Equations Handout for Robert Gallager's Minimum Delay Routing Algorithm Using Distributed Computation

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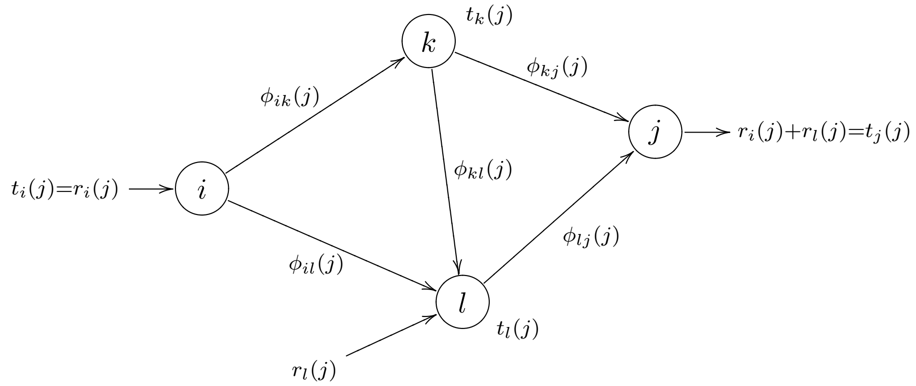
1 Model and Analysis



Model Variables

- Set of n nodes enumerated by $\{1, 2, \ldots, n\}$
- Set of links: $\mathcal{L} := \{(i, j) \text{ is existing link}\}$
- Input traffic set $\boldsymbol{r} := \{r_i(j)\}$
  $r_i(j)$ traffic entering at i and exiting at j.
- Node flow set $\boldsymbol{t} := \{t_i(j)\}$
  $t_i(j)$ traffic at node i destined for j.
- Routing variable set $\boldsymbol{\phi} := \{\phi_{ik}(j)\}$.
  $\phi_{ik}(j)$ fraction of traffic $t_i(j)$, which travels over the link (i, k).

Constraints on $\boldsymbol{\phi}$

1. No traffic on non-existing links and no loop-back traffic

$$\phi_{ik}(j) = 0 \quad \forall\, (i, j) \notin \mathcal{L} \text{ or } i = j$$

2. No loss of traffic is allowed.

$$\sum_{k=1}^{n} \phi_{ik}(j) = 1 \quad \forall\, i, j$$

3. All nodes are inter-connected.

$$\phi_{ik}(j) > 0, \phi_{kl}(j) > 0, \ldots, \phi_{mj}(j) > 0$$
$$\exists\, i, k, l, \ldots, m, j \,\forall\, i, j$$

Traffic Aggregation Equation

$$t_i(j) = r_i(j) + \sum_{l=1}^{n} t_l(j)\phi_{li}(j)$$

Total Traffic on a Link

$$f_{ik} = \sum_{j} t_i(j)\phi_{ik}(j)$$

Total Delay

$$D_T = \sum_{i,k} D_{ik}(f_{ik})$$

Partial Derivative Regarding Input Traffic

$$\frac{\partial D_T}{\partial r_i(j)} = \sum_{k} \phi_{ik}(j) \left(D'_{ik}(f_{ik}) + \frac{\partial D_T}{\partial r_k(j)} \right)$$

Partial Derivative Regarding Routing Variables

$$\frac{\partial D_T}{\partial \phi_{ik}(j)} = t_i(j) \left(D'_{ik}(f_{ik}) + \frac{\partial D_T}{\partial r_k(j)} \right)$$

Necessary Condition

$$\frac{\partial D_T}{\partial \phi_{ik}(j)} \begin{cases} = \lambda_{ij}, & \phi_{ik}(j) > 0 \\ \geq \lambda_{ij}, & \phi_{ik}(j) = 0 \end{cases} \quad \forall\, i \neq j\, \forall\, (i,k)$$

Sufficient Condition

$$\frac{\partial D_T}{\partial r_i(j)} \leq D'_{ik}(f_{ik}) + \frac{\partial D_T}{\partial r_k(j)}$$

Iterative Version

$$D'_{ik}(f_{ik}) + \frac{\partial D_T}{\partial r_k(j)} \geq \min_{(i,m)\in\mathcal{L}} \left(D'_{im}(f_{im}) + \frac{\partial D_T}{\partial r_m(j)} \right)$$

2 Algorithm

Difference to Best Link

$$a_{ik}(j) = \underbrace{D'_{ik}(f_{ik}) + \frac{\partial D_T}{\partial r_k(j)}}_{\text{on link } k} - \underbrace{\left(D'_{ib}(f_{ib}) + \frac{\partial D_T}{\partial r_b(j)} \right)}_{\text{on the best link}}$$

Reduction

$$\Delta_{ik}(j) = \min\left\{ \phi_{ik}(j),\ \frac{\eta}{t_i(j)} a_{ik}(j) \right\}$$

New Routing Variables

$$\phi^1_{ik}(j) = \begin{cases} \phi_{ik}(j) - \Delta_{ik}(j) & \text{if } (i,k) \text{ is not the best link} \\ \phi_{ib}(j) + \sum\limits_{\substack{(i,m)\in\mathcal{L} \\ m\neq b}} \Delta_{im}(j) & \text{if } (i,k) \text{ is the best link and therefore } k = b \end{cases}$$

Improper Routing Variable

$$\phi_{ik}(j) > 0 \quad \text{and} \quad \frac{\partial D_T}{\partial r_i(j)} \leq \frac{\partial D_T}{\partial r_k(j)}$$