 10^{12}B_{15,20}(X) + 1.38635\cdot 10^{12}B_{16,20}(X) + 1.52248\cdot 10^{12}B_{17,20}(X) + 1.65874\cdot 10^{12}B_{18,20}(X) + 1.79514\cdot 10^{12}B_{19,20}(X) + 1.93167\cdot 10^{12}B_{20,20}(X)$$



Degree reduction and raising:

$$q_3 = 4.19115\cdot 10^{7}X^{3} + 2.55676\cdot 10^{10}X^{2} + 2.67938\cdot 10^{12}X - 7.73315\cdot 10^{11}$$
$$= -7.73315\cdot 10^{11}B_{0,3} + 1.19811\cdot 10^{11}B_{1,3} + 1.02146\cdot 10^{12}B_{2,3} + 1.93167\cdot 10^{12}B_{3,3}$$

$$\widetilde{q}_3 = 7.76113\cdot 10^{12}X^{20} - 7.74124\cdot 10^{13}X^{19} + 3.54025\cdot 10^{14}X^{18} - 9.86359\cdot 10^{14}X^{17} + 1.87865\cdot 10^{15}X^{16} - 2.60616\cdot 10^{15}X^{15} + 2.74484\cdot 10^{15}X^{14} - 2.26196\cdot 10^{15}X^{13} + 1.49059\cdot 10^{15}X^{12} - 7.9498\cdot 10^{14}X^{11} + 3.4287\cdot 10^{14}X^{10} - 1.17682\cdot 10^{14}X^{9} + 3.10913\cdot 10^{13}X^{8} - 5.97612\cdot 10^{12}X^{7} + 7.56817\cdot 10^{11}X^{6} - 4.9072\cdot 10^{10}X^{5} - 9.12317\cdot 10^{8}X^{4} + 4.02822\cdot 10^{8}X^{3} + 2.55515\cdot 10^{10}X^{2} + 2.67938\cdot 10^{12}X - 7.73315\cdot 10^{11}$$
$$= -7.73315\cdot 10^{11}B_{0,20} - 6.39346\cdot 10^{11}B_{1,20} - 5.05243\cdot 10^{11}B_{2,20} - 3.71005\cdot 10^{11}B_{3,20} - 2.36631\cdot 10^{11}B_{4,20} - 1.02126\cdot 10^{11}B_{5,20} + 3.25201\cdot 10^{10}B_{6,20} + 1.6729\cdot 10^{11}B_{7,20} + 3.02207\cdot 10^{11}B_{8,20} + 4.37239\cdot 10^{11}B_{9,20} + 5.72436\cdot 10^{11}B_{10,20} + 7.07722\cdot 10^{11}B_{11,20} + 8.43196\cdot 10^{11}B_{12,20} + 9.78781\cdot 10^{11}B_{13,20} + 1.11451\cdot 10^{12}B_{14,20} + 1.25036\cdot 10^{12}B_{15,20} + 1.38635\cdot 10^{12}B_{16,20} + 1.52248\cdot 10^{12}B_{17,20} + 1.65874\cdot 10^{12}B_{18,20} + 1.79514\cdot 10^{12}B_{19,20} + 1.93167\cdot 10^{12}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 2.7052 \cdot 10^{7}$.
Bounding polynomials M and m:

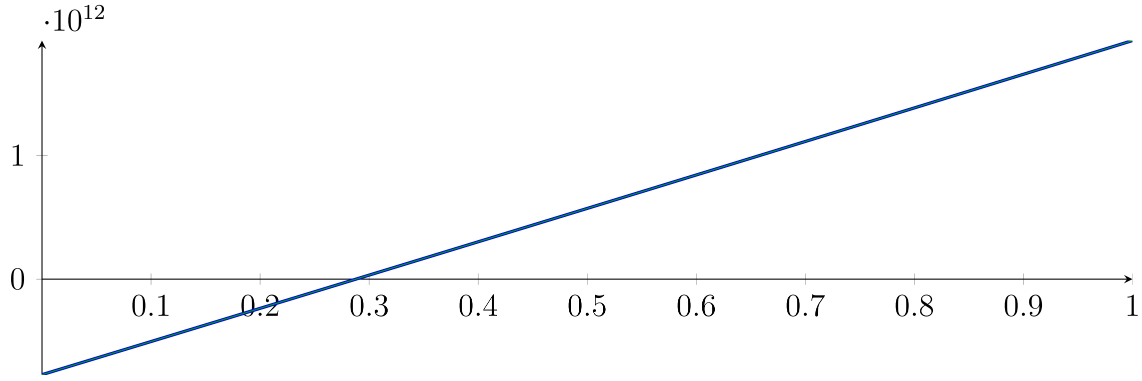
$$M = 4.19115 \cdot 10^{7} X^{3} + 2.55676 \cdot 10^{10} X^{2} + 2.67938 \cdot 10^{12} X - 7.73288 \cdot 10^{11}$$

$$m = 4.19115 \cdot 10^{7} X^{3} + 2.55676 \cdot 10^{10} X^{2} + 2.67938 \cdot 10^{12} X - 7.73342 \cdot 10^{11}$$

Root of M and m:

$$N(M) = \{-475.514, -134.812, 0.287817\} \qquad N(m) = \{-475.514, -134.812, 0.287837\}$$

Intersection intervals:



$[0.287817, 0.287837]$

Longest intersection interval: $2.00824 \cdot 10^{-05}$
$\Longrightarrow$ Selective recursion: interval 1: $[17.9554, 17.9554]$,

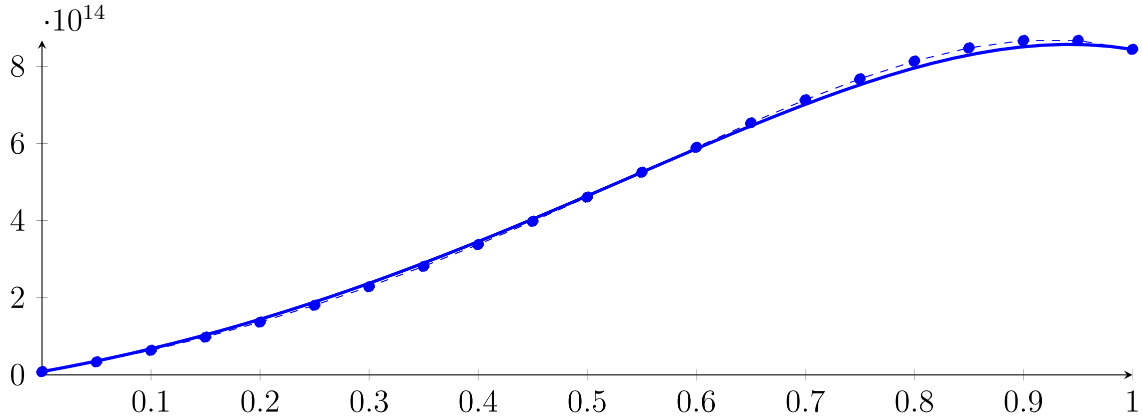
3.68 Recursion Branch 1 2 1 2 2 1 1 1 in Interval 1: $[17.9554, 17.9554]$

Found root in interval $[17.9554, 17.9554]$ at recursion depth 8!

3.69 Recursion Branch 1 2 1 2 2 2 on the Second Half $[17.9688, 18.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

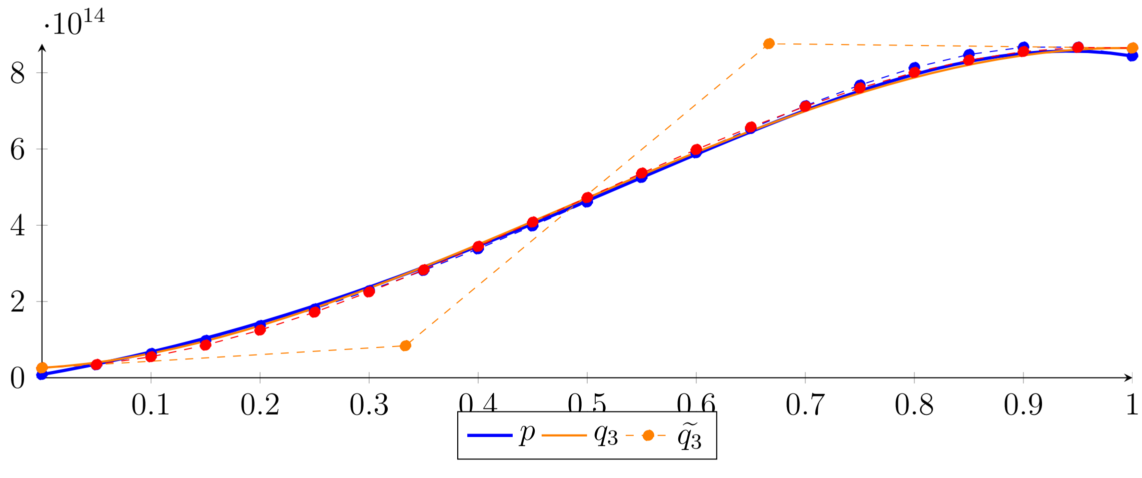
$$\begin{aligned} p = & -406735 X^{20} + 3.23454 \cdot 10^{6} X^{19} - 9.27253 \cdot 10^{6} X^{18} + 5.54286 \cdot 10^{7} X^{17} - 2.32388 \cdot 10^{8} X^{16} + 1.68297 \\ & \cdot 10^{8} X^{15} + 6.61085 \cdot 10^{7} X^{14} + 1.79234 \cdot 10^{9} X^{13} + 1.9977 \cdot 10^{10} X^{12} + 1.68452 \cdot 10^{11} X^{11} + 1.0336 \\ & \cdot 10^{12} X^{10} + 4.36677 \cdot 10^{12} X^{9} + 1.0574 \cdot 10^{13} X^{8} + 3.92297 \cdot 10^{11} X^{7} - 9.30488 \cdot 10^{13} X^{6} - 3.00689 \\ & \cdot 10^{14} X^{5} - 3.37885 \cdot 10^{14} X^{4} + 2.16815 \cdot 10^{14} X^{3} + 8.25489 \cdot 10^{14} X^{2} + 5.08276 \cdot 10^{14} X + 8.42928 \cdot 10^{12} \\ = & \, 8.42928 \cdot 10^{12} B_{0,20}(X) + 3.38431 \cdot 10^{13} B_{1,20}(X) + 6.36016 \cdot 10^{13} B_{2,20}(X) + 9.7895 \\ & \cdot 10^{13} B_{3,20}(X) + 1.36844 \cdot 10^{14} B_{4,20}(X) + 1.80479 \cdot 10^{14} B_{5,20}(X) + 2.28721 \cdot 10^{14} B_{6,20}(X) \\ & + 2.81356 \cdot 10^{14} B_{7,20}(X) + 3.38006 \cdot 10^{14} B_{8,20}(X) + 3.98106 \cdot 10^{14} B_{9,20}(X) + 4.60869 \\ & \cdot 10^{14} B_{10,20}(X) + 5.25254 \cdot 10^{14} B_{11,20}(X) + 5.89938 \cdot 10^{14} B_{12,20}(X) + 6.53281 \cdot 10^{14} B_{13,20}(X) \\ & + 7.13299 \cdot 10^{14} B_{14,20}(X) + 7.67635 \cdot 10^{14} B_{15,20}(X) + 8.13539 \cdot 10^{14} B_{16,20}(X) + 8.47861 \\ & \cdot 10^{14} B_{17,20}(X) + 8.67049 \cdot 10^{14} B_{18,20}(X) + 8.67168 \cdot 10^{14} B_{19,20}(X) + 8.43944 \cdot 10^{14} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_3 = -1.53794\cdot 10^{15}X^3 + 2.20298\cdot 10^{15}X^2 + 1.73728\cdot 10^{14}X + 2.60158\cdot 10^{13}$$
$$= 2.60158\cdot 10^{13}B_{0,3} + 8.3925\cdot 10^{13}B_{1,3} + 8.7616\cdot 10^{14}B_{2,3} + 8.64779\cdot 10^{14}B_{3,3}$$

$$\widetilde{q}_3 = -6.60507\cdot 10^{16}X^{20} + 6.63414\cdot 10^{17}X^{19} - 3.08649\cdot 10^{18}X^{18} + 8.82373\cdot 10^{18}X^{17} - 1.73374\cdot 10^{19}X^{16} + 2.48098\cdot 10^{19}X^{15} - 2.67308\cdot 10^{19}X^{14} + 2.2112\cdot 10^{19}X^{13} - 1.41943\cdot 10^{19}X^{12} + 7.1014\cdot 10^{18}X^{11} - 2.7668\cdot 10^{18}X^{10} + 8.35096\cdot 10^{17}X^9 - 1.93504\cdot 10^{17}X^8 + 3.40222\cdot 10^{16}X^7 - 4.48083\cdot 10^{15}X^6 + 4.35592\cdot 10^{14}X^5 - 3.07262\cdot 10^{13}X^4 - 1.53646\cdot 10^{15}X^3 + 2.20294\cdot 10^{15}X^2 + 1.73728\cdot 10^{14}X + 2.60158\cdot 10^{13}$$
$$= 2.60158\cdot 10^{13}B_{0,20} + 3.47022\cdot 10^{13}B_{1,20} + 5.4983\cdot 10^{13}B_{2,20} + 8.55104\cdot 10^{13}B_{3,20} + 1.2493\cdot 10^{14}B_{4,20} + 1.71911\cdot 10^{14}B_{5,20} + 2.25053\cdot 10^{14}B_{6,20} + 2.83129\cdot 10^{14}B_{7,20} + 3.44543\cdot 10^{14}B_{8,20} + 4.08356\cdot 10^{14}B_{9,20} + 4.72653\cdot 10^{14}B_{10,20} + 5.36738\cdot 10^{14}B_{11,20} + 5.98635\cdot 10^{14}B_{12,20} + 6.57519\cdot 10^{14}B_{13,20} + 7.11653\cdot 10^{14}B_{14,20} + 7.59927\cdot 10^{14}B_{15,20} + 8.00869\cdot 10^{14}B_{16,20} + 8.33184\cdot 10^{14}B_{17,20} + 8.55505\cdot 10^{14}B_{18,20} + 8.66486\cdot 10^{14}B_{19,20} + 8.64779\cdot 10^{14}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 2.08353\cdot 10^{13}$.

Bounding polynomials M and m:

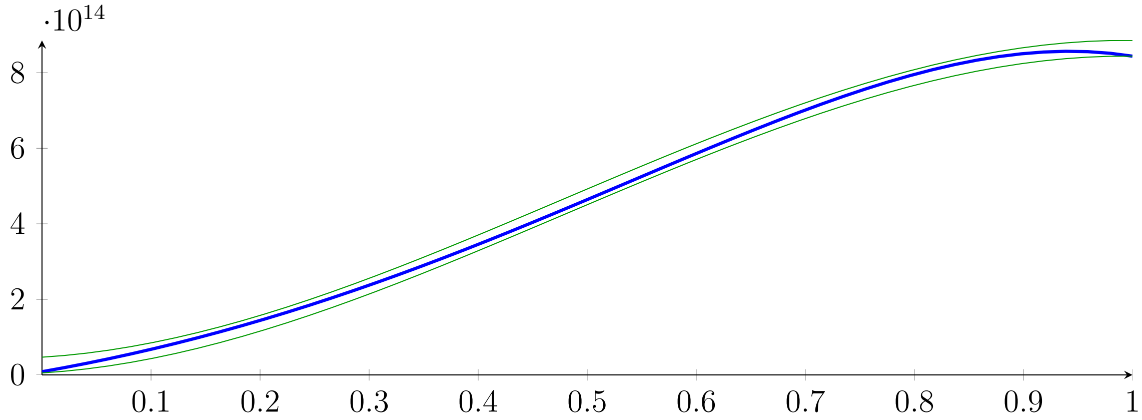
$$M = -1.53794\cdot 10^{15}X^3 + 2.20298\cdot 10^{15}X^2 + 1.73728\cdot 10^{14}X + 4.68511\cdot 10^{13}$$

$$m = -1.53794\cdot 10^{15}X^3 + 2.20298\cdot 10^{15}X^2 + 1.73728\cdot 10^{14}X + 5.18049\cdot 10^{12}$$

Root of M and m:

$$N(M) = \{1.51993\} \qquad N(m) = \{1.50877\}$$

Intersection intervals:

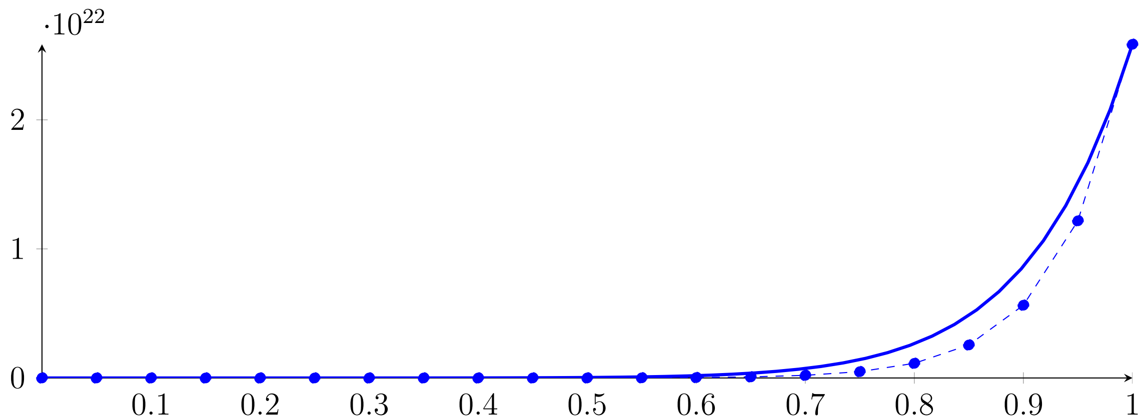


No intersection intervals with the x axis.

3.70 Recursion Branch 1 2 2 on the Second Half $[18.75, 25]$

Normalized monomial und Bézier representations and the Bézier polygon:

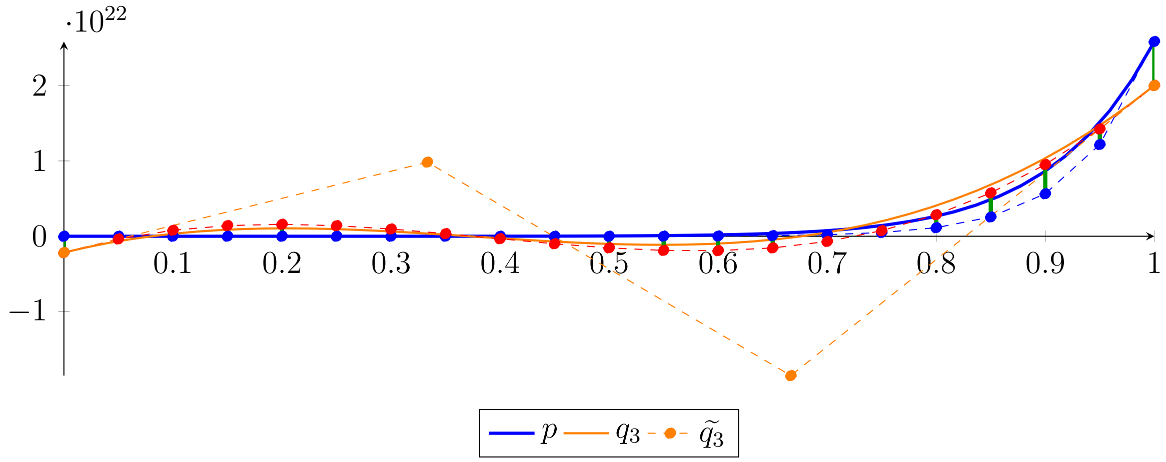
$$\begin{aligned} p &= 8.27177\cdot 10^{15}X^{20} + 2.18376\cdot 10^{17}X^{19} + 2.66802\cdot 10^{18}X^{18} + 2.00154\cdot 10^{19}X^{17} + 1.03147\cdot 10^{20}X^{16} \\ &\quad + 3.86992\cdot 10^{20}X^{15} + 1.09286\cdot 10^{21}X^{14} + 2.36814\cdot 10^{21}X^{13} + 3.97654\cdot 10^{21}X^{12} + 5.18646\cdot 10^{21}X^{11} \\ &\quad + 5.22867\cdot 10^{21}X^{10} + 4.02002\cdot 10^{21}X^{9} + 2.29598\cdot 10^{21}X^{8} + 9.25412\cdot 10^{20}X^{7} + 2.3318\cdot 10^{20}X^{6} + 2.12469 \\ &\quad \cdot 10^{19}X^{5} - 6.75399\cdot 10^{18}X^{4} - 2.49502\cdot 10^{18}X^{3} - 2.83854\cdot 10^{17}X^{2} - 3.71586\cdot 10^{15}X + 8.43944\cdot 10^{14} \\ &= 8.43944\cdot 10^{14}B_{0,20}(X) + 6.58151\cdot 10^{14}B_{1,20}(X) - 1.02161\cdot 10^{15}B_{2,20}(X) - 6.38396 \\ &\quad \cdot 10^{15}B_{3,20}(X) - 1.90115\cdot 10^{16}B_{4,20}(X) - 4.25105\cdot 10^{16}B_{5,20}(X) - 7.31244\cdot 10^{16}B_{6,20}(X) \\ &\quad - 7.43935\cdot 10^{16}B_{7,20}(X) + 9.63026\cdot 10^{16}B_{8,20}(X) + 8.81646\cdot 10^{17}B_{9,20}(X) + 3.50544 \\ &\quad \cdot 10^{18}B_{10,20}(X) + 1.11134\cdot 10^{19}B_{11,20}(X) + 3.13849\cdot 10^{19}B_{12,20}(X) + 8.23454\cdot 10^{19}B_{13,20}(X) \\ &\quad + 2.04998\cdot 10^{20}B_{14,20}(X) + 4.9022\cdot 10^{20}B_{15,20}(X) + 1.13504\cdot 10^{21}B_{16,20}(X) + 2.55855 \\ &\quad \cdot 10^{21}B_{17,20}(X) + 5.63734\cdot 10^{21}B_{18,20}(X) + 1.21777\cdot 10^{22}B_{19,20}(X) + 2.5852\cdot 10^{22}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 1.0719\cdot 10^{23}X^3 - 1.21006\cdot 10^{23}X^2 + 3.60039\cdot 10^{22}X - 2.16941\cdot 10^{21} \\ &= -2.16941\cdot 10^{21}B_{0,3} + 9.8319\cdot 10^{21}B_{1,3} - 1.85023\cdot 10^{22}B_{2,3} + 2.00179\cdot 10^{22}B_{3,3} \end{aligned}$$

$$\widetilde{q}_3 = 1.80398{\cdot}10^{24}X^{20} - 1.81562{\cdot}10^{25}X^{19} + 8.45864{\cdot}10^{25}X^{18} - 2.42016{\cdot}10^{26}X^{17} + 4.75759{\cdot}10^{26}X^{16} - 6.81126{\cdot}10^{26}X^{15} + 7.34525{\cdot}10^{26}X^{14} - 6.08765{\cdot}10^{26}X^{13} + 3.92201{\cdot}10^{26}X^{12} - 1.97436{\cdot}10^{26}X^{11} + 7.76669{\cdot}10^{25}X^{10} - 2.37621{\cdot}10^{25}X^{9} + 5.59694{\cdot}10^{24}X^{8} - 9.97598{\cdot}10^{23}X^{7} + 1.31059{\cdot}10^{23}X^{6} - 1.22427{\cdot}10^{22}X^{5} + 7.78735{\cdot}10^{20}X^{4} + 1.07158{\cdot}10^{23}X^{3} - 1.21006{\cdot}10^{23}X^{2} + 3.60039{\cdot}10^{22}X - 2.16941{\cdot}10^{21}$$
$$= -2.16941{\cdot}10^{21}B_{0,20} - 3.69217{\cdot}10^{20}B_{1,20} + 7.94107{\cdot}10^{20}B_{2,20} + 1.41456{\cdot}10^{21}B_{3,20} + 1.58629{\cdot}10^{21}B_{4,20} + 1.40284{\cdot}10^{21}B_{5,20} + 9.59703{\cdot}10^{20}B_{6,20} + 3.4743{\cdot}10^{20}B_{7,20} - 3.33169{\cdot}10^{20}B_{8,20} - 9.99274{\cdot}10^{20}B_{9,20} - 1.54131{\cdot}10^{21}B_{10,20} - 1.88324{\cdot}10^{21}B_{11,20} - 1.91354{\cdot}10^{21}B_{12,20} - 1.55262{\cdot}10^{21}B_{13,20} - 6.96384{\cdot}10^{20}B_{14,20} + 7.43271{\cdot}10^{20}B_{15,20} + 2.86331{\cdot}10^{21}B_{16,20} + 5.75656{\cdot}10^{21}B_{17,20} + 9.51744{\cdot}10^{21}B_{18,20} + 1.42399{\cdot}10^{22}B_{19,20} + 2.00179{\cdot}10^{22}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 5.83413{\cdot}10^{21}$.

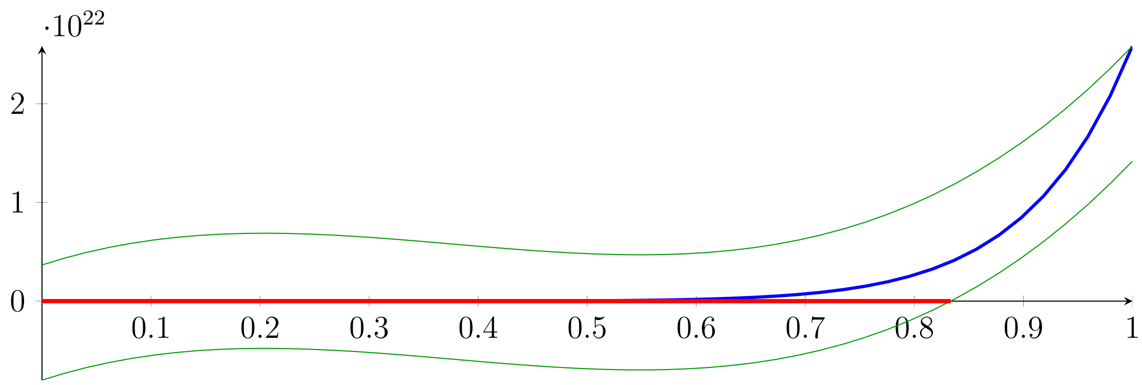
Bounding polynomials M and m:

$$M = 1.0719{\cdot}10^{23}X^3 - 1.21006{\cdot}10^{23}X^2 + 3.60039{\cdot}10^{22}X + 3.66471{\cdot}10^{21}$$
$$m = 1.0719{\cdot}10^{23}X^3 - 1.21006{\cdot}10^{23}X^2 + 3.60039{\cdot}10^{22}X - 8.00354{\cdot}10^{21}$$

Root of M and m:

$$N(M) = \{-0.0792161\} \qquad N(m) = \{0.833357\}$$

Intersection intervals:



$$[0, 0.833357]$$

Longest intersection interval: 0.833357

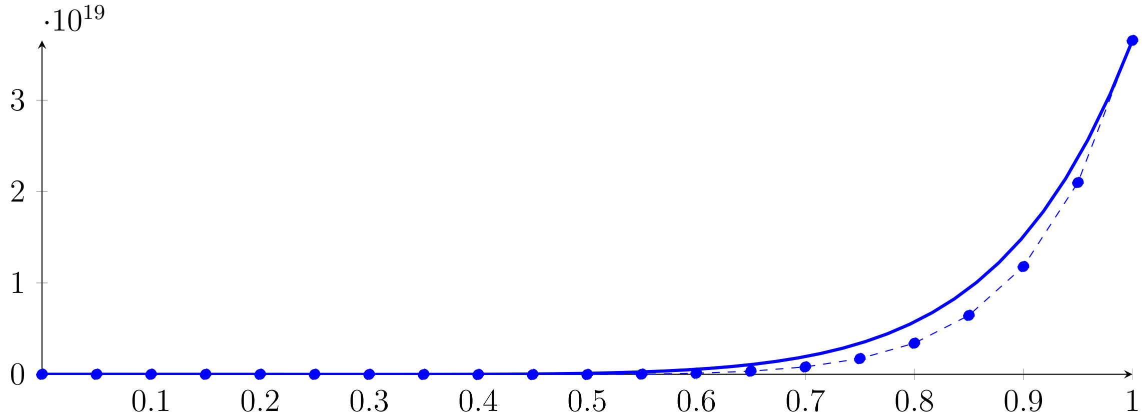
$\Longrightarrow$ Bisection: first half $[18.75, 21.875]$ und second half $[21.875, 25]$

3.71 Recursion Branch 1 2 2 1 on the First Half [18.75, 21.875]

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 7.80391\cdot10^{9}X^{20} + 4.16786\cdot10^{11}X^{19} + 1.01746\cdot10^{13}X^{18} + 1.52705\cdot10^{14}X^{17} + 1.57391\cdot10^{15}X^{16} + 1.18101\cdot10^{16}X^{15} + 6.67027\cdot10^{16}X^{14} + 2.8908\cdot10^{17}X^{13} + 9.70834\cdot10^{17}X^{12} + 2.53245\cdot10^{18}X^{11} + 5.10612\cdot10^{18}X^{10} + 7.8516\cdot10^{18}X^{9} + 8.96866\cdot10^{18}X^{8} + 7.22978\cdot10^{18}X^{7} + 3.64345\cdot10^{18}X^{6} + 6.63965\cdot10^{17}X^{5} - 4.22124\cdot10^{17}X^{4} - 3.11878\cdot10^{17}X^{3} - 7.09636\cdot10^{16}X^{2} - 1.85793\cdot10^{15}X + 8.43944\cdot10^{14}$$

$$= 8.43944\cdot10^{14}B_{0,20}(X) + 7.51047\cdot10^{14}B_{1,20}(X) + 2.84658\cdot10^{14}B_{2,20}(X) - 8.288\cdot10^{14}B_{3,20}(X) - 2.95003\cdot10^{15}B_{4,20}(X) - 6.48404\cdot10^{15}B_{5,20}(X) - 1.17433\cdot10^{16}B_{6,20}(X) - 1.86237\cdot10^{16}B_{7,20}(X) - 2.59286\cdot10^{16}B_{8,20}(X) - 3.00592\cdot10^{16}B_{9,20}(X) - 2.25952\cdot10^{16}B_{10,20}(X) + 1.40163\cdot10^{16}B_{11,20}(X) + 1.13942\cdot10^{17}B_{12,20}(X) + 3.40665\cdot10^{17}B_{13,20}(X) + 8.08159\cdot10^{17}B_{14,20}(X) + 1.71567\cdot10^{18}B_{15,20}(X) + 3.40411\cdot10^{18}B_{16,20}(X) + 6.44636\cdot10^{18}B_{17,20}(X) + 1.17905\cdot10^{19}B_{18,20}(X) + 2.09852\cdot10^{19}B_{19,20}(X) + 3.65302\cdot10^{19}B_{20,20}(X)$$

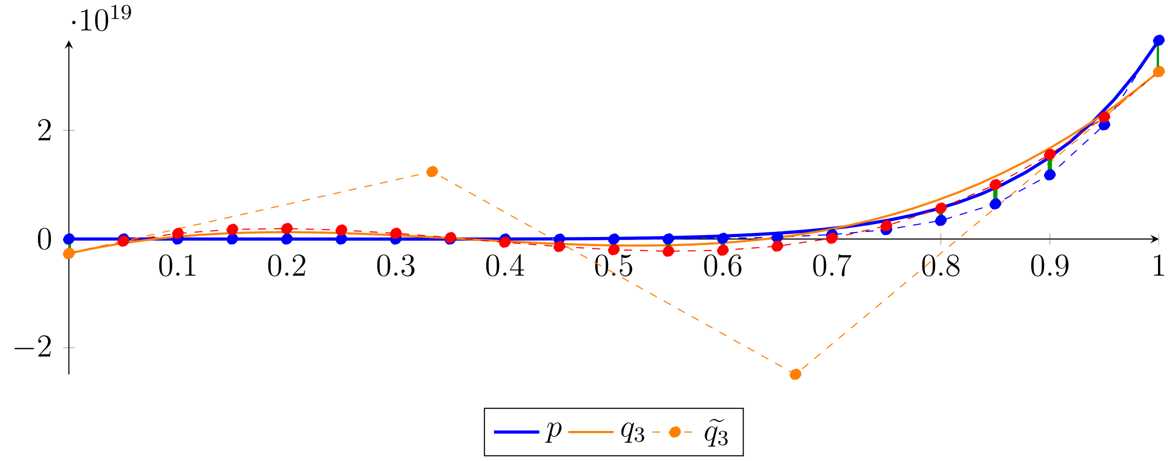


Degree reduction and raising:

$$q_3 = 1.45263\cdot10^{20}X^{3} - 1.56967\cdot10^{20}X^{2} + 4.51225\cdot10^{19}X - 2.64833\cdot10^{18}$$
$$= -2.64833\cdot10^{18}B_{0,3} + 1.23925\cdot10^{19}B_{1,3} - 2.48891\cdot10^{19}B_{2,3} + 3.07699\cdot10^{19}B_{3,3}$$

$$\widetilde{q}_3 = 2.38818\cdot10^{21}X^{20} - 2.4038\cdot10^{22}X^{19} + 1.11997\cdot10^{23}X^{18} - 3.20462\cdot10^{23}X^{17} + 6.30004\cdot10^{23}X^{16} - 9.02009\cdot10^{23}X^{15} + 9.7281\cdot10^{23}X^{14} - 8.06368\cdot10^{23}X^{13} + 5.1963\cdot10^{23}X^{12} - 2.61679\cdot10^{23}X^{11} + 1.02994\cdot10^{23}X^{10} - 3.1533\cdot10^{22}X^{9} + 7.43344\cdot10^{21}X^{8} - 1.32591\cdot10^{21}X^{7} + 1.74243\cdot10^{20}X^{6} - 1.62678\cdot10^{19}X^{5} + 1.03307\cdot10^{18}X^{4} + 1.45221\cdot10^{20}X^{3} - 1.56966\cdot10^{20}X^{2} + 4.51225\cdot10^{19}X - 2.64833\cdot10^{18}$$

$$= -2.64833\cdot10^{18}B_{0,20} - 3.92204\cdot10^{17}B_{1,20} + 1.03778\cdot10^{18}B_{2,20} + 1.76902\cdot10^{18}B_{3,20} + 1.92911\cdot10^{18}B_{4,20} + 1.6448\cdot10^{18}B_{5,20} + 1.04549\cdot10^{18}B_{6,20} + 2.53983\cdot10^{17}B_{7,20} - 5.9331\cdot10^{17}B_{8,20} - 1.3838\cdot10^{18}B_{9,20} - 1.96948\cdot10^{18}B_{10,20} - 2.24673\cdot10^{18}B_{11,20} - 2.06499\cdot10^{18}B_{12,20} - 1.31591\cdot10^{18}B_{13,20} + 1.41235\cdot10^{17}B_{14,20} + 2.42604\cdot10^{18}B_{15,20} + 5.66981\cdot10^{18}B_{16,20} + 9.99839\cdot10^{18}B_{17,20} + 1.55397\cdot10^{19}B_{18,20} + 2.24211\cdot10^{19}B_{19,20} + 3.07699\cdot10^{19}B_{20,20}$$



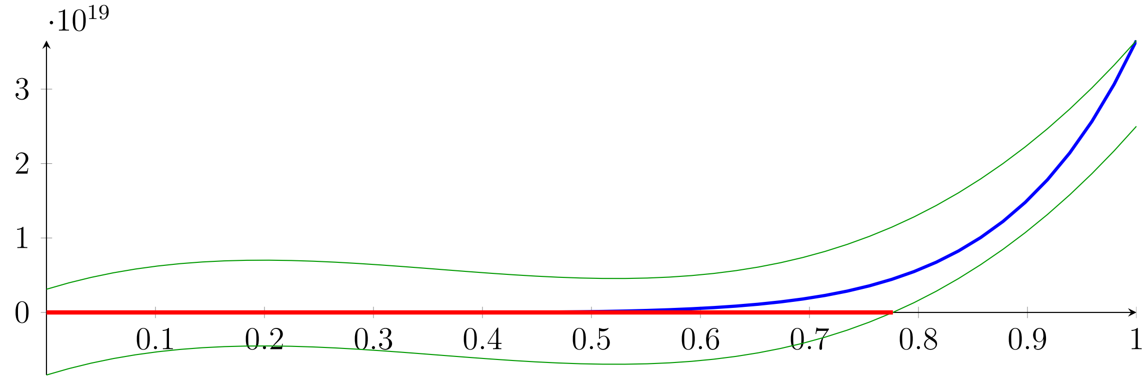
The maximum difference of the Bézier coefficients is $\delta = 5.76028 \cdot 10^{18}$.
Bounding polynomials M and m:

$$M = 1.45263 \cdot 10^{20} X^3 - 1.56967 \cdot 10^{20} X^2 + 4.51225 \cdot 10^{19} X + 3.11194 \cdot 10^{18}$$
$$m = 1.45263 \cdot 10^{20} X^3 - 1.56967 \cdot 10^{20} X^2 + 4.51225 \cdot 10^{19} X - 8.40861 \cdot 10^{18}$$

Root of M and m:

$$N(M) = \{-0.0570477\} \qquad N(m) = \{0.776558\}$$

Intersection intervals:



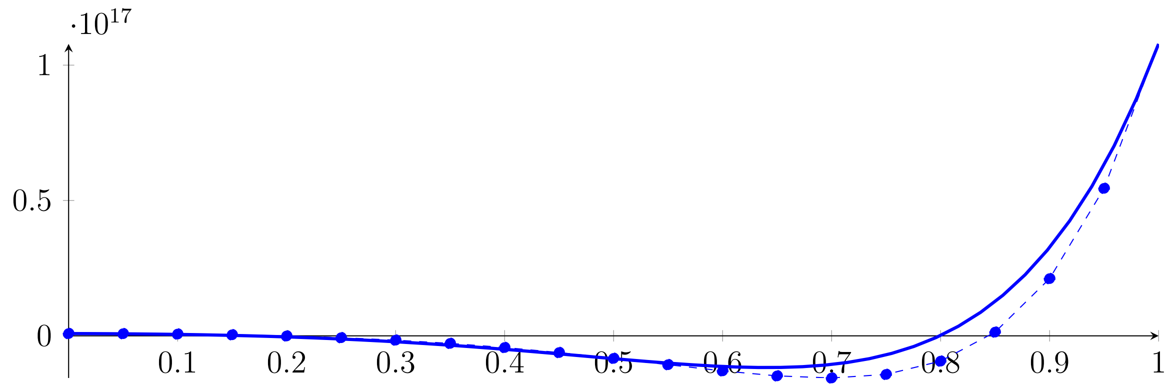
$[0, 0.776558]$

Longest intersection interval: 0.776558
$\Longrightarrow$ Bisection: first half $[18.75, 20.3125]$ und second half $[20.3125, 21.875]$

3.72 Recursion Branch 1 2 2 1 1 on the First Half $[18.75, 20.3125]$

Normalized monomial und Bézier representations and the Bézier polygon:

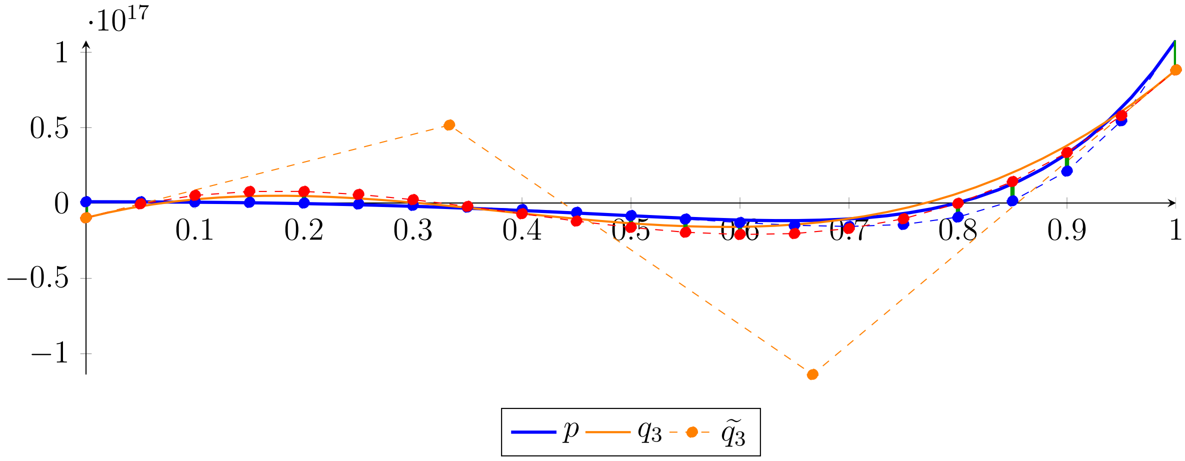
$$\begin{aligned} p = {} & 6.05184 \cdot 10^{6} X^{20} - 7.45545 \cdot 10^{7} X^{19} + 1.58964 \cdot 10^{8} X^{18} + 2.46862 \cdot 10^{8} X^{17} + 2.74811 \cdot 10^{10} X^{16} + 3.58417 \\ & \cdot 10^{11} X^{15} + 4.07176 \cdot 10^{12} X^{14} + 3.52881 \cdot 10^{13} X^{13} + 2.37021 \cdot 10^{14} X^{12} + 1.23655 \cdot 10^{15} X^{11} + 4.98645 \\ & \cdot 10^{15} X^{10} + 1.53352 \cdot 10^{16} X^{9} + 3.50338 \cdot 10^{16} X^{8} + 5.64827 \cdot 10^{16} X^{7} + 5.69288 \cdot 10^{16} X^{6} + 2.07489 \\ & \cdot 10^{16} X^{5} - 2.63828 \cdot 10^{16} X^{4} - 3.89847 \cdot 10^{16} X^{3} - 1.77409 \cdot 10^{16} X^{2} - 9.28966 \cdot 10^{14} X + 8.43944 \cdot 10^{14} \\ = {} & 8.43944 \cdot 10^{14} B_{0,20}(X) + 7.97496 \cdot 10^{14} B_{1,20}(X) + 6.57674 \cdot 10^{14} B_{2,20}(X) + 3.90283 \\ & \cdot 10^{14} B_{3,20}(X) - 4.43219 \cdot 10^{13} B_{4,20}(X) - 6.89889 \cdot 10^{14} B_{5,20}(X) - 1.59147 \cdot 10^{15} B_{6,20}(X) \\ & - 2.7904 \cdot 10^{15} B_{7,20}(X) - 4.31613 \cdot 10^{15} B_{8,20}(X) - 6.17337 \cdot 10^{15} B_{9,20}(X) - 8.32293 \\ & \cdot 10^{15} B_{10,20}(X) - 1.06535 \cdot 10^{16} B_{11,20}(X) - 1.29411 \cdot 10^{16} B_{12,20}(X) - 1.47922 \cdot 10^{16} B_{13,20}(X) \\ & - 1.55635 \cdot 10^{16} B_{14,20}(X) - 1.42523 \cdot 10^{16} B_{15,20}(X) - 9.34631 \cdot 10^{15} B_{16,20}(X) + 1.37971 \\ & \cdot 10^{15} B_{17,20}(X) + 2.11374 \cdot 10^{16} B_{18,20}(X) + 5.44898 \cdot 10^{16} B_{19,20}(X) + 1.07836 \cdot 10^{17} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_3 = 5.94419{\cdot}10^{17}X^3 - 6.8086{\cdot}10^{17}X^2 + 1.84498{\cdot}10^{17}X - 9.80248{\cdot}10^{15}$$
$$= -9.80248{\cdot}10^{15}B_{0,3} + 5.16969{\cdot}10^{16}B_{1,3} - 1.13757{\cdot}10^{17}B_{2,3} + 8.82541{\cdot}10^{16}B_{3,3}$$

$$\widetilde{q}_3 = 1.05364{\cdot}10^{19}X^{20} - 1.06027{\cdot}10^{20}X^{19} + 4.93921{\cdot}10^{20}X^{18} - 1.41317{\cdot}10^{21}X^{17} + 2.77808{\cdot}10^{21}X^{16} - 3.97734{\cdot}10^{21}X^{15} + 4.28899{\cdot}10^{21}X^{14} - 3.55409{\cdot}10^{21}X^{13} + 2.28891{\cdot}10^{21}X^{12} - 1.15149{\cdot}10^{21}X^{11} + 4.52501{\cdot}10^{20}X^{10} - 1.38243{\cdot}10^{20}X^9 + 3.25067{\cdot}10^{19}X^8 - 5.78605{\cdot}10^{18}X^7 + 7.60346{\cdot}10^{17}X^6 - 7.13188{\cdot}10^{16}X^5 + 4.58897{\cdot}10^{15}X^4 + 5.94228{\cdot}10^{17}X^3 - 6.80856{\cdot}10^{17}X^2 + 1.84498{\cdot}10^{17}X - 9.80248{\cdot}10^{15}$$
$$= -9.80248{\cdot}10^{15}B_{0,20} - 5.77572{\cdot}10^{14}B_{1,20} + 5.06388{\cdot}10^{15}B_{2,20} + 7.64313{\cdot}10^{15}B_{3,20} + 7.68239{\cdot}10^{15}B_{4,20} + 5.70019{\cdot}10^{15}B_{5,20} + 2.22646{\cdot}10^{15}B_{6,20} - 2.23755{\cdot}10^{15}B_{7,20} - 7.13082{\cdot}10^{15}B_{8,20} - 1.19974{\cdot}10^{16}B_{9,20} - 1.62251{\cdot}10^{16}B_{10,20} - 1.93975{\cdot}10^{16}B_{11,20} - 2.08912{\cdot}10^{16}B_{12,20} - 2.0269{\cdot}10^{16}B_{13,20} - 1.69503{\cdot}10^{16}B_{14,20} - 1.04488{\cdot}10^{16}B_{15,20} - 2.25491{\cdot}10^{14}B_{16,20} + 1.42338{\cdot}10^{16}B_{17,20} + 3.34528{\cdot}10^{16}B_{18,20} + 5.79524{\cdot}10^{16}B_{19,20} + 8.82541{\cdot}10^{16}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.95817{\cdot}10^{16}$.

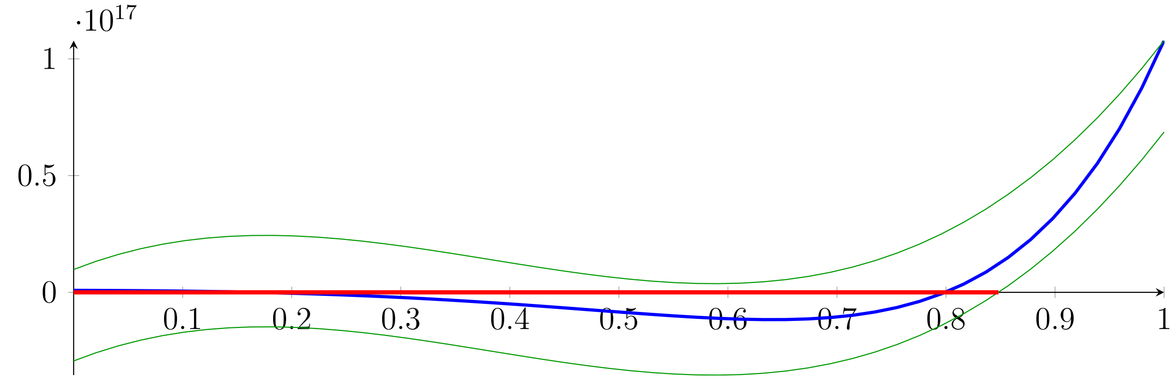
Bounding polynomials M and m:

$$M = 5.94419{\cdot}10^{17}X^3 - 6.8086{\cdot}10^{17}X^2 + 1.84498{\cdot}10^{17}X + 9.77921{\cdot}10^{15}$$
$$m = 5.94419{\cdot}10^{17}X^3 - 6.8086{\cdot}10^{17}X^2 + 1.84498{\cdot}10^{17}X - 2.93842{\cdot}10^{16}$$

Root of M and m:

$$N(M) = \{-0.0451759\} \qquad N(m) = \{0.8482\}$$

Intersection intervals:



$[0, 0.8482]$

Longest intersection interval: 0.8482

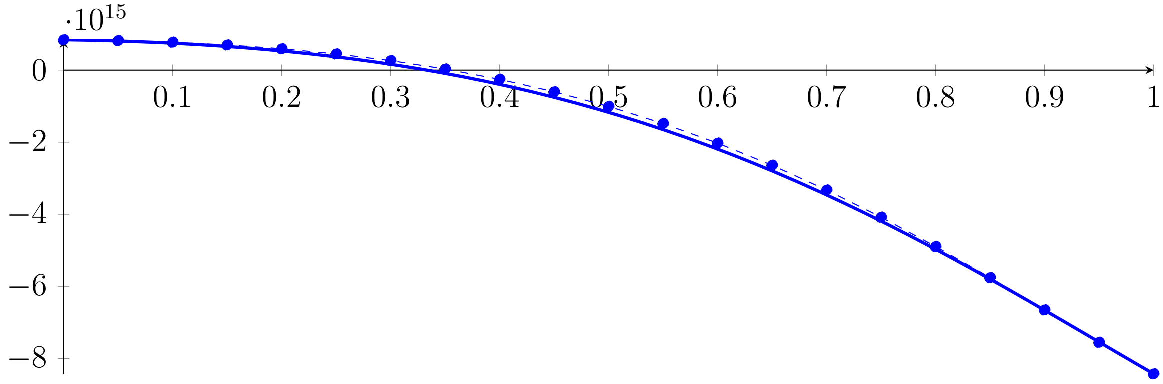
$\Longrightarrow$ Bisection: first half $[18.75, 19.5312]$ und second half $[19.5312, 20.3125]$

Bisection point is very near to a root?!?

3.73 Recursion Branch 1 2 2 1 1 1 on the First Half $[18.75, 19.5312]$

Normalized monomial und Bézier representations and the Bézier polygon:

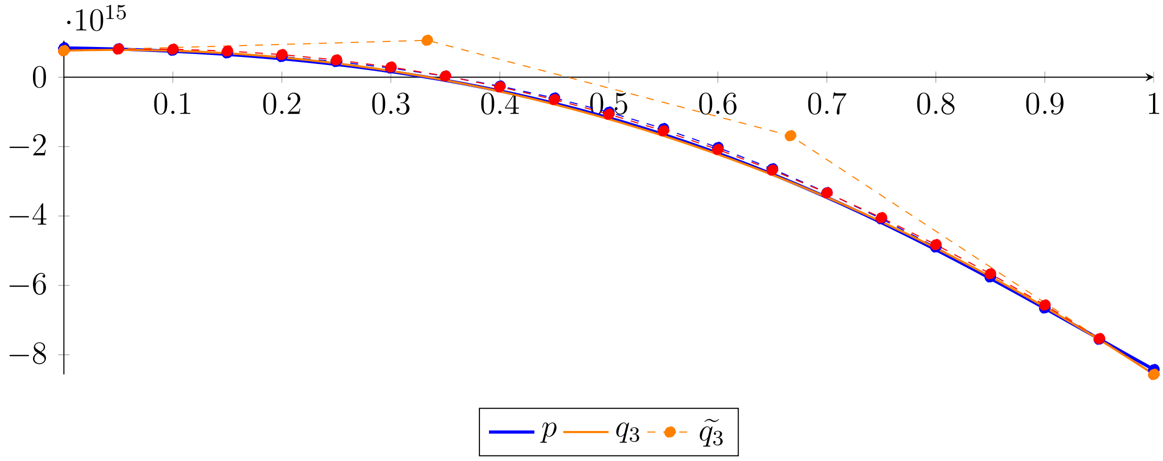
$$p = 671857X^{20} - 1.12793\cdot 10^{7}X^{19} + 1.00397\cdot 10^{7}X^{18} - 1.19008\cdot 10^{8}X^{17} + 2.98863\cdot 10^{8}X^{16} - 1.11857\cdot 10^{8}X^{15} + 2.53287\cdot 10^{8}X^{14} + 4.24137\cdot 10^{9}X^{13} + 5.7751\cdot 10^{10}X^{12} + 6.037\cdot 10^{11}X^{11} + 4.86952\cdot 10^{12}X^{10} + 2.99515\cdot 10^{13}X^{9} + 1.36851\cdot 10^{14}X^{8} + 4.41271\cdot 10^{14}X^{7} + 8.89513\cdot 10^{14}X^{6} + 6.48403\cdot 10^{14}X^{5} - 1.64892\cdot 10^{15}X^{4} - 4.87309\cdot 10^{15}X^{3} - 4.43522\cdot 10^{15}X^{2} - 4.64483\cdot 10^{14}X + 8.43944\cdot 10^{14}$$
$$= 8.43944\cdot 10^{14}B_{0,20}(X) + 8.2072\cdot 10^{14}B_{1,20}(X) + 7.74152\cdot 10^{14}B_{2,20}(X) + 6.99967\cdot 10^{14}B_{3,20}(X) + 5.93549\cdot 10^{14}B_{4,20}(X) + 4.49984\cdot 10^{14}B_{5,20}(X) + 2.64126\cdot 10^{14}B_{6,20}(X) + 3.06864\cdot 10^{13}B_{7,20}(X) - 2.55633\cdot 10^{14}B_{8,20}(X) - 5.99971\cdot 10^{14}B_{9,20}(X) - 1.00708\cdot 10^{15}B_{10,20}(X) - 1.48104\cdot 10^{15}B_{11,20}(X) - 2.02487\cdot 10^{15}B_{12,20}(X) - 2.64013\cdot 10^{15}B_{13,20}(X) - 3.32625\cdot 10^{15}B_{14,20}(X) - 4.07993\cdot 10^{15}B_{15,20}(X) - 4.89422\cdot 10^{15}B_{16,20}(X) - 5.75748\cdot 10^{15}B_{17,20}(X) - 6.65215\cdot 10^{15}B_{18,20}(X) - 7.55318\cdot 10^{15}B_{19,20}(X) - 8.42625\cdot 10^{15}B_{20,20}(X)$$



Degree reduction and raising:

$$q_3 = -1.07844\cdot 10^{15}X^{3} - 9.14551\cdot 10^{15}X^{2} + 8.9405\cdot 10^{14}X + 7.65325\cdot 10^{14}$$
$$= 7.65325\cdot 10^{14}B_{0,3} + 1.06334\cdot 10^{15}B_{1,3} - 1.68715\cdot 10^{15}B_{2,3} - 8.56458\cdot 10^{15}B_{3,3}$$

$$\widetilde{q}_3 = 5.66658\cdot 10^{16}X^{20} - 5.68031\cdot 10^{17}X^{19} + 2.64391\cdot 10^{18}X^{18} - 7.57613\cdot 10^{18}X^{17} + 1.49341\cdot 10^{19}X^{16} - 2.1425\cdot 10^{19}X^{15} + 2.30756\cdot 10^{19}X^{14} - 1.89709\cdot 10^{19}X^{13} + 1.19891\cdot 10^{19}X^{12} - 5.82412\cdot 10^{18}X^{11} + 2.16288\cdot 10^{18}X^{10} - 6.08647\cdot 10^{17}X^{9} + 1.2901\cdot 10^{17}X^{8} - 2.08596\cdot 10^{16}X^{7} + 2.70922\cdot 10^{15}X^{6} - 3.00297\cdot 10^{14}X^{5} + 2.79757\cdot 10^{13}X^{4} - 1.08026\cdot 10^{15}X^{3} - 9.14546\cdot 10^{15}X^{2} + 8.9405\cdot 10^{14}X + 7.65325\cdot 10^{14}$$
$$= 7.65325\cdot 10^{14}B_{0,20} + 8.10027\cdot 10^{14}B_{1,20} + 8.06596\cdot 10^{14}B_{2,20} + 7.54083\cdot 10^{14}B_{3,20} + 6.51546\cdot 10^{14}B_{4,20} + 4.98031\cdot 10^{14}B_{5,20} + 2.92618\cdot 10^{14}B_{6,20} + 3.42782\cdot 10^{13}B_{7,20} - 2.77724\cdot 10^{14}B_{8,20} - 6.4471\cdot 10^{14}B_{9,20} - 1.06712\cdot 10^{15}B_{10,20} - 1.54645\cdot 10^{15}B_{11,20} - 2.08315\cdot 10^{15}B_{12,20} - 2.6786\cdot 10^{15}B_{13,20} - 3.33338\cdot 10^{15}B_{14,20} - 4.04868\cdot 10^{15}B_{15,20} - 4.82531\cdot 10^{15}B_{16,20} - 5.66428\cdot 10^{15}B_{17,20} - 6.56651\cdot 10^{15}B_{18,20} - 7.53296\cdot 10^{15}B_{19,20} - 8.56458\cdot 10^{15}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.38328{\cdot}10^{14}$.

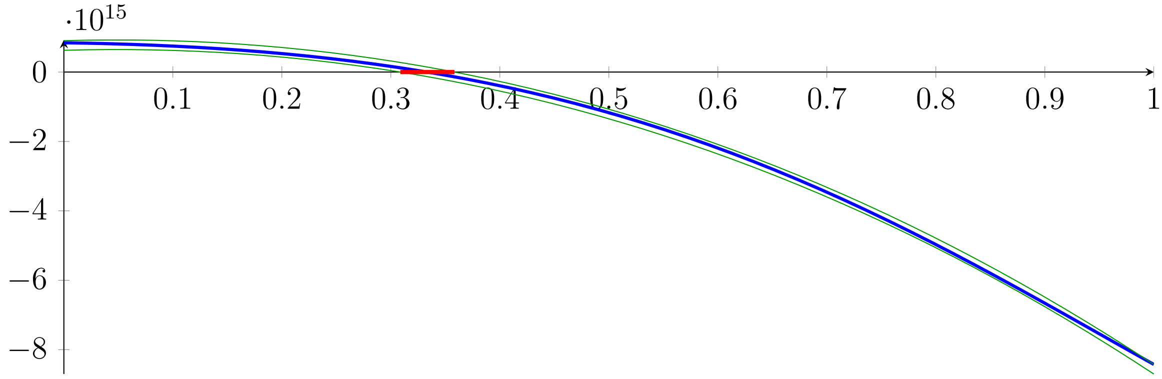
Bounding polynomials M and m:

$$M = -1.07844{\cdot}10^{15}X^3 - 9.14551{\cdot}10^{15}X^2 + 8.9405{\cdot}10^{14}X + 9.03653{\cdot}10^{14}$$
$$m = -1.07844{\cdot}10^{15}X^3 - 9.14551{\cdot}10^{15}X^2 + 8.9405{\cdot}10^{14}X + 6.26997{\cdot}10^{14}$$

Root of M and m:

$$N(M) = \{-8.56569, -0.272985, 0.358348\} \qquad N(m) = \{-8.56915, -0.219821, 0.308648\}$$

Intersection intervals:



$$[0.308648, 0.358348]$$

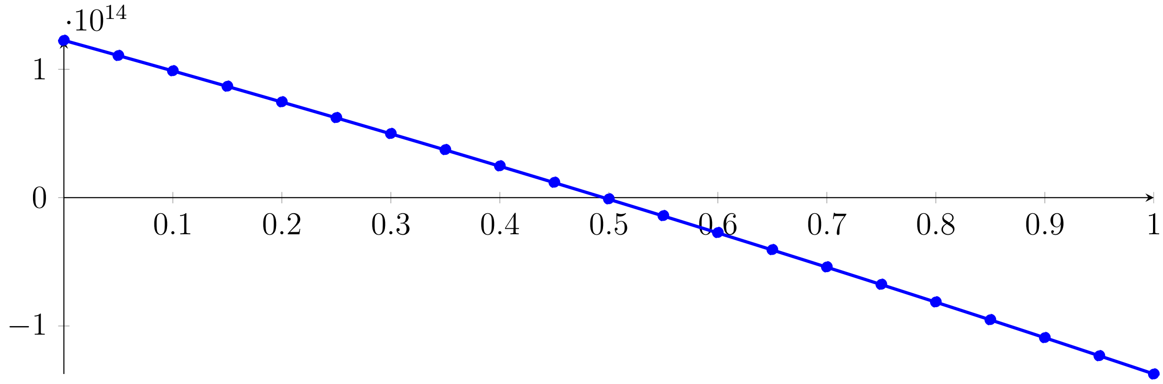
Longest intersection interval: 0.0497003

$\Longrightarrow$ Selective recursion: interval 1: $[18.9911, 19.03]$,

3.74 Recursion Branch 1 2 2 1 1 1 1 in Interval 1: $[18.9911, 19.03]$

Normalized monomial und Bézier representations and the Bézier polygon:

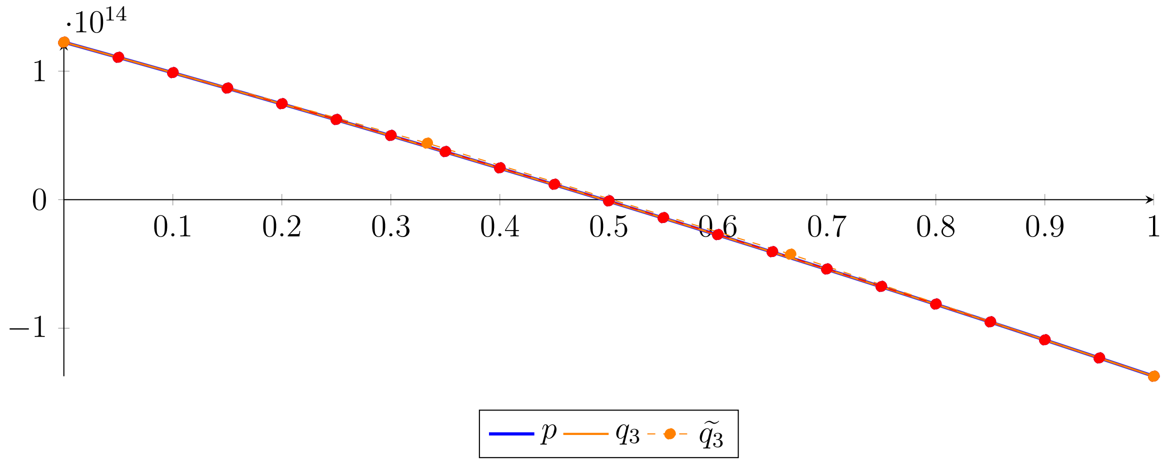
$$\begin{aligned} p = &-12147.6X^{20} - 184148X^{19} - 904513X^{18} - 69433.1X^{17} - 1.26084{\cdot}10^{7}X^{16} + 1.26954{\cdot}10^{7}X^{15} \\ &- 8.99595{\cdot}10^{6}X^{14} - 6.00053{\cdot}10^{6}X^{13} - 2.41351{\cdot}10^{7}X^{12} - 7.29051{\cdot}10^{6}X^{11} - 9.18584 \\ &{\cdot}10^{6}X^{10} - 1.8738{\cdot}10^{6}X^{9} - 74794.7X^{8} + 675878X^{7} + 3.44734{\cdot}10^{7}X^{6} + 1.0437{\cdot}10^{9}X^{5} \\ &+ 7.17142{\cdot}10^{9}X^{4} - 6.87992{\cdot}10^{11}X^{3} - 2.35863{\cdot}10^{13}X^{2} - 2.35659{\cdot}10^{14}X + 1.22535{\cdot}10^{14} \\ = &\ 1.22535{\cdot}10^{14}B_{0,20}(X) + 1.10752{\cdot}10^{14}B_{1,20}(X) + 9.88447{\cdot}10^{13}B_{2,20}(X) + 8.68129 \\ &{\cdot}10^{13}B_{3,20}(X) + 7.46557{\cdot}10^{13}B_{4,20}(X) + 6.23726{\cdot}10^{13}B_{5,20}(X) + 4.9963{\cdot}10^{13}B_{6,20}(X) \\ &+ 3.74262{\cdot}10^{13}B_{7,20}(X) + 2.47617{\cdot}10^{13}B_{8,20}(X) + 1.19688{\cdot}10^{13}B_{9,20}(X) - 9.52921 \\ &{\cdot}10^{11}B_{10,20}(X) - 1.40042{\cdot}10^{13}B_{11,20}(X) - 2.71856{\cdot}10^{13}B_{12,20}(X) - 4.04976{\cdot}10^{13}B_{13,20}(X) \\ &- 5.39409{\cdot}10^{13}B_{14,20}(X) - 6.75161{\cdot}10^{13}B_{15,20}(X) - 8.12237{\cdot}10^{13}B_{16,20}(X) - 9.50643 \\ &{\cdot}10^{13}B_{17,20}(X) - 1.09038{\cdot}10^{14}B_{18,20}(X) - 1.23147{\cdot}10^{14}B_{19,20}(X) - 1.3739{\cdot}10^{14}B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_3 = -6.70632\cdot10^{11}X^3 - 2.35981\cdot10^{13}X^2 - 2.35656\cdot10^{14}X + 1.22535\cdot10^{14}$$
$$= 1.22535\cdot10^{14}B_{0,3} + 4.39826\cdot10^{13}B_{1,3} - 4.24354\cdot10^{13}B_{2,3} - 1.3739\cdot10^{14}B_{3,3}$$

$$\widetilde{q}_3 = -8.91733\cdot10^{15}X^{20} + 8.93804\cdot10^{16}X^{19} - 4.14542\cdot10^{17}X^{18} + 1.18044\cdot10^{18}X^{17} - 2.30927\cdot10^{18}X^{16} + 3.29069\cdot10^{18}X^{15} - 3.5341\cdot10^{18}X^{14} + 2.91984\cdot10^{18}X^{13} - 1.87773\cdot10^{18}X^{12} + 9.4489\cdot10^{17}X^{11} - 3.71933\cdot10^{17}X^{10} + 1.13863\cdot10^{17}X^9 - 2.68018\cdot10^{16}X^8 + 4.76615\cdot10^{15}X^7 - 6.24707\cdot10^{14}X^6 + 5.82462\cdot10^{13}X^5 - 3.6501\cdot10^{12}X^4 - 5.27986\cdot10^{11}X^3 - 2.36012\cdot10^{13}X^2 - 2.35656\cdot10^{14}X + 1.22535\cdot10^{14}$$
$$= 1.22535\cdot10^{14}B_{0,20} + 1.10752\cdot10^{14}B_{1,20} + 9.88447\cdot10^{13}B_{2,20} + 8.6813\cdot10^{13}B_{3,20} + 7.46554\cdot10^{13}B_{4,20} + 6.23738\cdot10^{13}B_{5,20} + 4.99604\cdot10^{13}B_{6,20} + 3.74314\cdot10^{13}B_{7,20} + 2.47536\cdot10^{13}B_{8,20} + 1.19797\cdot10^{13}B_{9,20} - 9.65174\cdot10^{11}B_{10,20} - 1.39933\cdot10^{13}B_{11,20} - 2.71933\cdot10^{13}B_{12,20} - 4.04928\cdot10^{13}B_{13,20} - 5.39433\cdot10^{13}B_{14,20} - 6.75152\cdot10^{13}B_{15,20} - 8.12239\cdot10^{13}B_{16,20} - 9.50641\cdot10^{13}B_{17,20} - 1.09038\cdot10^{14}B_{18,20} - 1.23147\cdot10^{14}B_{19,20} - 1.3739\cdot10^{14}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.22534\cdot10^{10}$.

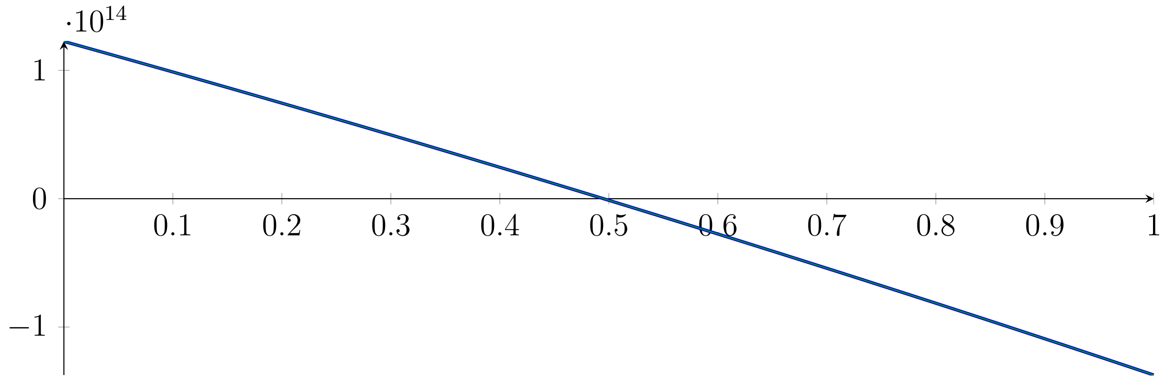
Bounding polynomials M and m:

$$M = -6.70632\cdot10^{11}X^3 - 2.35981\cdot10^{13}X^2 - 2.35656\cdot10^{14}X + 1.22547\cdot10^{14}$$
$$m = -6.70632\cdot10^{11}X^3 - 2.35981\cdot10^{13}X^2 - 2.35656\cdot10^{14}X + 1.22522\cdot10^{14}$$

Root of M and m:

$$N(M) = \{0.49513\} \qquad N(m) = \{0.495035\}$$

Intersection intervals:



$$[0.495035, 0.49513]$$

Longest intersection interval: $9.44327\cdot10^{-05}$

$\Longrightarrow$ Selective recursion: interval 1: $[19.0104, 19.0104]$,

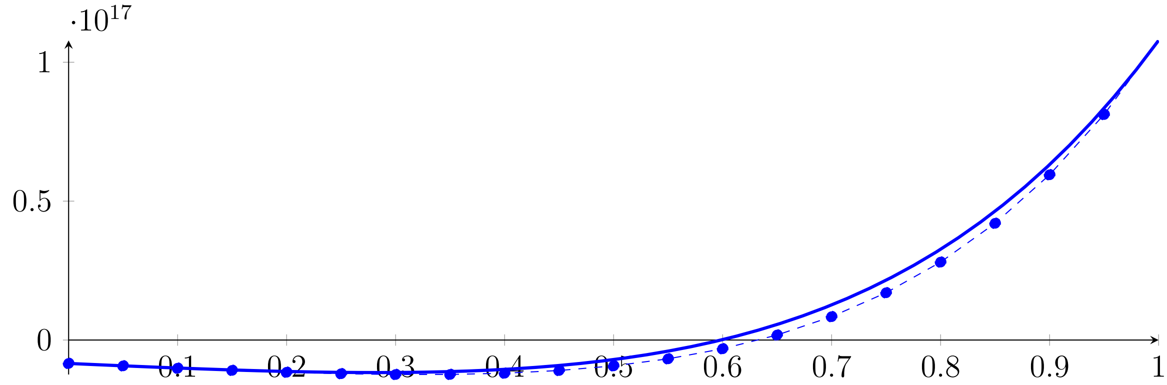
3.75 Recursion Branch 1 2 2 1 1 1 1 1 in Interval 1: $[19.0104, 19.0104]$

Found root in interval $[19.0104, 19.0104]$ at recursion depth 8!

3.76 Recursion Branch 1 2 2 1 1 2 on the Second Half $[19.5312, 20.3125]$

Normalized monomial und Bézier representations and the Bézier polygon:

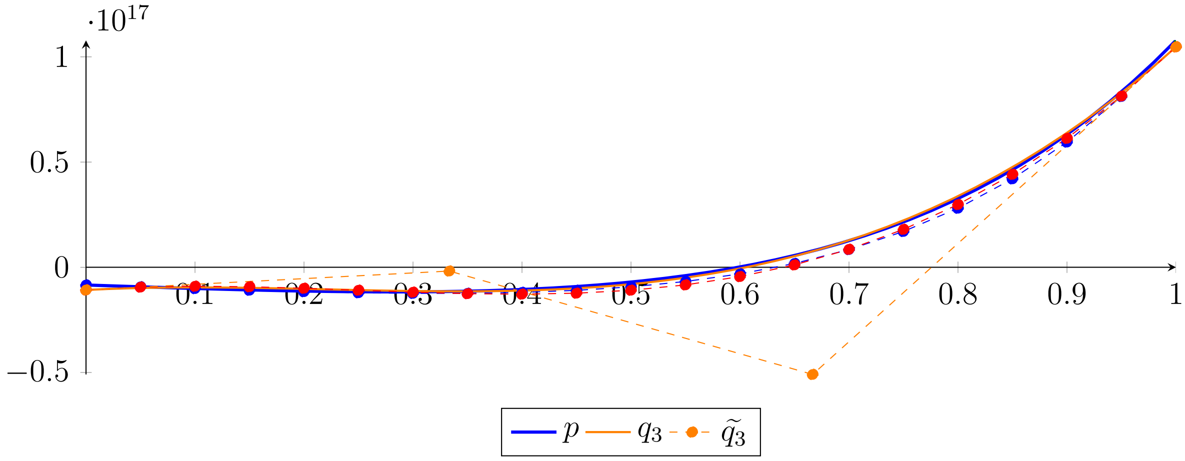
$$p = 9.60014\cdot10^{6}X^{20} - 1.47184\cdot10^{7}X^{19} + 2.89235\cdot10^{8}X^{18} - 1.01982\cdot10^{9}X^{17} + 6.86881\cdot10^{9}X^{16} - 5.88208\cdot10^{9}X^{15} + 2.56494\cdot10^{9}X^{14} + 1.03059\cdot10^{10}X^{13} + 1.4806\cdot10^{11}X^{12} + 1.7421\cdot10^{12}X^{11} + 1.68488\cdot10^{13}X^{10} + 1.28223\cdot10^{14}X^{9} + 7.60179\cdot10^{14}X^{8} + 3.45208\cdot10^{15}X^{7} + 1.16894\cdot10^{16}X^{6} + 2.82477\cdot10^{16}X^{5} + 4.49876\cdot10^{16}X^{4} + 3.91276\cdot10^{16}X^{3} + 5.31198\cdot10^{15}X^{2} - 1.74614\cdot10^{16}X - 8.42625\cdot10^{15}$$
$$= -8.42625\cdot10^{15}B_{0,20}(X) - 9.29932\cdot10^{15}B_{1,20}(X) - 1.01444\cdot10^{16}B_{2,20}(X) - 1.09273\cdot10^{16}B_{3,20}(X) - 1.16042\cdot10^{16}B_{4,20}(X) - 1.21206\cdot10^{16}B_{5,20}(X) - 1.24084\cdot10^{16}B_{6,20}(X) - 1.2384\cdot10^{16}B_{7,20}(X) - 1.19451\cdot10^{16}B_{8,20}(X) - 1.09678\cdot10^{16}B_{9,20}(X) - 9.30216\cdot10^{15}B_{10,20}(X) - 6.76818\cdot10^{15}B_{11,20}(X) - 3.15062\cdot10^{15}B_{12,20}(X) + 1.80691\cdot10^{15}B_{13,20}(X) + 8.40872\cdot10^{15}B_{14,20}(X) + 1.70147\cdot10^{16}B_{15,20}(X) + 2.80495\cdot10^{16}B_{16,20}(X) + 4.20121\cdot10^{16}B_{17,20}(X) + 5.94882\cdot10^{16}B_{18,20}(X) + 8.11628\cdot10^{16}B_{19,20}(X) + 1.07836\cdot10^{17}B_{20,20}(X)$$



Degree reduction and raising:

$$q_3 = 2.62964\cdot10^{17}X^{3} - 1.74434\cdot10^{17}X^{2} + 2.70042\cdot10^{16}X - 1.07969\cdot10^{16}$$
$$= -1.07969\cdot10^{16}B_{0,3} - 1.7955\cdot10^{15}B_{1,3} - 5.09388\cdot10^{16}B_{2,3} + 1.04737\cdot10^{17}B_{3,3}$$

$$\widetilde{q_3} = 5.01769\cdot10^{18}X^{20} - 5.04799\cdot10^{19}X^{19} + 2.35074\cdot10^{20}X^{18} - 6.72284\cdot10^{20}X^{17} + 1.32103\cdot10^{21}X^{16} - 1.89067\cdot10^{21}X^{15} + 2.03868\cdot10^{21}X^{14} - 1.69007\cdot10^{21}X^{13} + 1.08971\cdot10^{21}X^{12} - 5.49373\cdot10^{20}X^{11} + 2.16585\cdot10^{20}X^{10} - 6.64448\cdot10^{19}X^{9} + 1.56952\cdot10^{19}X^{8} - 2.80486\cdot10^{18}X^{7} + 3.69542\cdot10^{17}X^{6} - 3.47035\cdot10^{16}X^{5} + 2.23424\cdot10^{15}X^{4} + 2.62871\cdot10^{17}X^{3} - 1.74432\cdot10^{17}X^{2} + 2.70042\cdot10^{16}X - 1.07969\cdot10^{16}$$
$$= -1.07969\cdot10^{16}B_{0,20} - 9.4467\cdot10^{15}B_{1,20} - 9.01455\cdot10^{15}B_{2,20} - 9.26988\cdot10^{15}B_{3,20} - 9.98163\cdot10^{15}B_{4,20} - 1.09205\cdot10^{16}B_{5,20} - 1.18518\cdot10^{16}B_{6,20} - 1.25545\cdot10^{16}B_{7,20} - 1.2779\cdot10^{16}B_{8,20} - 1.23257\cdot10^{16}B_{9,20} - 1.09209\cdot10^{16}B_{10,20} - 8.38398\cdot10^{15}B_{11,20} - 4.43546\cdot10^{15}B_{12,20} + 1.11514\cdot10^{15}B_{13,20} + 8.52653\cdot10^{15}B_{14,20} + 1.80128\cdot10^{16}B_{15,20} + 2.98129\cdot10^{16}B_{16,20} + 4.41541\cdot10^{16}B_{17,20} + 6.12682\cdot10^{16}B_{18,20} + 8.13856\cdot10^{16}B_{19,20} + 1.04737\cdot10^{17}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 3.0988\cdot 10^{15}$.

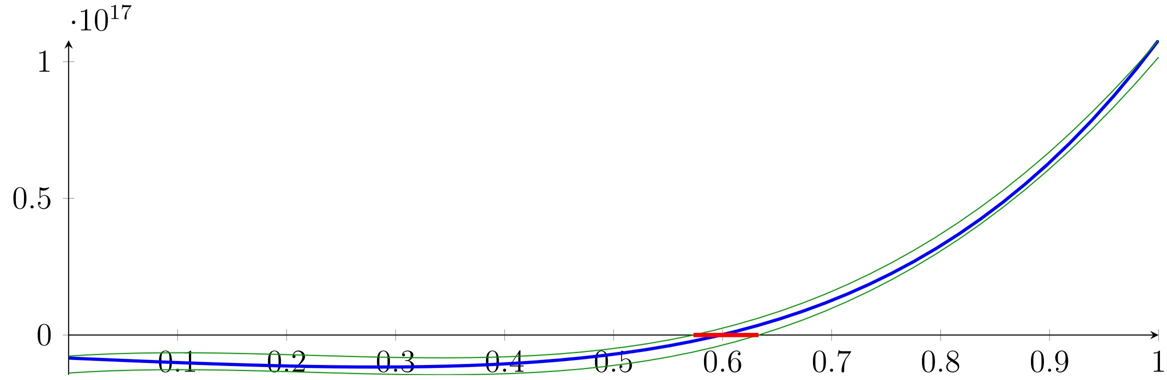
Bounding polynomials M and m:

$$M = 2.62964\cdot 10^{17}X^3 - 1.74434\cdot 10^{17}X^2 + 2.70042\cdot 10^{16}X - 7.69811\cdot 10^{15}$$
$$m = 2.62964\cdot 10^{17}X^3 - 1.74434\cdot 10^{17}X^2 + 2.70042\cdot 10^{16}X - 1.38957\cdot 10^{16}$$

Root of M and m:

$$N(M) = \{0.573283\} \qquad N(m) = \{0.63299\}$$

Intersection intervals:



$[0.573283, 0.63299]$

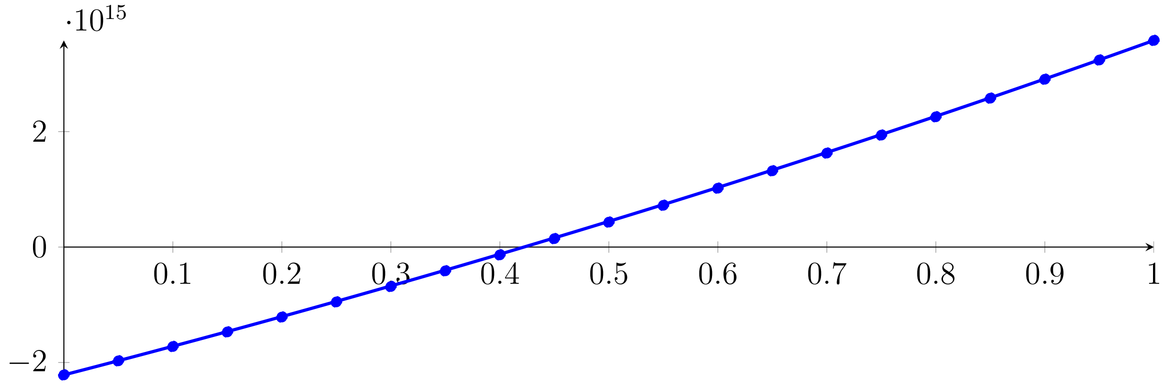
Longest intersection interval: 0.0597064

$\Longrightarrow$ Selective recursion: interval 1: $[19.9791, 20.0258]$,

3.77 Recursion Branch 1 2 2 1 1 2 1 in Interval 1: $[19.9791, 20.0258]$

Normalized monomial und Bézier representations and the Bézier polygon:

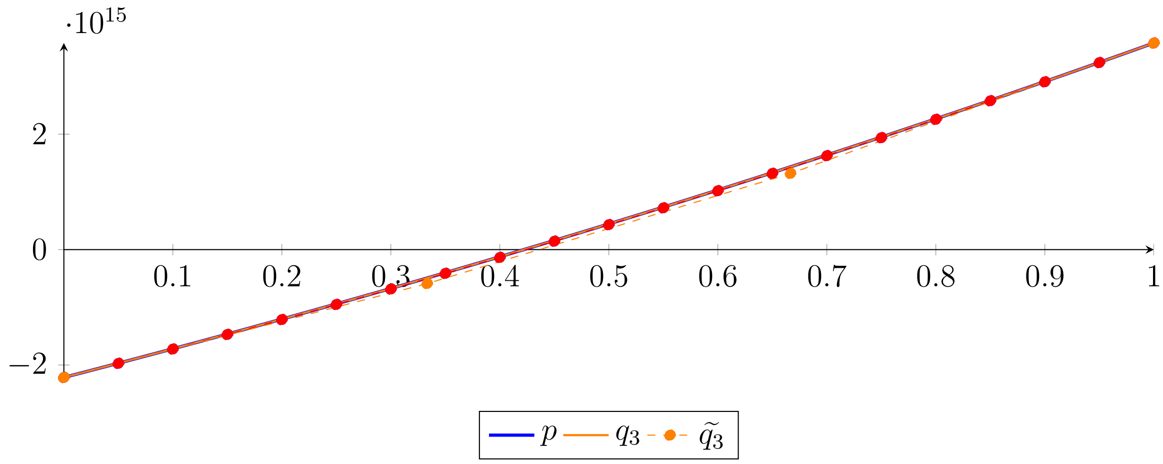
$$\begin{aligned}p = &-134475X^{20} + 6.21317\cdot 10^{6}X^{19} + 6.90636\cdot 10^{6}X^{18} + 5.31545\cdot 10^{7}X^{17} + 1.09364\cdot 10^{7}X^{16} - 7.87797\\ &\cdot 10^{7}X^{15} + 1.11987\cdot 10^{8}X^{14} + 1.0347\cdot 10^{8}X^{13} + 3.09036\cdot 10^{8}X^{12} + 1.29455\cdot 10^{8}X^{11} + 1.38428\\ &\cdot 10^{8}X^{10} + 3.00228\cdot 10^{7}X^{9} + 2.42492\cdot 10^{6}X^{8} + 2.37793\cdot 10^{7}X^{7} + 1.58696\cdot 10^{9}X^{6} + 7.76518\\ &\cdot 10^{10}X^{5} + 2.70982\cdot 10^{12}X^{4} + 6.28506\cdot 10^{13}X^{3} + 8.51381\cdot 10^{14}X^{2} + 4.87843\cdot 10^{15}X - 2.21528\cdot 10^{15}\\ = &-2.21528\cdot 10^{15}B_{0,20}(X) - 1.97136\cdot 10^{15}B_{1,20}(X) - 1.72296\cdot 10^{15}B_{2,20}(X) - 1.47002\\ &\cdot 10^{15}B_{3,20}(X) - 1.21249\cdot 10^{15}B_{4,20}(X) - 9.50311\cdot 10^{14}B_{5,20}(X) - 6.83428\cdot 10^{14}B_{6,20}(X)\\ &- 4.11783\cdot 10^{14}B_{7,20}(X) - 1.35317\cdot 10^{14}B_{8,20}(X) + 1.46027\cdot 10^{14}B_{9,20}(X) + 4.3231\\ &\cdot 10^{14}B_{10,20}(X) + 7.2359\cdot 10^{14}B_{11,20}(X) + 1.01993\cdot 10^{15}B_{12,20}(X) + 1.32138\cdot 10^{15}B_{13,20}(X)\\ &+ 1.62802\cdot 10^{15}B_{14,20}(X) + 1.9399\cdot 10^{15}B_{15,20}(X) + 2.25709\cdot 10^{15}B_{16,20}(X) + 2.57964\\ &\cdot 10^{15}B_{17,20}(X) + 2.90763\cdot 10^{15}B_{18,20}(X) + 3.24112\cdot 10^{15}B_{19,20}(X) + 3.58017\cdot 10^{15}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$q_3 = 6.84913{\cdot}10^{13}X^3 + 8.47707{\cdot}10^{14}X^2 + 4.87925{\cdot}10^{15}X - 2.21532{\cdot}10^{15}$$
$$= -2.21532{\cdot}10^{15}B_{0,3} - 5.88907{\cdot}10^{14}B_{1,3} + 1.32008{\cdot}10^{15}B_{2,3} + 3.58013{\cdot}10^{15}B_{3,3}$$

$$\widetilde{q}_3 = 1.24472{\cdot}10^{17}X^{20} - 1.24737{\cdot}10^{18}X^{19} + 5.78157{\cdot}10^{18}X^{18} - 1.64471{\cdot}10^{19}X^{17} + 3.21369{\cdot}10^{19}X^{16} - 4.57423{\cdot}10^{19}X^{15} + 4.90881{\cdot}10^{19}X^{14} - 4.05576{\cdot}10^{19}X^{13} + 2.61166{\cdot}10^{19}X^{12} - 1.31819{\cdot}10^{19}X^{11} + 5.21475{\cdot}10^{18}X^{10} - 1.6074{\cdot}10^{18}X^9 + 3.81281{\cdot}10^{17}X^8 - 6.81997{\cdot}10^{16}X^7 + 8.92376{\cdot}10^{15}X^6 - 8.15871{\cdot}10^{14}X^5 + 4.81127{\cdot}10^{13}X^4 + 6.6856{\cdot}10^{13}X^3 + 8.47737{\cdot}10^{14}X^2 + 4.87925{\cdot}10^{15}X - 2.21532{\cdot}10^{15}$$
$$= -2.21532{\cdot}10^{15}B_{0,20} - 1.97136{\cdot}10^{15}B_{1,20} - 1.72294{\cdot}10^{15}B_{2,20} - 1.46999{\cdot}10^{15}B_{3,20} - 1.21246{\cdot}10^{15}B_{4,20} - 9.5031{\cdot}10^{14}B_{5,20} - 6.83385{\cdot}10^{14}B_{6,20} - 4.11862{\cdot}10^{14}B_{7,20} - 1.35226{\cdot}10^{14}B_{8,20} + 1.45848{\cdot}10^{14}B_{9,20} + 4.32448{\cdot}10^{14}B_{10,20} + 7.23399{\cdot}10^{14}B_{11,20} + 1.02001{\cdot}10^{15}B_{12,20} + 1.32131{\cdot}10^{15}B_{13,20} + 1.62806{\cdot}10^{15}B_{14,20} + 1.93991{\cdot}10^{15}B_{15,20} + 2.25712{\cdot}10^{15}B_{16,20} + 2.57967{\cdot}10^{15}B_{17,20} + 2.90765{\cdot}10^{15}B_{18,20} + 3.24112{\cdot}10^{15}B_{19,20} + 3.58013{\cdot}10^{15}B_{20,20}$$



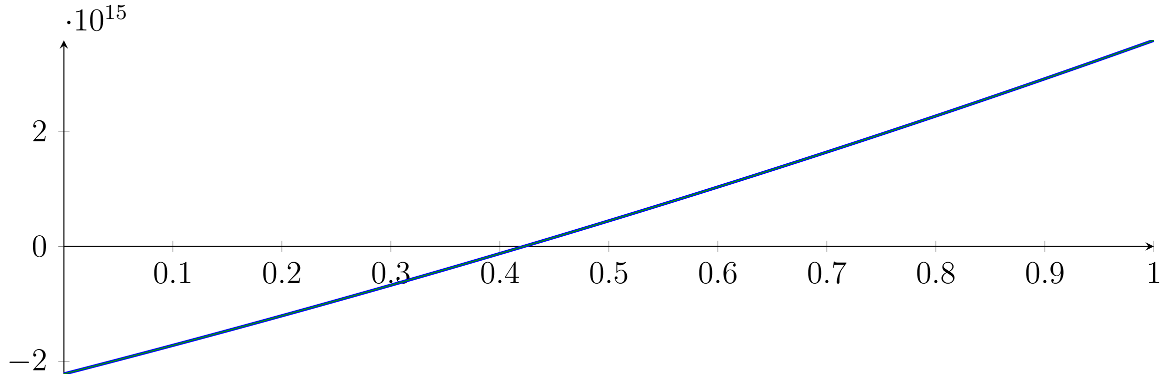
The maximum difference of the Bézier coefficients is $\delta = 1.90706{\cdot}10^{11}$.
Bounding polynomials M and m:

$$M = 6.84913{\cdot}10^{13}X^3 + 8.47707{\cdot}10^{14}X^2 + 4.87925{\cdot}10^{15}X - 2.21513{\cdot}10^{15}$$
$$m = 6.84913{\cdot}10^{13}X^3 + 8.47707{\cdot}10^{14}X^2 + 4.87925{\cdot}10^{15}X - 2.21551{\cdot}10^{15}$$

Root of M and m:

$$N(M) = \{0.421996\} \qquad N(m) = \{0.422064\}$$

Intersection intervals:



$[0.421996, 0.422064]$

Longest intersection interval: $6.773 \cdot 10^{-05}$
$\implies$ Selective recursion: interval 1: $[19.9988, 19.9988]$,

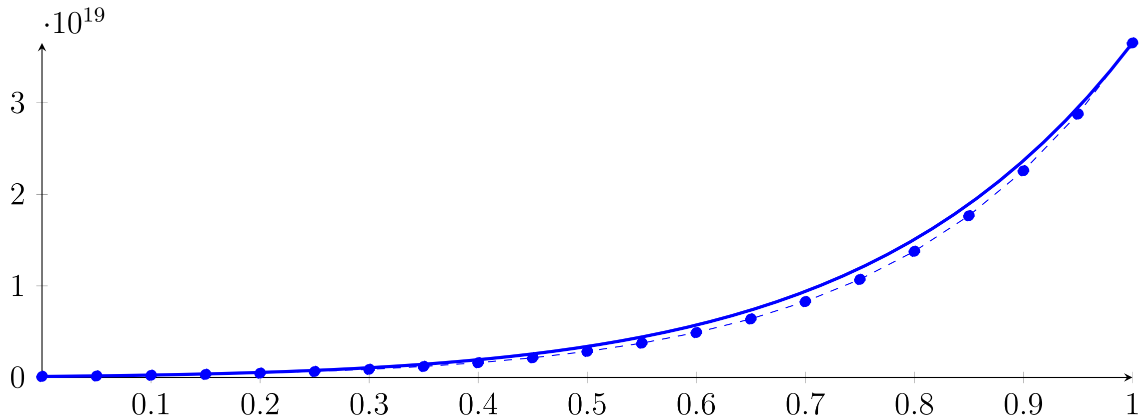
3.78 Recursion Branch 1 2 2 1 1 2 1 1 in Interval 1: $[19.9988, 19.9988]$

Found root in interval $[19.9988, 19.9988]$ at recursion depth 8!

3.79 Recursion Branch 1 2 2 1 2 on the Second Half $[20.3125, 21.875]$

Normalized monomial und Bézier representations and the Bézier polygon:

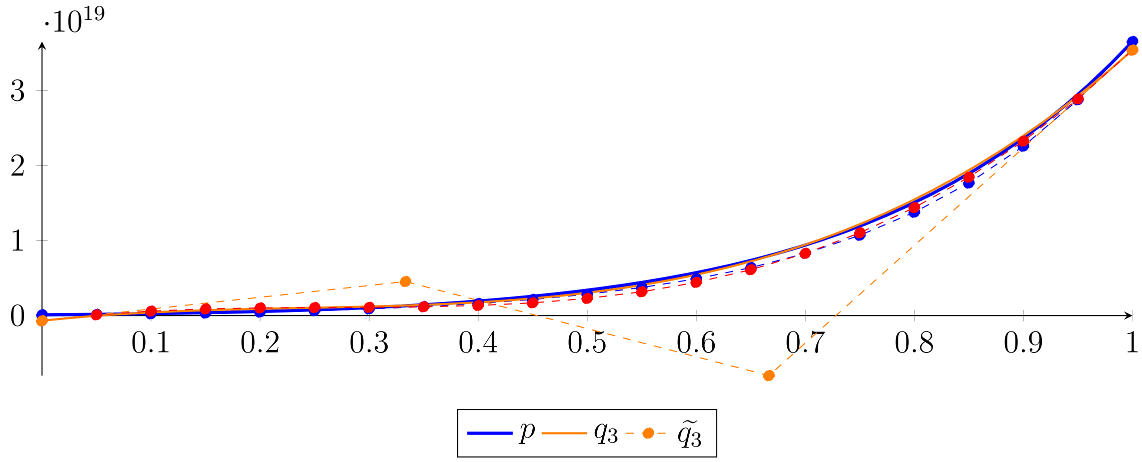
$$\begin{aligned} p = & -2.3982 \cdot 10^{9} X^{20} + 2.73788 \cdot 10^{10} X^{19} - 6.19725 \cdot 10^{10} X^{18} + 3.56643 \cdot 10^{11} X^{17} - 1.25055 \cdot 10^{12} X^{16} \\ & + 1.77598 \cdot 10^{12} X^{15} + 1.30077 \cdot 10^{13} X^{14} + 1.46709 \cdot 10^{14} X^{13} + 1.28145 \cdot 10^{15} X^{12} + 8.9278 \cdot 10^{15} X^{11} \\ & + 4.96984 \cdot 10^{16} X^{10} + 2.20845 \cdot 10^{17} X^{9} + 7.79096 \cdot 10^{17} X^{8} + 2.16012 \cdot 10^{18} X^{7} + 4.63365 \cdot 10^{18} X^{6} + 7.51293 \\ & \cdot 10^{18} X^{5} + 8.89517 \cdot 10^{18} X^{4} + 7.29479 \cdot 10^{18} X^{3} + 3.79879 \cdot 10^{18} X^{2} + 1.06692 \cdot 10^{18} X + 1.07836 \cdot 10^{17} \\ = & \ 1.07836 \cdot 10^{17} B_{0,20}(X) + 1.61182 \cdot 10^{17} B_{1,20}(X) + 2.34521 \cdot 10^{17} B_{2,20}(X) + 3.34253 \\ & \cdot 10^{17} B_{3,20}(X) + 4.68613 \cdot 10^{17} B_{4,20}(X) + 6.48155 \cdot 10^{17} B_{5,20}(X) + 8.86361 \cdot 10^{17} B_{6,20}(X) \\ & + 1.20039 \cdot 10^{18} B_{7,20}(X) + 1.61199 \cdot 10^{18} B_{8,20}(X) + 2.14872 \cdot 10^{18} B_{9,20}(X) + 2.84528 \\ & \cdot 10^{18} B_{10,20}(X) + 3.74538 \cdot 10^{18} B_{11,20}(X) + 4.90381 \cdot 10^{18} B_{12,20}(X) + 6.38923 \cdot 10^{18} B_{13,20}(X) \\ & + 8.28736 \cdot 10^{18} B_{14,20}(X) + 1.0705 \cdot 10^{19} B_{15,20}(X) + 1.37752 \cdot 10^{19} B_{16,20}(X) + 1.7663 \\ & \cdot 10^{19} B_{17,20}(X) + 2.25728 \cdot 10^{19} B_{18,20}(X) + 2.87577 \cdot 10^{19} B_{19,20}(X) + 3.65302 \cdot 10^{19} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = & \ 7.36639 \cdot 10^{19} X^{3} - 5.32582 \cdot 10^{19} X^{2} + 1.5664 \cdot 10^{19} X - 6.86781 \cdot 10^{17} \\ = & -6.86781 \cdot 10^{17} B_{0,3} + 4.53454 \cdot 10^{18} B_{1,3} - 7.99688 \cdot 10^{18} B_{2,3} + 3.53829 \cdot 10^{19} B_{3,3} \end{aligned}$$

$$
\begin{aligned}
\widetilde{q}_3 &= 7.86232\cdot 10^{20}X^{20} - 7.92928\cdot 10^{21}X^{19} + 3.69956\cdot 10^{22}X^{18} - 1.05958\cdot 10^{23}X^{17} + 2.08451\cdot 10^{23}X^{16} \\
&\quad - 2.98682\cdot 10^{23}X^{15} + 3.22548\cdot 10^{23}X^{14} - 2.68016\cdot 10^{23}X^{13} + 1.73459\cdot 10^{23}X^{12} - 8.79661\cdot 10^{22}X^{11} \\
&\quad + 3.4985\cdot 10^{22}X^{10} - 1.08622\cdot 10^{22}X^{9} + 2.60232\cdot 10^{21}X^{8} - 4.70557\cdot 10^{20}X^{7} + 6.19405\cdot 10^{19}X^{6} - 5.64298 \\
&\quad \cdot 10^{18}X^{5} + 3.32962\cdot 10^{17}X^{4} + 7.36524\cdot 10^{19}X^{3} - 5.3258\cdot 10^{19}X^{2} + 1.5664\cdot 10^{19}X - 6.86781\cdot 10^{17} \\
&= -6.86781\cdot 10^{17}B_{0,20} + 9.64169\cdot 10^{16}B_{1,20} + 5.99309\cdot 10^{17}B_{2,20} + 8.86504\cdot 10^{17}B_{3,20} + 1.02268 \\
&\quad \cdot 10^{18}B_{4,20} + 1.07221\cdot 10^{18}B_{5,20} + 1.10042\cdot 10^{18}B_{6,20} + 1.17033\cdot 10^{18}B_{7,20} + 1.34954\cdot 10^{18}B_{8,20} \\
&\quad + 1.69779\cdot 10^{18}B_{9,20} + 2.28649\cdot 10^{18}B_{10,20} + 3.17238\cdot 10^{18}B_{11,20} + 4.42781\cdot 10^{18}B_{12,20} \\
&\quad + 6.1111\cdot 10^{18}B_{13,20} + 8.29104\cdot 10^{18}B_{14,20} + 1.10299\cdot 10^{19}B_{15,20} + 1.43934\cdot 10^{19}B_{16,20} \\
&\quad + 1.84458\cdot 10^{19}B_{17,20} + 2.32517\cdot 10^{19}B_{18,20} + 2.88759\cdot 10^{19}B_{19,20} + 3.53829\cdot 10^{19}B_{20,20}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 1.14736\cdot 10^{18}$.

Bounding polynomials M and m:

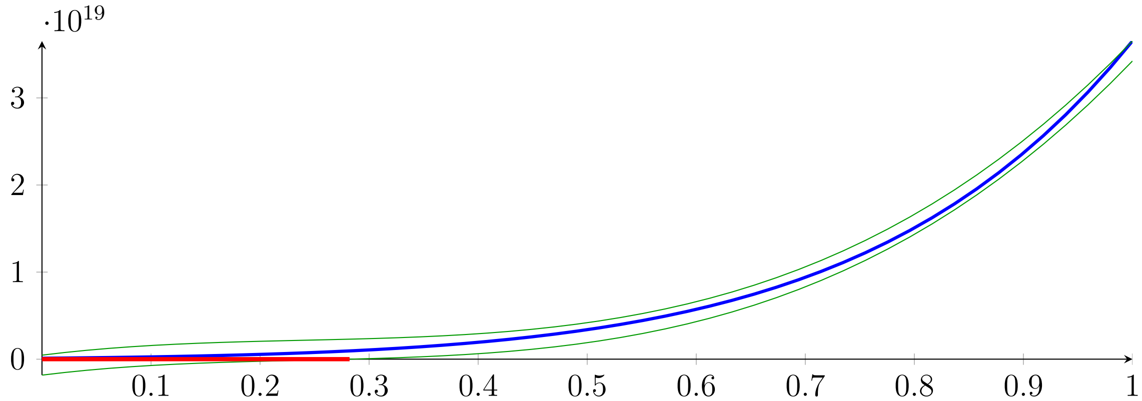
$$M = 7.36639\cdot 10^{19}X^3 - 5.32582\cdot 10^{19}X^2 + 1.5664\cdot 10^{19}X + 4.60579\cdot 10^{17}$$

$$m = 7.36639\cdot 10^{19}X^3 - 5.32582\cdot 10^{19}X^2 + 1.5664\cdot 10^{19}X - 1.83414\cdot 10^{18}$$

Root of M and m:

$$N(M) = \{-0.0268597\} \qquad N(m) = \{0.282075\}$$

Intersection intervals:



$$[0, 0.282075]$$

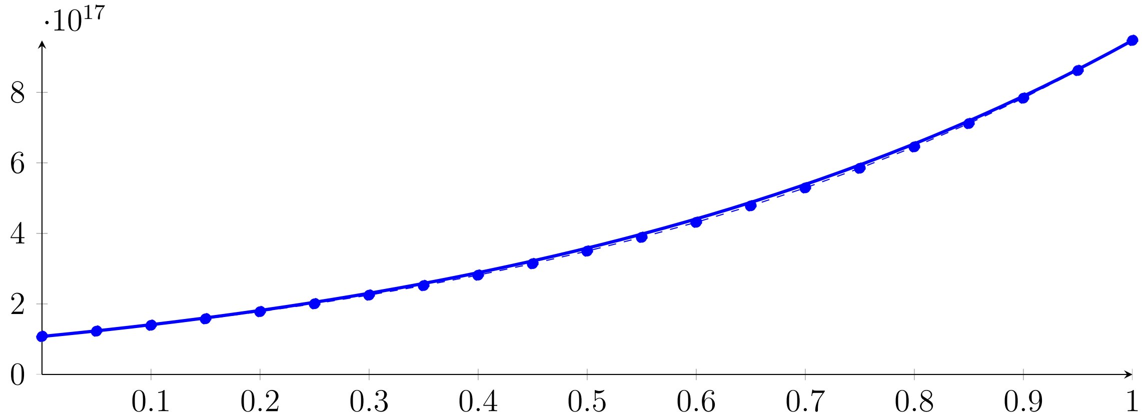
Longest intersection interval: 0.282075

$\Longrightarrow$ Selective recursion: interval 1: $[20.3125, 20.7532]$,

3.80 Recursion Branch 1 2 2 1 2 1 in Interval 1: $[20.3125, 20.7532]$

Normalized monomial und Bézier representations and the Bézier polygon:

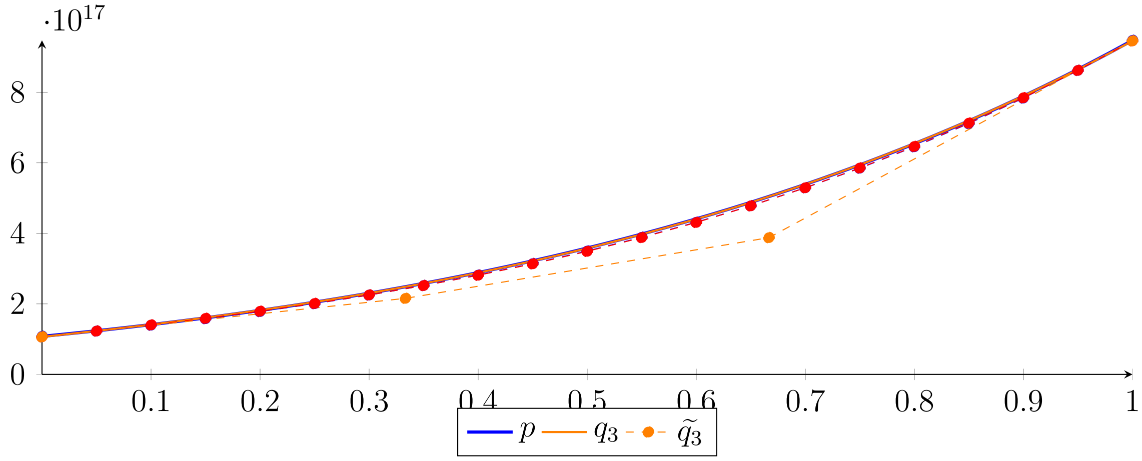
$$p = -3.12958\cdot10^{8}X^{20} + 2.29013\cdot10^{9}X^{19} - 8.50075\cdot10^{9}X^{18} + 4.15617\cdot10^{10}X^{17} - 1.90133\cdot10^{11}X^{16} + 1.41606\cdot10^{11}X^{15} - 5.02348\cdot10^{10}X^{14} - 1.76287\cdot10^{10}X^{13} - 1.12569\cdot10^{11}X^{12} + 7.60523\cdot10^{8}X^{11} + 1.28901\cdot10^{11}X^{10} + 2.49403\cdot10^{12}X^{9} + 3.12252\cdot10^{13}X^{8} + 3.06926\cdot10^{14}X^{7} + 2.33407\cdot10^{15}X^{6} + 1.34164\cdot10^{16}X^{5} + 5.63138\cdot10^{16}X^{4} + 1.63723\cdot10^{17}X^{3} + 3.02256\cdot10^{17}X^{2} + 3.00952\cdot10^{17}X + 1.07836\cdot10^{17}$$
$$= 1.07836\cdot10^{17}B_{0,20}(X) + 1.22883\cdot10^{17}B_{1,20}(X) + 1.39522\cdot10^{17}B_{2,20}(X) + 1.57895\cdot10^{17}B_{3,20}(X) + 1.78157\cdot10^{17}B_{4,20}(X) + 2.00477\cdot10^{17}B_{5,20}(X) + 2.25036\cdot10^{17}B_{6,20}(X) + 2.52028\cdot10^{17}B_{7,20}(X) + 2.81666\cdot10^{17}B_{8,20}(X) + 3.14176\cdot10^{17}B_{9,20}(X) + 3.49805\cdot10^{17}B_{10,20}(X) + 3.88816\cdot10^{17}B_{11,20}(X) + 4.31494\cdot10^{17}B_{12,20}(X) + 4.78147\cdot10^{17}B_{13,20}(X) + 5.29105\cdot10^{17}B_{14,20}(X) + 5.84724\cdot10^{17}B_{15,20}(X) + 6.45386\cdot10^{17}B_{16,20}(X) + 7.11502\cdot10^{17}B_{17,20}(X) + 7.83515\cdot10^{17}B_{18,20}(X) + 8.61902\cdot10^{17}B_{19,20}(X) + 9.47171\cdot10^{17}B_{20,20}(X)$$



Degree reduction and raising:

$$q_3 = 3.22672\cdot10^{17}X^{3} + 1.8909\cdot10^{17}X^{2} + 3.27395\cdot10^{17}X + 1.06473\cdot10^{17}$$
$$= 1.06473\cdot10^{17}B_{0,3} + 2.15605\cdot10^{17}B_{1,3} + 3.87766\cdot10^{17}B_{2,3} + 9.4563\cdot10^{17}B_{3,3}$$

$$\widetilde{q}_3 = -3.54112\cdot10^{19}X^{20} + 3.54951\cdot10^{20}X^{19} - 1.64846\cdot10^{21}X^{18} + 4.70532\cdot10^{21}X^{17} - 9.23232\cdot10^{21}X^{16} + 1.31933\cdot10^{22}X^{15} - 1.41935\cdot10^{22}X^{14} + 1.17189\cdot10^{22}X^{13} - 7.50321\cdot10^{21}X^{12} + 3.73966\cdot10^{21}X^{11} - 1.44884\cdot10^{21}X^{10} + 4.33784\cdot10^{20}X^{9} - 9.94861\cdot10^{19}X^{8} + 1.73289\cdot10^{19}X^{7} - 2.28246\cdot10^{18}X^{6} + 2.26635\cdot10^{17}X^{5} - 1.67691\cdot10^{16}X^{4} + 3.23535\cdot10^{17}X^{3} + 1.89066\cdot10^{17}X^{2} + 3.27395\cdot10^{17}X + 1.06473\cdot10^{17}$$
$$= 1.06473\cdot10^{17}B_{0,20} + 1.22843\cdot10^{17}B_{1,20} + 1.40208\cdot10^{17}B_{2,20} + 1.58852\cdot10^{17}B_{3,20} + 1.79054\cdot10^{17}B_{4,20} + 2.01108\cdot10^{17}B_{5,20} + 2.25271\cdot10^{17}B_{6,20} + 2.51889\cdot10^{17}B_{7,20} + 2.81113\cdot10^{17}B_{8,20} + 3.13445\cdot10^{17}B_{9,20} + 3.48867\cdot10^{17}B_{10,20} + 3.88012\cdot10^{17}B_{11,20} + 4.30827\cdot10^{17}B_{12,20} + 4.77876\cdot10^{17}B_{13,20} + 5.29231\cdot10^{17}B_{14,20} + 5.85307\cdot10^{17}B_{15,20} + 6.46319\cdot10^{17}B_{16,20} + 7.12579\cdot10^{17}B_{17,20} + 7.84361\cdot10^{17}B_{18,20} + 8.61951\cdot10^{17}B_{19,20} + 9.4563\cdot10^{17}B_{20,20}$$



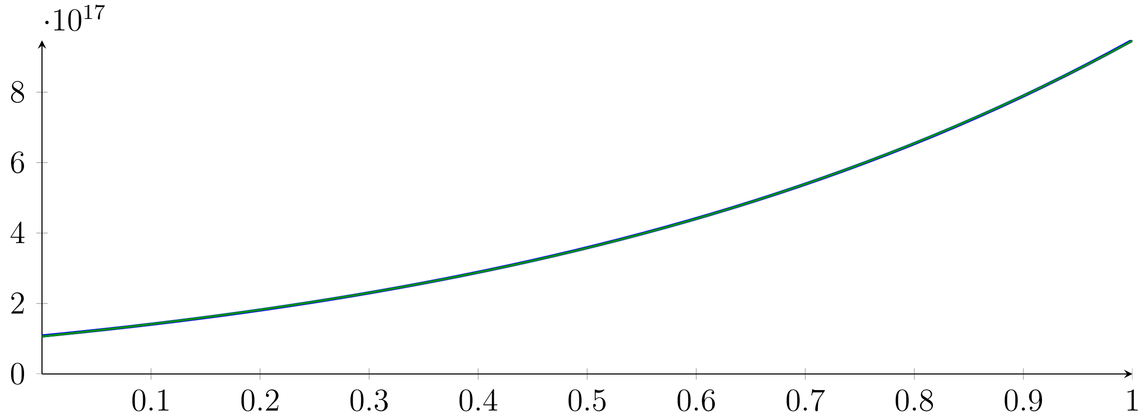
The maximum difference of the Bézier coefficients is $\delta = 1.54104 \cdot 10^{15}$.
Bounding polynomials M and m:

$$M = 3.22672 \cdot 10^{17} X^3 + 1.8909 \cdot 10^{17} X^2 + 3.27395 \cdot 10^{17} X + 1.08014 \cdot 10^{17}$$
$$m = 3.22672 \cdot 10^{17} X^3 + 1.8909 \cdot 10^{17} X^2 + 3.27395 \cdot 10^{17} X + 1.04932 \cdot 10^{17}$$

Root of M and m:

$$N(M) = \{-0.358747\} \qquad N(m) = \{-0.348956\}$$

Intersection intervals:

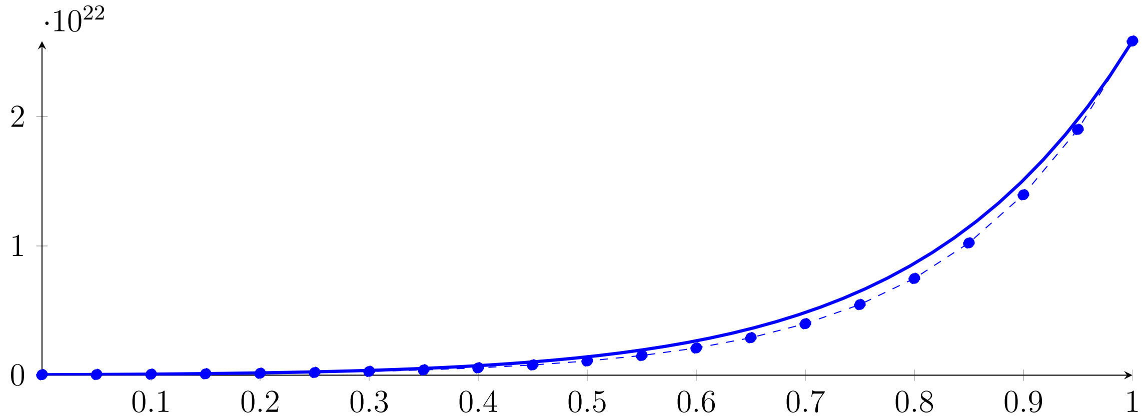


No intersection intervals with the x axis.

3.81 Recursion Branch 1 2 2 2 on the Second Half $[21.875, 25]$

Normalized monomial und Bézier representations and the Bézier polygon:

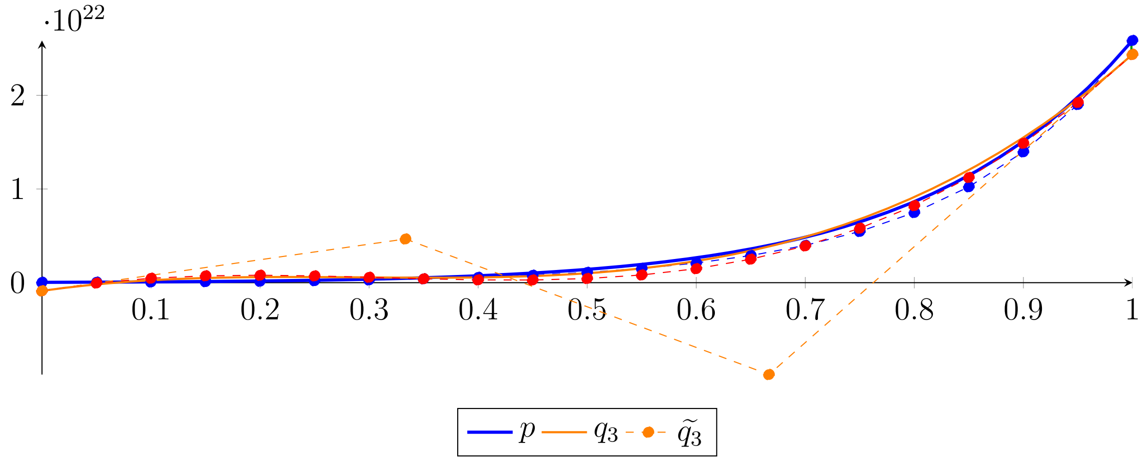
$$\begin{aligned} p = & -9.43719 \cdot 10^{11} X^{20} + 1.17887 \cdot 10^{13} X^{19} - 7.98914 \cdot 10^{12} X^{18} + 5.47338 \cdot 10^{14} X^{17} + 5.6893 \cdot 10^{15} X^{16} \\ & + 6.80909 \cdot 10^{16} X^{15} + 5.72765 \cdot 10^{17} X^{14} + 3.80691 \cdot 10^{18} X^{13} + 2.01926 \cdot 10^{19} X^{12} + 8.62526 \cdot 10^{19} X^{11} \\ & + 2.98009 \cdot 10^{20} X^{10} + 8.33374 \cdot 10^{20} X^{9} + 1.88062 \cdot 10^{21} X^{8} + 3.40128 \cdot 10^{21} X^{7} + 4.87441 \cdot 10^{21} X^{6} \\ & + 5.4405 \cdot 10^{21} X^{5} + 4.6091 \cdot 10^{21} X^{4} + 2.84983 \cdot 10^{21} X^{3} + 1.20656 \cdot 10^{21} X^{2} + 3.109 \cdot 10^{20} X + 3.65302 \cdot 10^{19} \\ = & \ 3.65302 \cdot 10^{19} B_{0,20}(X) + 5.20752 \cdot 10^{19} B_{1,20}(X) + 7.39705 \cdot 10^{19} B_{2,20}(X) + 1.04716 \\ & \cdot 10^{20} B_{3,20}(X) + 1.47763 \cdot 10^{20} B_{4,20}(X) + 2.07864 \cdot 10^{20} B_{5,20}(X) + 2.91553 \cdot 10^{20} B_{6,20}(X) \\ & + 4.07786 \cdot 10^{20} B_{7,20}(X) + 5.68821 \cdot 10^{20} B_{8,20}(X) + 7.91397 \cdot 10^{20} B_{9,20}(X) + 1.09833 \\ & \cdot 10^{21} B_{10,20}(X) + 1.52065 \cdot 10^{21} B_{11,20}(X) + 2.10052 \cdot 10^{21} B_{12,20}(X) + 2.89506 \cdot 10^{21} B_{13,20}(X) \\ & + 3.98159 \cdot 10^{21} B_{14,20}(X) + 5.46453 \cdot 10^{21} B_{15,20}(X) + 7.48476 \cdot 10^{21} B_{16,20}(X) + 1.0232 \\ & \cdot 10^{22} B_{17,20}(X) + 1.39612 \cdot 10^{22} B_{18,20}(X) + 1.90149 \cdot 10^{22} B_{19,20}(X) + 2.5852 \cdot 10^{22} B_{20,20}(X) \end{aligned}$$



Degree reduction and raising:

$$q_3 = 6.86472{\cdot}10^{22}X^3 - 6.00267{\cdot}10^{22}X^2 + 1.66112{\cdot}10^{22}X - 8.7336{\cdot}10^{20}$$
$$= -8.7336{\cdot}10^{20}B_{0,3} + 4.66371{\cdot}10^{21}B_{1,3} - 9.80813{\cdot}10^{21}B_{2,3} + 2.43583{\cdot}10^{22}B_{3,3}$$

$$\widetilde{q}_3 = 9.32268{\cdot}10^{23}X^{20} - 9.39093{\cdot}10^{24}X^{19} + 4.37784{\cdot}10^{25}X^{18} - 1.25314{\cdot}10^{26}X^{17} + 2.4643{\cdot}10^{26}X^{16} - 3.52944{\cdot}10^{26}X^{15} + 3.80858{\cdot}10^{26}X^{14} - 3.16015{\cdot}10^{26}X^{13} + 2.04004{\cdot}10^{26}X^{12} - 1.03029{\cdot}10^{26}X^{11} + 4.07247{\cdot}10^{25}X^{10} - 1.25406{\cdot}10^{25}X^9 + 2.97612{\cdot}10^{24}X^8 - 5.33913{\cdot}10^{23}X^7 + 7.0231{\cdot}10^{22}X^6 - 6.49642{\cdot}10^{21}X^5 + 4.01501{\cdot}10^{20}X^4 + 6.86318{\cdot}10^{22}X^3 - 6.00264{\cdot}10^{22}X^2 + 1.66112{\cdot}10^{22}X - 8.7336{\cdot}10^{20}$$
$$= -8.7336{\cdot}10^{20}B_{0,20} - 4.27986{\cdot}10^{19}B_{1,20} + 4.71834{\cdot}10^{20}B_{2,20} + 7.30741{\cdot}10^{20}B_{3,20} + 7.9421{\cdot}10^{20}B_{4,20} + 7.22189{\cdot}10^{20}B_{5,20} + 5.75686{\cdot}10^{20}B_{6,20} + 4.13083{\cdot}10^{20}B_{7,20} + 2.98114{\cdot}10^{20}B_{8,20} + 2.85212{\cdot}10^{20}B_{9,20} + 4.42633{\cdot}10^{20}B_{10,20} + 8.21283{\cdot}10^{20}B_{11,20} + 1.49046{\cdot}10^{21}B_{12,20} + 2.50294{\cdot}10^{21}B_{13,20} + 3.92402{\cdot}10^{21}B_{14,20} + 5.81097{\cdot}10^{21}B_{15,20} + 8.22546{\cdot}10^{21}B_{16,20} + 1.12271{\cdot}10^{22}B_{17,20} + 1.48764{\cdot}10^{22}B_{18,20} + 1.92334{\cdot}10^{22}B_{19,20} + 2.43583{\cdot}10^{22}B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 1.49368{\cdot}10^{21}$.

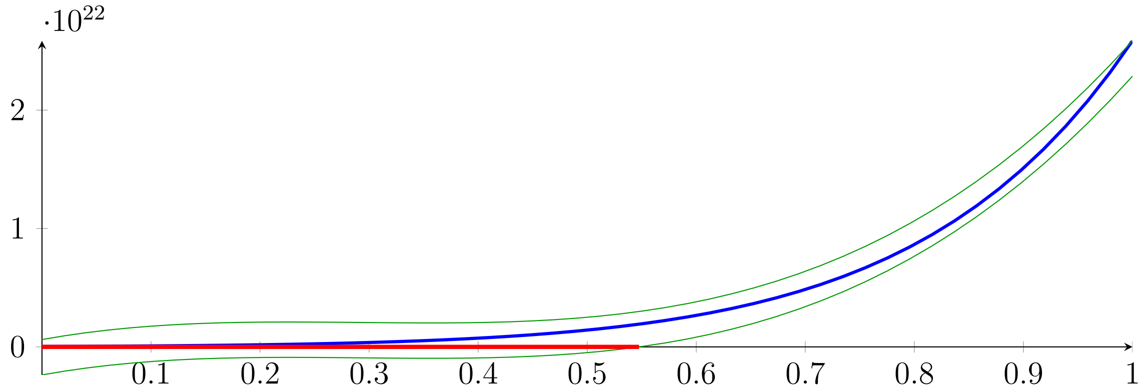
Bounding polynomials M and m:

$$M = 6.86472{\cdot}10^{22}X^3 - 6.00267{\cdot}10^{22}X^2 + 1.66112{\cdot}10^{22}X + 6.2032{\cdot}10^{20}$$
$$m = 6.86472{\cdot}10^{22}X^3 - 6.00267{\cdot}10^{22}X^2 + 1.66112{\cdot}10^{22}X - 2.36704{\cdot}10^{21}$$

Root of M and m:

$$N(M) = \{-0.0332073\} \qquad N(m) = \{0.547472\}$$

Intersection intervals:



$$[0, 0.547472]$$

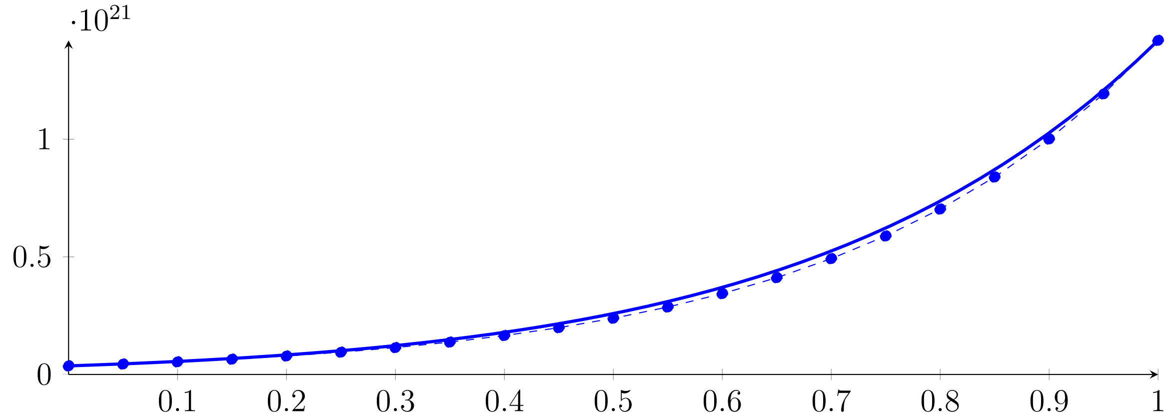
Longest intersection interval: 0.547472

$\Longrightarrow$ Bisection: first half $[21.875, 23.4375]$ und second half $[23.4375, 25]$

3.82 Recursion Branch 1 2 2 2 1 on the First Half $[21.875, 23.4375]$

Normalized monomial und Bézier representations and the Bézier polygon:

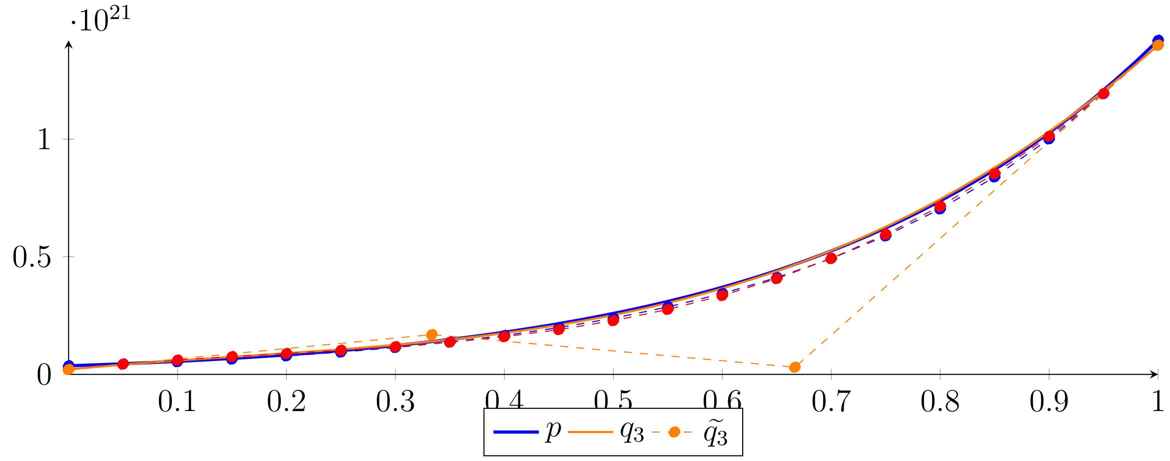
$$p = -2.01933\cdot10^{11}X^{20} + 1.74162\cdot10^{12}X^{19} - 5.05078\cdot10^{12}X^{18} + 2.75484\cdot10^{13}X^{17} - 1.18885\cdot10^{14}X^{16} + 8.19896\cdot10^{13}X^{15} + 7.11535\cdot10^{12}X^{14} + 4.61935\cdot10^{14}X^{13} + 4.87343\cdot10^{15}X^{12} + 4.21153\cdot10^{16}X^{11} + 2.91009\cdot10^{17}X^{10} + 1.62768\cdot10^{18}X^{9} + 7.34619\cdot10^{18}X^{8} + 2.65725\cdot10^{19}X^{7} + 7.61627\cdot10^{19}X^{6} + 1.70015\cdot10^{20}X^{5} + 2.88069\cdot10^{20}X^{4} + 3.56228\cdot10^{20}X^{3} + 3.01641\cdot10^{20}X^{2} + 1.5545\cdot10^{20}X + 3.65302\cdot10^{19}$$
$$= 3.65302\cdot10^{19}B_{0,20}(X) + 4.43027\cdot10^{19}B_{1,20}(X) + 5.36628\cdot10^{19}B_{2,20}(X) + 6.49229\cdot10^{19}B_{3,20}(X) + 7.84551\cdot10^{19}B_{4,20}(X) + 9.47016\cdot10^{19}B_{5,20}(X) + 1.14188\cdot10^{20}B_{6,20}(X) + 1.37539\cdot10^{20}B_{7,20}(X) + 1.65495\cdot10^{20}B_{8,20}(X) + 1.98935\cdot10^{20}B_{9,20}(X) + 2.389\cdot10^{20}B_{10,20}(X) + 2.86622\cdot10^{20}B_{11,20}(X) + 3.43561\cdot10^{20}B_{12,20}(X) + 4.11441\cdot10^{20}B_{13,20}(X) + 4.92302\cdot10^{20}B_{14,20}(X) + 5.88551\cdot10^{20}B_{15,20}(X) + 7.03032\cdot10^{20}B_{16,20}(X) + 8.39098\cdot10^{20}B_{17,20}(X) + 1.0007\cdot10^{21}B_{18,20}(X) + 1.1925\cdot10^{21}B_{19,20}(X) + 1.41998\cdot10^{21}B_{20,20}(X)$$



Degree reduction and raising:

$$q_3 = 1.79435\cdot10^{21}X^3 - 8.59945\cdot10^{20}X^2 + 4.43952\cdot10^{20}X + 2.11048\cdot10^{19}$$
$$= 2.11048\cdot10^{19}B_{0,3} + 1.69089\cdot10^{20}B_{1,3} + 3.04247\cdot10^{19}B_{2,3} + 1.39946\cdot10^{21}B_{3,3}$$

$$\widetilde{q}_3 = -9.11784\cdot10^{19}X^{20} - 1.53574\cdot10^{20}X^{19} + 4.37506\cdot10^{21}X^{18} - 1.98813\cdot10^{22}X^{17} + 4.99805\cdot10^{22}X^{16} - 8.7689\cdot10^{22}X^{15} + 1.2118\cdot10^{23}X^{14} - 1.39605\cdot10^{23}X^{13} + 1.33819\cdot10^{23}X^{12} - 1.03292\cdot10^{23}X^{11} + 6.19415\cdot10^{22}X^{10} - 2.79094\cdot10^{22}X^{9} + 9.07282\cdot10^{21}X^{8} - 1.9956\cdot10^{21}X^{7} + 2.6325\cdot10^{20}X^{6} - 1.44364\cdot10^{19}X^{5} - 8.94211\cdot10^{17}X^{4} + 1.79453\cdot10^{21}X^{3} - 8.59953\cdot10^{20}X^{2} + 4.43952\cdot10^{20}X + 2.11048\cdot10^{19}$$
$$= 2.11048\cdot10^{19}B_{0,20} + 4.33025\cdot10^{19}B_{1,20} + 6.0974\cdot10^{19}B_{2,20} + 7.56936\cdot10^{19}B_{3,20} + 8.90353\cdot10^{19}B_{4,20} + 1.02572\cdot10^{20}B_{5,20} + 1.17881\cdot10^{20}B_{6,20} + 1.36532\cdot10^{20}B_{7,20} + 1.60099\cdot10^{20}B_{8,20} + 1.90158\cdot10^{20}B_{9,20} + 2.28284\cdot10^{20}B_{10,20} + 2.76048\cdot10^{20}B_{11,20} + 3.35031\cdot10^{20}B_{12,20} + 4.06806\cdot10^{20}B_{13,20} + 4.92934\cdot10^{20}B_{14,20} + 5.95004\cdot10^{20}B_{15,20} + 7.14578\cdot10^{20}B_{16,20} + 8.53239\cdot10^{20}B_{17,20} + 1.01256\cdot10^{21}B_{18,20} + 1.19411\cdot10^{21}B_{19,20} + 1.39946\cdot10^{21}B_{20,20}$$



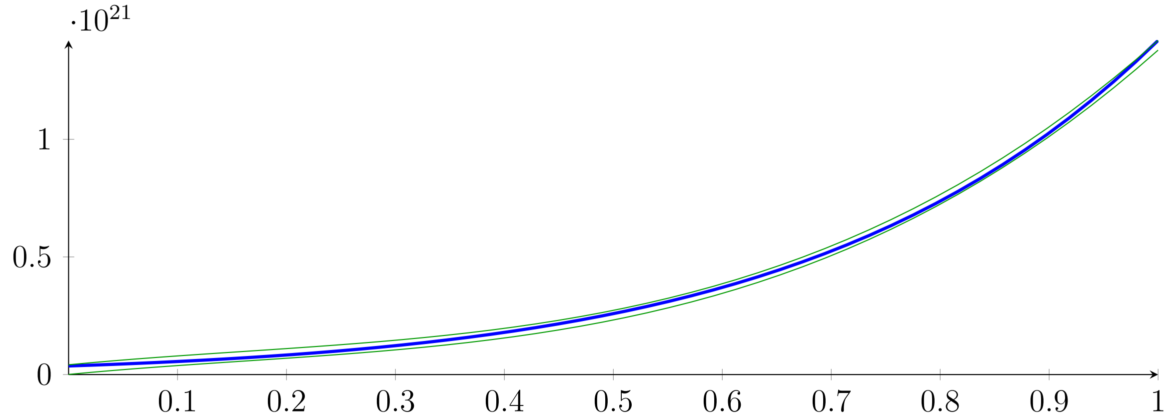
The maximum difference of the Bézier coefficients is $\delta = 2.05188{\cdot}10^{19}$.
Bounding polynomials M and m:

$$M = 1.79435{\cdot}10^{21}X^3 - 8.59945{\cdot}10^{20}X^2 + 4.43952{\cdot}10^{20}X + 4.16237{\cdot}10^{19}$$
$$m = 1.79435{\cdot}10^{21}X^3 - 8.59945{\cdot}10^{20}X^2 + 4.43952{\cdot}10^{20}X + 5.86025{\cdot}10^{17}$$

Root of M and m:

$$N(M) = \{-0.0794883\} \qquad N(m) = \{-0.00131665\}$$

Intersection intervals:

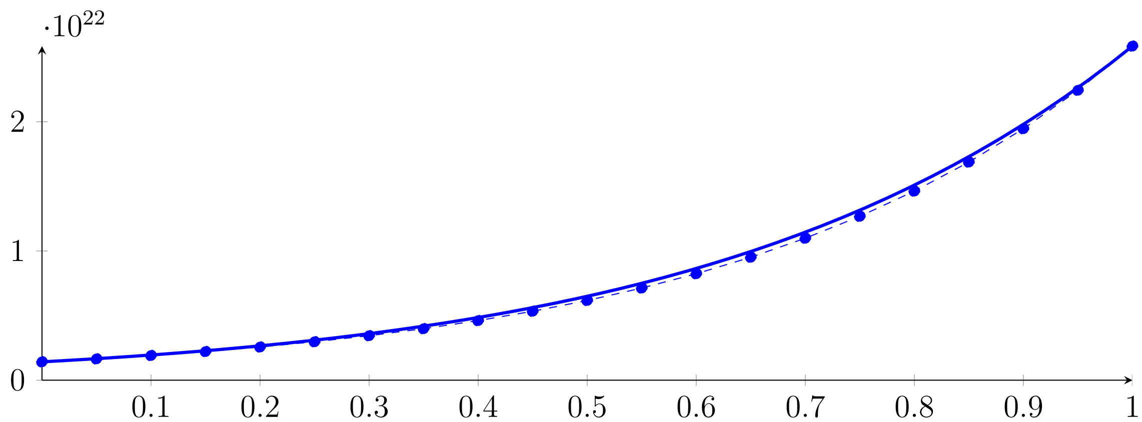


No intersection intervals with the x axis.

3.83 Recursion Branch 1 2 2 2 2 on the Second Half $[23.4375, 25]$

Normalized monomial und Bézier representations and the Bézier polygon:

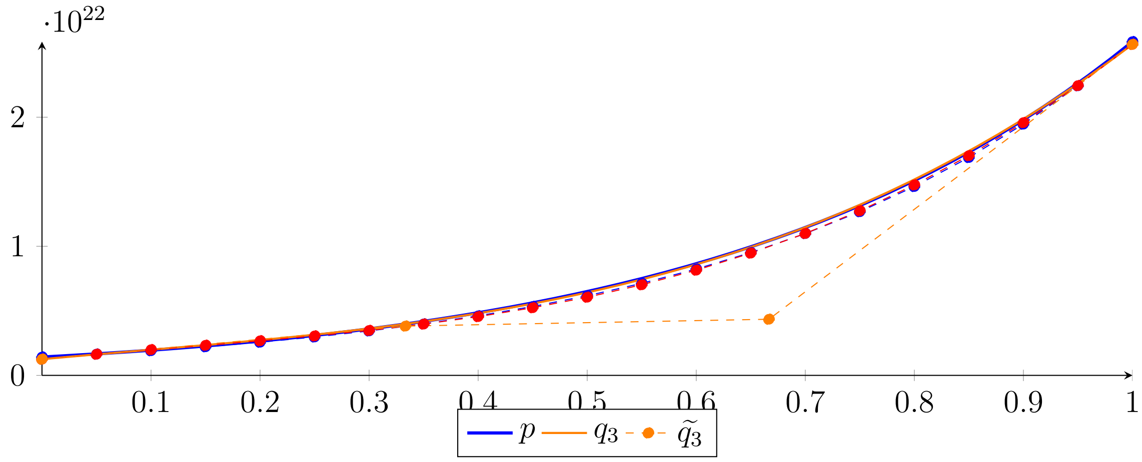
$$\begin{aligned}p = &-5.54586{\cdot}10^{12}X^{20} + 4.52218{\cdot}10^{13}X^{19} - 1.44038{\cdot}10^{14}X^{18} + 7.39598{\cdot}10^{14}X^{17} - 3.2688{\cdot}10^{15}X^{16}\\ &+ 2.3812{\cdot}10^{15}X^{15} - 7.8844{\cdot}10^{14}X^{14} + 1.10835{\cdot}10^{15}X^{13} + 1.34875{\cdot}10^{16}X^{12} + 1.53483{\cdot}10^{17}X^{11}\\ &+ 1.25414{\cdot}10^{18}X^{10} + 8.35262{\cdot}10^{18}X^{9} + 4.51985{\cdot}10^{19}X^{8} + 1.97596{\cdot}10^{20}X^{7} + 6.9063{\cdot}10^{20}X^{6} + 1.89886\\ &{\cdot}10^{21}X^{5} + 4.00777{\cdot}10^{21}X^{4} + 6.25317{\cdot}10^{21}X^{3} + 6.77942{\cdot}10^{21}X^{2} + 4.54961{\cdot}10^{21}X + 1.41998{\cdot}10^{21}\\ = &\ 1.41998{\cdot}10^{21}B_{0,20}(X) + 1.64746{\cdot}10^{21}B_{1,20}(X) + 1.91062{\cdot}10^{21}B_{2,20}(X) + 2.21495\\ &{\cdot}10^{21}B_{3,20}(X) + 2.56676{\cdot}10^{21}B_{4,20}(X) + 2.97331{\cdot}10^{21}B_{5,20}(X) + 3.44295{\cdot}10^{21}B_{6,20}(X)\\ &+ 3.98528{\cdot}10^{21}B_{7,20}(X) + 4.61135{\cdot}10^{21}B_{8,20}(X) + 5.33384{\cdot}10^{21}B_{9,20}(X) + 6.16731\\ &{\cdot}10^{21}B_{10,20}(X) + 7.12849{\cdot}10^{21}B_{11,20}(X) + 8.23659{\cdot}10^{21}B_{12,20}(X) + 9.51366{\cdot}10^{21}B_{13,20}(X)\\ &+ 1.0985{\cdot}10^{22}B_{14,20}(X) + 1.26796{\cdot}10^{22}B_{15,20}(X) + 1.46307{\cdot}10^{22}B_{16,20}(X) + 1.68765\\ &{\cdot}10^{22}B_{17,20}(X) + 1.94607{\cdot}10^{22}B_{18,20}(X) + 2.24334{\cdot}10^{22}B_{19,20}(X) + 2.5852{\cdot}10^{22}B_{20,20}(X)\end{aligned}$$



Degree reduction and raising:

$$q_3 = 2.27982 \cdot 10^{22} X^3 - 6.10522 \cdot 10^{21} X^2 + 7.69433 \cdot 10^{21} X + 1.25361 \cdot 10^{21}$$
$$= 1.25361 \cdot 10^{21} B_{0,3} + 3.81839 \cdot 10^{21} B_{1,3} + 4.3481 \cdot 10^{21} B_{2,3} + 2.5641 \cdot 10^{22} B_{3,3}$$

$$\widetilde{q}_3 = -3.63416 \cdot 10^{23} X^{20} + 3.63118 \cdot 10^{24} X^{19} - 1.68246 \cdot 10^{25} X^{18} + 4.79457 \cdot 10^{25} X^{17} - 9.39601 \cdot 10^{25} X^{16} + 1.34101 \cdot 10^{26} X^{15} - 1.43975 \cdot 10^{26} X^{14} + 1.18434 \cdot 10^{26} X^{13} - 7.5329 \cdot 10^{25} X^{12} + 3.71315 \cdot 10^{25} X^{11} - 1.41382 \cdot 10^{25} X^{10} + 4.12744 \cdot 10^{24} X^9 - 9.16548 \cdot 10^{23} X^8 + 1.54974 \cdot 10^{23} X^7 - 2.03995 \cdot 10^{22} X^6 + 2.15429 \cdot 10^{21} X^5 - 1.81043 \cdot 10^{20} X^4 + 2.2809 \cdot 10^{22} X^3 - 6.10555 \cdot 10^{21} X^2 + 7.69434 \cdot 10^{21} X + 1.25361 \cdot 10^{21}$$
$$= 1.25361 \cdot 10^{21} B_{0,20} + 1.63833 \cdot 10^{21} B_{1,20} + 1.99091 \cdot 10^{21} B_{2,20} + 2.33137 \cdot 10^{21} B_{3,20} + 2.67967 \cdot 10^{21} B_{4,20} + 3.05588 \cdot 10^{21} B_{5,20} + 3.4798 \cdot 10^{21} B_{6,20} + 3.97201 \cdot 10^{21} B_{7,20} + 4.55117 \cdot 10^{21} B_{8,20} + 5.23953 \cdot 10^{21} B_{9,20} + 6.05402 \cdot 10^{21} B_{10,20} + 7.01819 \cdot 10^{21} B_{11,20} + 8.14866 \cdot 10^{21} B_{12,20} + 9.46834 \cdot 10^{21} B_{13,20} + 1.09949 \cdot 10^{22} B_{14,20} + 1.27498 \cdot 10^{22} B_{15,20} + 1.47523 \cdot 10^{22} B_{16,20} + 1.70227 \cdot 10^{22} B_{17,20} + 1.95809 \cdot 10^{22} B_{18,20} + 2.2447 \cdot 10^{22} B_{19,20} + 2.5641 \cdot 10^{22} B_{20,20}$$



The maximum difference of the Bézier coefficients is $\delta = 2.11062 \cdot 10^{20}$.

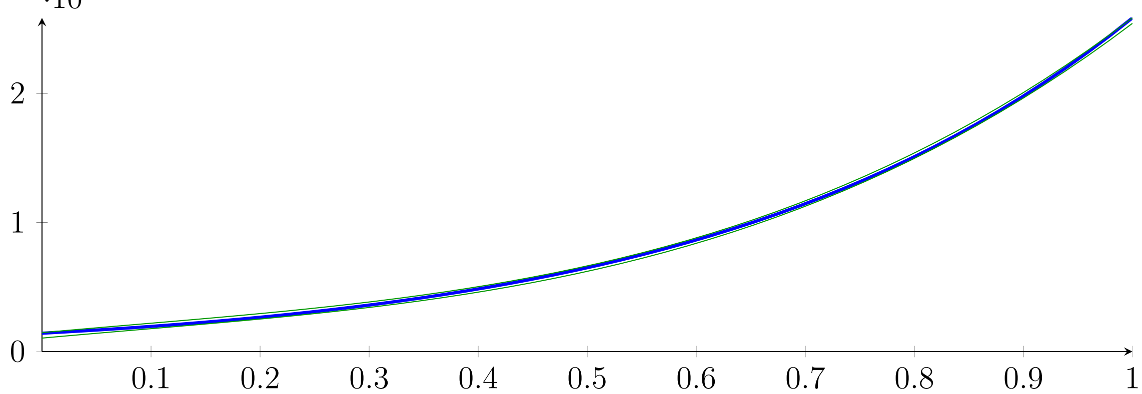
Bounding polynomials M and m:

$$M = 2.27982 \cdot 10^{22} X^3 - 6.10522 \cdot 10^{21} X^2 + 7.69433 \cdot 10^{21} X + 1.46467 \cdot 10^{21}$$
$$m = 2.27982 \cdot 10^{22} X^3 - 6.10522 \cdot 10^{21} X^2 + 7.69433 \cdot 10^{21} X + 1.04255 \cdot 10^{21}$$

Root of M and m:

$$N(M) = \{-0.158585\} \qquad N(m) = \{-0.119203\}$$

Intersection intervals:

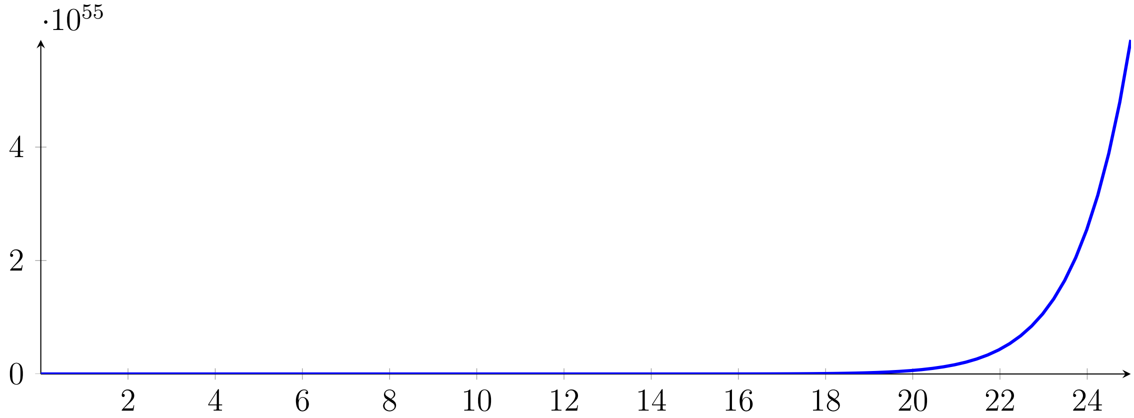


No intersection intervals with the x axis.

3.84 Result: 20 Root Intervals

Input Polynomial on Interval $[0, 25]$

$$p = 9.09495\cdot10^{27}X^{20} - 7.63976\cdot10^{28}X^{19} + 2.99988\cdot10^{29}X^{18} - 7.31583\cdot10^{29}X^{17} + 1.24164\cdot10^{30}X^{16} - 1.55743\cdot10^{30}X^{15} + 1.49652\cdot10^{30}X^{14} - 1.12669\cdot10^{30}X^{13} + 6.74145\cdot10^{29}X^{12} - 3.2326\cdot10^{29}X^{11} + 1.24696\cdot10^{29}X^{10} - 3.86898\cdot10^{28}X^{9} + 9.61774\cdot10^{27}X^{8} - 1.90023\cdot10^{27}X^{7} + 2.94592\cdot10^{26}X^{6} - 3.5156\cdot10^{25}X^{5} + 3.13977\cdot10^{24}X^{4} - 2.01108\cdot10^{23}X^{3} + 8.62735\cdot10^{21}X^{2} - 2.18824\cdot10^{20}X + 2.4329\cdot10^{18}$$



Result: Root Intervals

$[0.999999, 1]$, $[2, 2]$, $[3, 3]$, $[3.99975, 4.00021]$, $[4.99999, 5.00001]$, $[5.99996, 6.00007]$, $[7.00004, 7.00006]$, $[7.99947, 7.99948]$, $[9.00306, 9.00308]$, $[9.98807, 9.98808]$, $[11.0363, 11.0363]$, $[11.9255, 11.9255]$, $[13.1501, 13.1501]$, $[13.8088, 13.8088]$, $[15.2175, 15.2176]$, $[15.8089, 15.8093]$, $[17.0973, 17.0973]$, $[17.9554, 17.9554]$, $[19.0104, 19.0104]$, $[19.9988, 19.9988]$

with precision $\varepsilon = 0.001$.